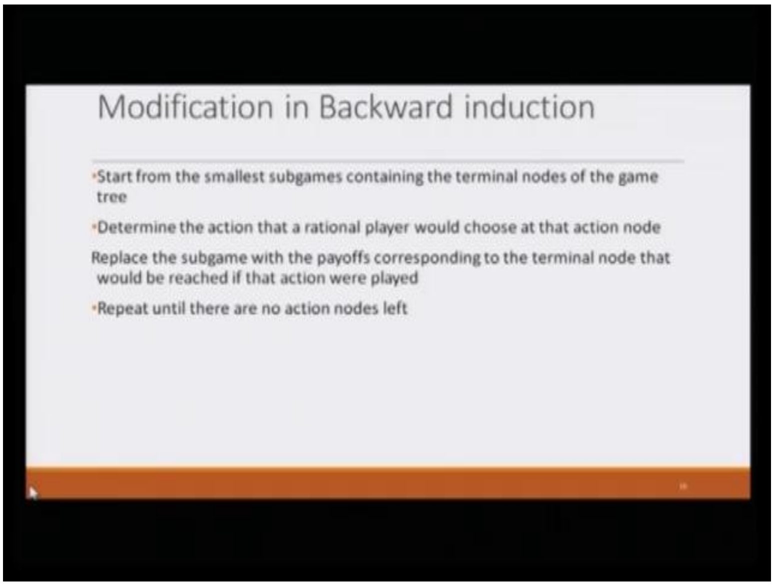
Strategy: An Introduction to Game Theory
Prof. Vimal Kumar
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture – 28

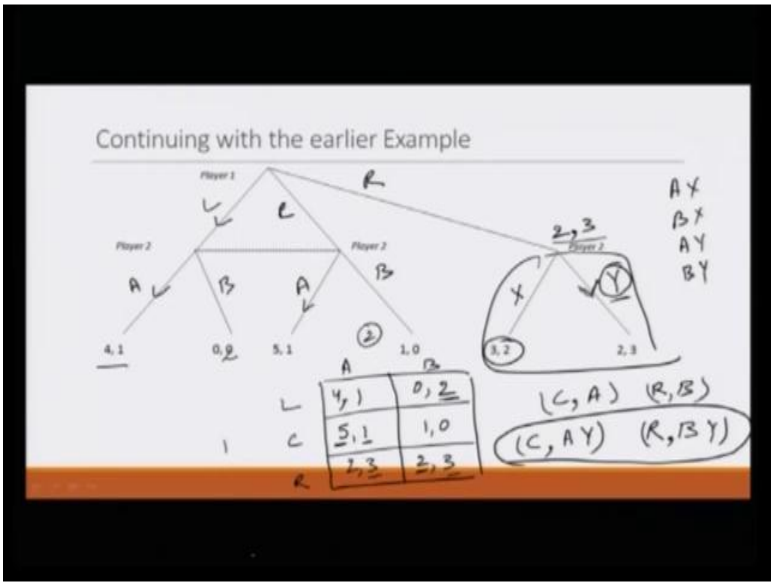
(Refer Slide Time: 00:21)



So, now as I discussed right, I said right in the beginning that we are interested in getting the solution concept, how to solve a general extensive form game. So, for that, of course, we have backward induction but we are going to modify it little bit. What do we have, that start from the smallest subgame containing the terminal nodes of the game tree. Remember, what happens when we have this smallest subgame with the terminal nodes? In most of the cases there is no strategic interaction left, applier decides and game ends. So, there player can very easily decides.

Now, of course, what we will have, determine the action that a rational player would choose at that action node, just before the terminal node. Replace the subgame with the payoff corresponding to the terminal node that would be reached if the action were played. And, repeat until there are no action left. So, this would be clear when we do it.

(Refer Slide Time: 01:13)



Let us do it for this game. For example, here, we know this is the subgame. So, we will do it for the, we will do it here; player 2 would take this game in this direction. So, this subgame can be replaced by here, p (2, 3), why? Because player 1 knows player 2 is rational and given a chance, player 2 is going to take, decides in this direction.

Now, we have the whole game, how would we solve it? Ok. So, very simple, we will do the normal form of the game. We will do the normal form of the game, and I had already modified this. I had put it as 2. We can get the normal form game. And, what would be the normal form game? Again, let me point out here; let us say, x; let us give them name – left, centre and right; and here, A B, A B.

And, we can make the table, as we have already learned in the one of the previous modules, that player 1 has how many action now? L C R 3, here we do not have to worry about. So, player 1 is going to take action L C R. How many strategies player 2 has? In fact, 2 here and 2 here, 2 multiplied by 2, so, 4. But we do not have to take care of the 4 because we know if game reaches to this point, this is what player 2 is going to do. So, we will take a reduced game in which this game tree is replaced by the outcome from this smallest sub game.

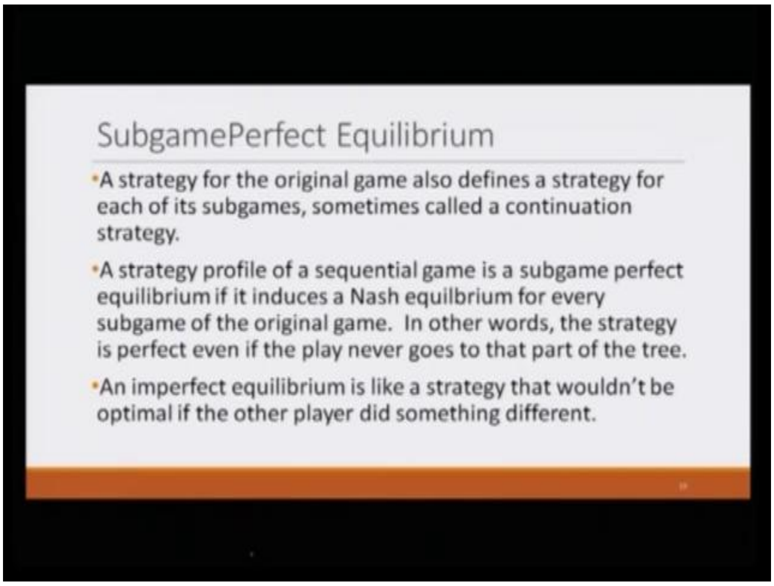
So, what we will do? Here, in this, in the reduced form, player 2 has either A or B. So, if player 1 plays L and player 2 plays A, game will reach to this. So, payoff would be 4, 1. And, similarly, let me write 0, 2; here 5,1; here 1, 0. And, if player 1 plays R, no matter what player 2 does. Here we will have 2, 3; 2, 3. Notice, I, we got rid of this one.

And then we solve. How do we solve? Just for your reason, we try to obtain the nash equilibrium, what do we get? If player 1 believes that player 2 is going to play A, the best response of player 1 is to play C. If player 1 thinks that player 2 is going to play B, the best response is R.

Now, if player, we let us do it for, from player 2's perspective. If player 2 thinks that player 1 is going to play L, then the best response is B. And similarly, we get here this, and here this. So, we get 2 Nash equilibrium, (C, A) and (R, B). Notice, that player 1, player 2 does not have any strategies named A or B. Player 2 has a strategy named, if we put here x and y, then player 2 has a strategies named, A x, B x, A y and B y.

So, how can we give the equilibrium? It is very clear that player 1, one equilibrium would be that player 1 plays C, and player 2 plays here A. But here we do no matter player 2 is always going to play y, so, we can say player 2 is going to play, A y. And similarly, here we can say, what? Player 1 plays R and player 2 plays B, and of course, here y, so, B y. These will be the 2 solutions that we can get, that player 1 takes the strategy C, and player 2 takes the strategy A y; or player 1 takes strategy R and player 2 take the strategy B y.

(Refer Slide Time: 05:37)



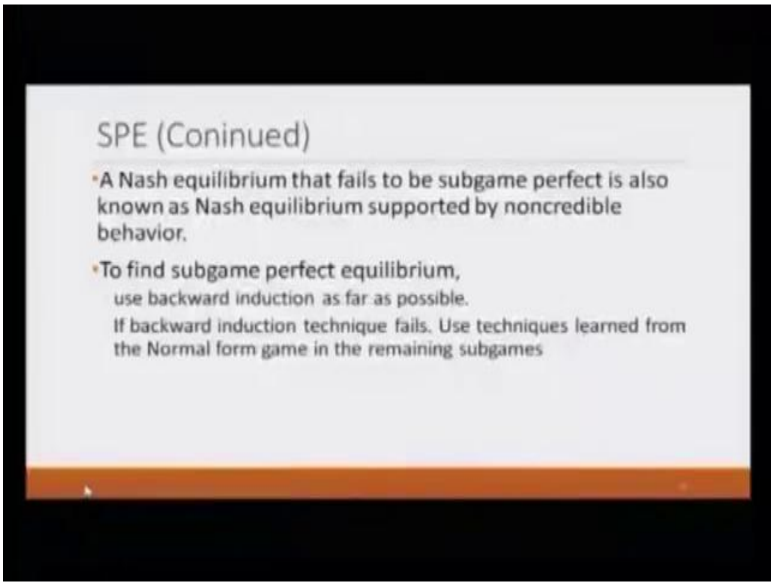
In fact, we did the modified backward induction, this is what we call subgame perfect equilibrium. What is subgame perfect equilibrium? First of all, we, to understand subgame perfect equilibrium let us understand the continuation strategy, what do we mean by a continuation strategy? A strategy for the original game also defines a strategy for each of its subgames, sometimes called a continuation strategy. So, what we meant here that when we have B y, and this particular subgame has reached, what would be the continuation strategy in this subgame, only y. Similarly, for A y, the continuation strategy would be only y.

Now, a strategy profile of a sequential game is a subgame perfect equilibrium. In short, it is called SPE; in some books it is also called SP any. It includes a Nash equilibrium for every subgame of the original game. So, if we treat subgames of the original games as independent game then subgame perfect equilibrium should induce Nash equilibrium not only in the whole game, but also in all the subgames. So, in other word, the strategy is perfect even if player never goes to the that part of tree.

An imperfect equilibrium is like a strategy that would not be optimal if the other player did something different. Remember, we talked about the mistakes, what a player makes a mistake, in that case if a strategy profile is subgame perfect equilibrium no problem, but if a Nash equilibrium is not subgame perfect equilibrium then you would not get the optimal result.

Of course, by now you must have noticed that any subgame perfect equilibrium is Nash equilibrium, but not all Nash equilibrium of the game are subgame perfect equilibrium. Although, wherever we can obtain result from backward induction technique, backward induction equilibrium definitely is subgame perfect equilibrium.

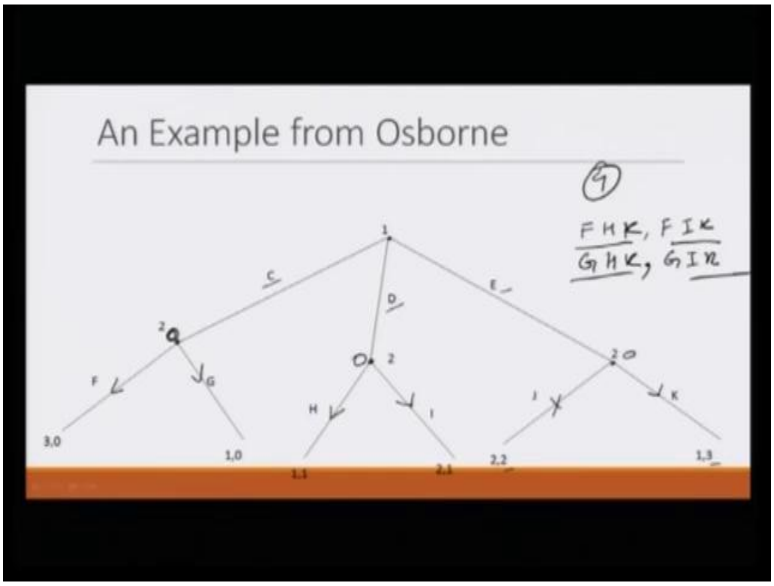
(Refer Slide Time: 08:05)



What, again, this is something I am saying, but I will revisit this statement in other module where we will talk about credibility. A Nash equilibrium that fails to be subgame perfect is also known as Nash equilibrium supported by non-credible equilibrium. So, if I want to give you the basic rule, how to obtain subgame perfect equilibrium, use backward induction first; if you can use backward induction well and good; the equilibrium that you are getting would be subgame perfect equilibrium, if you are mind full of the strategies of players; do not just write outcome as the equilibrium.

Equilibrium always means the strategy profile equilibrium should give 1 strategy for all of the players involved in the strategic interaction. And, if backward induction technique fails, use techniques learned from the normal form game in the remaining game, not subgame, in remaining games.

(Refer Slide Time: 09:10)

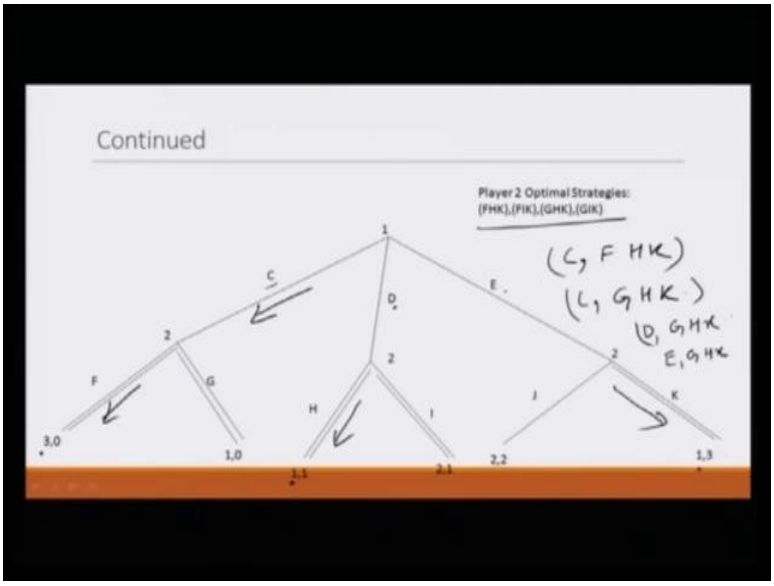


Let us take an example. This example is given in professor Martin Osborne’s a very famous book “An Introduction to Game Theory”. What we have here, that we have player 1; player 1 can take one of these 3 actions – C, D, or E. If player 1 takes action C, then player 2 can take action F or G. And similarly, I have written if player 1 takes action D, then player 2 is can take action H or I.

Similarly, if player 1 takes action E, then player 2 can take action J or K. Here, backward induction, the vanilla form, the earlier backward induction has a problem, why? For player 2, it is optimal to take any one of these 2 action. Even if player 2 takes F or he takes G, his payoff would remain same.

So, next, here player 2 again can take either H or I, in both cases his payoff would be 1. Here it is clear that player 2 would take K not J because K gives 3 and J gives 2. So, this is not possible; we can get rid of this right in the beginning. Mind you, how many subgames do we have in this game? One subgame starting here, the whole game; then, another subgame starting here; another subgame starting here; and another subgame starting here; so, we have a total of 4 subgames.

(Refer Slide Time: 10:44)

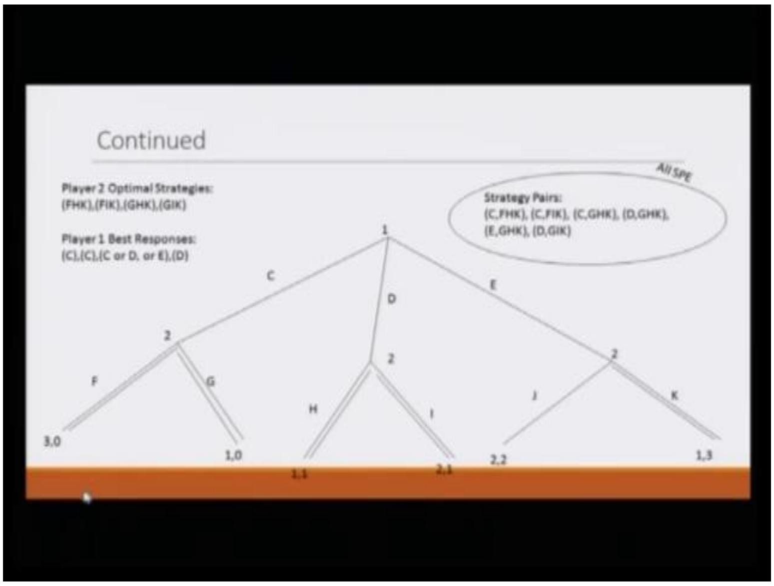


How should be proceed now? This we have already indicated. Notice, that we can say, the best response from player 2; what would be the best response from player 2, no matter what player 1 is doing, best response would be F H K. What is F H K? Play F if given a chance in this node, play H if given a chance in this node, play K given a chance is this node. But, this is not the only one, only best response. What are the other best response? F I K. What else? G H K and G I K. These are the 4 best responses from player 2, anyway I have written it here; these are the optimal strategies for player 2.

Given this optimal strategy, what would be the optimal strategy for player 1? If let us say, player 1 thinks that player 2 is going to play F H K. So, if you play C, he gets 3. If player 1 plays D, he gets 1; if he plays E he gets 1. So, of course, his better off by playing C. So, (C, F H K) can be unequilibrium. And, it is also subgame perfect that although the paths that game would follow is going to be this, but even in this subgame this is going to follow this path which is optimal, and here this path. So, this is subgame perfect.

I will give you one more subgame perfect and then request you to obtain all the subgame perfect equilibrium of this game. What, let us say, if player 2 plays G H K, what would happen? If player 1 plays C, then he gets 1; if player 1 plays D, then he gets 1; and if he plays E, then he gets 1. So, here we have, he is indifferent between playing C, D or E. So, all are equilibrium, (C, G H K), (D, G H K) and (E, G H K).

(Refer Slide Time: 13:16)



Here I have written all the (C, F H K), (C, G H K), (D, G H K), (E, G H K), and you can obtain all other subgame perfect equilibrium. So, I hope now, by now you have learnt how to figure out the subgame perfect equilibrium S P E or S P any of a unextensive form game. In the next module, we will start talking about some of the application of extensive form game.

Thank you.