Strategy: Introduction to Game Theory
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Lecture -17

Welcome to lecture on Strategy on Introduction to Game Theory. In this module, I am going to emphasize two assumptions which are basic foundation of game theory. Those two assumptions are rationality that players are rational, and they have common knowledge. So, it will become clear as what do you mean by these terms.

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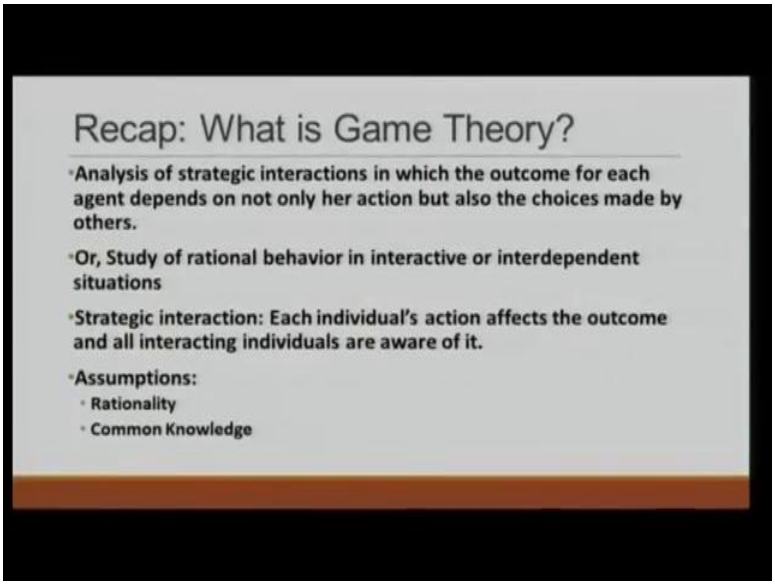

Before we do so, let us just think about what is game theory have been doing for two weeks. What do you mean by game theory? A game theory is analysis of strategic interactions in which outcome for each agent depends on not only his or her own action, but also choices made by others. So, the key term is that choices have to be choices that are made by others have to be taken into consideration. So, study of rational behavior in interactive or interdependent setting is game theory.

Now, we have been talking about Nash equilibrium and other solution concept, but all those solution concepts, for that we have to think about two concepts, first two assumptions. First those assumptions are rationality and common knowledge.

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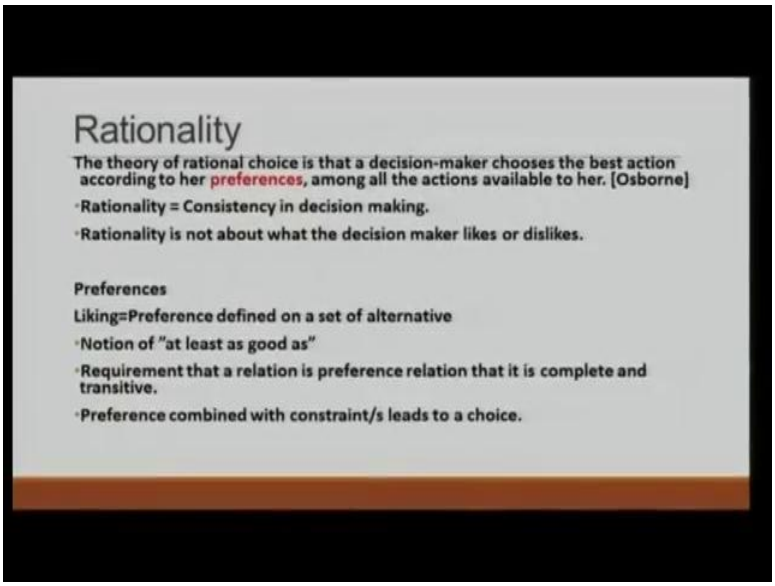

So, before we talk about rationality game theory, it is basically a study in rational choice theory according to economics. So, what do you mean by Rational Choice Theory? The theory of rational choice is that a decision-maker chooses the best action according to her preferences among all the actions available to her. You might be wondering that we are getting lot of new terms that when we learn a subject, it is important to understand the jargon that comes with that subject.

What do you mean by preference? We will soon come to that. What do you mean by preference? Basically, rationality will be to try to understand its day-to-day term. Rationality is all about consistency in decision making. If one person is consistent, rationality does not mean that he likes one thing over the other things. It is not about likes and dislikes; it is about consistency in decision making and also, the term preference. What do you mean by preference? Basically, preference gives us the notion of at least as good as like when I say I prefer coffee to tea, what I mean that I like coffee at least as much as tea, but not tea as much as coffee. So, preference is basically a relation that gives us the notion of at least as good as.

So, coming back to rationality, now we know what we mean by preference. Coming back to rationality, rationality simply says that preference has to satisfy two criteria, one is the completeness and another is the transitivity.

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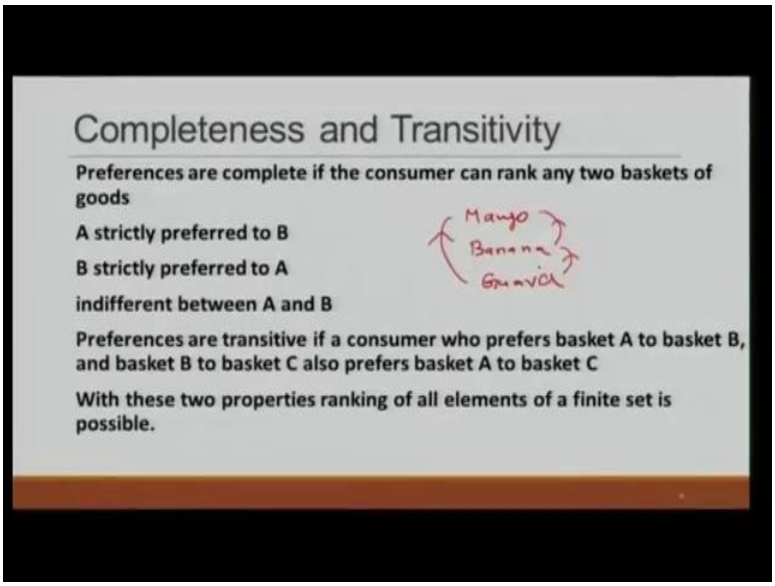

What do you mean by completeness? Let us say when you have two choices, choice A or choice B. You should be able to say that choice A is better than choice B or choice B is better than choice A, or you should be able to say that you are indifferent between choice A and choice B. When you could do that, then one can say that your preferences are complete. You might be wondering it to cover all the possibility.

Let me tell you one scenario in which your preference would not be complete given choice between. Let us say this A and this B when you are unable to decide, not because you like both of them equally, but you are unable to decide. You are not clear about what you like, then your preference is not complete. So, one thing that you should understand is rationality of your preference has to be complete. What does it mean? The given two options at a time, you should be able to decide which one you want.

The second requirement that we have is of transitivity. Completeness is about able to make decision. Transitivity is about consistency. So, what does it say? It says that preferences are transitive. If a consumer who prefers basket A or item A to item B and item B to item C, then he should prefer item A to item C. Fine? Let us take three examples. What we have? We have mango, then we have a banana and then, we have a guava, three things. What it says? If you prefer mango to banana and banana to guava, then you should prefer guava. You should prefer mango to guava. This is also possible that you may not prefer in this manner. What is also possible that you prefer mango to

banana and banana to guava, that given a choice between guava and mango, you pick guava. In this case, your choice would not be consistent. This is what we want to avoid. Why do we want to avoid? It is because if your decision making is not consistent, then it is impossible to figure out which decision you will make.

There is another problem also. Let us say that of course you like mango to banana and the difference you are willing to pay to exchange you to get mango in lieu of a banana. You are willing to get 1 rupee and again if you want to get banana in lieu of guava, you are willing to pay 1 rupee and again between guava and mango, again you are willing to pay 1 rupee. So, let us say that you have a mango at result I bring a guava. I say would you like to take guava because you are willing to pay at least as much as 1 rupee for guava. For that mango, I will give you my guava, what you will do? You will give me a mango and 1 rupee. Now, what do I do? I bring a banana and between banana and guava, you like banana more. So, again you will be willing to pay 1 rupee, you will give me guava as well as 1 rupee. Now, you will have a banana.

Remember I had your mango. I will bring that mango back to you and between mango and banana; you will take that mango and give me the banana and 1 rupee. Eventually now you again have the mango that you started with that you have 3 rupees less. So, that kind of situation we would like to avoid. So, we say that all the players should be rational. It means they are able to make decision, there decision making. So, these are the two requirements.

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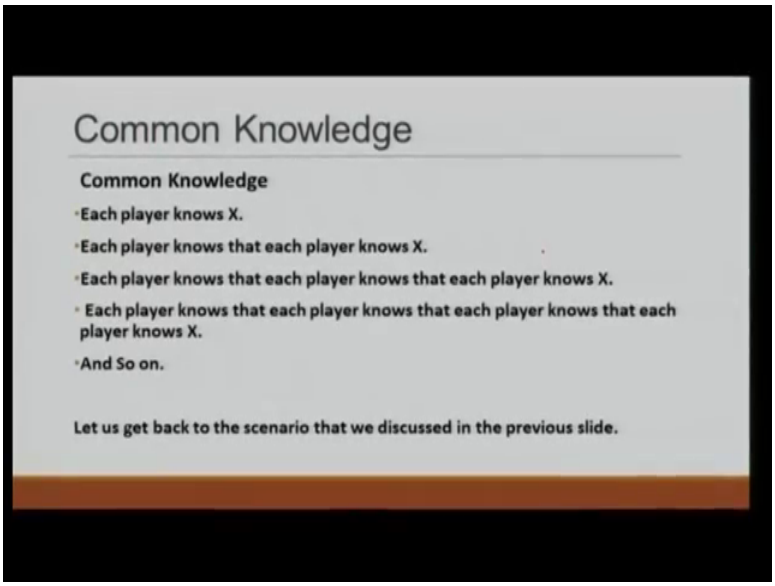

Now, what we have the second assumption that we require is a common knowledge. It sounds funny that bear with me. I will come to an example and you will understand it better. What is common knowledge? Let us say there is a fact that is known by each player. Each player knows X requirement. Next requirement for common knowledge is that each player knows that each player knows X. It does not stop here. It goes further that each player knows that X. Let me do one more time that each player knows that each player knows that each player knows that each player knows X, and we cannot stop here. We have to go on till infinity. We will have to keep on doing that it may sound funny that let us take an example.

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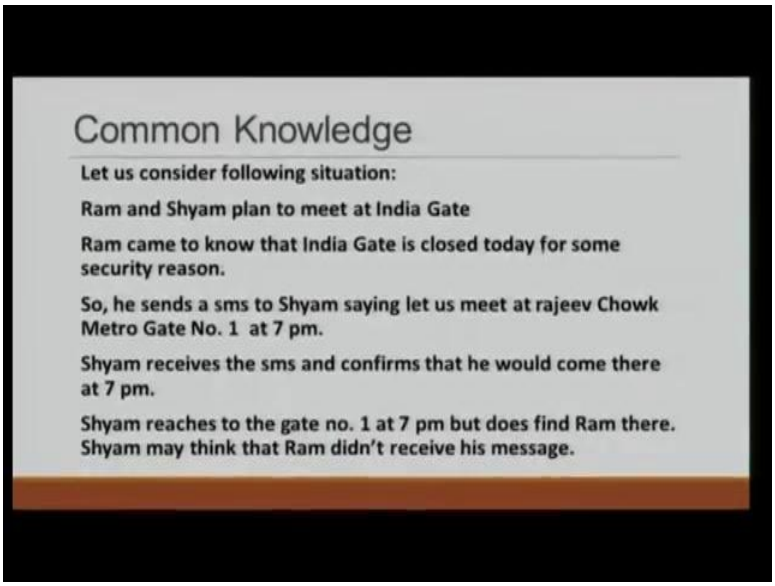

Let us consider the fact of some story that Ram and Shyam plan to meet at India Gate again. These are made up. This is the made up example. Ram and Shyam and India Gate are not important, it is just the fact.

Ram came to know that India Gate is closed today for some security reason. So, he sends an SMS to Shyam saying let us meet at Rajeev Chowk metro gate number 1 at 7 pm in the evening. Shyam receives the SMS and confirms that he would come there at 7 pm. Shyam reaches to gate number 1 at 7 pm, but does not find Ram there. Shyam may think that Ram did not receive the message.

Let us view the definition of common knowledge in this stating what the fact is. The fact is that they are supposed to meet. X is that they are supposed to meet at Rajeev Chowk metro gate number 1 at 7 pm. Let us consider this X. Ram knows that they are supposed to meet at metro gate number 1 at 7 pm. Why? It is because Ram is the one who messaged Shyam first. So, Ram knows this. Shyam also knows this. Why? It is because he receives the message. Let us go to the next step. Ram knows that Shyam knows this that they are supposed to meet at metro gate number 1 at 7 pm. Why? How can he say that Ram knows that Shyam knows because Ram sent this message to Shyam and Shyam replied back. How about Shyam? Shyam knows that Ram knows that they are supposed to meet at metro station. Yes. Shyam knows because Ram is the one who is sending the message. So, Shyam also knows. So, second step is clear.

How about the third step that Ram knows that Shyam knows that Ram knows that, they are supposed to meet at metro station at 7 pm. Yes, Ram knows because ram received the confirmation from Shyam, but let us look from Shyam's point of view. Does Shyam know that Ram knows that Shyam knows that they are supposed to meet at metro station? No. That is where it is a breakdown. So, it does not fulfill the requirement for common knowledge, so that we have to be careful. So, in game theory we assume that players have common knowledge. They all know the rules of the game, they all know the payoffs. Not only they know the payoffs and rules of the game, they know that the other players also know we cannot stop there. They know that all other player know that all other players know the payoffs and rule of the game and so on.

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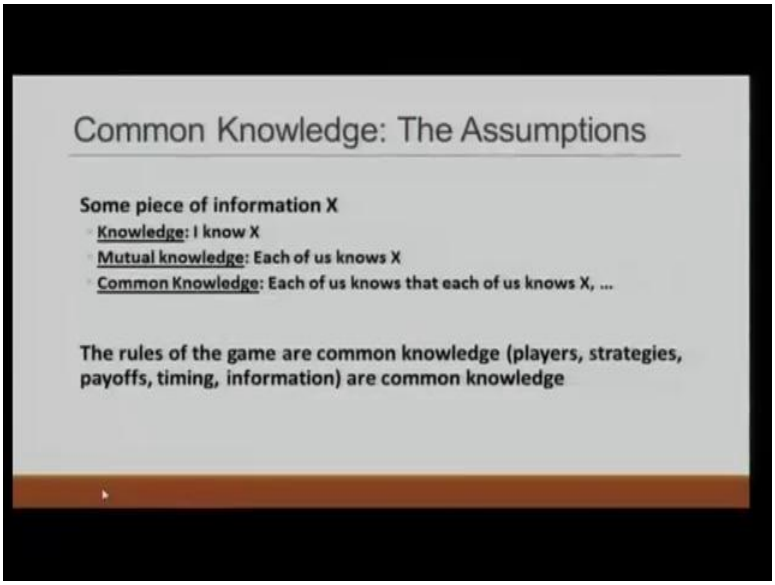

So, in game theory we have this requirement. Just to make it clear what you mean by knowledge that I know X. That is my knowledge, that is mutual knowledge that each of each of us know X, and from mutual knowledge we go to common knowledge that each of us knows that each of us knows that each of us knows to. You can keep on talking about it, fine. So, these are the two assumptions required for game theory rationality as well as common knowledge. Now, in the next module, we are going to use some of these things that we have learned.