Strategy:An Introduction to Game Theory
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Lecture - 26

Welcome to mooc lectures on Strategy, An Introduction to Game Theory. In this module, we are going to talk about extensive form game with simultaneous move and there normal form representation.

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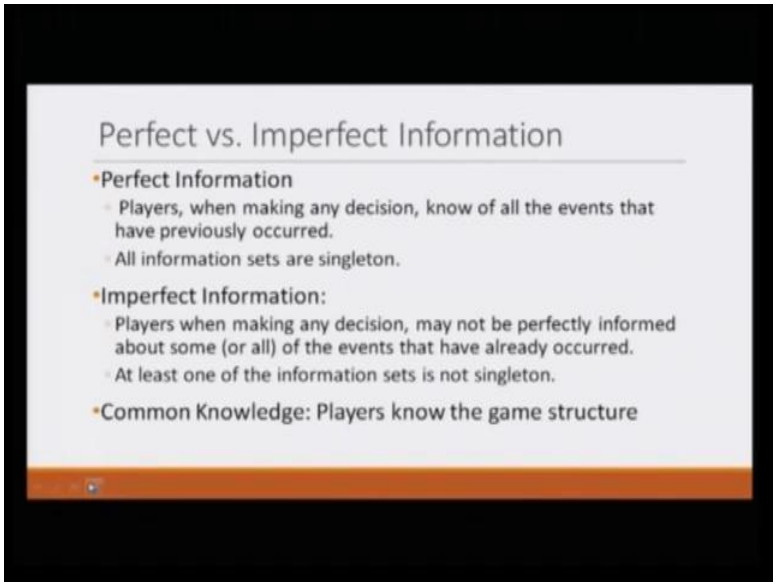

We have already learned two different kind of information structure presenting any strategic interaction, one is of perfect information and other is of imperfect information. What is perfect information, we already discussed, but just to recap, players when making any decision, know of all the events that have previously occurred. Then, the information structure, presenting information structure in the game is of perfect information.

And as oppose to this, when players making decision may not perfectly informed about, some of or all of the decisions, then it is called that game is of imperfect information. And how do we identify? We look at the information sets, all the information sets of that extensive form game. If all the information sets are single term, means they contain exactly one and only one node, then we call the game is of perfect information. Otherwise, game is of imperfect information that we did earlier, nothing new.

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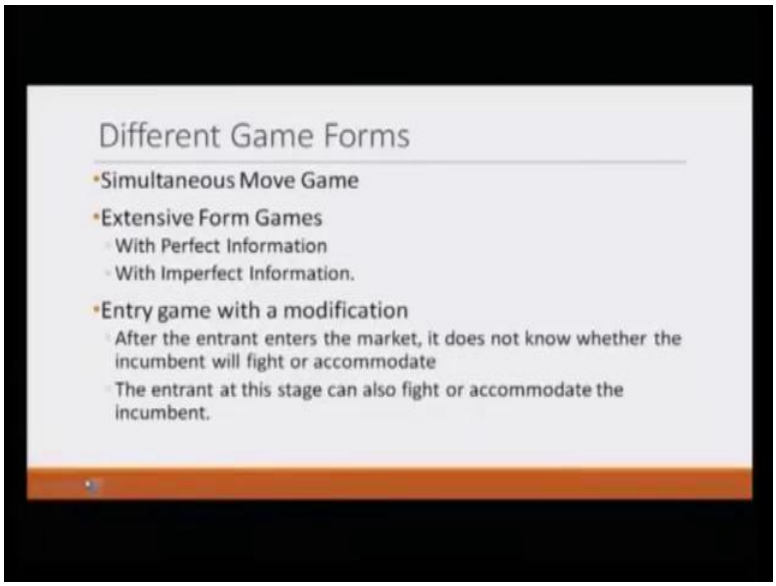

So, what we have learnt so far, we have learnt different forms of game, first we learnt normal form of the game or simultaneous move game. And then, we talked about that how the simultaneous move game is inadequate in representing some of the strategic interaction, so we learn extensive form game. So, extensive form game can be of two types, one with perfecting information another with imperfect information.

So, that we already know, but we are going to do, we are going to take a strategic interaction for which we need to draw game tree, but we also need to have, we will see that there might be some simultaneous move presenting in the game. For example, let us take an entry game with modification. What we have discussed in the entry game? That there is an entrant and there is also an incumbent.

Entrant decides whether to enter in the market or remain out of the market and if entrant decides to enter in the market, then incumbent can decide either fight or accommodate. Now, we are talking about a simple modification that once entrant decides to enter in the market, then both incumbent and entrant not just only incumbent, but both incumbent and entrant, both the players can engage in costly price war or advertisement war. So, they both can imply these two actions, fight or accommodate. How can we model that?

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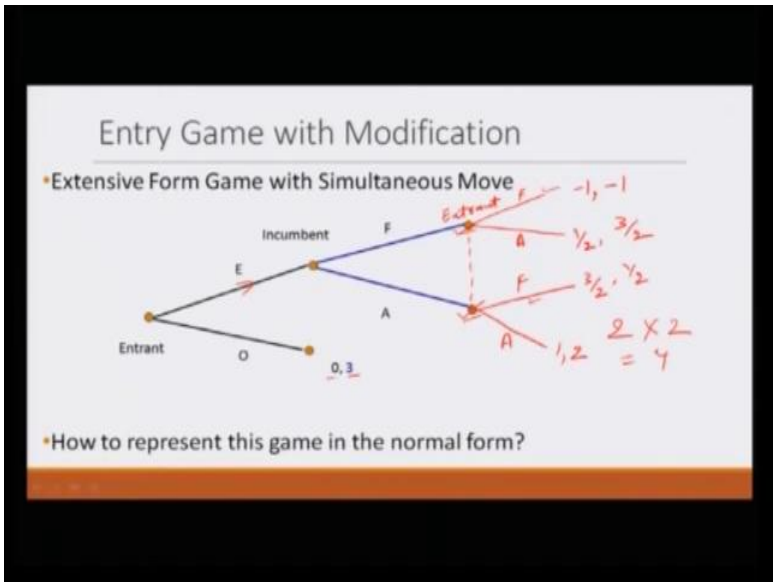

So, this game tree we are already familiar with, the modification how can we represent the modification, so what is happening. Now, after entrant decides to enter, incumbent and entrant both are simultaneously deciding, whether to fight or accommodate each other. So, we already know, we have seen how to represent prisoners dilemma in game tree, we can do that here.

Here again, entrant will get to move either fight or accommodate and here also fight or accommodate and we have to put a dotted line, because entrant does not know, whether incumbent has decided to fight or accommodate. So, this is the way we can represent extensive form game with simultaneous move. Notice, whenever we have extensive form game with simultaneous move, game is definitely of imperfect information, why because here are two nodes in the same information set, entrant when called upon to move in this game at this particular information node, does not know whether this particular node has reached or this node has reached.

So, in this information set, we have two nodes, so that is why game is of imperfect information. Whenever, there is an extensive form game with simultaneous move, we will have a game with imperfect information. Now, the next question is, how can we represent this game in the normal form.

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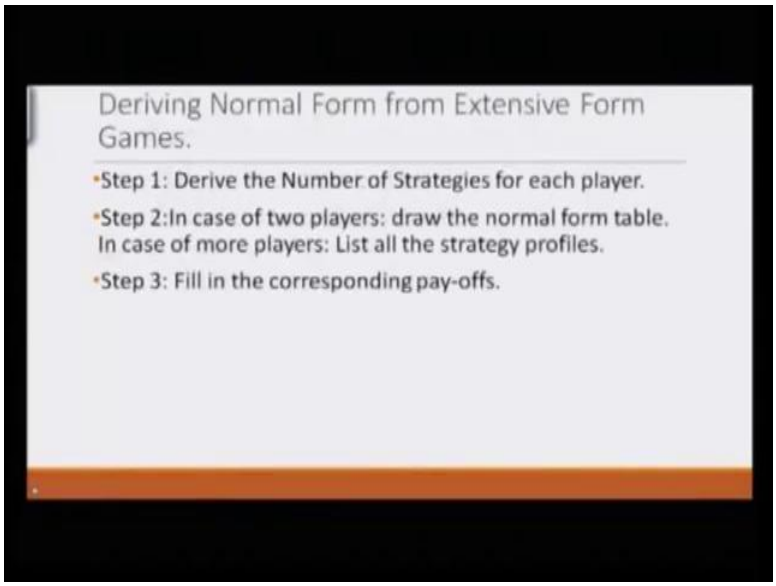

It is a very simple technique that we are going to follow, three step techniques to represent a normal form game, a three steps to represent an extensive form game in the normal form. The first step is that derive the number of a strategies for each player. Why we are doing that? Remember, that in to represent the normal form game, we have three things, first, list of the player and then, strategies set for all the players and then, corresponding payoff.

So, of course, we have list of a player; that is we can clearly get, what is the list of player by looking at the game tree. Now, we have to obtain the strategies of different player, in one of the previous module, I talked about how can we get a strategies for a particular player? So, that is the first stage. The second stage is simply, if we have two players, draw the normal formal table and if we have more players, then list all the strategies of different players and get all the strategy profiles.

And the last step is, once you get the table, fill the different boxes with corresponding payoff and of course, when you have more than two players, then of course then you will have to list all the payoff corresponding to all the strategy profiles.

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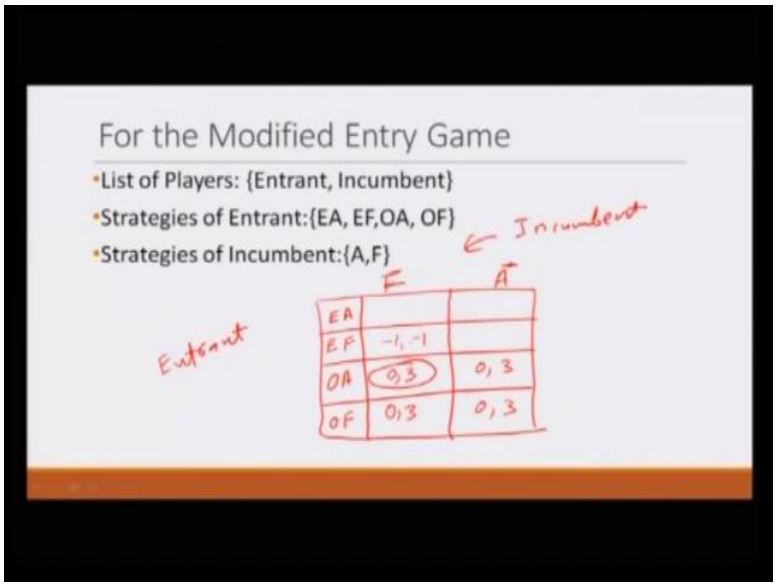

So, let us do that in this case, what do we have, first of all list of player entrant as well as incumbent. Second, we have to figure out the strategies of both the player, entrant gets to move at two different information set, once at the beginning and second, once he decides to enter in the market. So, in the beginning he has two strategies, two actions available to enter in the market or remain out and again he has two strategies, either fight or accommodate.

So, we can have 2 multiplied by 2, four strategies for entrant and what are those four strategies, enter in the beginning and fight after entering. Second, enter in the beginning or and accommodate after entering, third one is remain out in the beginning and if given a chance to move again fight. Fourth, remain out at the beginning and if given a chance again, accommodate.

So, these are the four strategies and how about for the incumbent, incumbent we will have just two strategies, because incumbent gets to move only at one information set and at that information set, he has only two actions, fight and accommodate. So, we got this list of strategies, now how to go about it, we will have to draw the table and how can we describe it, on row side we can put for entrant and here, we can have incumbent.

An entrant has how many strategies, 4, so we can have 1, 2, 3, 4 and these are E A, which in sort enter in the beginning and accommodate later. And similarly E F, O A, O F and then, two strategies for incumbent which are fight and accommodate and this is for

incumbent. Now, how can we fill the boxes? Let us take, how can we get here, O A that entrant is taking or adopting a strategy OA and incumbent is going to fight.

So, we have to go back to the table ((Refer Time: 09:01)), what we are saying entrant is going to remain out in the beginning and given a chance, entrant would accommodate and incumbent is let say going to fight. So, right in the beginning as soon as came begins, entrant enters to play O A, as soon as he plays O, game ends and what he will get, entrant will get 0 and incumbent will get 3.

So, we can say, whenever entrant is playing O A or O F, any strategy in which the action at the beginning is remain out. No, matter what action player incumbent takes, outcome is always going to be 0 comma 3, 0 comma 3, 0 comma 3. Similarly, I will fill for one more, if entrant is going to play E F as strategy and incumbent is going to play F, what will happen, how will the game proceed ((Refer Time: 10:09)), E entrant is going to play E, E at this point and F here.

Again, we do not know which whether this one will happen or this one will happen, we do not know, we have no idea which one will happened. And of course, anyway we have not written the payoff, so let me just also write the payoffs, what would be the payoff. Let us say just for example, here minus 1, minus 1 and here we have half, 3, 2 and here we have 3 by 2 comma half, again these payoff are immaterial, just for illustration I am writing some payoffs.

So, if entrant is playing E F and incumbent is playing F, so what is happening, here incumbent will take game in this direction and since, entrant is playing E F. So, here F will happen and the payoff will be minus 1, minus 1. Here, minus 1, minus 1 and similarly, you can fill the box and you will get normal form representation of this extensive form game.

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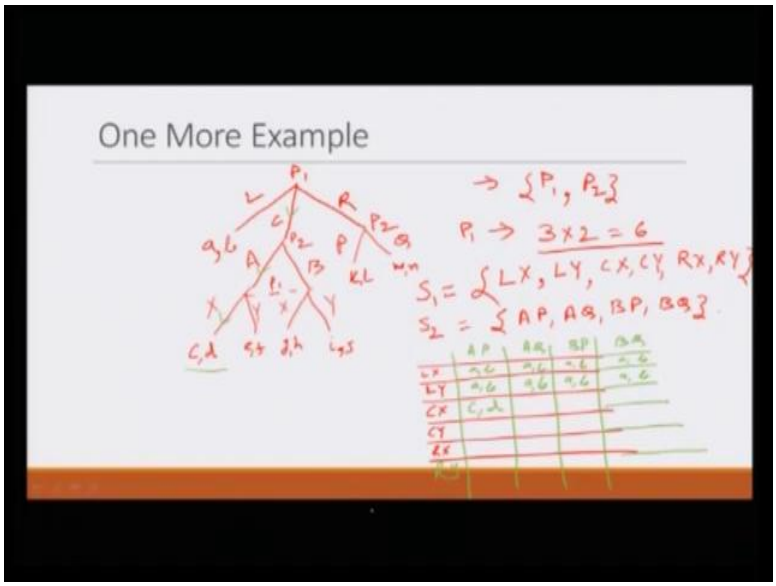

Let us do one more example which is slightly more complicated, it will give us little bit more idea. Let us say we have two players and player 1 can in the beginning, take one of the treats three action. Let us name them left, center and right, here let say we have player 1. If player 1 takes action L, then game ins and player 1 gets A and player 2 gets B and if player 1 takes action C, then player 2 gets to move and player 2 can take action either A or B.

Notice, that action I am representing by capital later and payoff am representing by small later. So, this a small, a is the payoff for player 1 in case player 1 decides to play L and this A it presents the action of player 2. And let us say if player 1 takes action C, then player 1 and player 2 move simultaneously and player 2 decides either A or B and player 1 decides either X or Y.

So, this we can present it like this, again capital, it should be capital and if player 1 decides two play R, player 2 gets to move and player 2 takes action either P or Q in all these cases game ends. So, what do we have here, let us say just random payoff I am writing C D, E F, G H, I J and K L, M N; this is just a general case. How can be represent this game in a normal form, what do we have, first we have to get the list of player; that is very simple list of player, only two players P 1, P 2.

Now, we have to get this strategies of player 1, what are the strategies of player 1, player 1 gets to move has a play at two different information or write at the beginning are here.

Write at the beginning, he has 3 at different actions available, at this information set, he has two actions available X and Y. So, the total number of the strategy for him would be 3 multiplied by 2, 6 and what are those 6 in sort I can write L X, L Y that says play L in the beginning.

And if you get to play the game here then play X, similarly we can interpret other also, CX, CY, RX, RY. This is the strategy set, let me write it as S 1 for player 1 and then, again we have for player 2, how many player 2 has a play, only at two information set, one here and one here and at both information sets, he has two actions each. So, he will have a total of 2 multiplied by 2, four strategies. So, which would be A P, A Q, B P and B Q?

Now, we can draw the table and the table would be, again it is 1, we have L X, L Y, we have then C X, C Y, R X and then, let me change the color of the and then, we have R Y and similarly, we have four for player and which are four for player 2; A P, A Q, B P, B Q. We can write the payoff one thing is very clear, whenever player 1 takes this decision to play L right at the beginning game ends. So, it does not matter, we can say that payoff are would be a comma b, a comma b, a comma b, no matter, what else player 2 is planning to do in this game or what player 1 has plan to do later.

So, we get like this, similarly if player 1 is playing C X, then let us see what happens, player 1 place C here and X here and then it player 1 plays A P, then this if player 1 is playing a c, a playing a strategy which has action c write in the beginning, then the second part is immaterial. Whenever, it is a, then we will get here C X and A P it is going to be c d, why player 1 is going to what player 1 is going to do, plays c here and X here and player 2 is going to play a here and P here.

So, because game is moving in this detection, game will end up at this c d and similarly, you can fill up all the box and just to remind you that you might be wondering that why we are considering L X or L Y, because as soon as player 1 plays L, then game ends. So, for that you will have to see one of the earlier module in which I discussed strategies, the different way that the ways is we can obtained a strategy for players in the strategic interaction and we have a reasons to represent a player's strategy in this matter. So, I believe now you know you should practice and you would be able to represents any extensive form game in the normal form.

Thank you.