**Strategy: An Introduction to Game Theory**
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**Lecture – 22**

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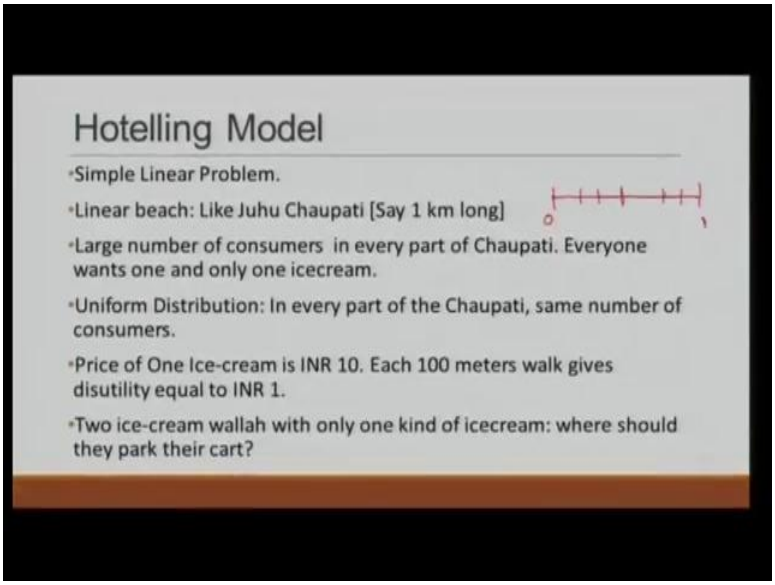


In this module, I am going to talk about one of the applications of game theory. So, here we have a model, which is simple linear in nature. Let us imagine that we are talking about Juhu Chaupati – a linear beach. By the way, do you remember the introductory video that we have put on the course website to for… so that you get some idea about the course. I was talking about ice-cream wallah problem. That is the problem I am going to discuss now. So, what we have is let say a Juhu Chaupati. Let us say for example, that, this is 1 kilometer long. This is just for example sake. So, do not worry about if it is incorrect. And, what we have is large number of consumers in every part of the Chaupati. And, we also assume that, everyone wants exactly one and only one ice cream; not more than one ice cream. Again, these are the assumptions that we are making. What we have that, these consumers are uniformly distributed on that Juhu Chaupati. What do we mean? In every part of Chaupati, we have same number of consumers.

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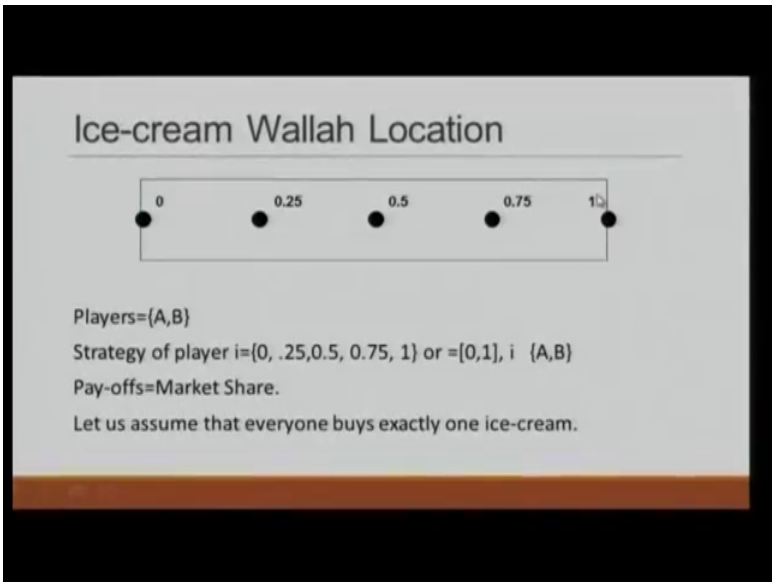


So, let us say if we have this say; this is the linear. Here, we say that, the beginning of Juhu Chaupati here. It is end of the Juhu Chaupati. If we take let say a segment here and a segment here; as long as the segment are… Say these segments are of equal size; they both will have the same number of consumers. That is what uniform distribution means. And, price of one ice cream is exactly 10 rupees. And also, because people are lazy, they do not want to walk. We assume that, each 100 meter walk gives this utility equal to 1 rupee. Or, in other word, you can say walking 100 meters is equivalent to 1 rupee cost.

So, let us say that, if I can get an ice cream… Let us say if I can get an ice cream and for that, I have to walk 500 meters; then, I am spending 10 rupee on the ice cream, because that is the cost of the ice cream; and, I have to walk for 500 meters. So, I also incur that 5 rupees of cost. So, total cost for me would be 15 rupees. And, for simplicity, let us say that, we have only 2 ice-cream wallahs and they are selling only one kind of ice cream. The question is very simple that, where should they park their cart; should they park their carts right in the middle both of them; or, should they park their carts; and, one should park at one end; and, other one should park at other end; or, they should park one here right in the middle of the first mid segment and second in the middle of the other half. So, we have to try to answer this.

Just to make our life simpler, what do I do here; I say that, Brihan Mumbai Municipal Corporation allows only these five locations to park an ice cream cart. Those locations

are 0, 1, 0.25 just in the middle and again at 0.75. Once you are done with this problem, you can also allow for that, any possible place they can park their cart. But, right now, we will solve the smaller question. This can be modeled as a game. As we have two players, both are ice-cream wallahs; they have the strategies are at their disposal. What are the strategies to park their cart either at 0 or at 2.5 or at 0.5 or 0.75 or 1. So, these are the strategies. Both the ice-cream wallahs have the same strategies and they also have payoffs. The payoff can simply be captured by market share, because we are assuming everyone wants one ice cream; the cost of ice cream is fixed; everything is fixed. So, the only thing that they can do is to capture the larger market share. So, we can say that, payoff is in terms of market share. How about the consumers – people who would buy ice cream? Are not they players also? They are not in a sense active players that, they do not have strategy consideration. So, we will not worry about them at present.

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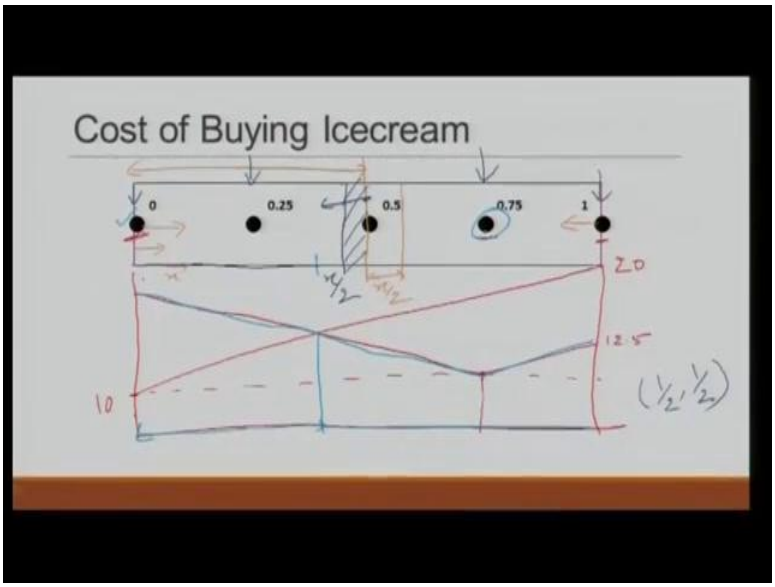


So, let us say if one ice-cream wallah is right here; how much would be the cost? Let us say here we have distance-wise we are doing. How much would be the cost? The person who is right here; it would cost him… The ice cream would cost him 10 rupees. And, as it will move in this direction, person who is standing here will have a cost of 20 rupees and it will linearly increase. So, we can say that, cost is 20 rupees. Here it is 10; here it is 20. Let us say if ice-cream wallah is found here. What would be the cost to people? At this point, let me just say so that this line gives us 10 rupees; at this point, the person who is at this point will have cost of 10 rupees. And, as I move, we move in this direction;

cost would linearly increase. And, for person here, the cost would be… because walk here is of 250 meters. So, that would translate into the cost of 2 rupees 50 paisa. So, it would be something of this sort – 12-50. And, here it will come around 17-50. So, this would be the cost. If I change the color; that I should have changed earlier; but I did not.

Let me just redraw it. So, here we have the cost from second ice-cream wallah. So, if this is the configuration, we see people would buy ice-cream from a cart, which would cost that person less amount. So, we see that, in this zone, people would buy ice cream from this ice-cream vendor. And, in this zone, people would buy ice-cream in this – with this ice-cream vendor. So, the question is now, where should they park their cart? One – we can think that, how about if they park – one parks his cart here at one end and other one parks at the other end? What would happen in that case?

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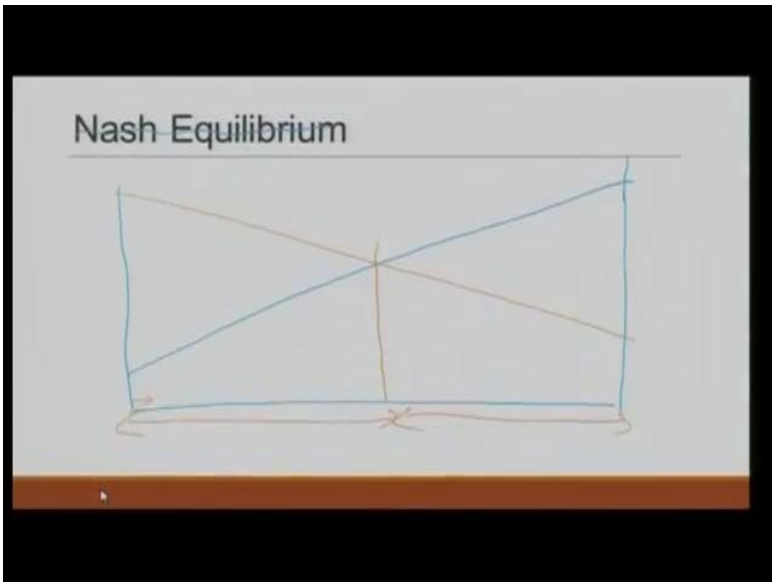


What would happen in this case? Although here I write nash equilibrium, but let us look at the graph. Cost would increase from 10 to 20; and, for other, it would be in the opposite direction. So, it would be like 20 to 10 here. And, both of them will capture exactly half of the market. Can we call this placement that, parking at one end and another person parking at the other end – can we call it a nash equilibrium? Let us say that, what if… What is the requirement for the nash equilibrium? That none of the players have any incentive to change the position of their cart. So, let us say if we look at this ice-cream wallah; if we move slightly in this direction, what happens?

Let us come back to the earlier graph. If he moves in this direction, what would happen? Earlier he was able to capture till here from 0 to 0.5. Let us say that, he moves x in this direction. Now, he would be able to capture all of this and plus here x by 2, because people would go to… A person would go to the ice-cream wallah who is located nearest to his position. So, also these people will start buying ice cream from here. So, he will have larger market share. In other word, he has incentive to deviate from his original position. Both ice-cream wallahs will have been incentive to deviate. In fact, this ice-cream wallah would like to move in this direction; and, this ice-cream wallah would like to move in this direction. So, there is only one strategy profile that gives nash equilibrium. And, that strategy profile is that, both of them stay in the middle. So, let me use this empty space to talk about it how it happens.

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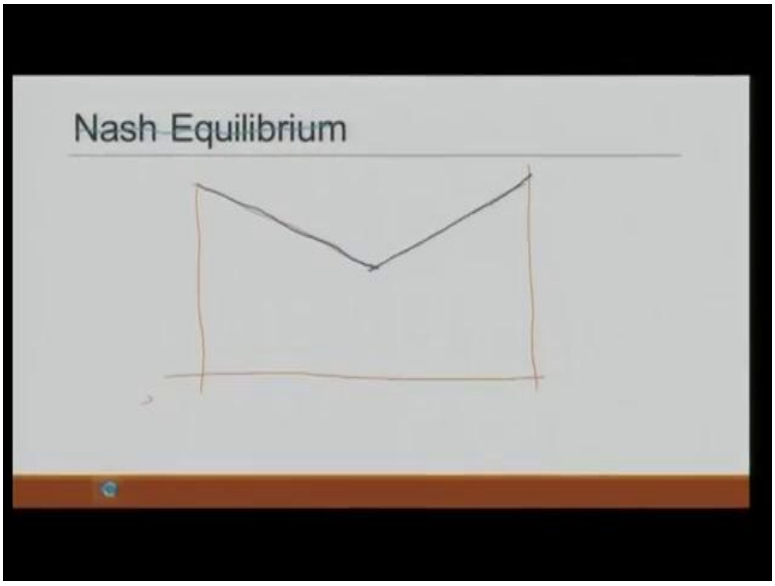


So, now, both of them are standing in middle. So, the cost for the people who is just at the middle would be 10 rupees and here it would be 15 rupees. So, both ice-cream wallah buying ice-cream from any of the ice-cream wallah would cost the same amount. So, we will say that, randomly, people would split and they both will capture the half of the market. And, that is nash equilibrium. In this case, none of the players have any incentive to change the location of his or her respective cart. Let us see what happens now if he moves. If he moves, let us say the other player is in the middle at this point. What happens if a player moves out of this mid position let us say in this x direction; what would happen? Now, he is able to capture all the people here and up to only x by 2

distance here. So, he will be losing this market share. So, he does not have any incentive to change his position of cart. By changing the position of his cart, he will be losing market share. So, the only nash equilibrium is that, both the ice-cream wallah park their cart right in the middle. But, is it socially optimal?

What is happening? Some people let us say the people who are at this end or at this end – they have to walk 500 meters. And, remember that, walking gives a cost. What if they park their carts at 0.25 and 0.75? Still they are able to capture half of the market. But, in this scenario, none of the consumers will have to walk more than 250 meters. So, this is better socially. But, unfortunately, that, it is not a nash equilibrium, because as we have learnt that, self-interest or the rationality, self… It terms the social good. Like we have seen in the prisoner's dilemma, and we see it in this also. So, none of the players will have… The players… None of the ice-cream wallahs will have incentive to park their cart at 0.25 and 0.75.

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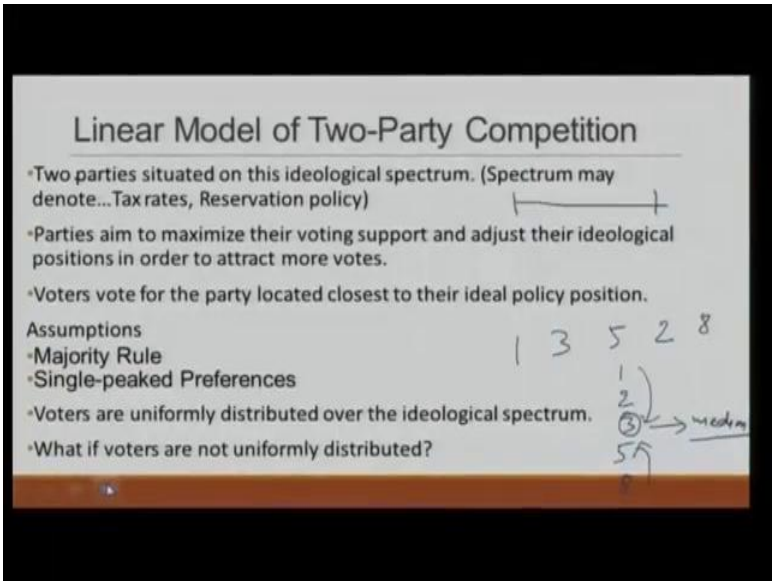


Now, why are we talking just about ice-cream wallahs? We can talk about electoral competition also. What we have now? We can say there are two parties situated on this ideological spectrum. What is the requirement? That this ideological spectrum is linear in nature; it can be… Let us say that, ideological spectrum is philosophical in nature. What we have? 0 represents the extreme left and 1 represents extreme right. Now, let us say that, parties are interested only in maximizing their voters' voting support; and, they are

willing to adjust their ideological position in order to attract more votes. And, the thing is very very simple that, voters vote for the party ((Refer Slide Time: 12:28)) policy position. We again make two rules that, winner is decided using majority rule; and, the voters have single-peaked preferences. Just hold on for a moment; I will talk about what do I mean by single-peaked preferences. And, again the assumption is that, voters are uniformly distributed over the ideological spectrum. In what sense they are ideologically distributed? We will talk about it and also the question we will try to tackle – what if the voters are not uniformly distributed?

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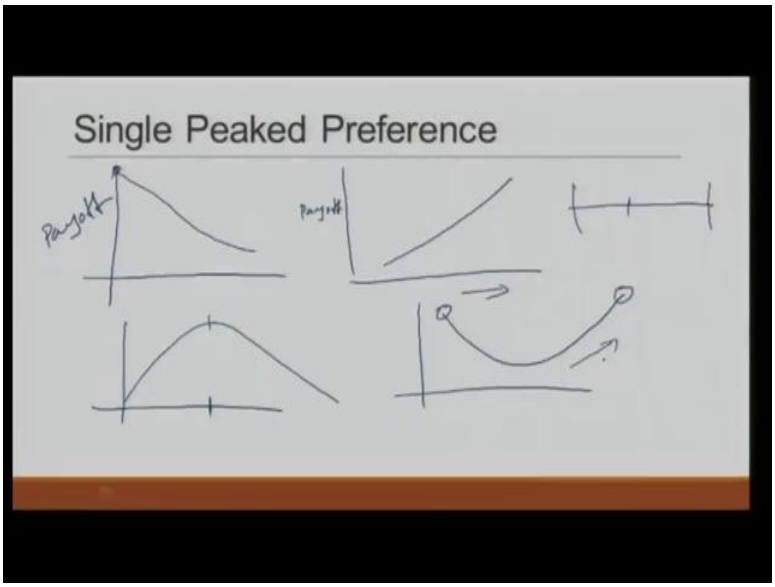


But, first, what is single peaked preference? Let us say this is the payoff; and, on x-axis, we have ideological preference. So, it is a possibility that, someone has this kind of payoff; he likes the extreme left and he does not like as he move towards right, he gets worse-off. So, this is single peaked, because highest is just at one point. Similarly, this is also single peaked. Again we have payoff and here ideological point. So, this is single peaked. We can also have a different kind of single peak like here – this person is centrist; he likes middle more than the extreme. So, this is also a single peaked. But, this one is not single peaked; it has two highest point. So, this is not what we allow. And, that is a reasonable assumption in politics. If I like extreme left, then I would not have any other peak. As I move away from my ideal position, I become worse-off. So, this is the requirement. Why is this requirement?

Remember the ice-cream wallah problem; same because they were located on a Juhu Chaupati at a particular point. And, further we said that, they have to move from their ideal policy point; they would be worse-off. Similarly, the same thing we want here. So, they move away from their ideal position point; they become worse-off. But, here if we allow for policy preference, which is not single-peaked, then this breaks down. So, in the beginning, it… Further the candidates policy is from his policy point, he becomes worse off; but, then he starts becoming better-off. So, we do not allow for non single-peaked preference.

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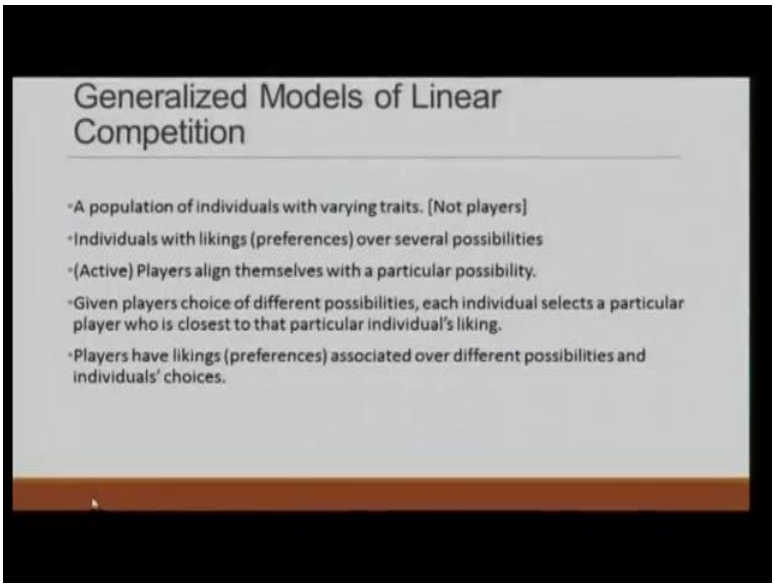


Again the attempt is to make this problem same as… – again same as the ice-cream wallah problem. What we are having is again same as ice-cream wallah problem. What should be the ideal policy point for the both political parties? Right in the middle. Why? Because then they do not have any incentive to deviate; they both will be able to capture half of the voters; fine? Any other policy point, one of the parties will have incentive to deviate. What if voters are not uniformly distributed? Then, we cannot talk about the mid-point what we have to talk about the median point. And, that is… What is median point? Let us say you have five numbers – 1 3 5 2 8; how do we get the median? We arrange them in increasing order or decreasing order, that is, 1 2 3 5 8. And, we figure out the number that is in the middle. So, if we move here – 3 and here also we move; so, we get… This is the median policy point. So, in case, when voters are not uniformly

distributed; in that case, is very very simple – instead of getting the mid-point as the outcome, both parties would announce median policy as their preferred outcome.

So, let us look at the generalized model of linear competition. What do we have? We talked about ice-cream wallah; we talked about electoral competition; but, several other problems can also be tackled. What do we have typically? A population of individual with varying traits. Earlier, what was the varying trait? The distance that they have to travel to get the ice cream in the electoral competition that, it was political distance, the ideological distance we talked about. So, they have to have some varying trait. Now, these individuals should have liking over different several possibilities. Of course, sometime we restrict ourselves to single peaked preference or something like that. Now, we also should have active players. What do we mean by active players? Players were engaged in the strategic interaction like two ice-cream vendors or two political parties. What do they do? Let us say that, they do that, they align themselves with a particular possibilities.

Given players choice of different possibilities, each individual selects a particular player who is closest to that particular individual's liking. Like in the case of ice-cream wallah, we talked about that shortest distance. In again political competition, we talked about the shortest ideological difference. And, players have liking associated over different possibilities and individual choices. This is much more general than what we have tackled. Earlier we have given very simple problem that, ice-cream wallah is interested in maximizing his market share; electoral parties are interested only in winning. But, there is also we can have much more general model that we are not dealing with here that, other than winning, they also care about their ideal policy point – what would have happen that, you should ponder over how would the outcome change. That is it for this module.

Thank you very much.