Strategy: An Introduction to Game Theory
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Lecture – 18

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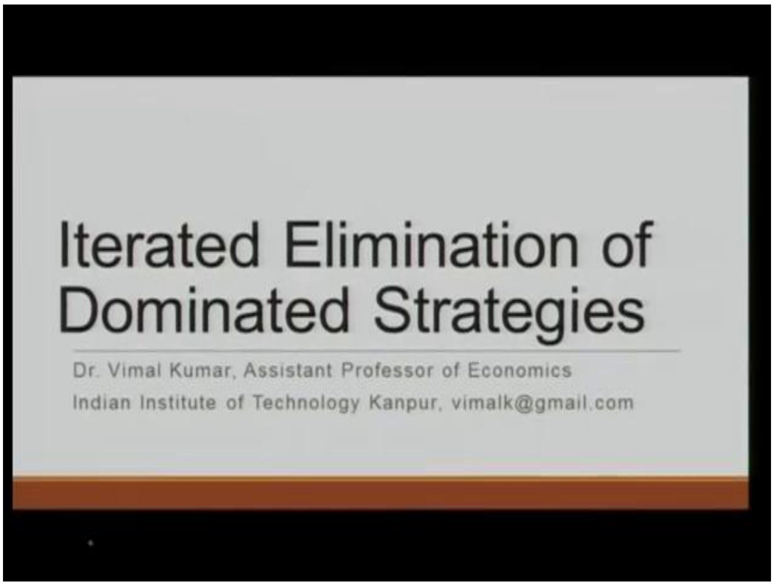

Welcome to next lecture on Strategy, An Introduction to Game Theory and in this module, we are going to talk about something called Iterated Elimination of Dominated Strategies. Before, we get into this, let us just for a moment let us take a moment just to think, what we have done so far. What we have done? We have learnt something called dominant strategy equilibrium, strictly dominant as well as weekly dominant strategy equilibrium, we have also done Nash equilibrium. Now, we are going to look at another solution concept that is, that we get using iterated elimination of dominated strategies.

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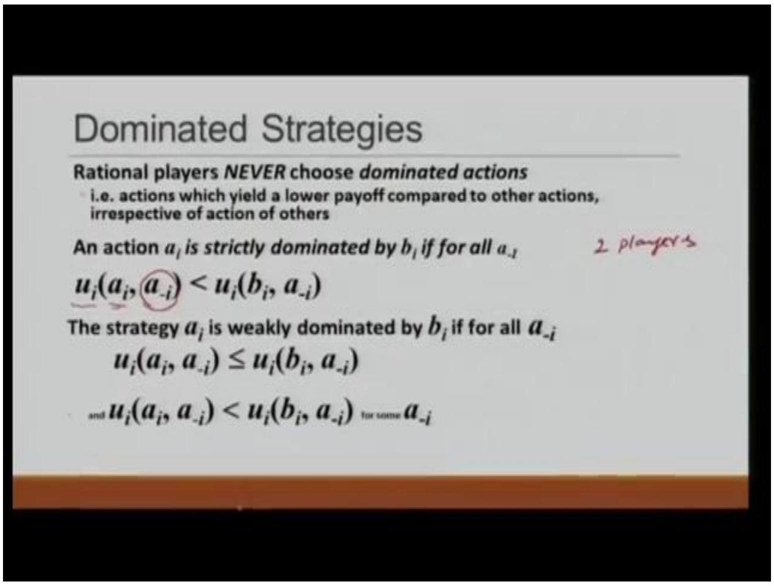

For that, we need to define, what do we mean by dominated strategy. The thing, the simple definition is u i, that is the payoff to player i given, that we takes action a i and all other player are taking action a minus i. For example, when we have two players, we have one is taking action a i and another player action is given by a minus i. When player i does worse than by playing a i in comparison to b i, no matter what all other players are doing, then we call that a i is strictly dominated by b i.

Similarly, we can also define the notion of weekly dominated. When do we call action a i weekly dominated by b i? That no matter, what other players are doing, player i can by playing b i, player i does at least as well as by playing a i, but in some situation he does better by playing b i.

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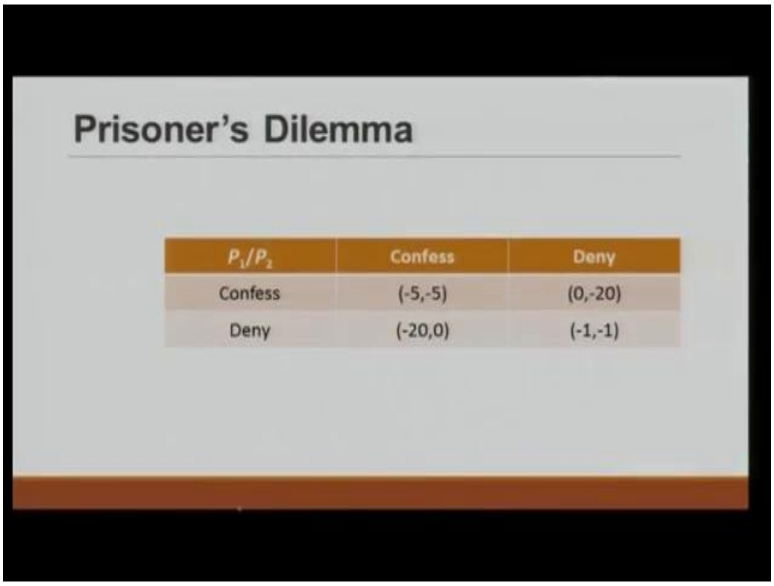

Let us take an example; the example is from Prisoner's dilemma, that you have already seen. What you have seen there? That confessing is strictly dominant strategy. Why it is strictly dominant? That no matter what other player is doing; confessing is better than deny. Now, as we have done a strictly dominant strategy, we can also look at the strictly dominated strategies. As confessing is strictly dominant, deny is strictly dominant. Why? Because no matter, what other player is doing, deny is worse than confess, so we have to be careful about it.

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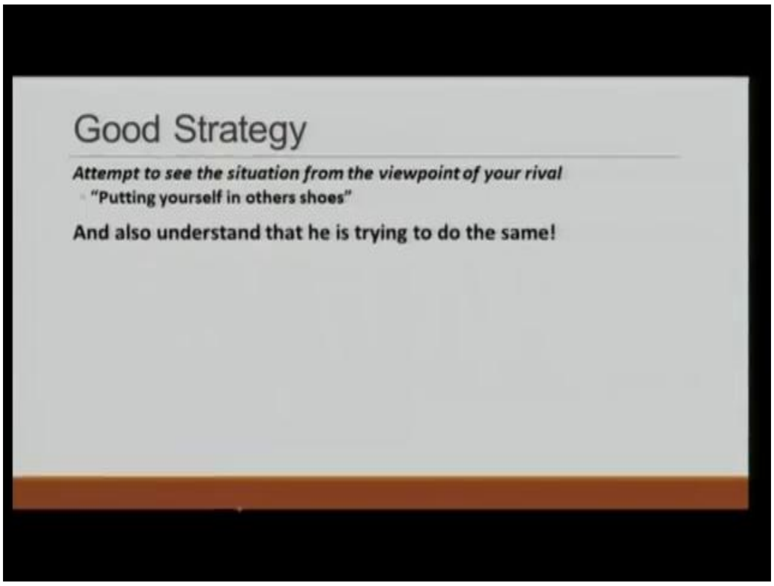

Now, game theory is all about that you put yourself in your other players shoe and also understand, that they are doing the same thing.

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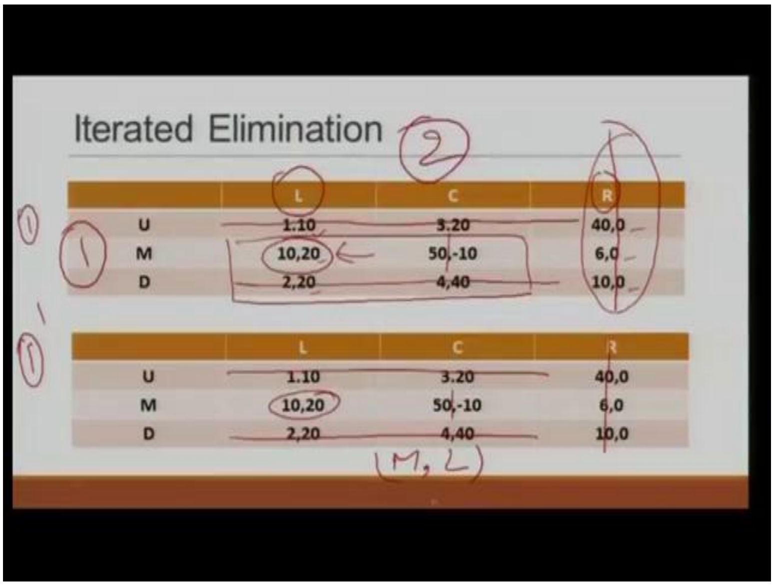

The key is very, very simple in this; that a player could never irrational player could never play a v at a dominated strategy. For example, let us take this game, we have the same game device that I have given, let us look at player 1. Here we have a player 1 and here we have player 2, 1 is the row player and 2 is the column player. If, we look at the action R and action L of player 2, what happens, let us look, let us compare 10 is greater than 0, 20 is greater than 0, 20 is again greater than 0.

So, no matter what player 1 is doing, L strictly dominates R or in other word, R is a strictly dominated by L. So, a rational player would never earlier, what did you say earlier in the strictly dominant strategy equilibrium, we talked about that this player will play L, right now we are not interested in, what would this player play, what we are interested in what this player would not be. So, what we know that this player will not play R, so we can take R.

Now, because of common, because we have resumed common knowledge, not only player 1 and 2 are rational, player 1 knows that player 2 is rational and player 2 knows that player 1 is rational. So, player 1 knows that player 2 is rational, so player 1 knows that player 2 is never going to play R, a rational player will never play dominated strategies. So, player 1 knows that player 2 is never going to play R, so we can take this whole thing out.

Now, player 1 is looking at a smaller game, in which player 2 has only two strategies L and C left and center. Player 1 does not need to consider, now this is strategy R of player 2. So, earlier player 1 did not had any dominant strategy or player 1 did not had dominated strategy.

Now, but let us look at in the reduced game, player 1 has a dominate strategy and player 1 has also a dominated strategy mean fact 2 dominated strategy. What do we see that in reduced game that M strictly dominates U, as well as M strictly dominates D. So, what we say that now we can eliminate, let us say we are eliminating U. Why we are eliminating? Again, let us pay attention that in the whole game player 2 is rational, so player 2 is not going to play R, but player 1 knows player 2 is rational, why because we are resuming common knowledge.

So, player 1 knows that player 2 is never going to play R, so we are taking this part out. Now, player 1 is rational, so we can say that because of presence of M he will never play U. Now, let us looking to the next step of the common knowledge that, not only player 2 is rational player 2 knows that player 1 is rational. So, we can say that player 2 knows that player 1 will not play U in the reduced game.

Now, we have much smaller game only involving two strategies from player 1 and 2 strategy from player 2, now let us look at it. What do we see? That again M strictly dominates D, so we can move D also. And, continuing with the process between, now we have only one strategy for player 1 and two strategies are player 2, given that L is strictly dominates C, so we can move, this would be the outcome.

Even, if you had used the Nash equilibrium concept, this was the outcome you would have obtained. This is another technique in which we can get equilibrium in some of the games, if not all games and it is iterated elimination of strictly dominated strategies. Again, I have the same game I just want to show you, in case of even you are removing the strictly dominated strategy, you do not have to worry about order.

So, let us see you remove this as we had done earlier, now first you remove this and again, then you remove this, you remove this and again the outcome would be same as M comma L. And this is the equilibrium that we obtained iterated eliminations of dominated strategies.

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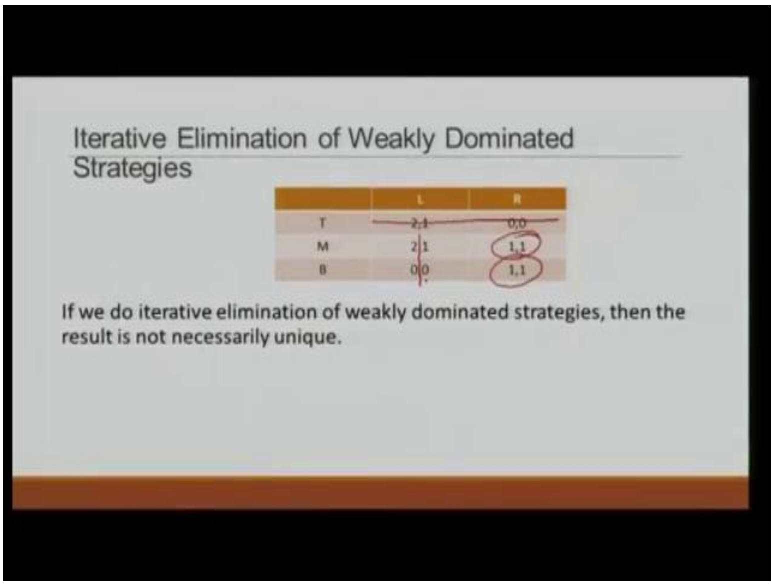

Now, if we are doing the iterated elimination of weekly dominated strategy, we have to be careful. We may end up removing one of the candidates or equilibrium, because of the order. So, let us see what happens here. What do we have here in this game? Do we have weekly dominated strategy? We have M weekly dominates T, M is at least as good as T no matter, what the other player is doing and strictly better than M, then player 2 is playing R, so M weekly dominates T, so we can remove T.

Now, what happens? That R, once we remove this and let me repeat again player 1 removes this T, because player 1 is rational and because of common knowledge player 2 knows that player 1 is rational. So, he does not need to consider that player 1 would never play T, now what happens look at player 2. Player 2 or player 2 R weekly dominates T, R weekly dominates L, so this can be removed and we have these two possibilities.

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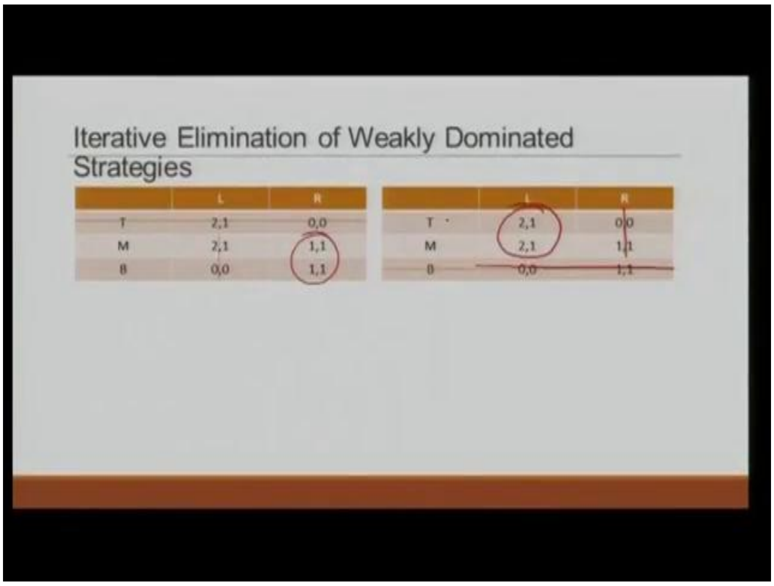

That I have in the next slide, in the next slide you see that these are the two possibilities I have talked about. Also if you pay attention that M not only weekly dominates T, but M also weekly dominates B, so in this time we are going to remove in this order, let us first remove B. Then, we see that L is, now L weekly dominate R is weekly dominated by L, so we will remove R and the possibility is here.

So, what I am saying that when we are removing weekly dominated strategies, then we have to be careful. But, when we are removing the strictly dominated strategies, then the order does not matter as long as we are removing only strictly dominated strategy. The same thing cannot be said about weekly dominated strategies.

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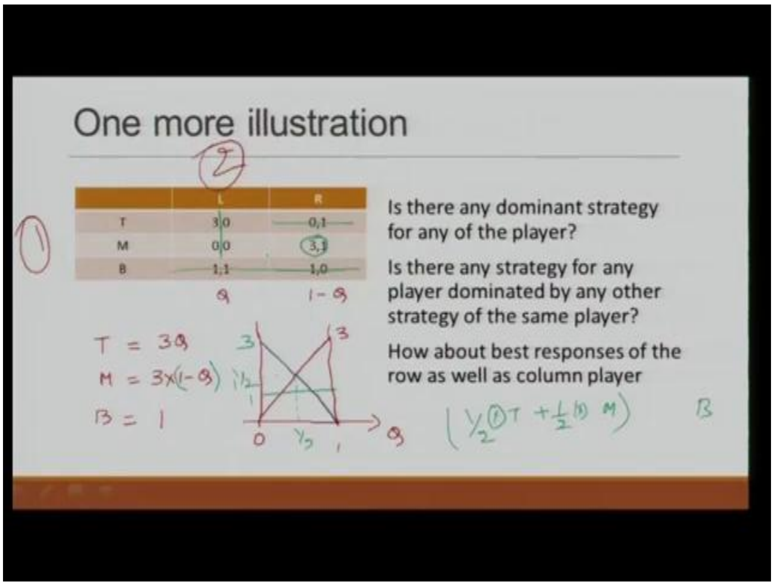

There is one more illustration that I want to give you, let us look at this game. Is there any dominant strategy for any of the player? Let us look at it. For player 1, can T B the dominant strategy? No, because when player 2 plays L is better than M, but player 2 plays R, M is better than T, so not the case. Again when we compare T B or M B, we see there is no dominant strategy for player 1 and similarly we can figure out, there is no dominant strategy for player 2.

So, in this game player of course, we have player 1 whenever I want say I mean that there are row player is player 1 and the column player is player 2. So, in this game none of the players are any dominant strategy is there strategy for any player dominated by any other strategy of the same player, notice I here you might be thinking, why did we learn this technique.

When, if a player has strictly dominant strategy the player would definitely have a strictly dominated strategy at it is not true other way around and that we will see. In this game none of the players are strictly dominant strategy, but if we allow for the mixed strategy that have learn by now, then we see that B is strictly dominated by half of T and half of M half of T and half of M, how can we say that, we can simply let us take the mixed strategy for player 2, let us say that player 2 is mixing L with probability Q and R with probability 1 minus Q if this player place first player place T, how much beyond 3 Q, 3 multiplied by Q, 0 multiplied by 1 minus Q.

If he place M, then how much is P of 0 multiplied by Q 3 multiplied by 1 minus Q and if you place B no matter with their player 2 is playing L or R is P of is always once of one. Now, we can plot player 1 is payoff with respect to Q let us say Q here is 0, Q here is 1 and Q can not be greater than once I can draw it like this, what do we have, let us draw first payoff from T, so it is going to be 3.

And similarly we will have, let us use a different color indicate the payoff from M it is going to be this and how much is payoff from B again, let us use green color lets going to be write 1 this point is R, R and this is again 3, what do we see if we next e and M half and half, what is happening the payoff would always be equal to 1 by 3 that is clear from this, it is always equal to 3 by 2, why payoff from B is equal to 1.

So, payoff from mets of t and m equal mets of t and m would always be higher than the payoff from B, so we can say that half of T plus half of m strictly dominates B. So, what we can do, we can eliminate this thing would not be with this can not be done, when we are using the strictly dominant strategy equilibrium concept, at this can be use in the case of elimination of strictly or weekly dominated strategies.

Then, we can say of course, now we see and between R and L, then L strictly dominates L and this also can be taken up and what do we get again player 1 will not play T because m gives strictly higher in the outcome maybe this. So, in this case what we get an outcome from iterative elimination of dominated strategies, but we cannot get any outcome using strictly dominated strategies, so this we have to be careful.

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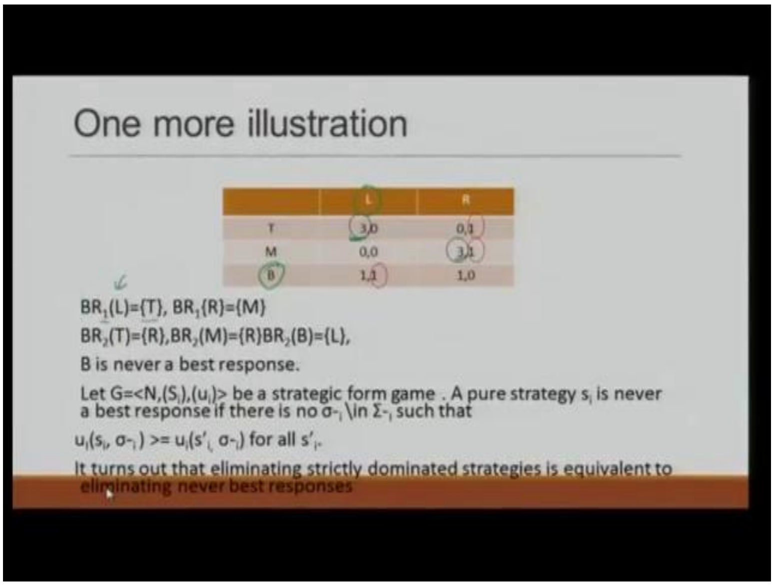

Before, we close we should be careful about one more idea, let us look at the best responses of player 1. So, player 2 player L, then player one best response is to play 3, 3 is higher than 0 and 1, so best response from player 1 is T that is, what I have written here, we are represents best response one represents from player one and this represents just strategy appear two, so best response from player 1 is T.

Similarly, what we have best response from player 2 is well player 1 is best response from player 1 is M, when player 2 is playing R, let us see here 3, which is higher than 0 and1, what do we get d is never a best response. Now, as a terms out I want to sate it without proving that eliminating a strictly dominated strategies equivalent to eliminating never the best response.

So, this is very very useful that we want have to, let us go back we do not have to draw this charge to see figure out that do we have a strictly a strategy which is strictly dominates any other strategy, we do not need to do that we only need to figure out the best responses and a strategy which is a never best response is strictly dominated we can remove that strategies.

So, this is also useful technique in figuring out the outcome, so now, you have a large set in of solution concept that one is a strictly dominated strategy equilibrium, then you have iterative elimination of strictly dominated or weekly dominated strategies, then you have nash equilibrium, notice that and a strictly dominant strategy equilibrium and strictly or the elimination of iterated elimination of dominated strategies, you are using only two assumptions, those assumptions are rationality and common knowledge.

But, when we are using nash equilibrium, we have to add one more assumption that is the clear of believe and their believe about other player actions or they will all the players are correct believe about other players action. Also I would like to add one more thing that, then we are able to solve a game using a strictly dominant strategy equilibrium notion or iterative elimination of dominated strategy or week dominant strategy equilibrium. Then, we say that game is dominants solve together otherwise the game is not dominants are that is it for this module we will meet again and talked about some applications.

Thank you.