Strategy: An Introduction to Game Theory
Prof. Vimal Kumar
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture – 24

Welcome to mooc lectures on Strategy, An Introduction to Game Theory. In this module, I am going to discuss game tree and information sets.

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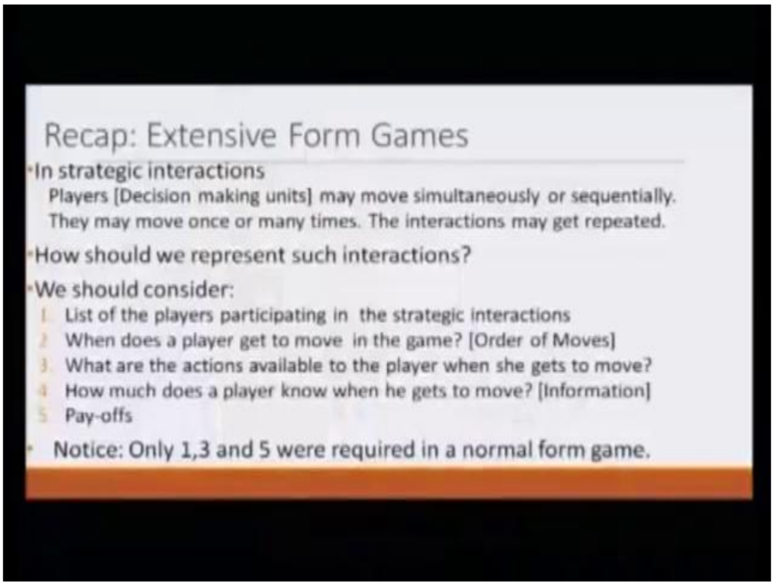

But, before we do that I just want to recap, what we did in the earlier module. In the earlier module, we introduced the concept of extensive form game, we said that in any strategic interaction, that we need to typically we need to describe five things. The list of players, also when does the player get to move and what would be his move and next, how much does that player know when we has an opportunity to move, make a move in that game and the payoffs.

And, then we said that of course, in strategic form game or normal form game that we also use to model strategic interaction. But, it is kind of a reduced form, in which we ignore the order of the move of the game and also how much players know about whatever has happened in the past. But, in the extensive form game we explicitly model these things also.

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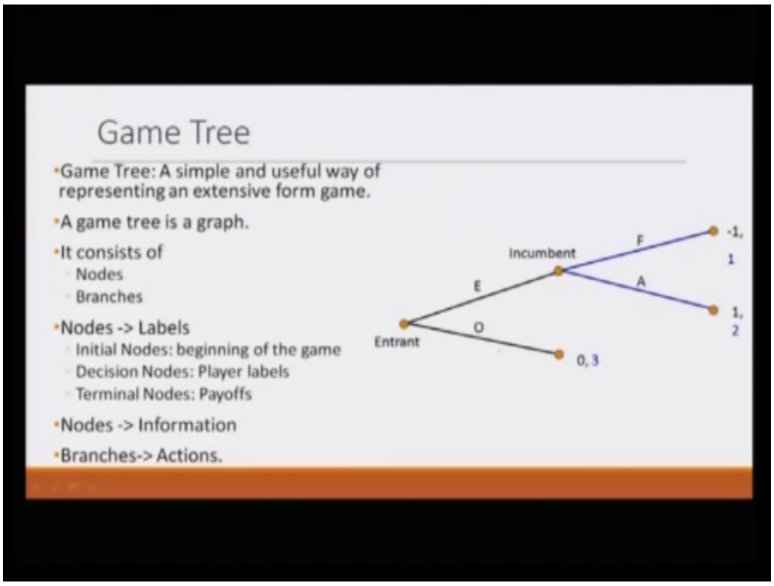

So, and we said that the game tree is the best way to represent an extensive form game and basically we said, that game tree is made of nodes and branches. Basically, it is a graph and nodes they represent basically one of the three things. Game theory as we talked about is in the last module, that it is basically a graph which is made of nodes as well as branches and there are three different kinds of nodes initial nodes, decision nodes and terminal nodes and branches, they indicate the actions taken at the node. Now, let us say that, so far we have been taking about the decision makers as players, but we have not tackle uncertainty in these strategic interactions.

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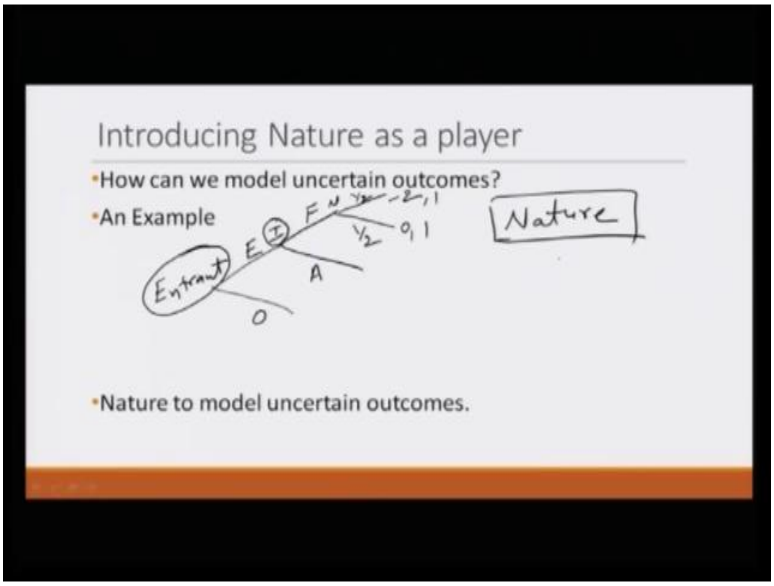

Let us take an example, that it may very well happen like in the earlier ((Refer Time:

02:35)) entry game, what we talked about, that there is an entrant which can enter or which can decide to remain out of the game. And if incumbent enters in the game, then incumbent to either fight or accommodate the incumbent. And, we say that in case of incumbent fights with the entrant either by going for advertisement or price cut, for whatever the reason, then the payoffs are retain here.

Let us consider a scenario in which payoff after incumbent decides to 5 is not certain. It depends on some random event, it may depend on government policies, that there can be two different kind of government policies one, for example it is conducive for incumbents business another, which is conducive for entrants business. So, how can we model such things? For that, what we simply can say, that as we can assume that government is not participating in this strategic interaction in a strategic manner, government is deciding it is parameter based on some other factors.

So, we can say that for these two players, those two factors would generate some sort of random possibilities and one of them would happened, that the entrant or the incumbent would have more control over which one would happen. So, we can say that here is another just extension of that graph, here is entrant decides to enter or remain out and then, here you have incumbent I am writing in short I and incumbent decides to fight or accommodate.

But, if incumbent decides to fight it depends on government policies, so we can introduce nature. Nature represents that chance that one of the things may happen. What these players, entrant or incumbent what do they need to know, they need to know the probability associated with those chances. So, let us say with probability half outcome is minus 2 comma 1, with probability remaining half the outcome is 0 comma 1.

So, what we are doing? Here we are introducing nature to model this uncertainty in strategic interaction. What we have to keep in mind, that this nature is not a strategic player, but nature is basically, they are just to figure out or to model the randomness in this strategic interaction. You can also think of a joint project, in which there are two players, both are farmers and the outcome depends on whether they work hard or they do not work hard, but also it depends on the climate, whether you had sufficient rain or not.

So, all those things can be modeled using nature, nature would be that player who would randomly decide that, which of the outcome is going to, which of the things going to happen. So, we will also consider nature in these games.

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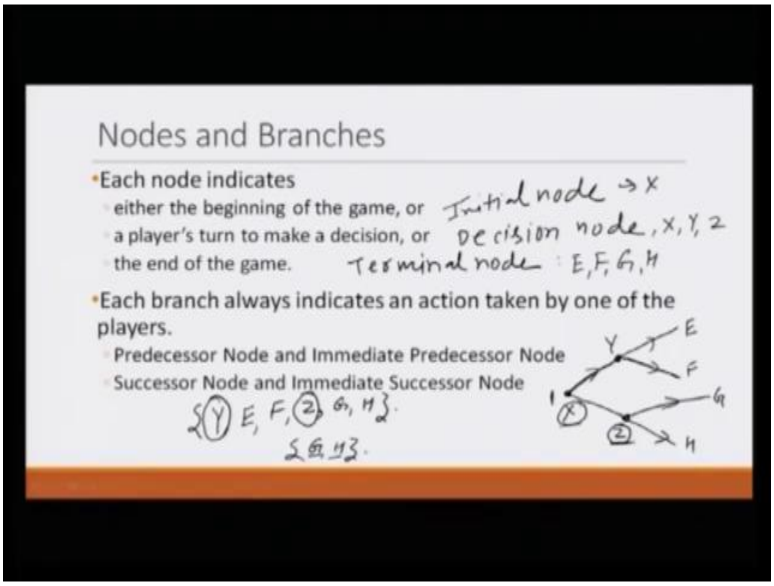

So, what we have seen so far that in nodes, node represents one of the three things, either it represents the beginning of the game or it represents a player's turn to make a decision and it may also represent in some cases, the end of the game. And, what we are saying this is initial node, this is decision node and this is terminal node. What we can say here, now nature can also start the game, natures action whether it is going to be sunny or it is going to be rainy, it nature would decide in the beginning and then the game would move on.

So, in the beginning either a player can start the strategic interaction or nature can start the strategic interaction. What do we have next? Then, we have branch, each branch always indicates an action taken by one of the players, based on these two things we can also define something called predecessor node or and successor nodes. And, we will also we should discuss the special case of predecessor node that is immediate predecessor node and also a special case of successor node that is immediate successor node.

So, what do you mean by successor node? Let us take a node, let us say this is the game tree. This is game tree, here player 1 moves this is one node, which is initial node, this is another node these three are decision node net. Let us represent them by X, Y, Z and here we have E, F, G, H, let us look in this game. X is initial node, what are the decision nodes, here we have X, Y, Z are decision nodes and E, F, G, H are terminal nodes.

So, what do we say that successor node of a node, let us say X are all the nodes that can be reached from X by progressing forward along a decision tree. So, here if we take X, then Y, because Y can be reached from X if game moves in this direction, we reach Y. Similarly, we can also reach E from X, so Y, E, F, because F can also be reached first, this action and then the first action from here to here and then, here we will reach to F.

And similarly, Z, G, H all these are successor nodes are X. We can take another example if we are taking node Z, then which are the successor nodes of node Z. Let us look at it, from Z if we move forward then we reach to G and H, so only G and H are the successor nodes. Similarly, we can define an immediate successor node, so of course, when there is a node a player gets to move at that node, so player can take one of the actions available to that particular node.

So, once if the player takes that action, then a node will be reached that node will be an immediate successor node. For example, for X here Y, this Y and Z are immediate successor node, Y for Z, G and H are the immediate successor nodes and those are the only successor node for Z. Then, we can also define predecessor node, if X node, if a note A, let us say for node A is a successor node of node B, then node B is a predecessor node of node A.

For example, if we take here Z which would be the predecessor node, so from X we can reach to Z. So, X is predecessor node and similarly, we can define immediate predecessor node.

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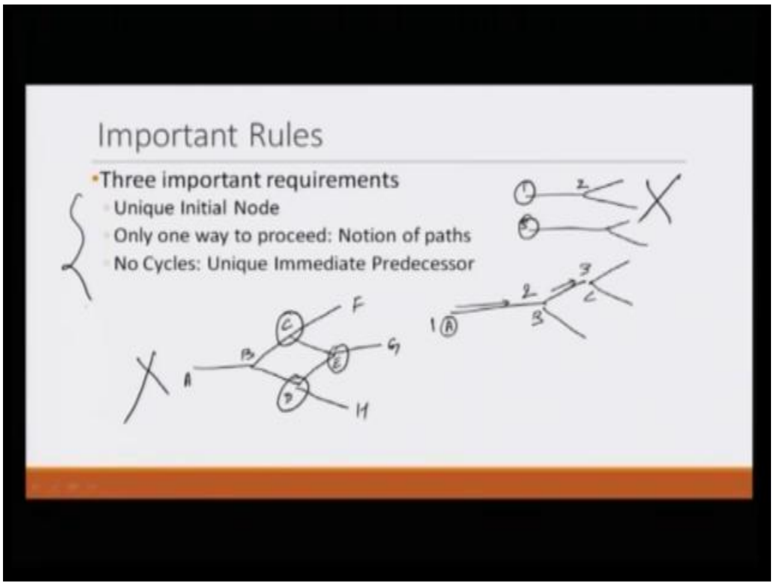

I have already given example, so in game tree, we have to keep three things in the mind, the first is that we should have only one initial node. For example, game should start, game should have only one beginning. What do I mean by only one beginning? Let us take an example, here a game is starting here, let us say a player move, here player 1 moves and then player 2 gets to move and then, we have here player 3 moves and again player 2 gets to move.

So, this is not allowed, this is not a game tree, why because game is beginning at two different points, so in a game tree we have to have a unique initial node, so this is not allowed. The second is that there is only one way to proceed. What do I mean by only one way to proceed? That from A, we should assume game tree, all the nodes are connected to other nodes. What do I mean by connected? That if we can take series of branches from one node, we will always be able to reach any other node in that tree, so in that sense game tree is connected.

Only one way to proceed is, that there should be only one path containing, when I say path I mean series of branches there should be only one series of branches, unique series of branches that should take from one node to another node. Let us take that example, here player 1 moves, then player 2 gets to move, then player 3 gets to move and if I want to reach from node A at which player 1 moves and to node, this node C, at which player C moves there should be only one path, like this is the path to each player, this is unique path.

There is no other path from node A to C, in that sense we should have only one way to proceed. The third thing that is a requirement is that there should not be any cycle in the game, in other word that each node except the initial node, notice that initial node would not have any predecessor node, because game is starting at the initial node.

So, except initial node all other nodes would have unique immediate predecessor. So, let me show you an example, where it is not true, let us say this is a graph, in which we have node A, B, C, D, E and also F, G, H. Let us look at node E, node E has two immediate predecessor nodes, which are C and D, so this graph is not a game tree. So, these are three important things that we have to keep in mind, when we are giving the game tree.

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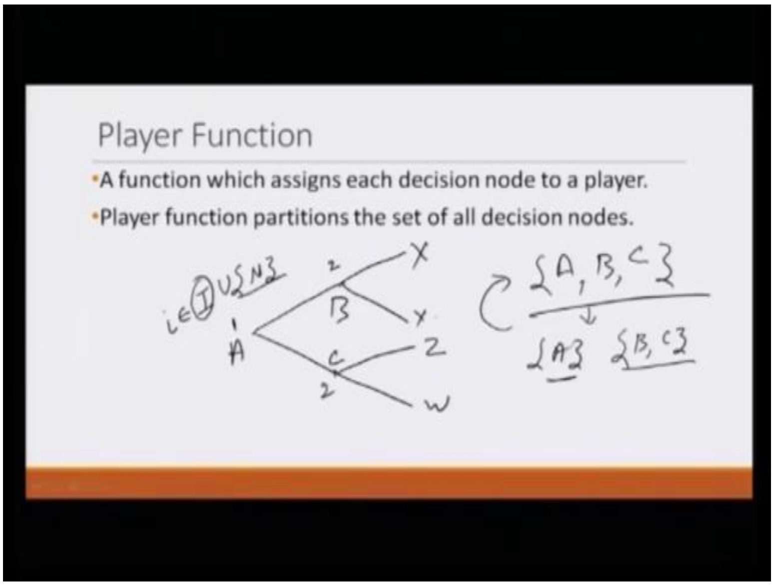

Now, let us move to something called player function, we have already said I already told you in the last module that each node gets, each decision node gets leveled by the player who would get to move. So, in other word, what do we have, we have a player function, a function which assigns each decision node to a player. What I want to emphasize that at one particular node only one player get to move. So, it is very, very clear, then if that node is reached which player will get to move.

So, in a word, let us say if we have a game tree a very simple game tree that I am drawing here, let us say we have A, B, C, X, Y, Z and W, these are noticed these are nodes. What are the decision nodes? Only A, B, C the set decision nodes we have A, B and C. What this player function would do? This player function would assign each decision node to one unique player. So, we can say that at A we can say either one of the player gets to move, let us say that one of the player is denoted by small i, we gets to move and capital I denotes the set of all player.

But, remember we also introduced nature as one of the players, nature can also make a move in this game. So, we should also allow for that nature, the player who is going to move with this node is either one of the active players or nature. So, what it does, the player function would partition the set of decision node, for example it is we can say here player 1 gets to move, here player 2 gets to move.

So, what we can say from here, we get a set A indicating that at all the nodes at which player 1 gets to move, this is the set and all the nodes at which player 2 gets to move this is the set. So, here we get the partition, just to remind you what do I mean by partition of a set, what we are basically doing, we are breaking a larger set into smaller sets in a very particular fashion. Such that, when we take two smaller sets, there is no common element in both of them and if we take all the smaller sets together, then we recover the larger sets.

So, let us see here if we have only two sets, so we take these two, we do not have any common element and if you take union, we get back the original set, so this is player function.

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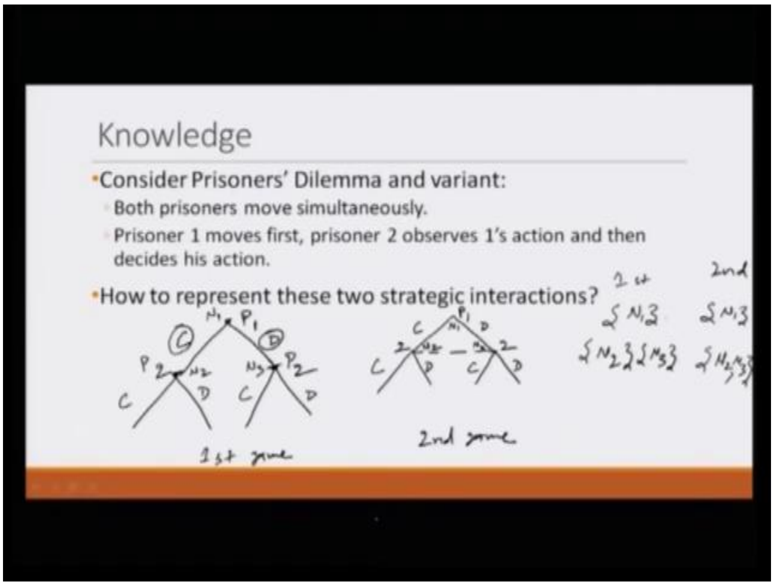

Now, let us come back to knowledge and to that I will take about prisoner's dilemma and it is variant. So, if you remember the prisoner's dilemma, what was the prisoner's dilemma, there were two prisoners. And then, move simultaneously, first question that I would like to answer can be model this prisoner's dilemma can we represents this prisoner's dilemma using game tree.

Let me show you how, let us say, what do we move by simultaneously that when a player make some move he or she is not aware of the actual action taken by all other players that what simultaneously move means. So, here, what is happening two prisoner's they move, they move simultaneously. So, when a prisoner's deciding his action his not he knows, what are the different actions the other player can take, but he does not no the actual action taken by the other player, so how can we represent introducing game tree.

So, let us see we have here prisoner's number 1, what he can do, he can either confess or he does not confess. And, let us take a variant, in which prisoner's 2 knows the actual move of prisoner 1, so in that case it is very simple, that prisoner 2, gets to move and here also prisoner 2, gets to move and he can take one of these two action C or D. Fine coming back to the original prisoner's dilemma their situation is different, here it clearly indicates that when player 2 gets to move at this particular node.

He knows the prisoner 2 nodes, that prisoner 1 has already taken action C and at this node when prisoner 2 gets to move he already he knows. This, fact that prisoner 1 has already taken action D, but in actual prisoner's dilemma that was discussed in the first week the prisoner 2 did in know the actual node. So, a very simple trick we can use to represent to bring this simultaneity, that we will connect this with doted line, here player 1 moves takes action C or D player 2 gets to move here.

But, player 2, basically is not aware of the node that has reached in this game. So, player 2 takes one of these to action. So, there is a difference here, here this is not connected clearly indicating that player 2 knows, that whether node, let us say this node has reached or this node has reached. But, in this case player 2 moves after player 1 has decided is actual move, but player 2 does not observe. So, player 2 nodes, that player 1 has decided, but player 2 is not able to distinguish between this node and this node.

So, we connected with the doted line to indicate this lack of information, lack of knowledge that prisoner 2 has. Now basically, so it is all about the level of knowledge here prisoner 2 nodes that prisoner 1 has taken action C here, prisoner 2 does not know, that whether prisoner 1 has taken action C or D he only knows that prisoner 1 has, what taken one of the action.

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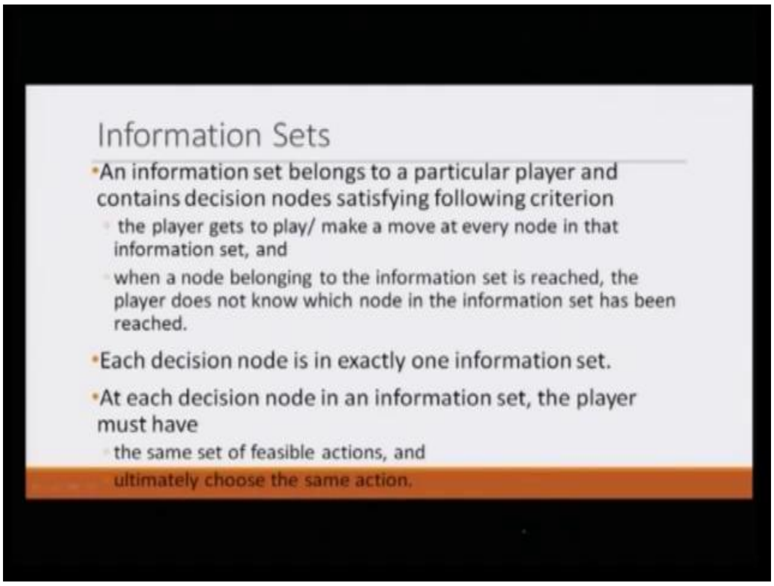

And this is model using information set, so what do we mean by an information set and information set, basically belongs to a particular player. And, contains this is a node satisfying following criteria these are the two criteria, that we have to keep in mind. The player 2 gets to play or make a move at all the nodes in that information set. So, information set contains only those decision nodes, at which the same player gets to make a move.

And, when a node that is the first criteria the second criteria is that, when a node belonging to the information set is reached the player does not know, which node in the information set has reached. ((Refer Time: 22:23)) So, let us take this example again, let us look at the prisoner's 1 information set, here, let us call this N 1 indicating node one this is N 2 this is N 3, again here we have N 1 N 2 N 3 for prisoner 1 is clear N 1 in this game is information set for prisoner 1. In the second game also here, let say for the first this is the first game, this is the second game, first here we have second, second also we have N.

How about for player 2, player 2 first criteria, we have to see, which are the nodes at which, player 2 get move N 2 and N 3. But, that is only the criteria 1, how about the criteria 2, when a not belonging to the information set is reached a player does not, which node in the information set has reached. But, here there is only one. So, we can say when game reached to N 2 player 2 knows that this node has reached.

So, we can say N 2 is only one information set and N 3, so N 2 in one information set and N 3 is another information set. But, about for the second game if game reaches here player 2 is not able to distinguish between N 2 and N 3, so using second criteria we will have information set, which will contain N 2 and N 3 both. So, further each decision node is in exactly one information set and at each decision node in an information set the player must have the same set of feasible action and ultimately ((Refer Time: 24:23)) chooses the same action.

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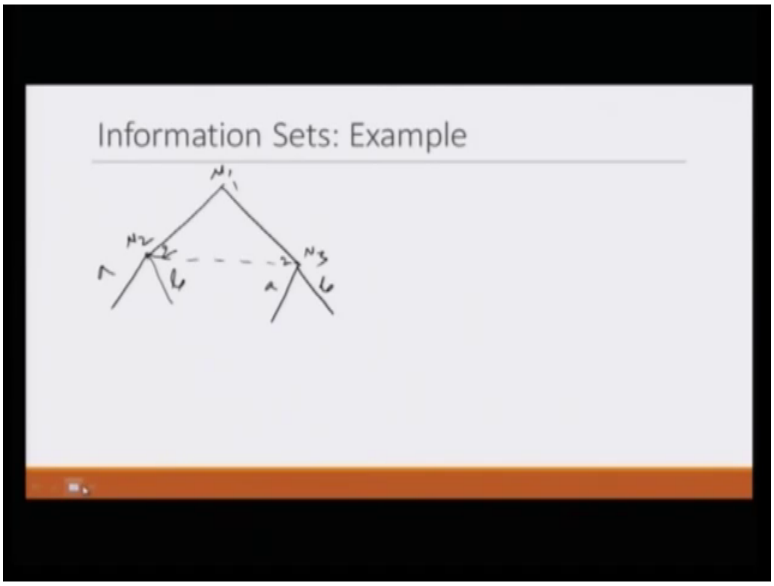

Let me indicate these last two term a little bit a more ((Refer Time: 24:29)) detail. Let us, taken tree, again let us consider as a scenario here again we have N 2 we have N 1 N 2and N 3. And here, we have three actions X Y and Z, here we have A and B first of all, what does the criteria say earlier that at each decision node in an information set the player must have same feasible sets the same feasible actions. Let us, look at it here, here we have A and B, but here we have X Y and Z.

So, what do we get if player C is that he has three action for at least disposal X Y and Z immediately he would in from that he is a N 2 and not an N 3, then put in this dotted line would not make sense. Because, this dotted line indicates that player 2 is not able to distinguish between node 2 and node 3. So, when we have a dotted line we cannot have situation like this that one place we have X Y and Z as a actions available and at other place we have A and B as actions available both the places we should have same. Otherwise, the player would understand, which node has reached this is what we get A N.

((Refer Time: 26:06)) The next thing it says that player ultimately chooses the same action that I cannot exactly show you, but let me tell you, because player does not the player who gets move at N 2 or N 3.He does not know whether the node that has reached is N 2 or N 3, he would decide he respective of the fact whether is that N 2 or N 3 he would only take N 2 a count that is act one of these two nodes. So, he will take the same action whether is actual restart N 2 or at N 3.

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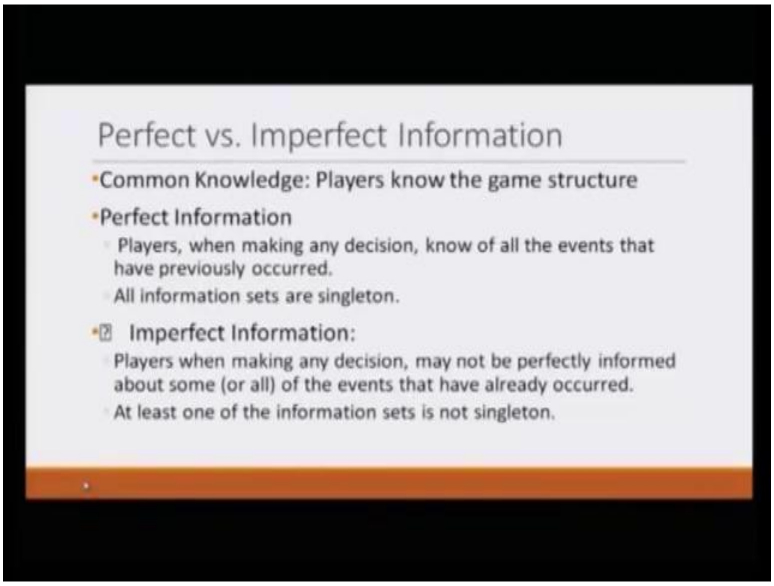

Now, we have talked about common knowledge, what does the common knowledge here, common knowledge says, that players know the game structure. But, then we are we should distinguish between perfect information and imperfect information. What is perfect information that players when making any decision know of all the events that has previously occurred, so his aware.

So, let us go back to the example that we add earlier, let us look at N 2 first game when player 2 is moving at N 2 he knows all the events that has previously occurred. That player 1 has move and he has taken action C or if you has reached to node N 3 he knows that player 1 has move to indicate action d. But, let us look at in the other game when player 2 gets to move he is not actually aware of all the previous moves.

When player is at N 2 he does not know whether player 1 has taken action C or action D or even when player 2 is at N 3, he does not know whether player 1 has taken action C or D. So, these are the two different situation that we are discussing one is called perfect information, when player knows all the events that has the previously occurred. and imperfect information when player does is not perfectly inform in the sense that he may not know all are some of the events that have already occurred.

One way to figure out how again is a perfect information when all the information set is singleton it means all the information set contains only one node. If, at least one of the information set is not singleton then we say game is of imperfect information, that is it for this module in the next module, we are going to talk about is strategies.

Thank you.