Strategy: An Introduction to Game Theory
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Lecture – 14

Hello everyone, welcome to another module in this online course Strategy, An Introduction to Game Theory. Now, in this module let us start looking at another aspect of the mixed strategy in Nash equilibrium in the battle of sexes game. Let us try to look at the battle of sexes game in terms of a best response dynamic.

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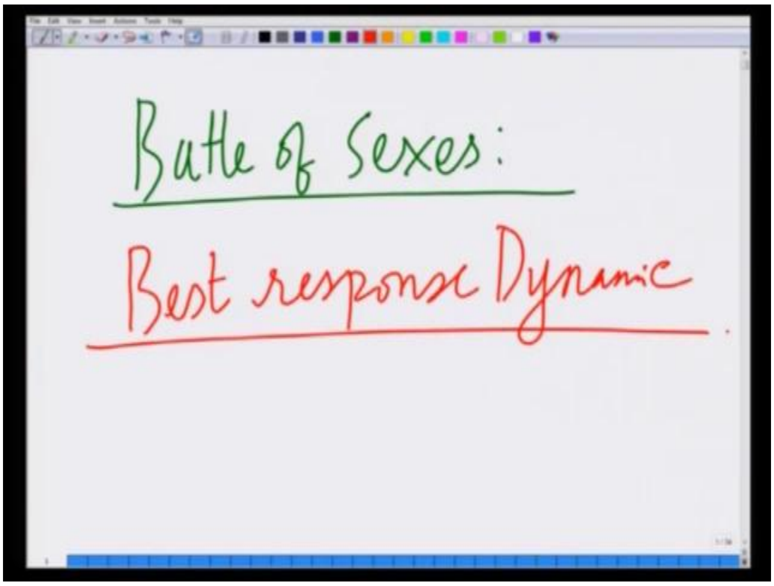

So, I am going to take another look at the battle of sexes game, this time employing the best response dynamic. So, that we try to understand it a little better.

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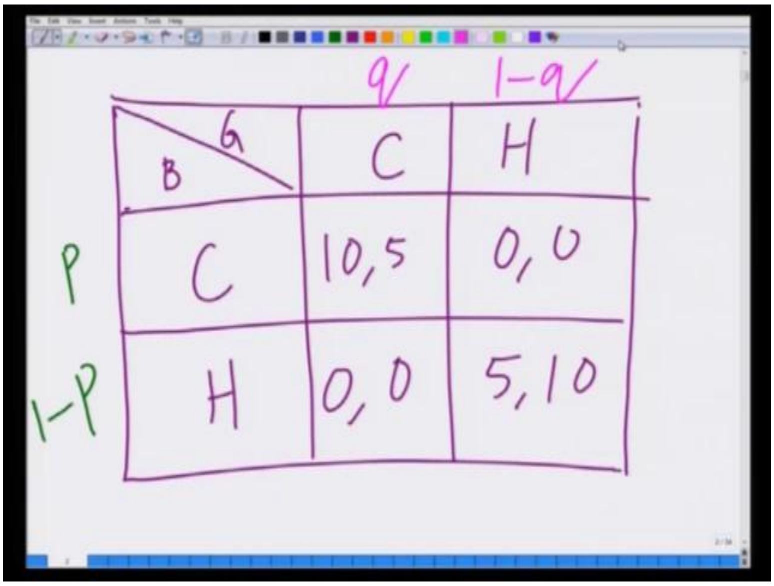

Let me redraw the game table again for this game, again that is we have two players, the boy and girl both can choose to watch either cricket or Harry Potter. If both choose different things, their payoff is 0, if they both watch cricket, their payoff is 10, 5. If they both watch the Harry Potter movie, their payoff is 5, 10. Let us now consider the boy mixing with probabilities p and 1 minus p.

Let us now consider the boy mixing with probabilities p and 1 minus p. We already said that, this is a mixed strategy employed by the boy, in which randomly fraction p of the time is choosing to watch cricket and fraction 1 minus p of the time is choosing to watch the Harry Potter movie.

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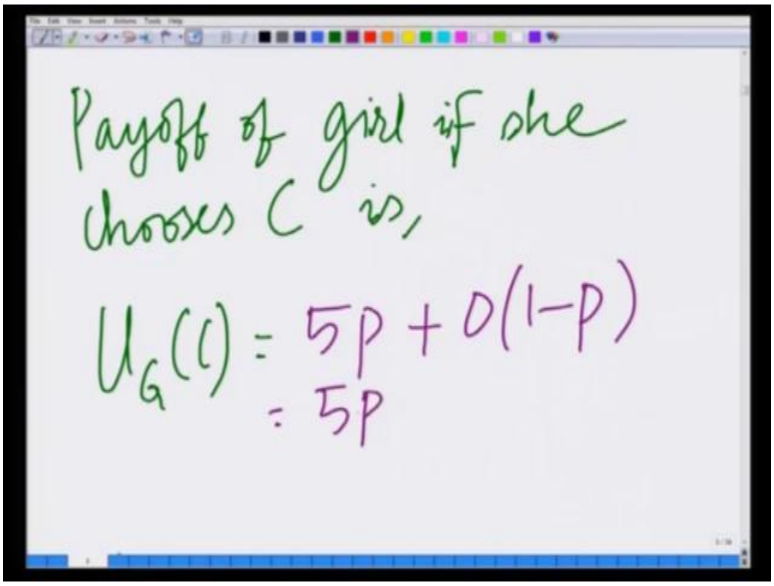

Now, the payoff of the girl, if she always chooses C is u girl of C equals; that is equals as we already said 5 times p plus 0 times 1 minus p. This is equal to 5 times p plus 0 times 1 minus p equals 5 times p.

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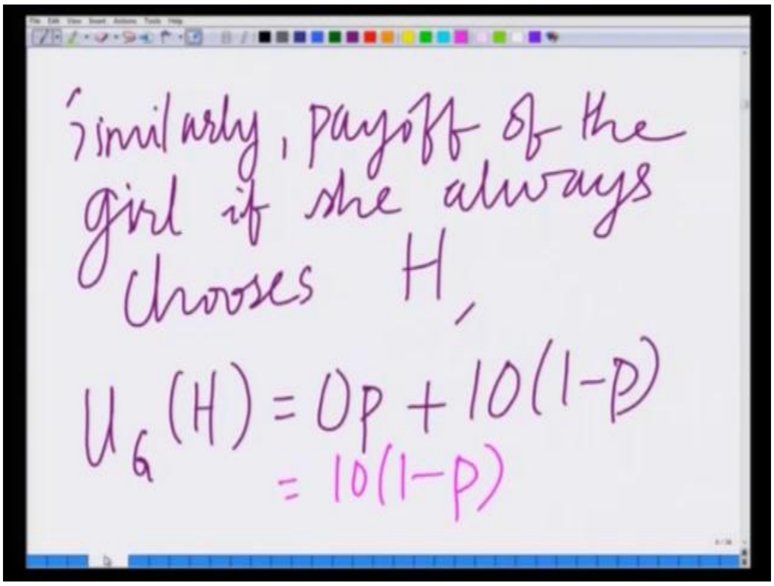

Similarly, payoff of the girl, if she always chooses to watch the Harry Potter movie; that is u G of H equals ((Refer Time: 03:06)) 0 times p plus 10 times 1 minus p, which is equal to 10 times 1 minus p. ((Refer Time: 03:21)) So, the payoff of the girl, if she always chooses to watch cricket is 5 times p, if she always chooses to watch the Harry

Potter movie, it is 10 times 1 minus p. Now, let the girl employee the mixed strategy ((Refer Time: 03:34)) q comma 1 minus q; that is she is choosing cricket fraction q of the time and Harry Potter movie fraction 1 minus q of the time.

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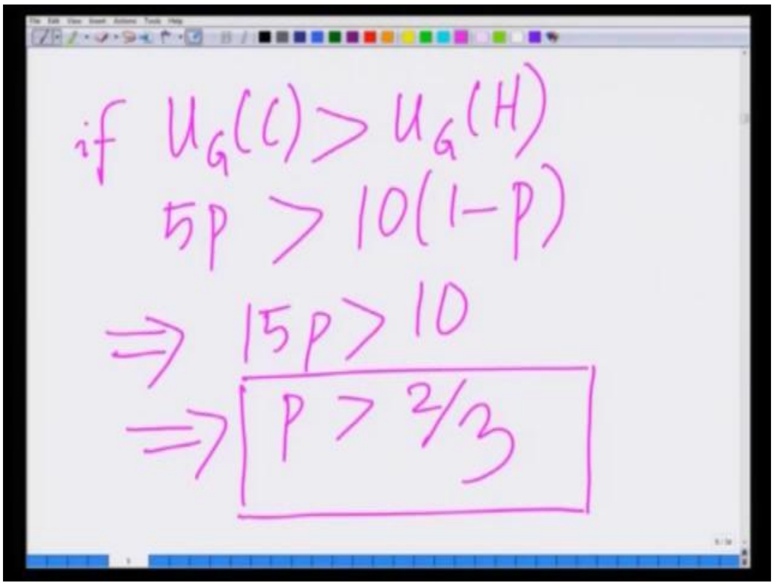

Now, let us consider different scenarios. Now, if her payoff from cricket, if u G C is always greater than u G H; that is payoff from cricket is always greater than payoff from Harry Potter movie. That is what we are saying is 5 times p is always greater than 10 times 1 minus p. If 5 times p is always greater than 10 times 1 minus p and when does this happen, this happens if when 15 p is greater than 10 implies, p is greater than 2 by 3.

So, follow this argument closely, if p is greater than 3, 2 by 3, if p is greater than 2 by 3, then what we are saying is 5 times p is greater than 10 times 1 minus p. Therefore, the payoff to the girl for choosing cricket is always greater than the payoff to the girl for choosing the Harry Potter movie. Therefore, she will always choose cricket, she will not employee a mixed strategy.

But, she will always choose cricket which means, fraction 1 of the time, always means fraction 1; that is with probability 1 she is choosing cricket, which means fraction 1 of the time; that is the whole of the time she is choosing cricket therefore, q is equal to 1. Her best response q, q star is equal to 1, so hope this is clear; that is best response.

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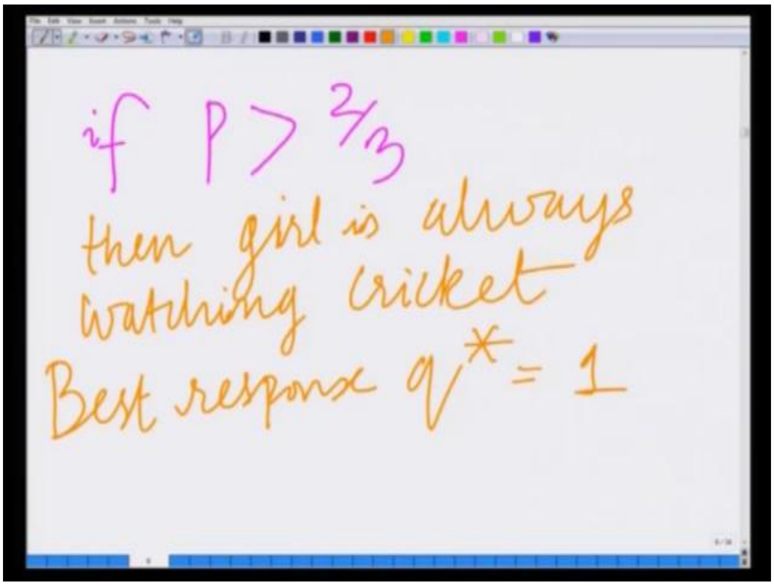

If what we are saying is, p is greater than 2 by 3, then girl is always watching cricket and therefore, best response q star is equal to 1. That is because of her payoff from cricket is more than payoff from the Harry Potter movie, she is always choosing to watch cricket and therefore, the best response q star is equal to 1. That is whole of time she is watching cricket, which means the fraction q is equal to 1, 1 minus q is equal to 0, q star is equal to 1.

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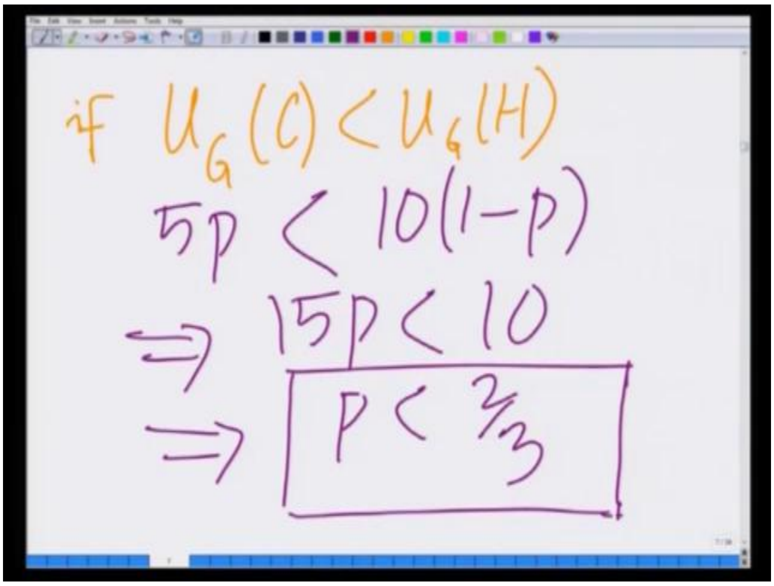

On the other hand, if payoff from cricket is strictly less than the payoff from the Harry Potter movie, if u girl of cricket is less than the payoff from the Harry Potter movie; that is to say that 5 p is strictly less than 10 times 1 minus p, which happens, when 15 p is less than 10 which happens, when p is less than 2 by 3. If p is less than 2 by 3, then Harry Potter movie, watching the Harry Potter movie always yields a strictly higher payoff compare to watching cricket.

Therefore, she is always going to choose the Harry Potter movie and therefore, the probability with which she chooses cricket is 0. That is she will not watch cricket at all and she will always watch the Harry Potter movie, in which case q is equal to 0 and 1 minus q equal to 1. So, in this case, the best response q star is 0.

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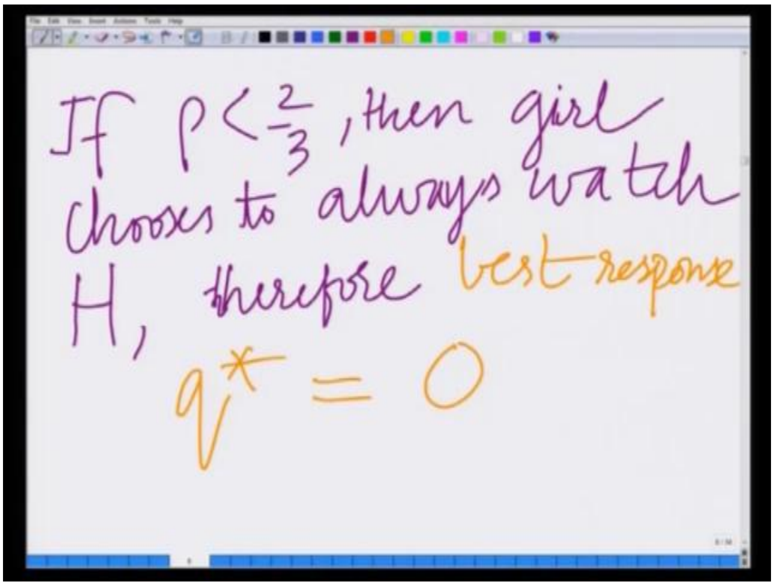

If p is less than 2 by 3, then girl chooses to always watch H therefore, best response q star equal to 0. If p is less than 2 by 3, then best response q star is equal to 0, which means, she is never watching cricket, she is always watching the Harry Potter movie. Because, watching the Harry Potter movie gives her a higher payoff compare to cricket. An interesting thing happens, when the payoff from cricket and Harry Potter are equal.

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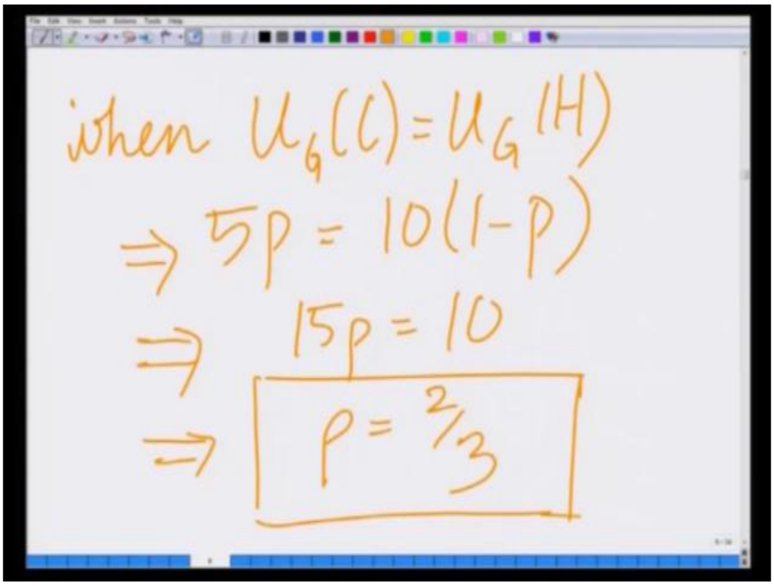

When u G of C that is the payoff to the girl from cricket is equal to the payoff to the girl from the Harry Potter movie implies 5 times p equals 10 times 1 minus p implies 15 p equals 10 implies p equals 2 by 3. If p equals 2 by 3 something interesting happens, if p equals 2 by 3, then 5 times p equals 10 times 1 minus p. Therefore, her payoff from watching cricket and her payoff from watching the Harry Potter movie are exactly identical.

In this case, she can choose anything, she can choose to watch cricket, she can choose to watch Harry Potter movie or she can randomly choose to watch either the cricket or the Harry Potter movie, her payoff is going to be exactly identical. So, in this case, she can use any value of the probability mixer q and each of them will yield exactly the same payoff, because cricket yields a payoff of 5 times p, p is equal to 2 by 3, which is 5 times 2 by 3; that is 10 times 3.

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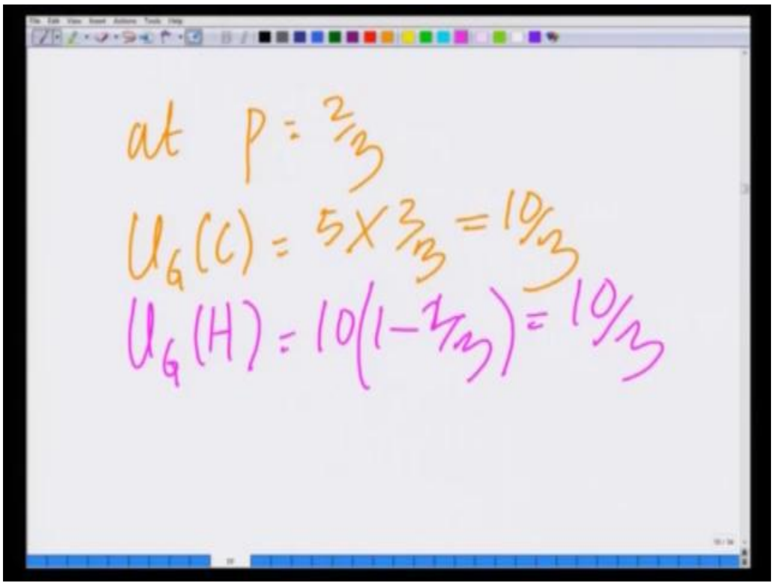

So, let us compute that, at p equal to 2 by 3 her payoff from cricket is equal to 5 times 2 by 3, which is 10 by 3. Her payoff from the Harry Potter movie is equal to 10 times 1 minus 2 by 3 equals 10 by 3. So, both cricket and Harry Potter movie yield exactly identical payoff of 10 by 3. So, she can choose to watch either and therefore, she can employee any probability mixer, she can randomly choose one or the other with any probability and that will yield exactly the same payoff.

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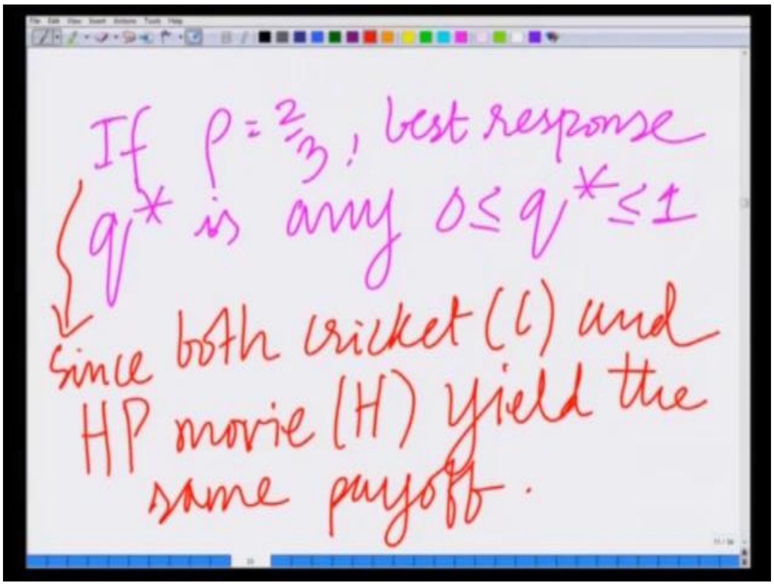

So, the best response q star, if p is equal to 2 by 3, best response q star is any 0 less than q star less than equal to 1. That is any probability q star which lies between 0 and 1 is a best response and that is since both cricket; that is C and the Harry Potter movie H yield the same payoff. Since, both of them yield the exactly same payoff, so she can use any probability 0 less than equal to q star less than equal to 1.

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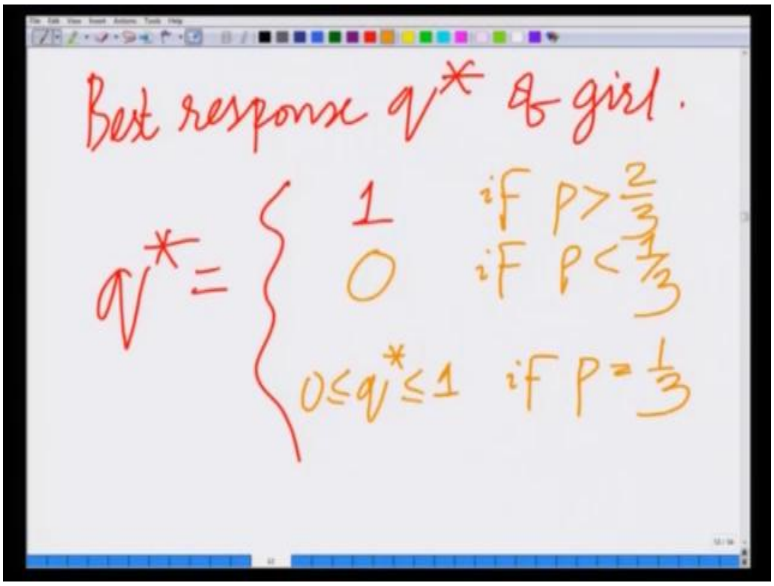

And therefore, let us now summaries the best response q star. Now, q star is equal to as we said, if p is greater than 2 by 3, then q star is 1; that is she always chooses to watch cricket. If p is less than 2 by 3, then q star is equal to 0, since she always chooses to watch the Harry Potter movie. On the other hand, if p is equal to 2 by 3, then any value of 0 less than equal to q star less than equal to 1 is acceptable. Because, p equal to 1 by 3, then both the Harry Potter movie and a game of cricket give her an exactly identical payoff, therefore any probability q star between 0 and 1 is acceptable. And let us try to draw this best response dynamic, let us now draw this best response dynamic and see how it looks like.

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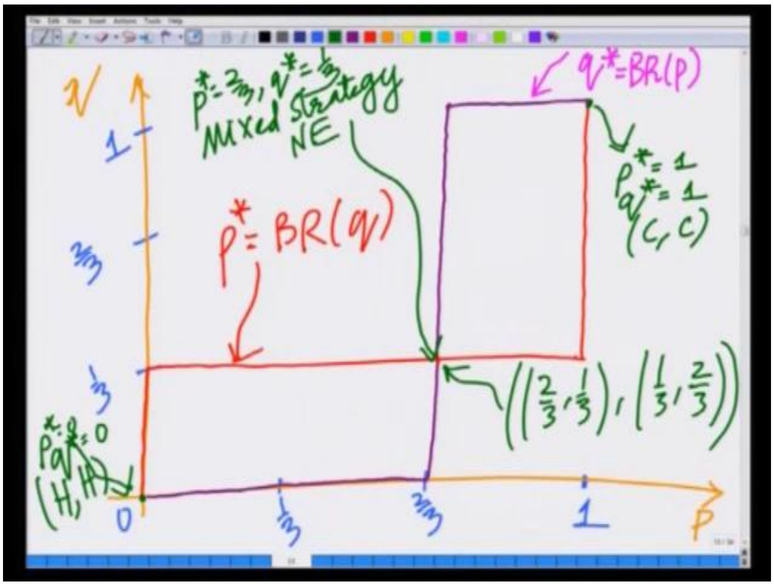

So, again I am going to draw my plot, on the x axis I have p, on the y axis I have q and q goes from 0 to 1 and p also, p goes from 0 to 1 and q also goes from 0 to 1, I will divide it into three equal parts 1 by 3, 2 by 3, 1 by 3, 2 by 3. Now, we said that if p is greater than 2 by 3, then the best response q star is equal to 1. So, if p is greater than 2 by 3, the best response q star is equal to 1.

On the other hand, if p is less than 2 by 3, the best response q star is equal to 0 and if p is exactly equal to 2 by 3, then the best response q star can lie anywhere between 0 and 1. So, try to understand this best response dynamic, what we are saying is, if p greater than 2 by 3, q star is equal to 1; if p is less than 2 by 3, then q star is equal to 0; but if p is exactly equal to 2 by 3, then any q star between 0 and 1 is the best response.

So, this is the best response q star equals, best response as a function of p; that is q star is the best response as a function of p. This is the best response q star as the function of the mixer probability p; that is employed by the boy. Now, let us try to look at it from the perspective of the mixer employed by the girl, So, let us now try to find n the best response p ((Refer Time: 14:43)) as a function of the mixture q employed by the girl. Now, the girl is employing the mixer q and 1 minus q, if the boy always chooses cricket, his payoff is 10 times q plus 0 times 1 minus q.

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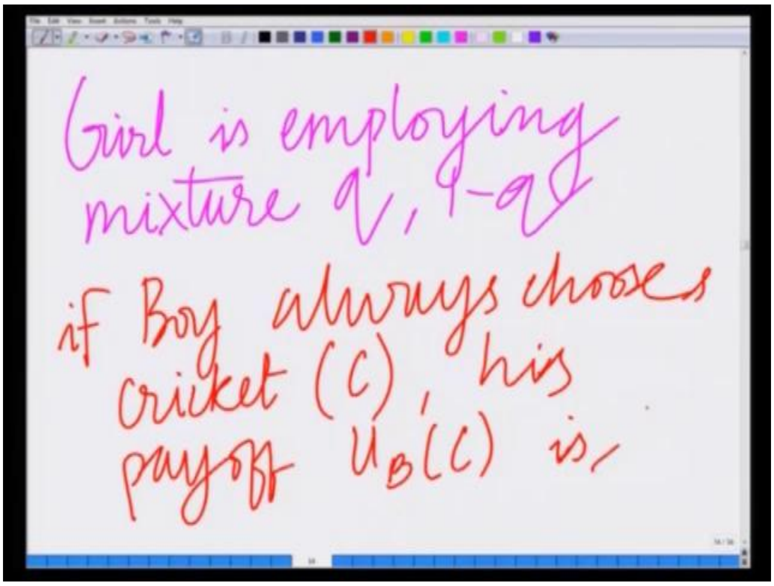

If the boy always chooses as cricket, girl is employing mixer q comma 1 minus q.

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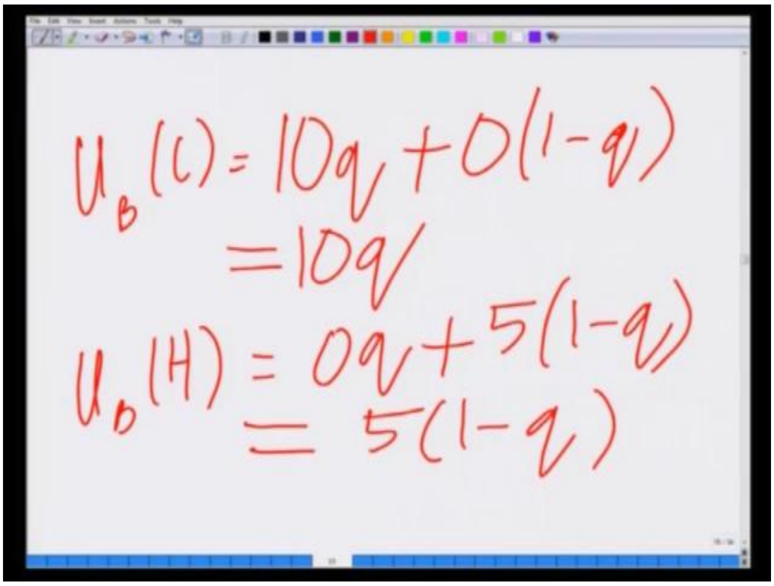

If boy always choose as cricket; that is C his payoff, well u B of C is equal to 10 times q plus 0 times 1 minus q, which is equal to 10 times q. On the other hand, ((Refer Time: 16:04)) if we always chooses towards the Harry Potter movie, his payoff 0 times q plus 5 times 1 minus q. So, as a function of q, his payoff are, if is always choosing to watch cricket, his payoff is 10 times q and if it is always choosing to watch the Harry Potter movie, his payoff is 5 times 1 minus q.

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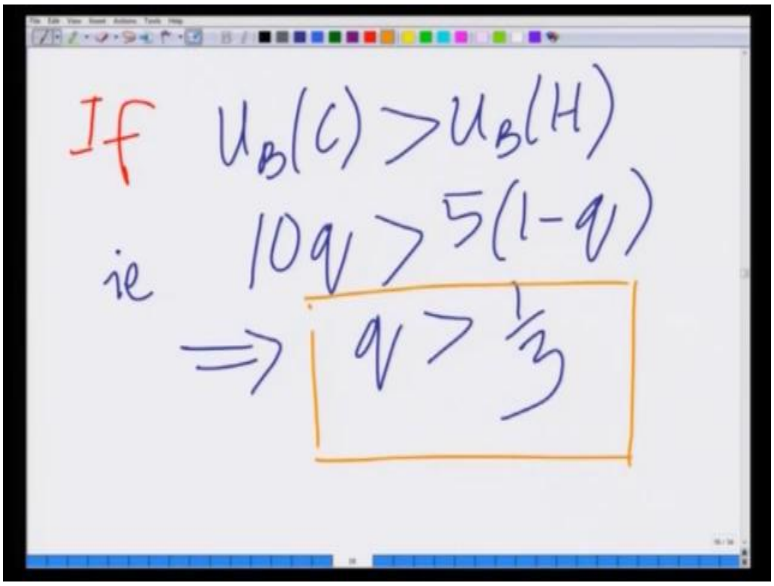

Now, if 10 times q, if the payoff to the boy from cricket is greater than the payoff to the boy from the Harry Potter movie; that is 10 times q is greater than 5 times 1 minus q, which implies that q is greater than 1 minus 3, 1 by 3. You can solve this and you will find that this implies that q is greater than 1 by 3. If q is greater than 1 by 3, then the best response of the boy, since the cricket is always yielding a higher payoff is to always watch cricket, which means, the whole of the time, he will just preferred to watch cricket, which means, the probability, the optimal best response probability p star is equal to 1.

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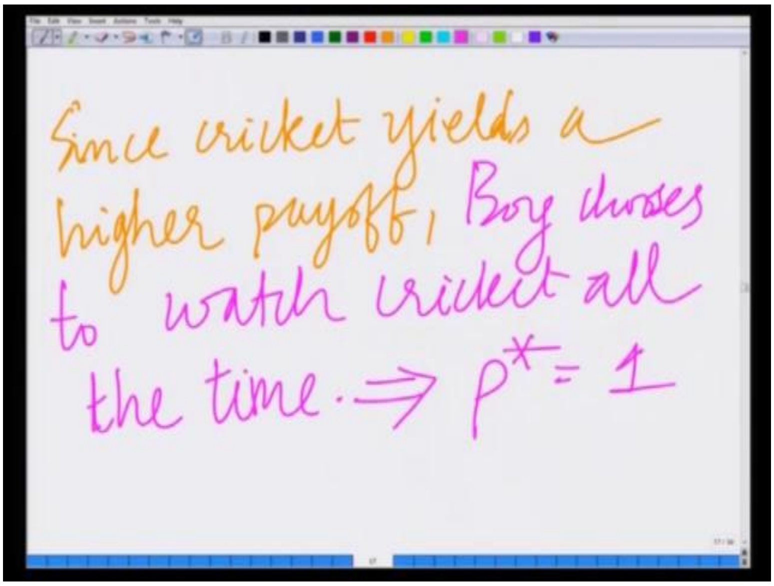

So, p star, since cricket is yielding a higher payoff, boy chooses to watch cricket all the time implies p star, best response p star is equal to 1, 1 minus p star is equal to 0. So, is not watching the Harry Potter movie at all, because cricket is yielding is strictly higher payoff.

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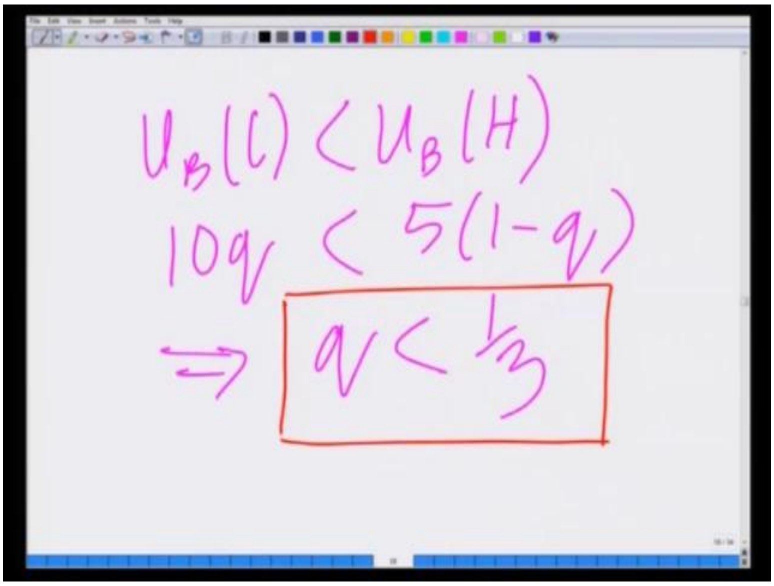

On the other hand, if the payoff from the Harry Potter movie u B of H is strictly greater than the payoff from cricket, which happens, when or let me write it an another way, just to be consistent, when the payoff from cricket is less than the payoff from the Harry Potter movie. That is to say that 10 q is strictly less than 5 times 1 minus q, which implies q is strictly less than 1 by 3. When q is straightly less than 1 minus 1 by 3, then is payoff from cricket is strictly less than payoff from watching the Harry Potter movie.

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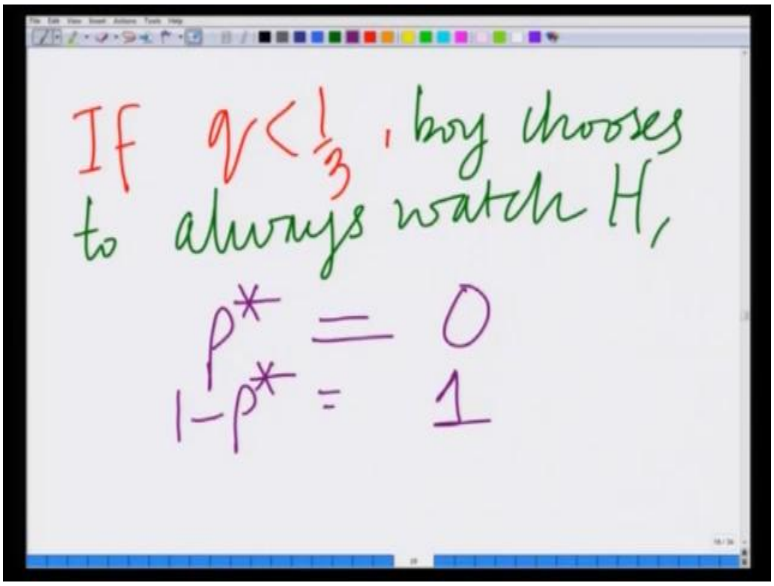

As a result boy choose as to, if q is less than 1 by 3, boy chooses to always watch Harry Potter movie in which case p star is equal to 0; that is not watching cricket any time; that is all of the time, fraction all of the time is choosing to watch the Harry Potter movie. Therefore, 1 minus p star is equal to, best response p star is equal to 0, 1 minus p star is equal to 1.

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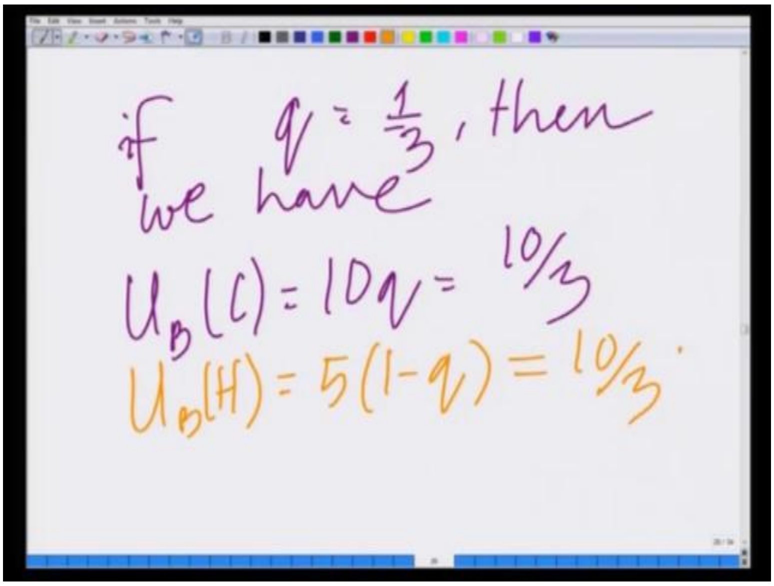

And now an interesting thing occurs, if q is equal to 1 by 3, then we have u of boy from cricket is 10 times q, which is 10 by 3 and payoff to boy from Harry Potter movie is 5 times 1 minus q, which is equal to 10 by 3. So, payoff from cricket and payoff from Harry Potter movie are exactly identical to 10 by 3. Therefore, he can choose one or the other or randomly choose one or the other he will get an identical payoff. Therefore, any piece star any probability which lies between 0 and 1 is a best response.

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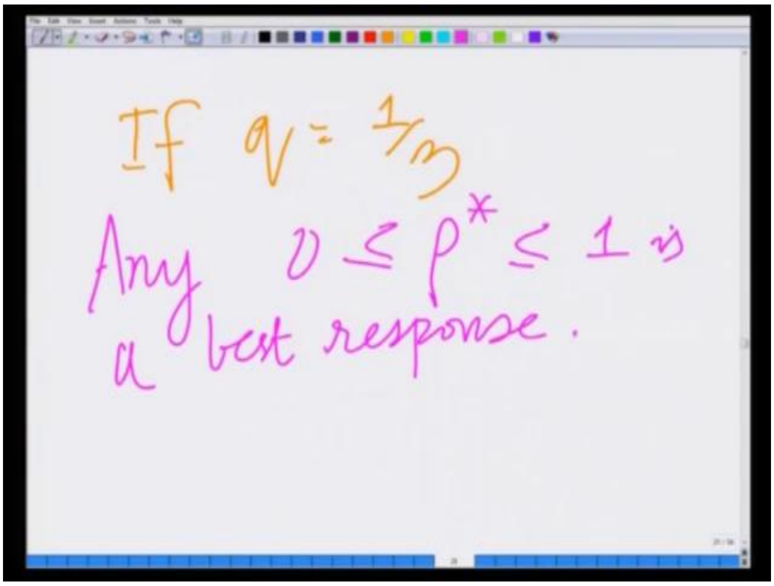

If q is equal to 1 by 3, therefore if q is equal to 1 by 3, any 0 less than equal to p star less than equal to equal to 1 is a best response any 0 less than or equal to p star less than are equal to 1 is a best response.

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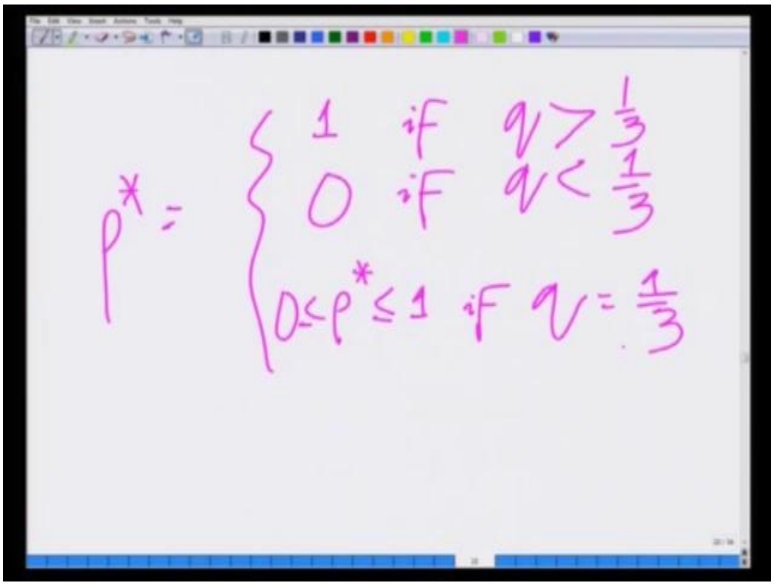

So, now if we can summaries the best response of the boy p star is equal to 1, if q is greater than 1 by 3; that is if use greater than 1 by 3 is always choosing to watch cricket 0. If q is less than 1 by 3; that is if is less than 1 by 3, which always choosing to watch the Harry Potter and it is any 0 less than or equal to p star, less than or equal to 1, if q is exactly equal 1 by 3.

And let us draw this again ((Refer Time: 21:59)) on our best response diagram, if q is greater than 1 by 3; that is somewhere over here, if q is greater than 1 by 3, best response p star is equal to 1. Let me draw the slightly different color, if q is greater than 1 by 3, best response p star is equal to 1. If q is less than 1 by 3, best response p star is equal to 0 and if q is equal to 1 by 3, best response p star is any probability between 0 and 1.

And therefore, this is the best response p star as a function of q and now, you can see where the best responses are intersecting. The best responses are intersecting at three points. This corresponds to p star equals 1, q star equals 1; that is both of them are always choosing to watch cricket, p star is equal to 1, which means, boys always choosing cricket, q star is equal to 1, means, girl is always choosing cricket. Therefore, this corresponds to the pure strategy Nash equilibrium C comma C.

And this point, where p star equal to 0 and q star equal 0, girl is never choosing to watch cricket, boys never choosing to watch cricket, which means, they are both always watching the Harry Potter movie, corresponds to the pure strategy Nash equilibrium H comma H. And this third point of intersection, which you can see corresponds to p star equals 2 by 3 and q star is equal to 1 by 3. This corresponds to the mixed strategy Nash equilibrium of the battle of sexes.

Mixed strategy as equilibrium, this corresponds to strategy equilibrium, where boy is watching cricket two-third over time and the Harry Potter one-third of the time and the girl is watching cricket one-third of the time and the Harry Potter movie one-third of the time. And therefore, this mixed strategy Nash equilibrium is 2 by 3, 1 by 3 comma 1 by 3 2 by 3; that is mixer of boy is 2 by 3 comma 1 by 3 and the mixer of the girl is on by 3 comma 2 by 3.

And therefore, this is the best response dynamic of the battle of sexes, considering mixed strategy Nash equilibrium. Therefore, this we can see there are three Nash equilibria of the battle of sexes game to pure strategy Nash equilibria, one mixed strategy in Nash equilibrium. And thus, we have now illustrated, how the best response dynamic and we used to basically solve a mixed at find out all the Nash equilibrium; that is pure strategy Nash equilibrium as well as the mixed strategy Nash equilibrium of the battle of sexes game. So, please go over this again to understand this thoroughly, thank you very much and we will continue with the next module.

Thank you.