Strategy: An Introduction to Game Theory
Prof. Vimal Kumar
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture - 20

Welcome back to the mooc course on Strategy, An Introduction to Game Theory.

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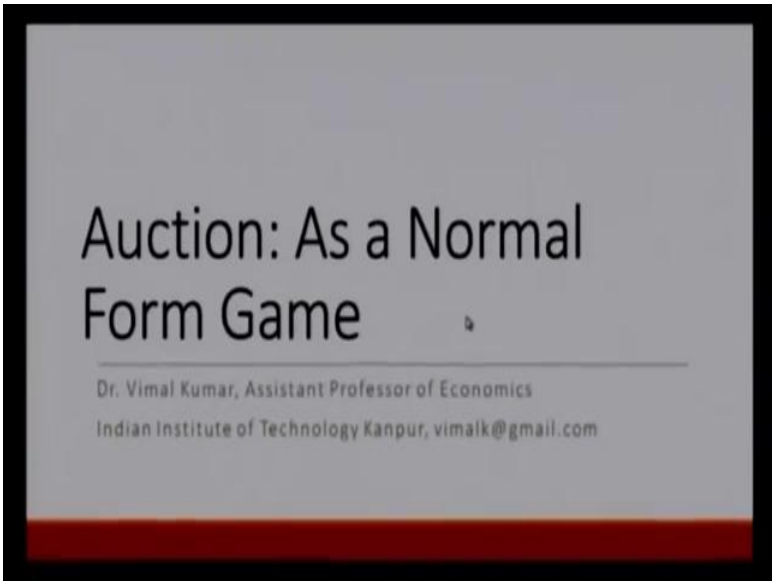

In one of the earlier modules, we talked about auction, we introduced the notion of auction and we talked about different types of auction. In this module, we are going to talk about auction as a game; the idea is to model auction as a normal form game. So, so far you have studied normal form or strategic form game. What we have learnt, that to describe a normal form game, we need to give three pieces of information.

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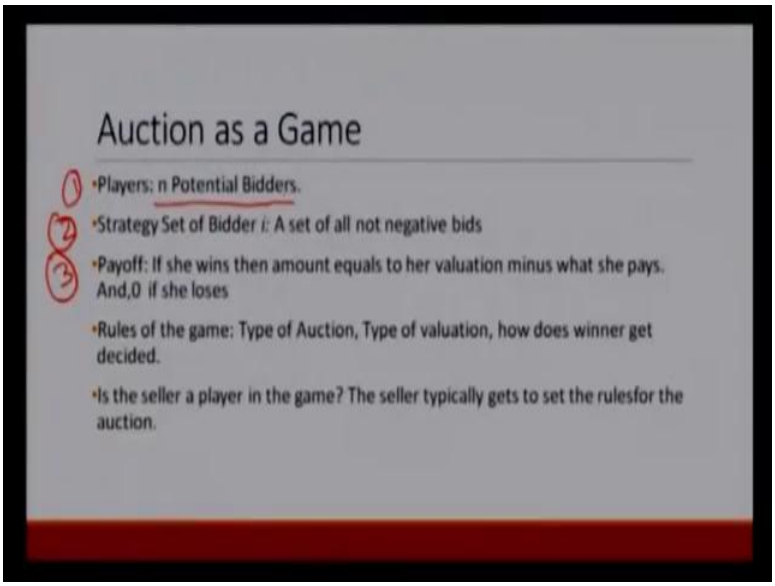

First, we need to give the list of players that is the first information required, the second piece of information is, that we need to give strategies set for all the players. And, third we need to give utility or payoff associated with all different strategy profile, so let us look at this auction a very simple auction, we will talk about. So, who can be the players in potential bidders? Since, we are looking at the auction for the first time, what we will assume that it is one sided auction and only buyers are interested, buyers bid; seller is not bidding, only one sided.

So, we have n potential buyers as players, then we also have to think about, what would be the strategy set of a particular bidder. So, how much money a bidder can bid? He can bid any number from 0 to let us say infinity, so that is what I am saying, that strategy set of a bidder i is a bid that is comes from a set of all non negative numbers. And what would be the payoff? Of course, the payoff would depend on the strategy profile, but one thing is very, very clear that if he loses, then he earns 0.

Let us ignore monitory value of time, the time that he would devote to participate in the process, so that we will ignore. So, if he loses, he does not pay anything, he does not get anything, so we will say he gets 0 and if he wins, then depending on the type of auction he will have to pay certain amount. So, his payoff would be the amount, the value that he associates with the goal minus the amount he pays, so we are able to describe all the three pieces of information, so auction can be modeled as a game.

Of course, I am not very right now I did not describe the exact payoff, the exact payoff will depend on the type of auction, type of valuation and also here, how does winner get decided and very clearly at this label, seller is not appear in the game. To describe auction as a normal form game, we need three pieces of information that I described players, strategy set of bidder i and payoffs. Of course, I said the winner wins, the winner gets the object and he pays certain amount.

But, of course, the rule of the auction would determine the game also, that how does the winner get determined. Of course, all the bidders valuation would play a role and type of auction is also very important to decide, how much would be the payoff. As in the first price sell bid auction, a bidder has to winner has to pay amount equal to his own bid, but in the second price sell bid auction he pays the amount equal to the highest from all remaining bids.

Now, the next question is, is the seller a player in the game. Typically, the seller set the, decide the format for the auction, he does not participate in the auction actively. So, if we are thinking about which auction format does the seller choose, then of course, we should consider seller as one of the players. But, once the seller decides at best he is passive player in the game, so we would not include seller in the list of active players or the players in the game.

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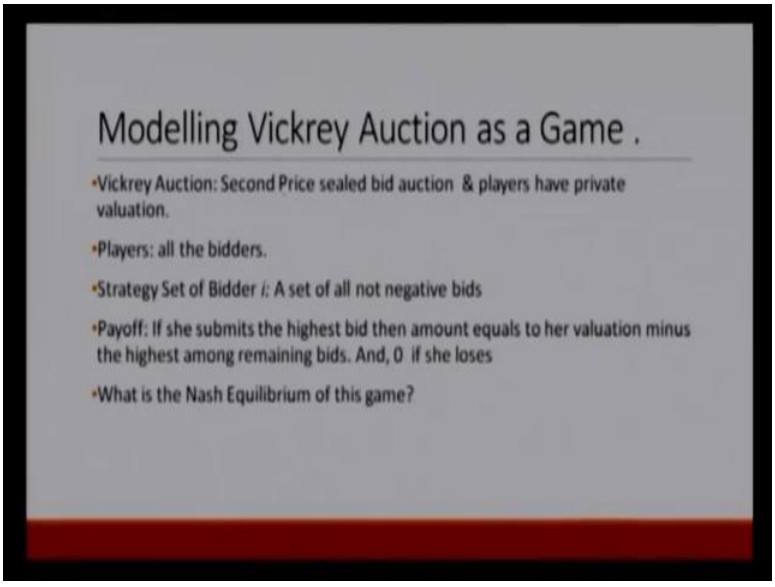

So, let us think about Vickrey auction and what is Vickrey auction, second price sell bid

auction and we will also assume the players have private valuation. Of course, as we discussed earlier, players would be all the bidders, strategy set of bidder i would be a set of all non indicative numbers. What would be the payoff? If you submit the highest bid, then amount equal to her valuation minus the highest amount all remaining bids and 0, if he does not submit the highest bid.

Can we calculate, can we figure out the Nash equilibrium of the game? By now, I hope you understand the importance of Nash equilibrium in game theory. So, how can we figure out the Nash equilibrium of this game? The technique that you have been using mostly is of drawing tables that would be difficult to use here, as we have infinite different bids coming from finite number of players. Let us say even between 0 to1, we have infinite real numbers, the different numbers a player can bid.

So, one way to do it is to restrict ourselves to whole number and say and make a smaller table that would work. But, if you want to do it little recursively, then we should allow that bidders can bid any non negative number. So, to figure out Nash, to before we figure out the Nash equilibrium, let us look at some other techniques that you have learned like a dominant strategy equilibrium. Do we have any dominant strategy equilibrium in this game?

How can we figure out? If all the players have either strictly dominant strategy or weakly dominant strategy, then we can say we have dominant strategy equilibrium in the game.

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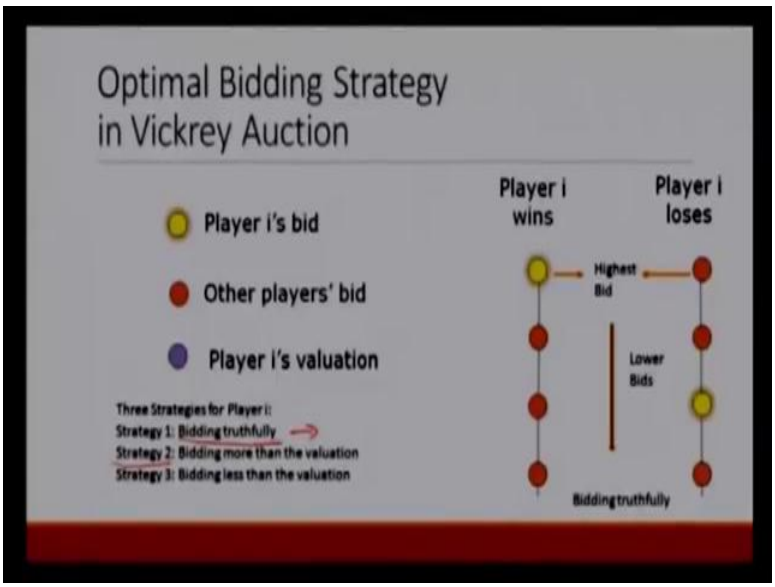

So, let us pay attention to bidding a strategy, optimal bidding strategy in this sort of auction. We will classify of course, we will classify a bidders strategies in three different groups. One that a bidder bids truthfully, means if it is valuation is, let say v i, he bids v i. Second category would be, the second set would be although I am writing here just a strategy 2, but basically it is a set; biding truthfully will have only one element, the bid equal to his true value.

But, when we are talking about second type of his strategy, bidding more than the evaluation, then of course we are talking about a whole set of strategies characterized by, where all the bids are higher than his true value. So, like if you values the object 5, he is bidding something more than 5 and third category is bidding less than the valuation. So, again going with the same example, if his valuation is 5, he is bidding something less than 5.

Let us see, what happens if he bids truthfully. Here is a case, in yellow color it is his bid and in orange color, we have other players bid. Since, player i is bidding truthfully, he wins only if his bid is highest or in or otherwise he loses, if his bid is highest, it means his evaluation is also highest, because let us say that we not only for player i, we are doing it for all the players. So, everyone is bidding truthfully, then what happens that player i wins, only if his valuation is the highest, otherwise he loses.

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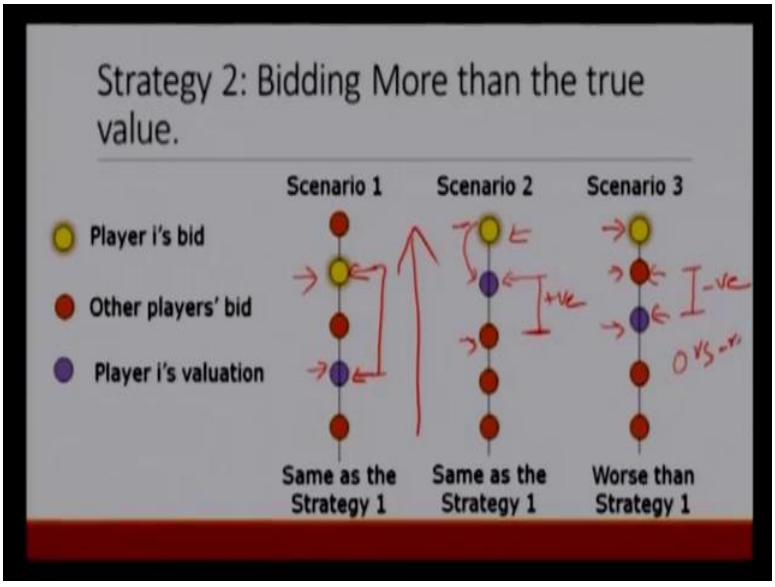

Next we are taking the second category, in which a player i is bidding more than his true

value. What happens, here is player i's valuation and he is bidding this much, this is increasing in valuation, so he bids this much more. In this scenario, there can be three different scenarios, in one scenario that his bid is not the highest. So, in this case what happens he does not win, as his bid he is not submitting the highest bid, here if he had bid truthfully he would have lost.

So, in this case it does not matter whether he bids truthfully or he bids higher than his actual valuation. Let us look at the scenario 2, in which his bid is the highest and his valuation is also higher than all other bids. What happens in this case? He bids this much amount, second highest bid is this much, so how much he pays, he pays second highest. Remember, it is Vickery auction, he does not have to pay amount equal to his own bid, he has to pay amount equal to the highest among all remaining bid.

Let us, say what would have happened if he had bid truthfully. In this case, this would have been his bid and still he would have owned the auction, because this would have been the highest bid. So, we are talking about moving this to this and how much he had to pay, same as this level. So, in both the cases his payoff would have been this much positive amount.

So, whether he is bidding more than the true value or he is bidding truthfully, in scenario 2 it would not have made any difference. But, let us look at scenario 3, here also his bid is the highest among all other bid, but someone else is bidding a value higher than the true value of player i. What happens, in this case player i wins, as he is submitting the highest bid. How much he has to pay? The highest bid among all remaining bid, so this much he has to pay.

How much value he gains from this, winning this option? This match, so basically he is losing this much. If, he had bid it truthfully, then what would have happened, he would have bid this much rather than this amount and someone else would have own the auction, because someone else as submitted this higher bid. So, in this case his payoff would have been 0, so 0 versus some negative amount of course, 0 is preferable to some negative amount.

So, in scenario 3, if he had bid truthfully he would have done better. So, what do we see, when we compare strategy 1 and second type of strategy. What happens, that bidding truthfully is always at least as good as bidding more than the valuation and in some of

the cases, bidding truthfully is strictly better than bidding more than the valuation. What does it mean? That bidding truthfully, weakly dominates any strategy in which player bids more than his true value.

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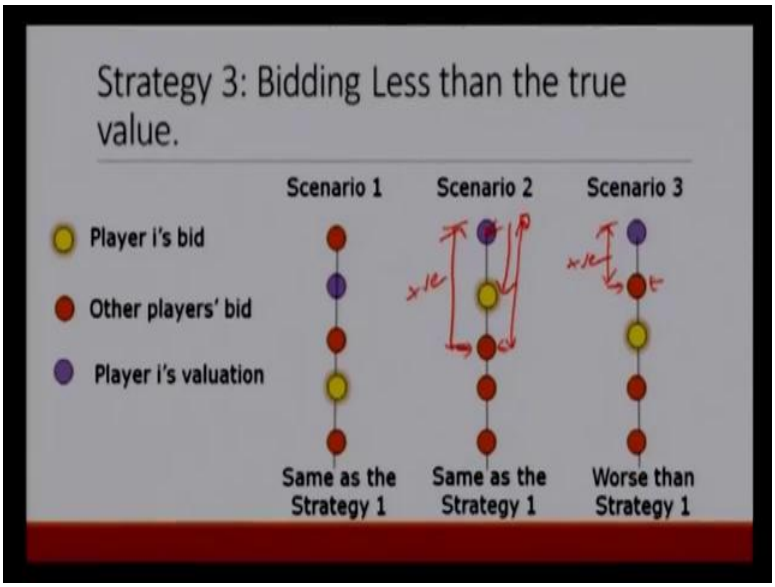

Now, let us move to the third type of bidding strategy, in which players a player bids less than the true value. Again, we have three scenario, in the scenario 1 his bid is not the highest, but in this case also we have a taking that is valuation is also lower than the highest bid. In this scenario bidding less than the true value or bidding equal to true value both would have given the same outcome to player i.

For a example his payoff is 0, even if you had bid truthfully his payoff would have remain equal to 0. Now, let us look at scenario 2, in which player i's valuation is higher than all other bids, but he is bidding less than his true values, so he is bidding less than his true value. What would happen? Because, he is use he is bidding less than his true value, but it still greater than all other bids, so he would win and how much he will have to pay he will have to pay this amount.

So, his payoff would be equal to this much positive, because we has to pay this amount and his gaining this much by winning the auction, so he is winning this much of positive amount. If, we had bid truthfully it means he would have bid this much again he would have to pay the second highest amount that remains same as this again he is payoff would have been the same. So, bidding truthfully or bidding less than the true value his

same as bidding truthfully in scenario 2.

Let us, look at scenario 3, in which player i's valuation is more than all other bits, but his bidding less than his true value and in this particulars case what happens he bids less than someone else. So, in this case he loses, because here is the highest bid and does not belong to player i and he gets 0. If, we had bid truthfully, then he had to pay amount equal to this bid, because then this would become second highest and his gain would have been a positive amount.

So, bidding less than the true value thus as well as bidding truthfully in scenario 1 and scenario 2, but does worse than scenario does worse than bidding truthfully in scenario number 3. So, what do we see again bidding truthfully is as good as bidding less than that true value in all the cases, but in and in some cases, bidding less than that true value bidding truthfully is as good as bidding less than the true value in all the scenarios. But, in some scenario bidding truthfully gives strictly better results than bidding less than the true values.

So, what do we see that bidding truthfully again weekly dominates any strategies, in which player i bids less than his true value. So, if you take all three together, what do we see, bidding truthfully is weekly dominant strategy for player i. So, what do we see there is nothing is special about player i the women that we just discussed would be true for all the players, so for all the players bidding truthfully is weekly dominant strategy.

So, what do we get, we get a strategy profile, in which all the players bid truthfully and this is strategy profile would constitute weekly dominant strategy equilibrium. And, we know that weekly dominant strategy equilibrium is a Nash equilibrium, so we obtain a Nash equilibrium for this game.

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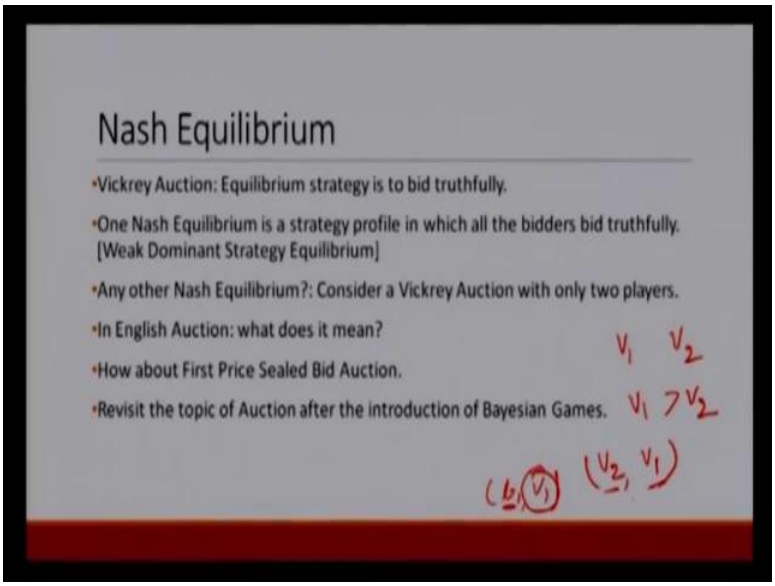

Now, can be talk about any other Nash equilibrium in this game, as we have discussed we have figured out one Nash equilibrium for this game. But, the fact is that in this game we can have many more other Nash equilibrium, for example let us take for simplicity only two players. And, let us say that player 1 has valuation V 1 and player 2 has a valuation of V 2.Where V 1 is greater than V 2, if player 1 bits V 2 and player 1 player 2 bids V 1 is this a Nash equilibrium.

Let us see, when can be call this as a Nash equilibrium when by deviating, when unify unilaterally deviating none of the player can make themselves better half. So, let us look at for player 1 can player 1 make himself better half by deviating, so instead of V 2 he bids more than V 2. If, his bid is still less than V 1 and he still loses and he gets 0 as in this case, where he is bidding V 2, because V 1 is a strictly greater than b 2, so player 2 wins.

But, if you bids more than V 1, then what happens he wins the auction and then the new would be b 1 and V 1 other b 1 is greater than V 1. In this case he wins the auction and he has to pay amount equal to V 1, so his net payoff would be V 1 as V 1 is the valuation for this person and minus V 1 as this amount he has to pay is payoff would be 0 same as the case when he loses the auction. So, player 1 does not have any incentive to bid to change his bid.

How about by bidding less than V 2, if you bids less than V 2 he definitely uses and he

makes his he will have a payoff of 0, so he does not have any incentive to derivative. How about player 2, if he bids more than V 1 again he wins he has to pay amount equal to V 2 is payoff would be same as the earlier case in both cases he would have 0. So, he does not have any incentive to bid more than V 1, how about less than V 1, it we keeps he bids a number between V 1 and V 2 he wins still we will have to pay V 2 and his payoff would be 0, but if he bids less than V 2 he loses and his payoff would be 0.

So, by deviating he is not able to increases his payoff so we can say he does not have any incentive to derivative and V 2 comma V 1 would be another Nash equilibrium my request you would be to try to figure out all the Nash equilibrium in this simple game this would be a very interesting exercise. By the way, so for we have been talking about sell bid second price auction.

What would happen in the English auction is it possible that player 1 will bid V 2, which is less than his valuation and player 2 would bid V 1 probably not, we are going to come back and revisit this, when we talk about extensive form game that is a other form of game that we are suppose to discuss. How about first price seller bid auction try to, obtain Nash equilibrium in the first price sell bid auction.

But, see so we have been talking about this sell bid auction here of course, seller is conducting the auction as seller does not know the evaluation of different players. But, then thing is all players know each other valuation first. We have assumed here the player 1 knows player 2 valuation players two most players 1 valuation that is bid funny that is not practical at it is a good intellectual exercise to understand the basic auction format.

What happens, if bidders or buyers, they know each other's valuation. Later on we are going to revisit the topic again and we will discuss an auction, where player is a particular bidder is aware of his own valuation. But, he does not know the valuation of other players for sure, that would be much more interesting to study, but for that you will have to wait.

Thank you very much.