An Introduction to Game Theory
Prof. Aditya K. Jagannatham
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture – 02
Strategy

Hello everyone, welcome to another module in this online course Strategy, An Introduction to Game Theory. So, today we are going to start looking at simple games.

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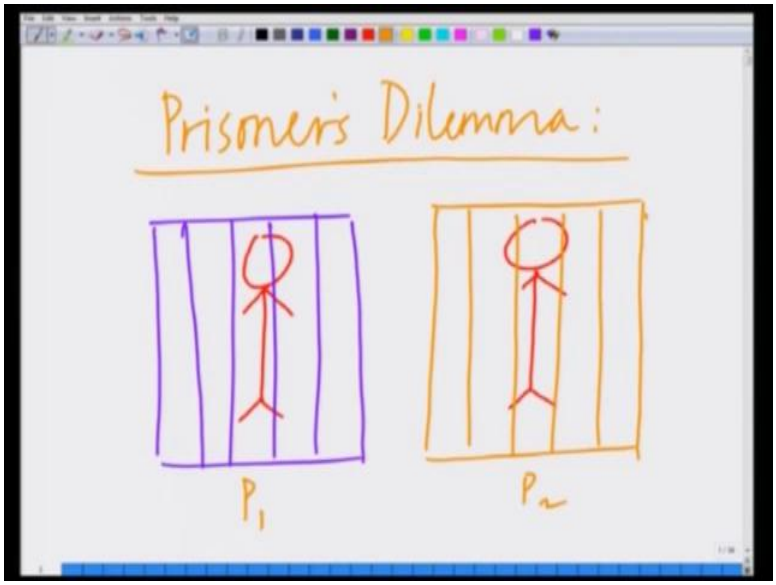

We are going to start looking at our first game, which is the game of, which is the very popular game, known as Prisoners Dilemma, which involves a simple game formulation of a simple game, a cartoon game between two prisoners. So, let me start by drawing a cartoon of these two prisoners over here. So, we have prisoner 1, prisoner 2 and they are actually in a prison. So, let me try to draw my little boxes.

And we are try to simulate as closely as possible, a prison although fortunately, thankfully I have no idea, how it really looks like. So, these are my two prisoners P 1 and P 2, who are two sort of figurate of metaphorical prisoners and well, these prisoners the game is as follows, these two prisoners are accused of a major crime.

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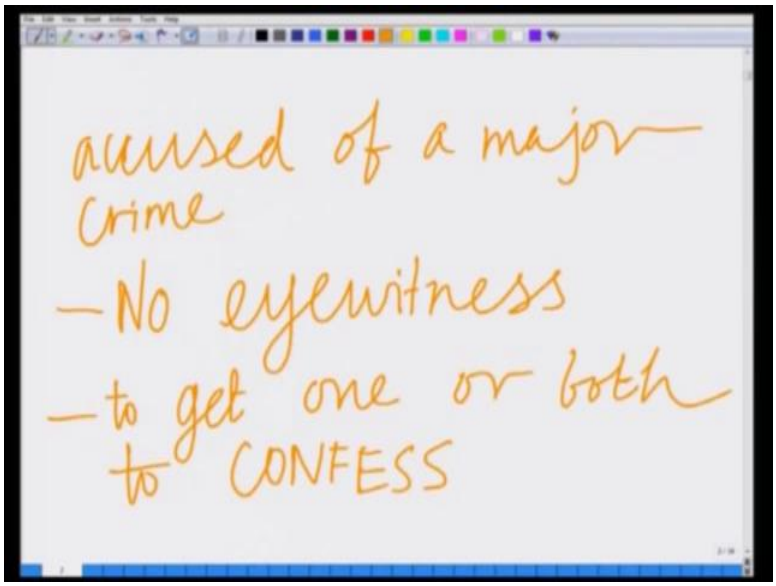

So, these two prisoners are accused of a major crime and the important thing here is, there is no evidence or there is no eye witness account of seeing them commit the crime. So, there is no eye witness. So, these two prisoners, so have been caught or accused of a major crime and an unfortunately, there is no eye witness and conveniently for our game, there is no eye witness.

So, the only possibility is to use one as an evidence or one of them as to confess to the crime and act as an evidence against other or both of them can confess to their crime. So, only possibility of getting convection is for one of them, at least one of them to confess. So, the idea is to get at least to get one or both to confess to this. So, the idea is to get one or both to confess this major.

So, the investigators divides apply or the interrogators divides a plan and the plan is follows, obviously if you ask both of them together, both of them are going to deny having to do anyway the anything with that. So, both of them are interrogated separately. So, the both the prisoners have interrogated in separate rooms, such that, they cannot communicate with each other.

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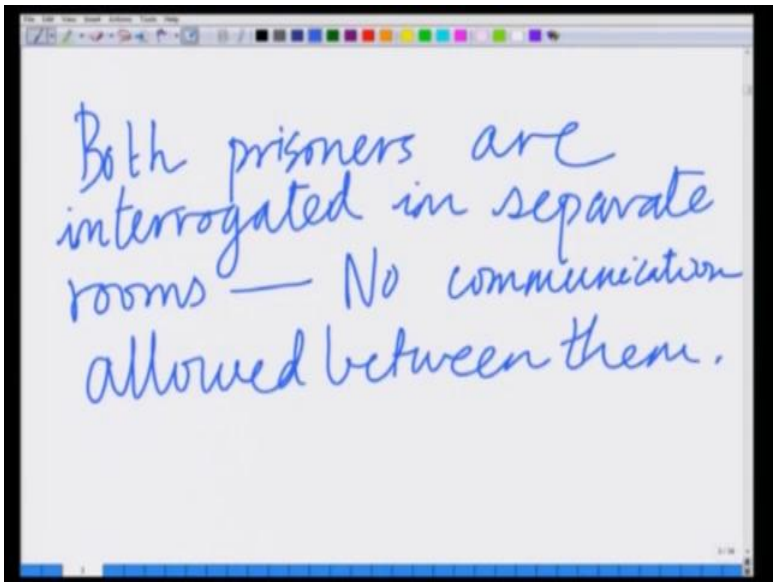

So, both and this is the key aspect of the game, this is an important is the rule or the framework of the game that we are talking about. That is both prisoners are interrogated in separate rooms, in the sense to say, there is no communication allowed between them. That is one cannot see, one of the prisoners cannot see decipher, what the other is trying to do or what the action of the other prisoners is.

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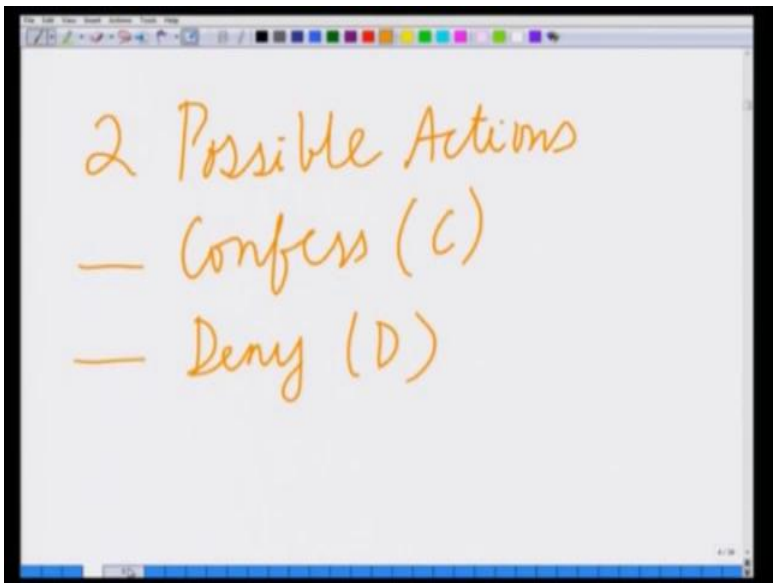

And as we already said, there are two possible actions available for each prisoner. There are two possible actions or two possible strategies, either to confess which we will

denote by C or to deny, which we will denote by D. So, each prisoner has two possible actions, either they confess or they can deny. Now, the utility or what they get as the result of their actions are as follows. If the both deny, then of course, there is no evidence of the crime. So, both of them get a sentence for a miner crime, which is 1 year in both.

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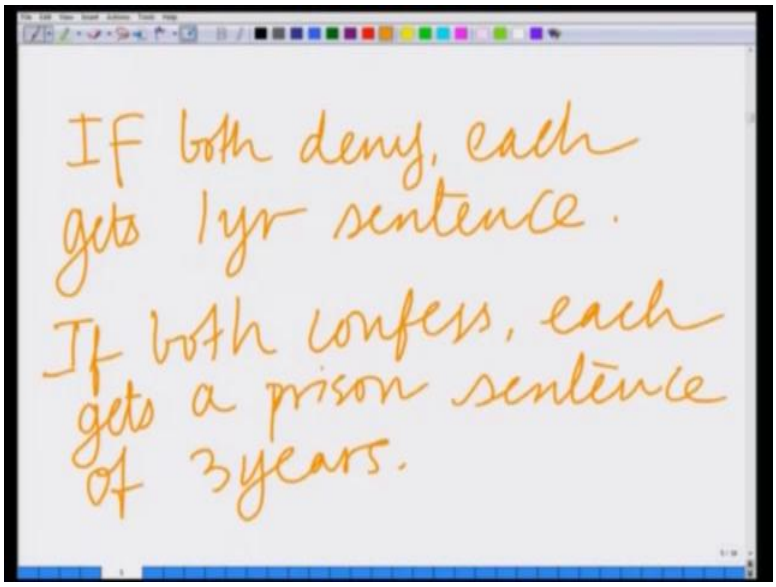

If both deny each gets a 1 year sentence, if the both confess, of course, there is evidence of a major crime, if both of them confess, then each of them gets a prison sentence of 3 years. If both confess each gets a prison sentence of 3 years in that sense, it is symmetric, both of them deny, they get prison sentence of 1 year each, if both of them confess, they get a prison sentence of 3 years each, because now that is evidence of the crime.

An interesting situation occurs, when one of them chooses to confess and the other denies. If one of them chooses to confess, then of course, as an incentive, he is allowed to walks free. The other person now is in trouble, because he is not cooperating and also, there is evidence against him for the crime.

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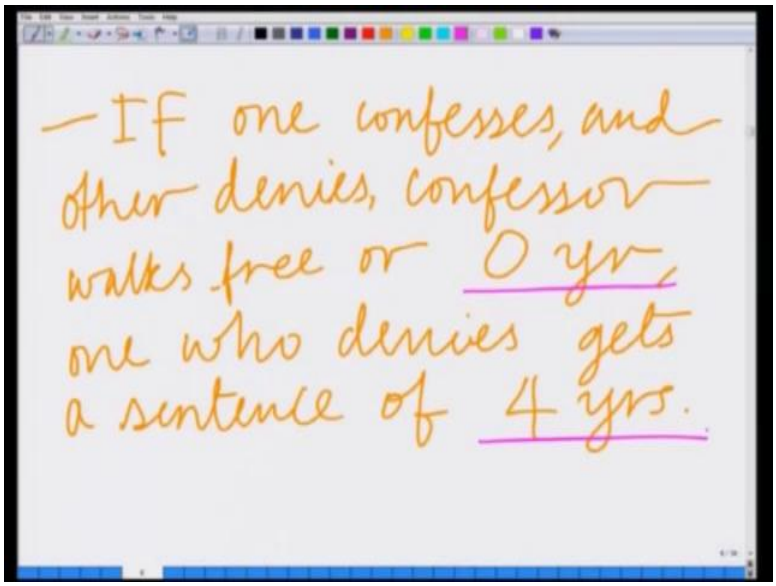

For instance, if P 1 confesses and P 2 denies, then P 2 gets a harsher sentence, because now there is evidence against P 2 and moreover, P 2 is not cooperating. So, if one confesses and other denies, then the confessor walks free or 0 year prison sentence. While, the person would denies gets a harsher sentence or one who denies gets a sentence of 4 years.

So, involve of the prisoner confesses, let us say P 1 confesses and P 2 denies, then P 1 gets 0 years and P 2 gets harsher punishment for 4 years. On the other hand, if P 1 denies and P 2 confesses, then P 2 gets the 0 year prison sentence, he walks free as an incentive and P 1 gets the harsher; that is 4 year prison sentence. So, that is the give. So, let me just repeat, if both of them are two possible actions, they can confess or deny, if both of them deny, there is no evidence. So, each gets a lighter sentence 1year each, if both of them confess each gets a slights a harsher sentence, 3 years each.

But, if one confesses at other denies, then the person was confessing box free; that is he gets 0 years and the person whose deny gets a 4 year sentence. So, obviously, you can see this is an interesting game, now let us first try to see, let us try to fit it into our model of a game.

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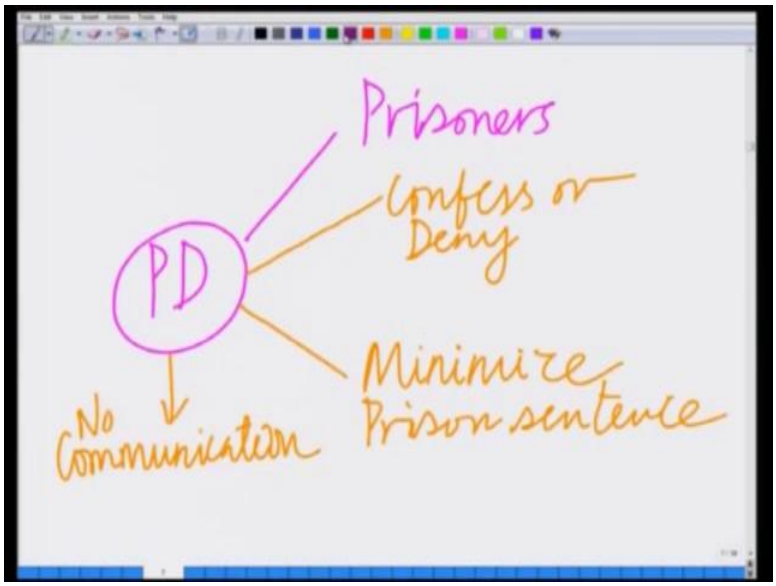

Let us try to see, this is the Prisoners Dilemma, which I am going to call abbreviate as PD, who are the players in this game. Of course, this, the different players here, it is obvious that the players are the different prisoners, the different prisoners are the players. They have a strategy, it is a competition between them to either confess or deny. These are the possible actions that these prisoners can take and what is the utility, the utility; obviously is each one is looking out for himself, each one is trying to minimize is prison sentence.

His utility can be thought of us as the negative of the prison sentence, the smaller is prison sentence, the larger is this profit, so we are trying. So, each one is trying to minimize his prison sentence, it is of course, we already set, there are some rules, what is the rule, they cannot communicate with each other. The rule is no communication between the prisoners. So, this is an example of a game, it has multiple players, prisoners, it is a competitive activity, each one can choose to either confess or deny.

And each one is trying to minimize his prison sentence, thereby trying to maximize his utility or trying to maximize his comfort or profit. And the rules are, they can be know, there is no communication between these different prisoners. So, this is fits into our definition of a game.

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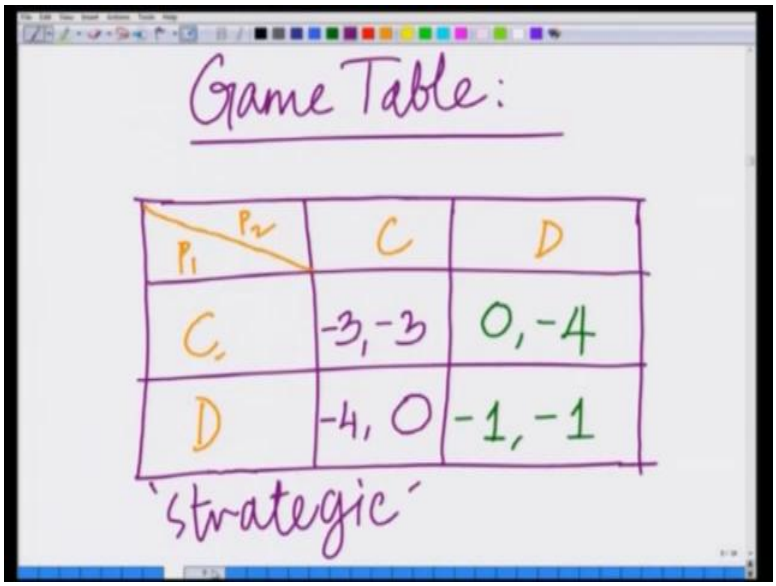

Now, how do we analyze this game? To analyze this game frequently such simple games can be represented in the form of what is known as a game table. So, to begin analyzing this game, we will take the help of the game table, we will cost this game in the form of a game of a convenient game table. Well, what does the game table have? The game table has well along bunch of columns and rows. It is become, you now become clear, what I am going to put, I am going to set the rows for P 1, the columns for P 2, P 1 can confess or deny P 2 can either confess or deny.

So, the row basically show the actions that are available to P 1 or the prisoner or prisoner 1 and therefore, P 1 can also be called as a row player, frequently is also known as the row player. The columns show the actions that are available to the column player; that is confess, one column is confess, one column is deny. Of course, here we see that the actions available for P 1 and P 2 are identical, but that need not be the case in each game, this game is of course, symmetric between P 1 and P 2.

And now, what we can look, we are going to fill each box with the corresponding payoff for each of the players depending on the output. So, if you look at C C; the box corresponding to C C; that means, prisoner 1 is choosing C, prisoner 2 is also choosing C and as we said, if both of them confess, each of them gets 3 years a prison. So, I am going to write a minus 3 comma minus 3; that is I am representing 3 years in prison as a

payoff of minus 3, because, remember each one is trying to, each one is trying to minimize his present term.

So, the lower is prison time, the greater is as profit. So, the payoff is really the negative of the prison time. So, I look at the box corresponding to C C, which means player 1 has confessed, player 2 has confessed, I am writing the payoffs of, the first payoff is that of player 1. The second payoff is that of player 2, each player gets 3 years in prison. So, it is a payoff of minus 3 for player 1 minus 3 for prisoner.

Similarly, now you can figure out, if I look at the D D box, where the first prisoner is denying the second prisoner is also denying, each one gets 1 year in prison. So, it is minus 1 for each one of them. So, each one gets a payoff; that is prisoner 1 gets a payoff of minus 1, prisoner 2 gets a payoff of minus 1. Now, an interesting thing of course happens, when prisoner 1 confesses, but prisoner 2 denies and in that case, we said the prisoner 1, who is confessing get 0 years in prison; that is he walks free.

While, prisoner 2, who is denying gets a horsiest sentence of 4 years in prison; that is minus 4 and if on other hand, P 1 denies doing the crime and P 2 confesses to the crime. Therefore, P 2 acts as evidence against P 1; therefore, P 1 gets minus 4 for not cooperating and P 2 gets to walk free for cooperating and acting as evidence against P 1. So, this summarizers, this short of elegantly and very conveniently captures this game of this Prisoners Dilemma.

So, if refuse the rows to represent the actions of prisoner 1, the columns to represent the actions of prisoner 2 and each box which corresponds to the intersection of one action of prisoner 1 and other action of prisoner 2 represents the payoff. And you can see interestingly, the payoff of prisoner 1 not only dependents on the action of prisoner 1, but also dependents on the action of prisoner 2.

For instance, if P 2 choose a C, if P 1 choose a C to confess, he get three as in prison, while it we chooses to deny, he gets 4 years in prison. So, the payoff of prisoner 1 or the utility of prisoner 1 is not only did determine by his own action, but it is also determine the action of his opponent or his competitor and that is part makes this a competition or that is what makes this interaction, a strategic interaction.

This is a competition or a strategic interaction, because payoff is not only determined by your own action, payoff is also determine your action in congestion take in together with actions of your competitors and that is an important aspect of a game.

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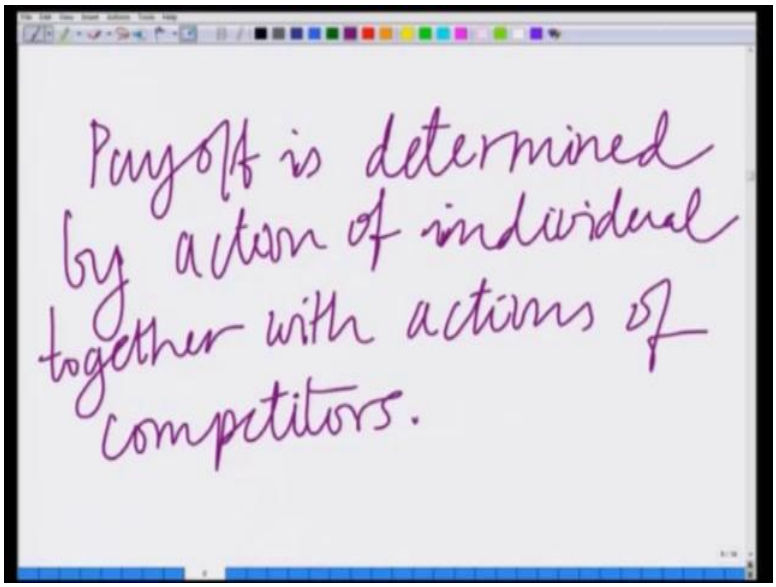

Payoff is determined by action of individual together with actions of competitors. That is an important part of this strategic interaction, when my it is not my one actions; that it mind the payoff, it is my one actions in relation to the actions of all my other competitors. So, this is a simple game Prisoners Dilemma in which there are two prisoners, each can another confess or deny and we also listed the various payoffs.

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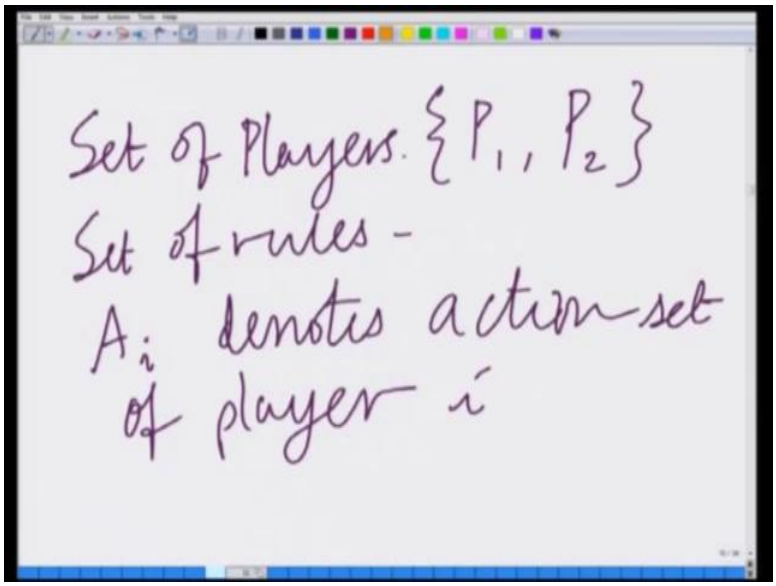

Now, let us formally extract the mathematical rotation of this game, of course, as we set each game as a set of players, multiple players. So, that can be represented by the set of players P 1 comma P 2. So, this is a set of players, I have to write it as a set, this is a set of players P 1 comma P 2 denoting prisoner 1 comma prisoner 2. There are a set of rules which we already stated, the set of rules which are implicit with this game. There is a set of actions, each player i as an action set A i. The action set, there is a set of actions A i denotes action set of player i for a example here.

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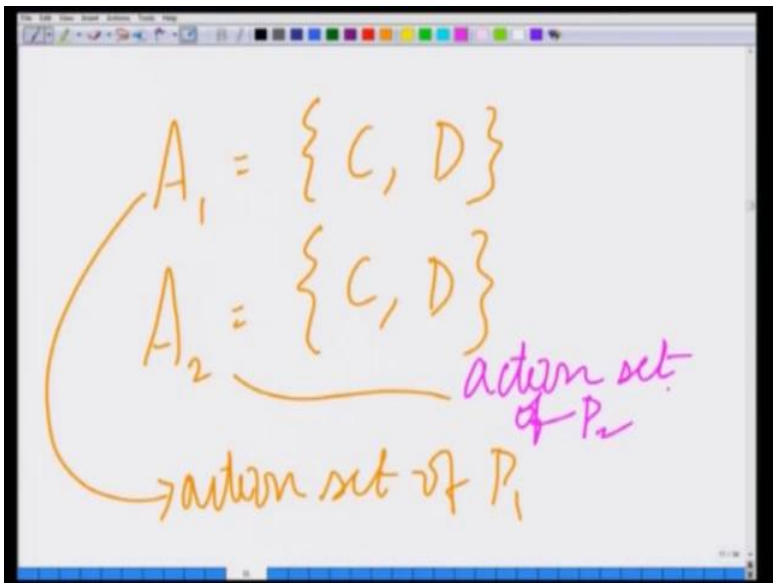

A 1 denotes the action set of player 1, which is C comma D confess or deny, A 2 which denotes the action set of player 2 is C comma D, which is to confess or deny. Of course, here it turns out that the action set of each of the players are same, but that need not be the case for any general games. So, this is the action set of P 1, this is action set of player P 2. So, these are the action set of the different players and of course, a game is characterized by a set of outcome.

What are the different outcomes of this game, for instance, where both prisoners are confessing is one outcome, one is confessing, other is denying is another outcome or both of them denying as another. So, there are different, what is the different outcome, these different outcomes nothing but, a combination of the actions of the different player.

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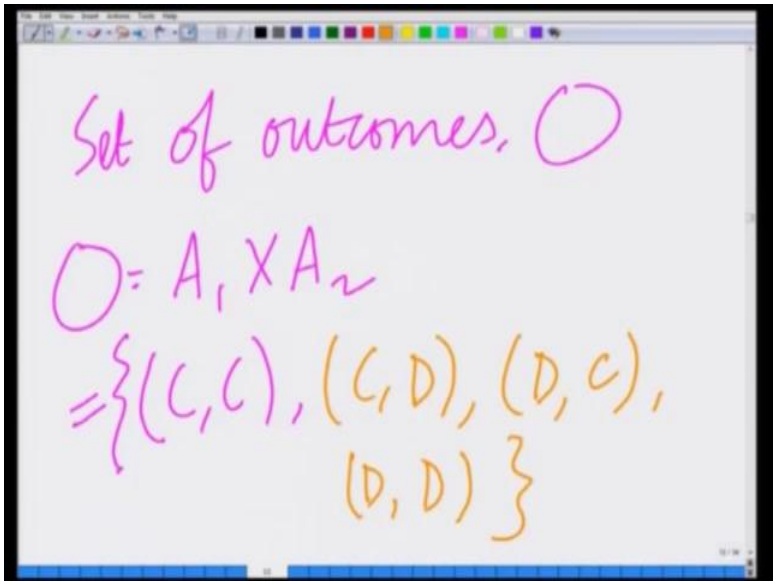

So, the set of outcomes is O, which is nothing but, the cross product or the Cartesian product between these two action sets, because each remember each agent or each player is choosing an actions from is action set. So, therefore, it is the Cartesian product between these action sets, which is C C both confessing C D player 1 confessing player 2 denying D C player 1 denying player 2 confessing or D D player 1 and player 2 both, then I will show I was set of outcomes.

And that is what we are saying, remember the payoff to each player depends not only any actions, but it depends on the outcome which is his action together with actions of all in the other players. So, the possible set of outcomes in this game are C C at both players

are confessing, C D, where prisoner 1 is confessing, prisoner 2 is denying, D C, where prisoner 1 is denying, prisoner 2 is confessing and D D, where prisoner 1 and prisoner 2 are both denying.

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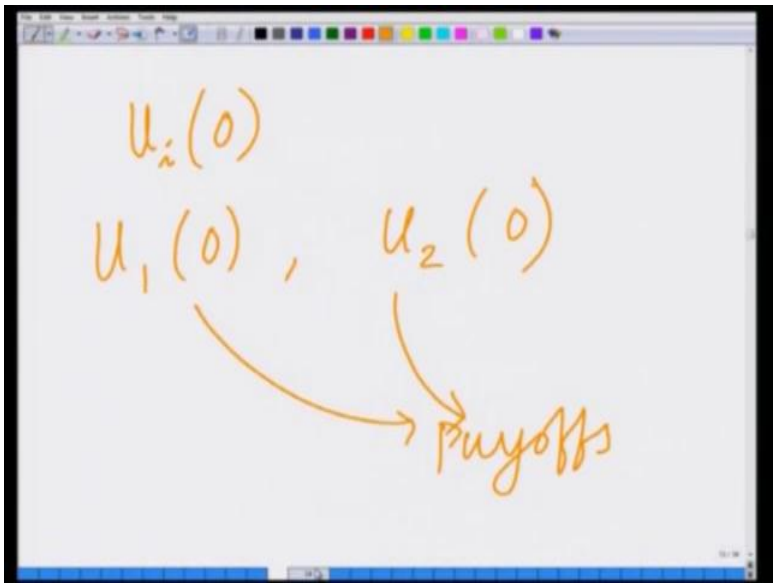

And what is the payoff, the payoff can be represented as a function u i of the outcome. For instance I have u 1 of the outcome, u 2 of the outcome O, for the represent the payoff of each player, these are the payoff. Payoffs are utility functions of the different players. However, in game theory we use slightly different and a slightly complicated notation and this is where I would like to draw you are special attention.

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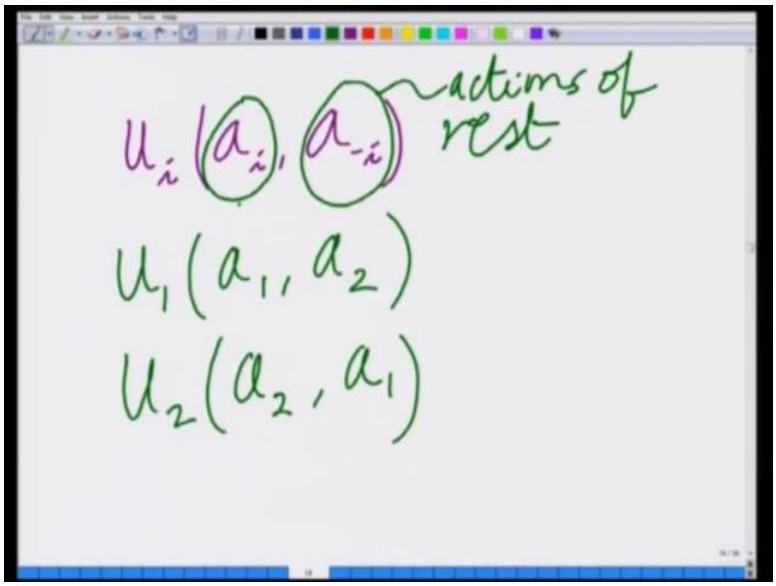

When we mention the utility of each player, we represent it by a utility function u of i and we first give the action of the i th player followed by the actions of all the other players, which is denoted by a of minus i. So, this is the convenient notation in game theory, where the first action a i belongs to the i th player and then, we give the actions of the rest of the players.

They give actions of rest, for instance if I am talking about player 1 by notation would be u 1 a 1 comma a 2; that is what is the action of player 1 followed by action of player 2. While, I am talking about utility of player 2, I will first mention the action a 2 of player 2 and action a 1 of player. So, game theory uses slightly different notation.

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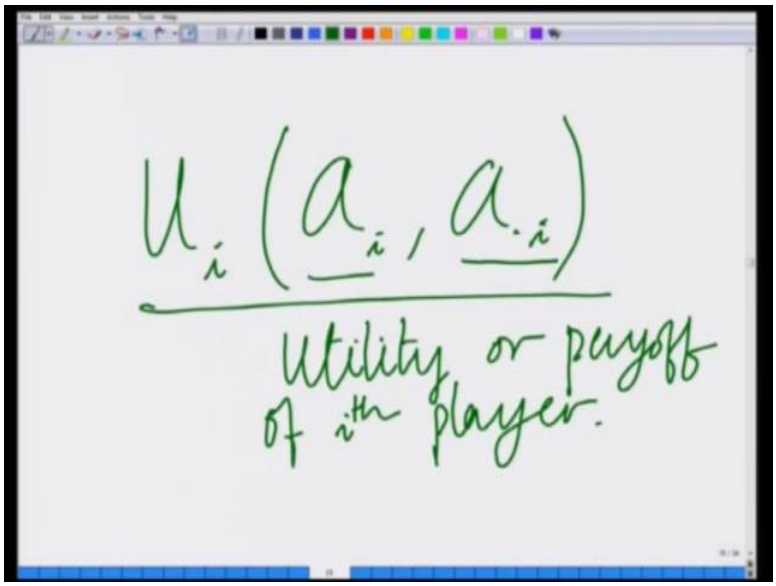

And this notation is denoted by u of i, a of i, a of minus i, a i is the action of the i th player a minus i denotes the actions of all the other players other there in the i th player. This is the notation used what denoting the utility or payoff of this is the notation used to denoting the utility or the payoff of the i th player of each player.

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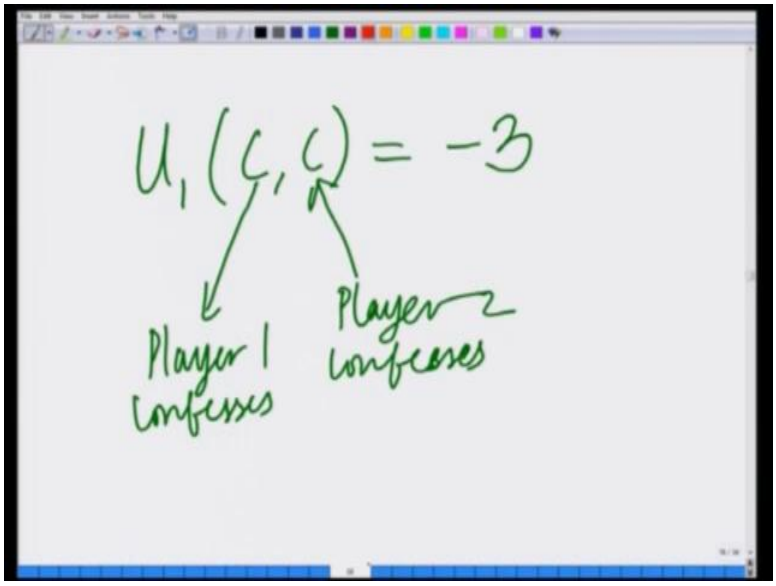

For instance, if I take go back to our example of the prisoners dilemma, ((Refer Time: 22:34)) if you look at it, if I look at u 1 of C comma C. Let us look at u 1 of C comma C is utility of player 1, player 1 confesses, player 2 also confesses. And therefore, we are

talking about the utility of player 1, when player 1 confesses and player 2 also confesses and as we know, this is equal to minus 3, because in this case, player 1 gets 3 years in prison.

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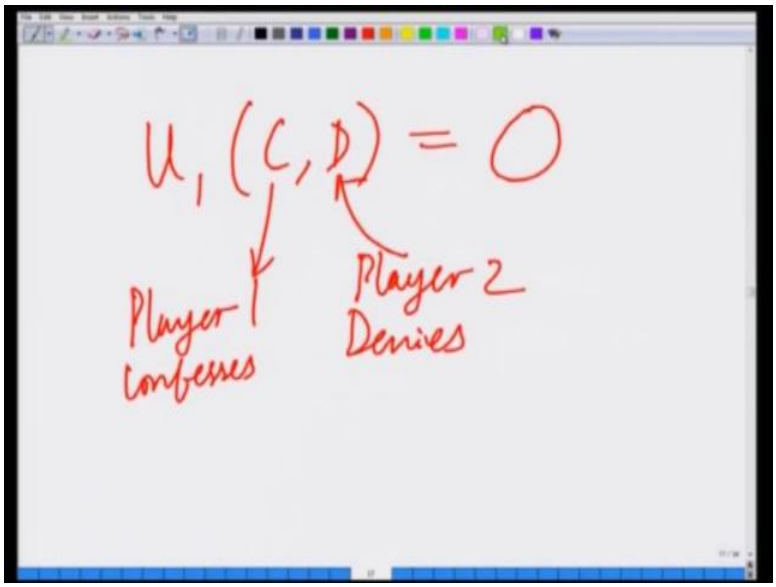

On the other hand if I look at utility of player 1 C comma D, this means player 1 confesses, player 2 denies and in this case, u 1 is a utility of player 1 and player 1 confesses, player 2 denies. This as we know is 0, because in this scenario player 1 gets 0 years in prison; that is he gets to walk free.

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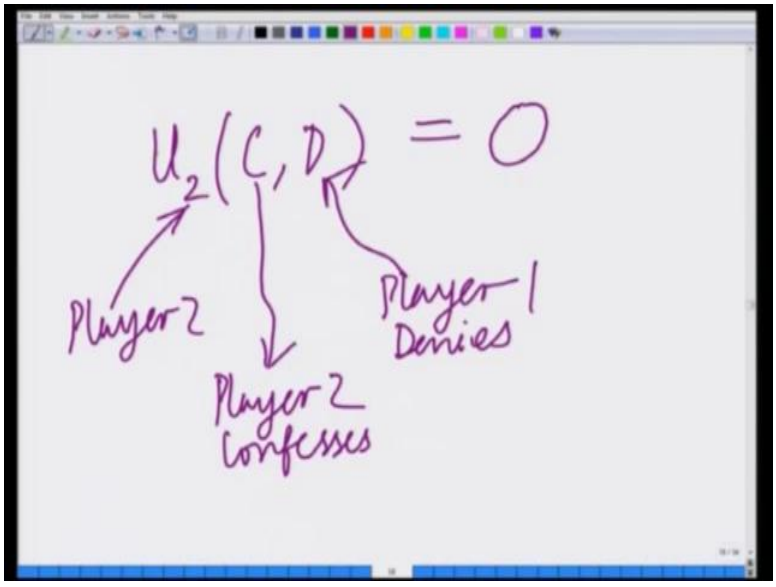

On the other hand, when we write, if you write u 2 of C comma D, remember we are talking about player 2. So, we are looking at the first action refers to player 2. So, this is when player 2 confesses and player 1, the other players, the rest of the player in this case, resolve one other player; that is player 1 denies. And this is equal to we are talking about the utility of 2, when player 2 confesses and player 1 denies, this is also again 0, because if there two is confessing, the other player is denying entire 2 gets two walk free. So, there is slight ambiguity in, this is slightly more shuttle certainness involved in this notation which I hope everyone is going to appreciate.

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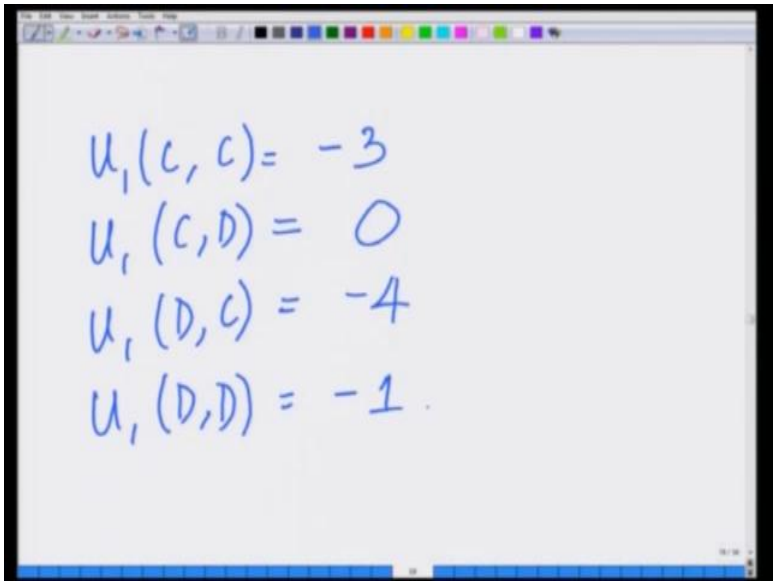

And now, if you therefore, the full set of utilities, you can verify that u 1 of C comma C, where player 1 is confessing, player 2 is also confessing is minus 3 u 1 of C comma D, where player 1 is confessing. Player 2 is denying is 0, u 1 of D comma C, where player 1 is denying, player 2 is confessing is minus 4 u 1 of D comma D, where player 1 is denying player 2 is also denying is minus 1. Because, he gets in both of them deny, you gets 1 year in prison.

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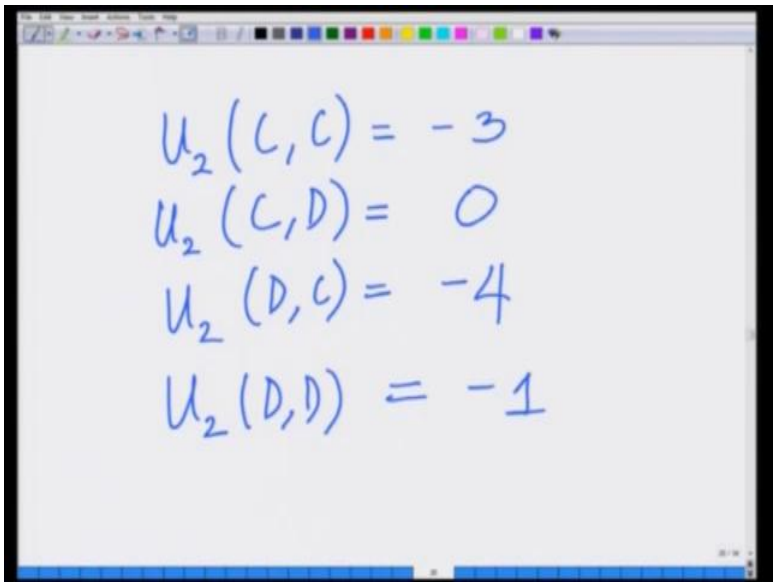

On the other hand, again if I have to write it four player 2 u 2 C comma, C where player 2 is confessing and player 1 is confessing is minus 3 u 2 of C comma D, where player 2 is confessing player 1 is denying is 0, u 2 of D comma C, where player 2 is denying player 1 is confessing is minus 4 and u 2 of D comma D. We are both players are denying is minus 1. Therefore, this is how, we mention the payoff.

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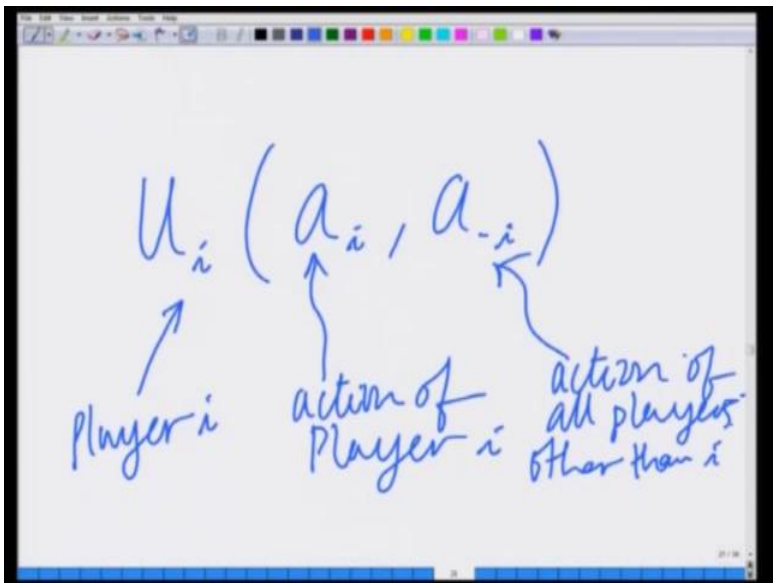

The payoff the important thing to notice here is that, the notation that we use is u of i, a of I, a of minus i, this is this i denotes, u denotes the utility, i denotes u of player i, the

action immediately following denotes the action of player i, a of minus i denotes actions of all other players. Action of all other players or let me write it this way action of all players other than i, this is the important aspect of the notation, one of the important aspect of the notation slightly complicated. So, I hope is suspend some time trying to understand this thing and therefore, why is the Prisoner Dilemma rather thing that I would like to motivate, before we start about how to analyze this game or how to interpreted the behavior of the different agents.

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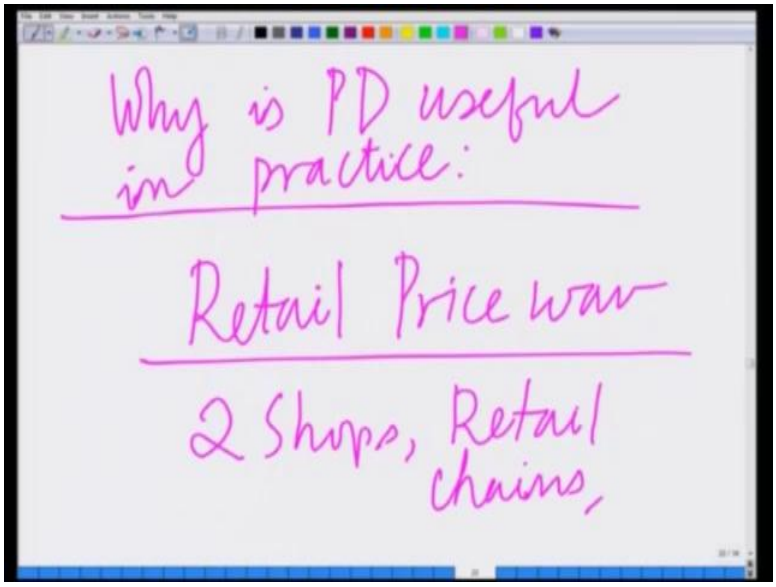

In this game, why is this Prisoners Dilemma is important are varied as why is PD, why is prisoners dilemma useful how is PD useful in practice. So, if you look at a prisoners dilemma for instance consider a simple price war, a retail price war. A retail price war that is a competition between two different shops or two different retail chains and these two different retail chains have an option either to set prices high.

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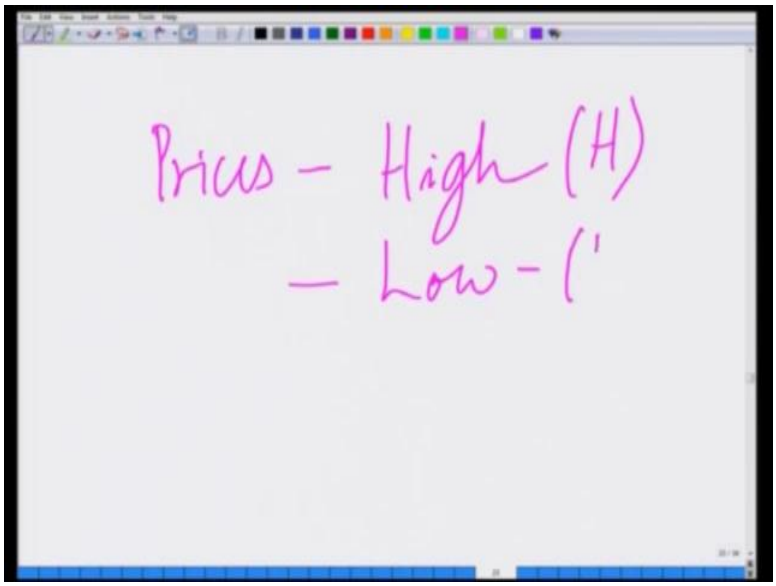

Or, to set prices low that you either, they can choose to set prices high or set prices low.

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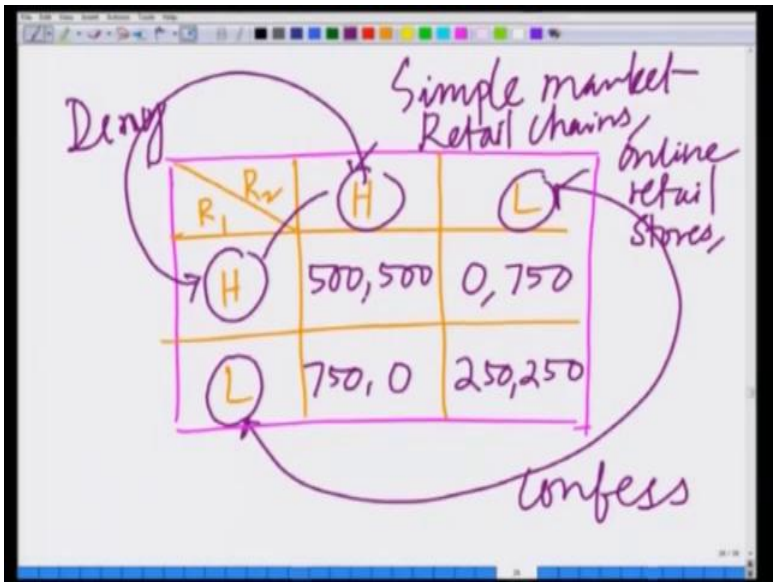

And therefore, I can model it as a game between two retail chains, let say and let us call them R 1, my row player is R 1, my column player is R 2. They can set prices either high or low R 1 or R 1 set either high price or a low price. If both of them set a high price both of them get high profit of 500. If both of them set a low price, both of them get a low profit of 250 each.

But, in the other hand, if one sets a high price and other sets are low price, then obviously everyone is going to block to the formal the retail chains that is set a low price if the person whose set a high price is left high and try and the person was set a low price is going to capture the market. And get higher profit 750, even if the lower price, because he has higher share of the market.

On the other hand, if retail chain 1 sets a low price, while retail chain 2 sets high price, retail chain 1 ends up getting a profit of 750, retail chain 2 ends up getting 0, because everyone is retail chain 1. Therefore, the payoffs are 750 comma 0, let us you can see both of them set high price, both of them get payoff of 500 each, go to them set a low price, you get a payoff of 250.

One of them sets a low price other sets a high price, the person the retail chain which set a low price; that is retail chain 1 in the row entry L and column entry H, retail chain 1 gets 750, retail chain 2, which is a set high price gets 0. In H comma L, which corresponds to retail chain 1 setting a high price, retail chain 2 setting a low price, retail chain 1 gets 0 and retail chain 2 which is set a low price gets 750.

And you can clearly see at least if you not able to realize this now, this is very similar to a Prisoners Dilemma kind of a situation, where you can thing of the low price, you can thing of at the low price as basically being a confess kind of strategy, this is out of setting a low price is basically sort of can be thought of as equivalent to confess. We are going to look at this later and sitting the high price can be sort of seen has a deny strategy.

If you try to closely analyze this game, closely look at this game, trying to if we are going to analyze this game shortly starting in the next couple of module. We are going to see that this game can be used equivalently model does the prisoners dilemma, where the low is equivalent to a confess and the high is equivalent to deny. And thus, this is the simple example of a market, which is equivalent to a short of equivalent to a Prisoners Dilemma.

And the simple of example of market helps us understand is going to help us understand some of the principle of course, we are going to look at more complicated examples of markets later on. But, this simple example itself yields valuable insides. So, this is a simple market competition between two retail chains or sort of online. You can also

thing of as online retail stores, etcetera; all can be used model a simple marketing example between 2.

So, what I have done in this module, in this module, we have dealt with very interesting on a basic game or fundamental game of prisoners dilemma, which is an interesting example. We modeled it as a game between two prisoners, we have included in the set of what are the rules of this game, how this game as displayed description of this game followed by the formulation of a game table, how to represent this game as a game table.

And we introduced a bunch of notation related into them same set of players, the set of actions in the set of outcomes and how this is a strategic interaction, because the payoff of each player depends not only your action. But, the actions of all the other players as well also we have given notation, ((Refer Time: 32:28)) which is an important notation related to the utility of each player.

Please take a look at this notation, ((Refer Time: 33:32)) again you also mention the utilities and we try to see how this Prisoners Dilemma. Even though a simple sort of a fabricated game can be sort of yield valuable insides in practices or can be used to model real life scenarios, such as a competition between two retail chains or competition between two forms in a market base. So, we stop this module here and in the next module, we will start trying to analyze these games trying to predict the behavior of try to predict. What are the possible outcomes of these kinds of games?

Thank you very much.