Strategy: An Introduction to Game Theory
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Lecture – 27

Welcome to Mooc lecture on Strategy, An Introduction to Game Theory.

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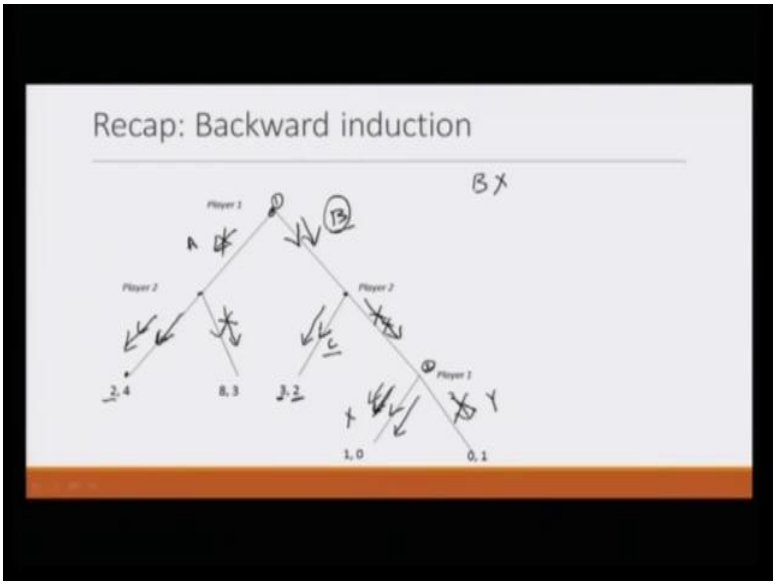

In this module, we are going to discuss a solution concept named sub game perfect equilibrium. By the way, why do we need a new solution concept, let us go back to the technique that we learnt to solve an extensive form game. The name of the technique was backward induction. So, just to recap, let us see, how do we do? We start towards the end and we go back towards the beginning.

So, what happens if player 1 given a chance, player 1 would play, if player 1 moves in this direction, if player 1 move in this direction player 1 gets 1, if player 1 moves in this direction, he gets 0. Notice here, there is no strategic interaction, as soon as the player 1 decides the game ends, so he does not have to worry about anything. So, of course, what should you do, he is a rational player, he is interested in maximizing his payoff, he does not care about the payoff that player 2 receives. If he cares, then we have to build it in the model.

But, once we have return the payoff, then what he cares about is maximizing his payoff, he does not care about the payoff of other player. So, as 1 is greater than 0, he will move in this direction not in this direction, so game will proceed here. Now, player 2 at this point if player 2 gets a chance to play, player 2 knows that player 1 is rational and he will move in this direction.

So, if player 1 decides to move in this direction game will reach here and player 2 will get 0. But, if player 2 decides to move in this directions player 2 will get 2, of course, 2 is greater than 0, so game will proceed in this direction as player 2 is not only rational, player 2 knows player 1 is rational, so game will not proceed in this direction. Similarly, here we can see player 2 has to decide between two actions, one action is to move in this direction and another is to move in this direction.

Moving in this direction gives player 2, 4, moving in this direction gives player 2, 3 of course, 4 is greater than 3, so game will proceed in this direction not in this direction, coming back to here. Why can we go back? Because, not only player 1 is rational, player 1 knows that player 2 is rational, player 2 is rational and player 2 knows, player 1 is rational and so on. So, it is the implication of common knowledge, so player 1 sees that if he moves in this direction, game will eventually reach to this point and his payoff will be 2.

And, if you proceeds in this direction game will reach here, and his payoff would be 3, so of course, 3 is greater than 2, so game will proceed in this direction not in this direction. So, we know the outcome, let us say that, if we name this A and B and then here C, then we can say the right in the beginning, player 1 will take action B and player 2 will take action C.

But, if you want to describe the strategy that player 1 will take right in the beginning, we cannot simply say B, because B is not his strategy, player 1 gets to play game at two nodes, one here and another time here. So, his strategy should describe his action at both of these decision nodes. So, what should he do? He should decide to play B here and let us again, let us give them name X and Y.

And of course, he should decides to play X here, so X strategy for player 1 is that he should play B X. How about player 2? Similarly, we have to see that player 2 decides to

move here and at this node here. So, his strategy should give act this particular action here and this particular action here and that is how, we get the equilibrium.

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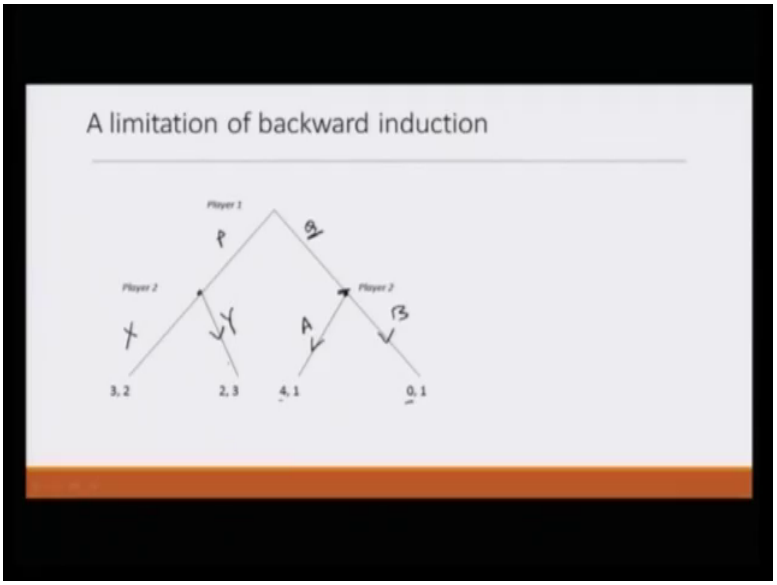

But, let us look at a particular limitation of this backward induction technique. At this, again there is no strategic interaction, player 2 either has to decide to play X or to play Y. Playing Y gives him higher, so of course game will move in this direction, at this point also player 1 has two actions A and B. Which actions should he take? Taking action A means that player 2 will get 1 and taking action B means he would get 1.

Of course, if player 2 takes action A here, and game has reach to this point, the payoff for player 1 could be 4 and if player 2 takes action B, payoff of for player 2 would be 0. But, notice that here player 2 does not player about how much player 1 is getting, player 2 is interested in only maximizing his payoff. So, but he is indifferent, he can go in this direction, he can go in this direction, then how should we, what should be do here, what should we take.

If, we think player 2 is going to take action A, then let us say, if we have action P for player 1 and Q for player 1. Of course, if player 1 thinks that player 2 is going to play A, then he is better off playing Q, because 4 is greater than 3 or 2. But, if player 1 thinks that player 2 is going to play B, then he is better off by playing P. So, how should here backward induction plays? It does not give as any answer and, we will see, because we have we may have multiple equilibrium in this games, so that is the reasoning.

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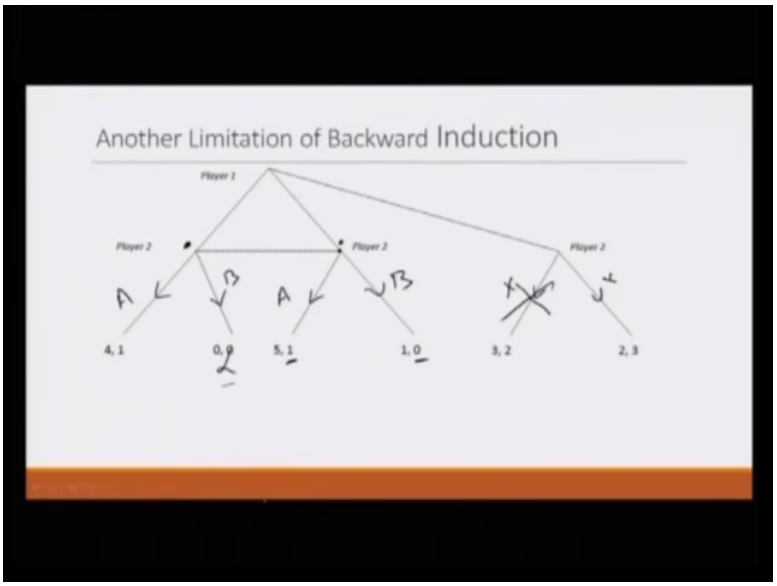

Again, coming back to another limitation of backward induction, again let us say player 2 has two actions X and Y. If, player 2 takes action X player 2 gets 2, if player 2 takes action Y, player 2 gets 3, player 2 should take action Y, so for all practical purposes, we can delete this part. Because, player 1 knows that player 2 is rational, so if game reaches to this point, player 2 is going to play Y. How about here? Can we say player 2 is going 2?

Let us say, this is A and this is B and then, if this is the case, we have to have A and B present here. We cannot have different actions at the different nodes of the same information's set. Again, you will have to refer if you are having difficulty in understanding this, then you should look at a module called strategies. So, player 2, can we say that player 2 decides do take this actions A, we do not know. Let me tell you that, let me change this payoff slightly, then it would become more interesting, let us make it 2.

So, we now we cannot say that player 2 will take action A or action B, because player 2 does not know whether he, game has reach to this node or this node. If he thinks this node has reached, he is better off by taking action A, because 1 is greater than 0. If, he thinks this node has reached, then he is better off by taking action B, but the problem is here, game is off in prefect information game is off imperfect information.

So, we cannot, the player 2 does not know, where the game has reached. So, we cannot decide, we cannot reduce, we cannot take out one of the branches and say this is what player 2 would do, so backward induction would not work here.

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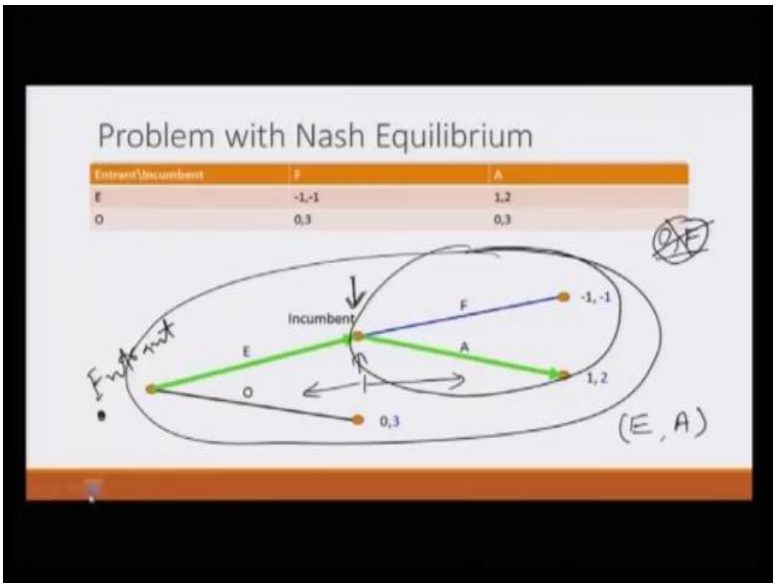

We should also, but you may say that why are you taking about a new solution concept, as we already have a solution concept known as Nash equilibrium. So, for that I want to remind you, that this was the entrant that we discussed earlier. What did we see? That, if we use backward induction, fortunately backward induction works here and what did we see that entrant inters, here we have entrant and incumbent accommodates, this is the outcome we get.

But, when we use the solution concept Nash equilibrium, we get two different possibilities. E and A is of course, one of the possibilities, but we also get O comma F as another possibility, this is really upset, because it does not satisfy sequential rationality, because game is of common knowledge, entrant would know if game reaches to this point player 2 has no option, but to accommodate. So, saying O comma F it may be rational at this point, but it is not rational, when game reaches to this point.

So, in that sense O comma F is an observed outcome recommended by Nash equilibrium. Can we have a technique to get rid of this obsolete outcome? Of course, we have that is backward induction, but we saw that backward induction does not work in some of the cases. So, what do we really need, backward induction is a good concept for extensive

form game, but we need some modification in backward induction to obtain a solutions in more in all of the extensive form games.

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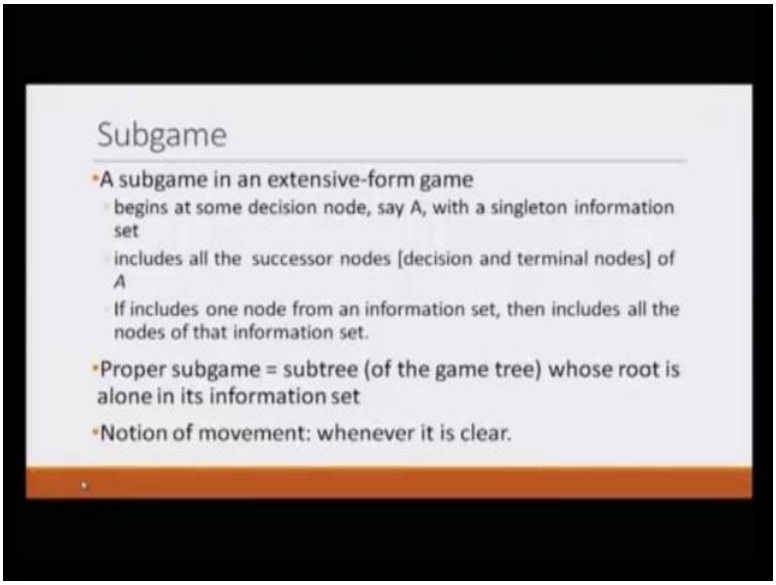

So, let us define something called sub game. What do you mean by a sub game? Again, let us go back before I define sub game. Let us, look at this entry game, because game is moving in a linear fashion, first entrant gets to decide, whether to inter or remain out of the game and then, incumbent gets to reside. When at look at the game at this point, one can say that a new game is beginning here, whatever happen in the past that incumbent takes as given and now, he has, he does not need to consider, what has happened in this part. Only he needs to consider, what is going to happen from now onwards, so that is how we get the concept of sub game.

We can say, this is the new game which is beginning at this point, when this node has reached and this is a sub game. So, every moment you know as we are moving in the game, we do not we of course, we have to consider the past, but only in the sense that we have to take it as given. If game has reached to this particular node, then nothing can be done for the past nodes that can only be taken as given, so that is why we define sub game.

So, what do we have a sub game, what we say that sub game begins at some decision node. Say A I am just saying one of the, I am giving that name A, which has a single

term information set. So, very, very clear a sub game begins at the single term in formation set. Why not an information set, which has more than one element?

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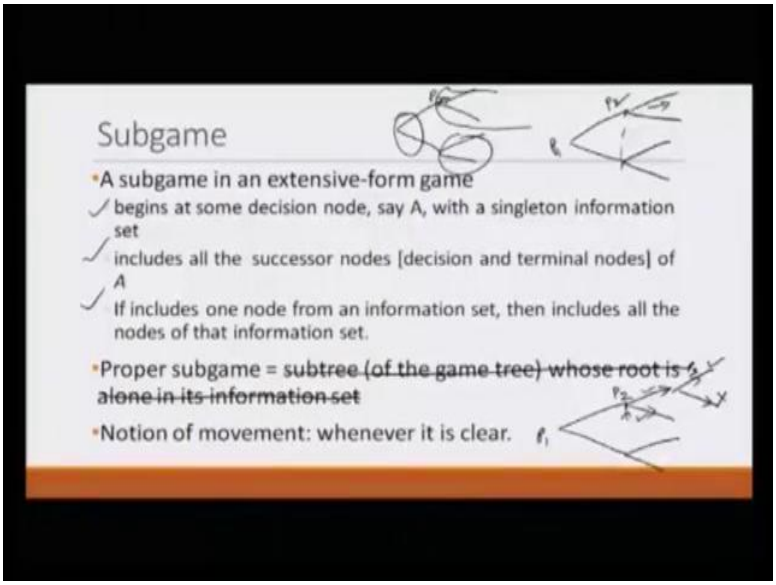

Let us look at it, here game cannot a sub game cannot begin at this information said, why because play at, let say here we have player 1 here, we have player 2, player 2 does not know whether this has reached or this has reached. So, he does not have a clear cut picture form the past, so that we cannot say again begins here, but incase in case, we have situation like this, where clear to here, as clarity that this node has rest if it have reached. Then, he does not needs to consider, what is happening here or what is happening here, he will consider a new beginning of a new game from this on word.

So, that is what I am saying that sub game begins only at the information sets, which are single term, second requirement is that it includes all the successor are note. Again, it would be, let us say, let us take another example, this is the case, here we have player 1, here we have player 2 here we have player 3. What we are saying? That, if we say that sub game is beginning here, then all the successor nodes should belong to this sub game why, because notice we are deciding now, game is at this is stage.

So, we do not know whether game will proceed in this direction are in this direction or in this direction. So, we cannot consider, let say this, this, this and not this, so what we say that once we a node is included in the sub game all the successor node of this node would be included in the sub game. And, I have already said I just want to emphasize that if it

includes one node from the information said, then it includes all the nodes of that information said.

Notice, that whole game is also a sub game of itself, because whole game would satisfy all the criteria. So, we define again for the shack that we have something called proper sub game it is the part of the game tree whose root is I think I have written it incorrect here, proper sub game is all the sub game of the game excluding the whole game itself. And of course, here everything is that we have notion of moment, if game is moving in this direction it should be clear.

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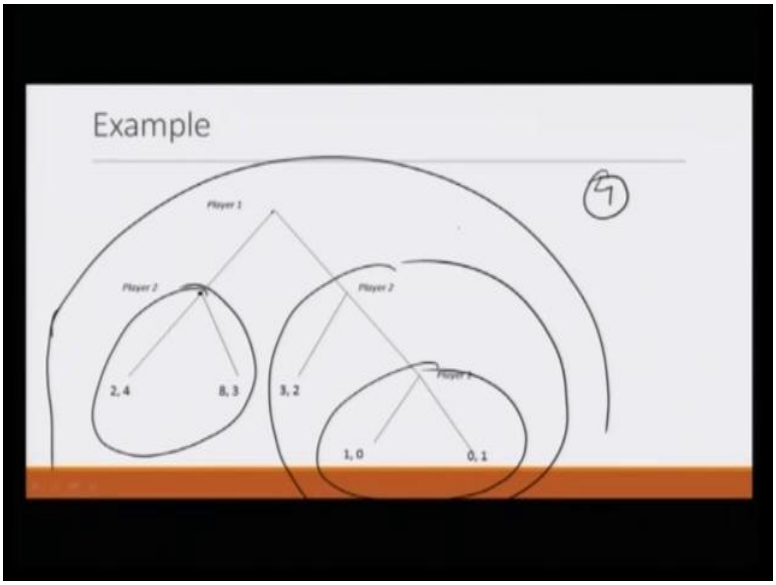

So, for example, how many sub games, do we have in this game one of course, beginning write at the initial node, because whole game is sub game of itself. So, all the whole game is not proper sub game of itself, but it a sub game of itself, another is beginning here. So, we can say, if we start drawing this is one sub game this is another sub game this is another sub game and then, the whole game is sub game of itself. So, how many we get 1 2 3 4 this game has 4 sub games 3 proper sub games here.

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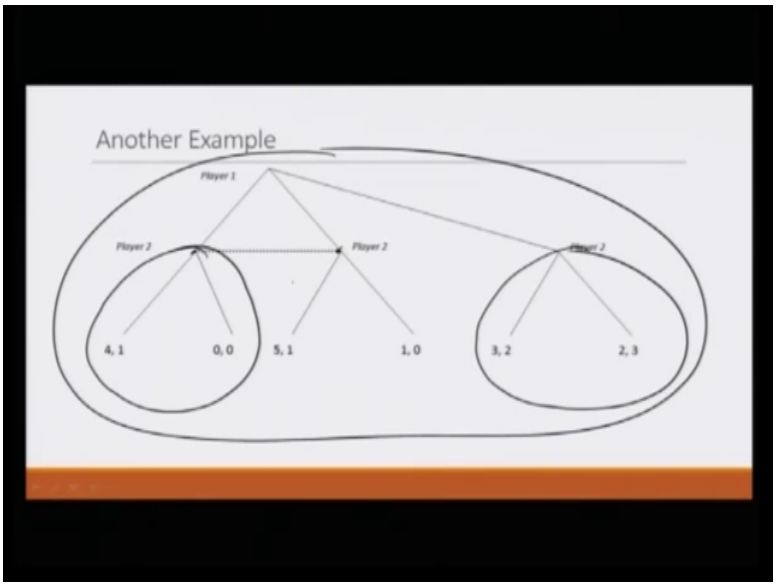

Another example, how many sub games do we have in this game, you should pass and think before you say I suggest that you write it in your notebook, that how many sub game do we have in this game of course, one sub game is starting here. And, another the whole game is also sub game of itself, but can we say a of sub game is starting here, no we cannot say, why because the third criteria says that if a node is included in the sub game the node belonging to information's all the nods belonging to the those information said present in this sub game should also be in the sub game.

For example, let me also, let me say it again that, if let us say this node is present in the sub game this node also belongs to the same information said the information to the information said that this node belongs to, but if we say this is the sub game, we are excluding this, so this cannot be a sub game. In fact, this game has only two sub games, so that is it about the sub game, now we are going to learn the solution concepts.