Strategy: An Introduction to Game Theory

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Lecture-13

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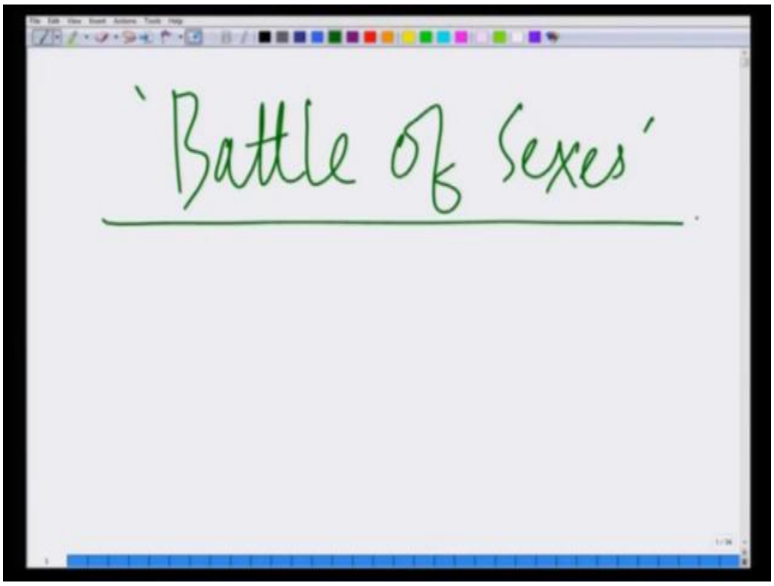

Hello, welcome to another module in this online course strategy an introduction to game theory and a in the last module we started looking at games with mixed strategy and we look at the game of matching pennies and derived the mixed strategy Nash equilibrium. Let us continue to look at games with mixed strategy and let us try to look at some other examples of games with mixed strategies and mixed strategy Nash equilibrium. And let us go back to one of the game that we have already looked at the context at pure strategies that is the battle of sexes game. So, let us go back and look at our battle of sexes game. But, this time in the context of mixed strategy let us try to look at the battle of sexes game in the context of the mixed strategy Nash equilibrium right.

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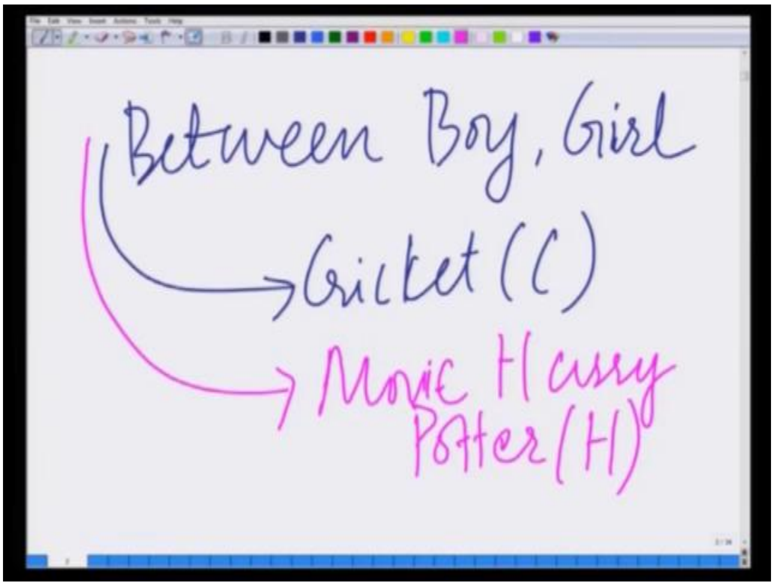

And just to remind you has already know the battle of sexes game is an interesting game between a couple. Between a boy and a girl with they have an option on either choosing to watch a game of cricket (C) or watching the movie that is our Harry Potter movie (H) to spend the evening together. So, both of them can either choose (C) or both of them either choose (H) in which case this spend time together, but each of them choose a different thing then the end of spending time apart in which case there the utility or there payoff a zero at we also drew the game table for this game.

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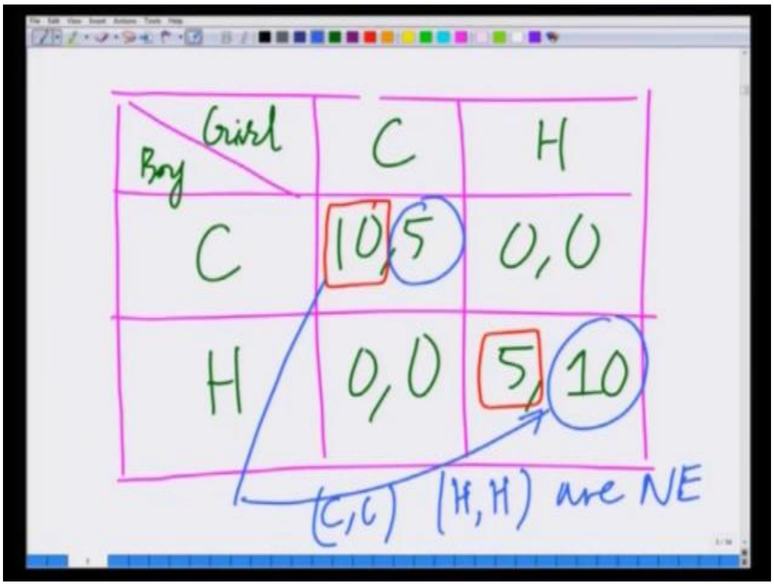

The game table for this can be represented as well there are two players and each player has two possible actions. So, we have the boy who is the row player the girl who is the column player. Each can choose either Cricket or Harry Potter. If, both of them choose different thing that is boy chooses Cricket and girl chooses the Harry Potter movie they both get 0, 0. Similarly, if boy chooses Harry Potter movie and girl chooses to watch Cricket they get 0, 0. On, the other hand if the boy and girl both choose to watch the game of Cricket then the boy gets 10 and the girl gets 5. On, the other hand if boy and girl both choose to watch the Harry Potter movie then the boy gets 5 and the girl gets the payoff of 10 right. And we have already looked at the pure strategy Nash equilibrium of this game again, let us do it once again really quick. So, as to refresh the memory regarding that if the girl chooses (C) the best response of the boy is to choose (C). If the girl chooses (H) best response of the boy is to choose (H). If the boy chooses (C) best response of girl is to choose (C). If the boy chooses (H) best response of the girl is to choose (H). Therefore, we have two Nash equilibria in pure strategies that is C, C and H, H are the Nash equilibria of this game. As, you remember both are Pareto optimal right. And, at the point we also said since each prefers a different Nash equilibrium because the boy prefers the C, C Nash equilibrium the girl prefers the H, H Nash equilibrium right. So, hidden in this game there is a third Nash equilibrium which is the compromise Nash equilibrium whereas, set of fraction of the time the boy is choosing H other times. And, the girl is choosing C set in fraction of time and the rest of the time she is choosing H and both of them are making these choices randomly. That is there is another mixed strategy Nash equilibrium in this game.

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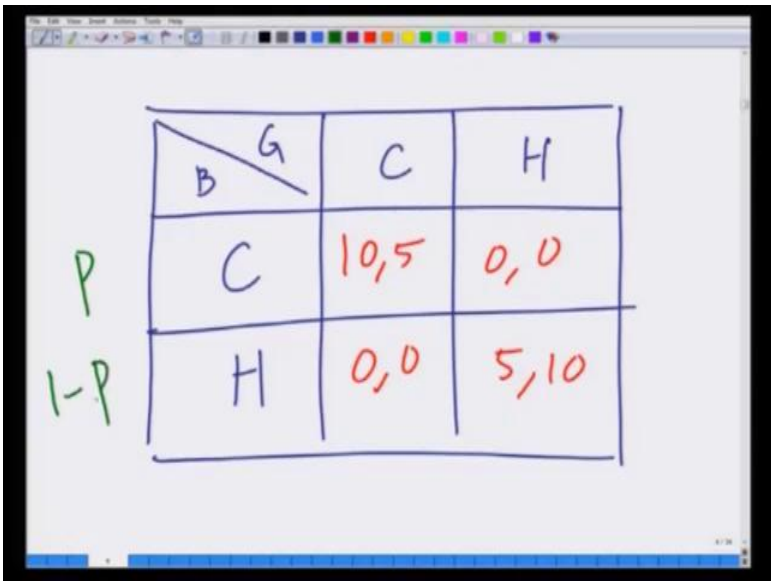

And, now let us try to find the mixed strategy Nash equilibrium let we redraw the game table clearly once again so, as to illustrate the mixed strategy Nash equilibrium for this game. So, we have a boy, girl they can either choose Cricket or Harry Potter and the payoffs of 10, 5; 5, 10; 0, 0; 0, 0. Let us now assume that the boy and the girl are both employee mixed strategy right. Let us assume now that the boy is employing the mixed strategy P one minus P that is to say the boy is playing C with probability P. And is playing H with probability 1 - P that is fraction of P of the time is choosing C fraction 1-P of that time is using H.

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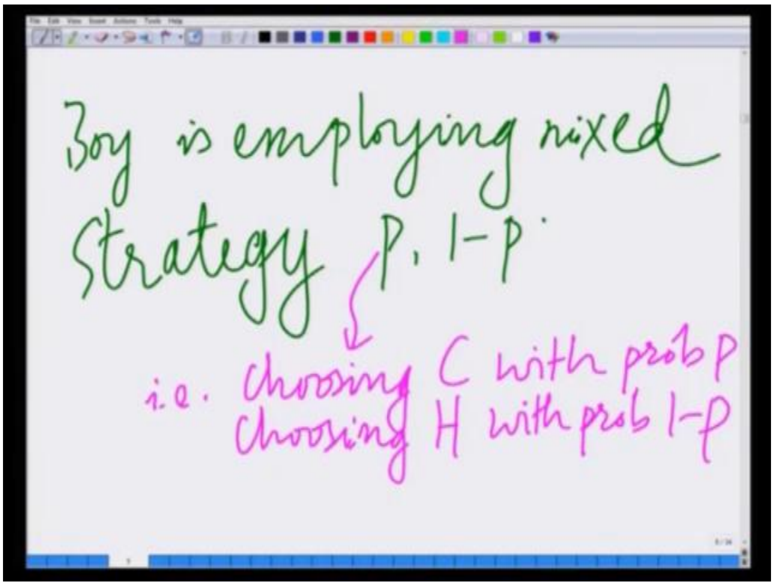

So, we start with assumption similar to the matching pennies game. That the boy is employing a mixed strategy in P that is choosing C with probability P and choosing H with probability 1 - P right. So, fraction P of the time is choosing C that is to watch the game of cricket. And fraction 1 - P of the time is choosing H that is to watch the Harry Potter movie. Now, let us look at the payoffs to the girl right. Now, if the girl chooses to always watch a game of Cricket then a payoff is fraction P of the time she encounters the boy watching cricket in which case her payoff is five. So, the average payoff corresponding to this is 5 times P. And, fraction 1- P of the time she encounters the boy watching the Harry Potter movie in which case her payoff is zero. So, her average payoff in this is 1 - P times 0.

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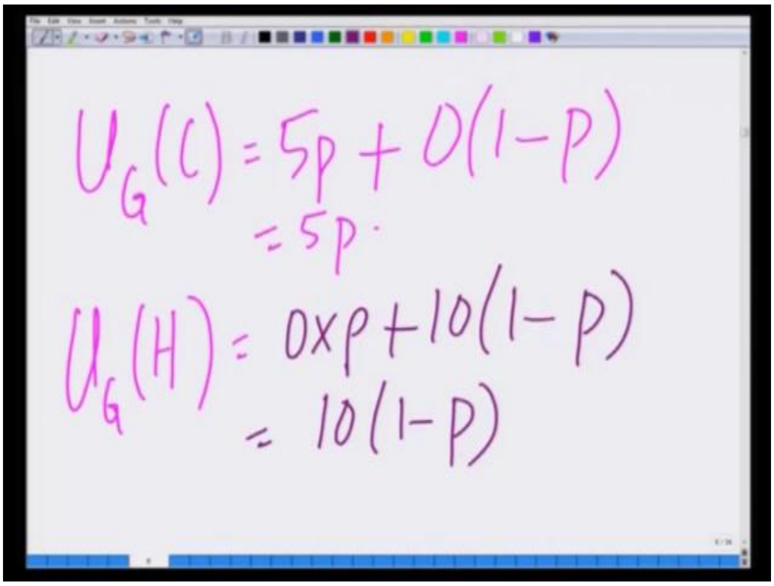

So, if the girl chooses to watch a game of Cricket. So, utility of the girl to watching the game of cricket you can see this it is 5 times P plus 0 times 1 -P. This is 5 times P plus 0 times 1-P equals 5 times P. Similarly, if the girl chooses to watch the Harry Potter movie always then with probability P she encounters the boy watching Cricket in which case her payoff is 0. So, the average payoff is P times zero and fraction 1 - P times she encounters the boy watching the Harry Potter movie in which case her payoffs is 10. So, it is 1 - P times 10 to the net average payoff if the girl chooses to watch the Horry Potter movie. Or, net average payoff is P times 0 + 1 - P times 10 that is 0 times P + 10 times 1-P equals ten times 1-P.

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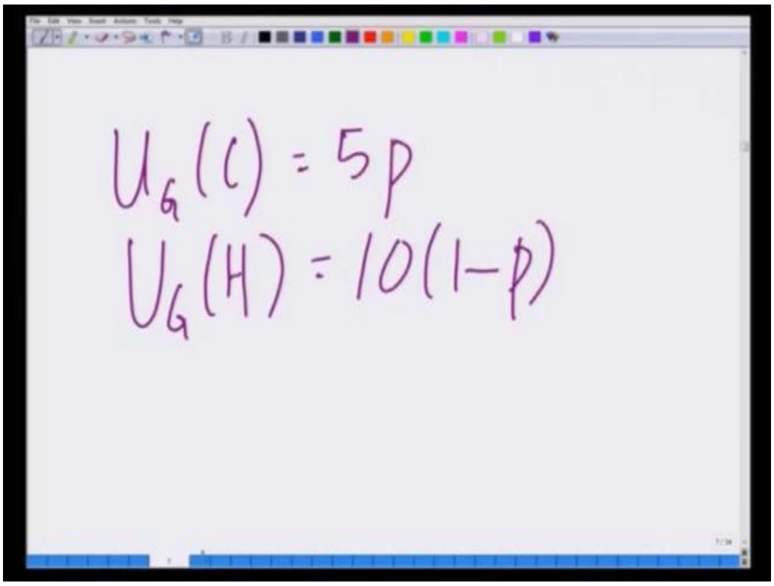

And, therefore, what we have is that the payoff to the girl watching cricket is 5 times P and the payoff to the girl watching the Harry Potter movie is 10 times 1 - P and therefore, if 5 p is greater than 10 times 1 -P then the girl will always choose to watch cricket there is no need for the girl to randomly choose between Cricket and Harry potter if cricket always yields a higher. Similarly, if watching the Harry potter movie always yields a higher pay that is 10 times 1 - P is greater than 5 times p she will always prefer watching the Harry Potter movie right. Therefore, she will randomly choose between watching either Cricket or the Harry potter movie only when 5 times P equals 10 times 1-P.

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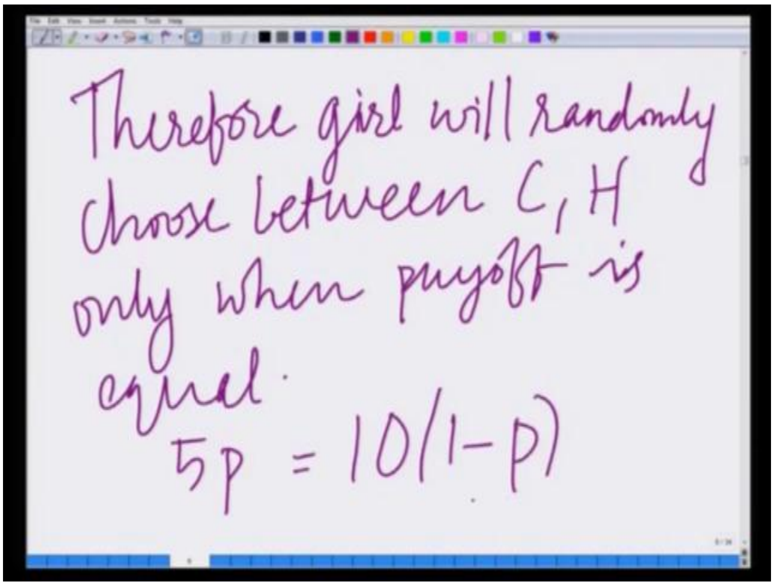

Therefore, girl will randomly choose between C, H only when payoff is equal and, that means, we must have 5 P equals 10 times 1 - P.

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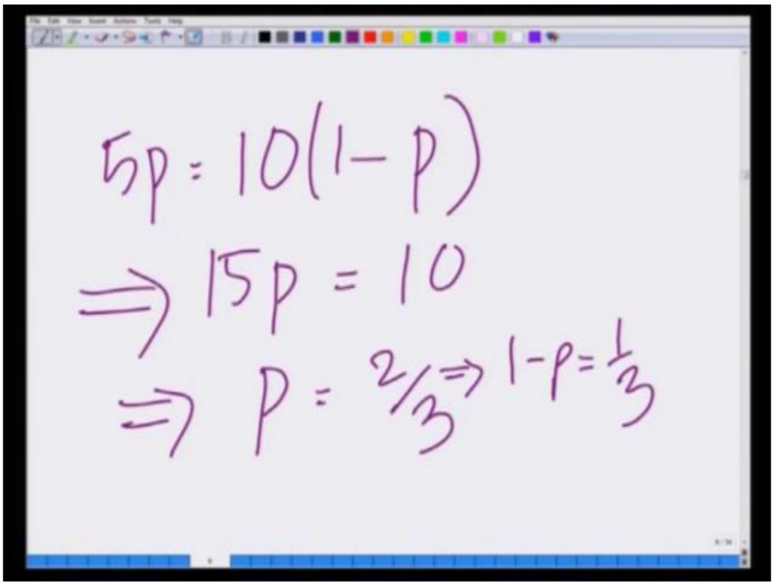

Therefore, we have 5 P equals 10 times 1 – P implies 15 p equals 10. Implies P equals 2 by 3 and therefore, 1 - p equals 1 by 3 which means the mixture of the boy that is P equals 2 by 3=> 1 - p equals 1 by 3 is the probability mixture of the boy.

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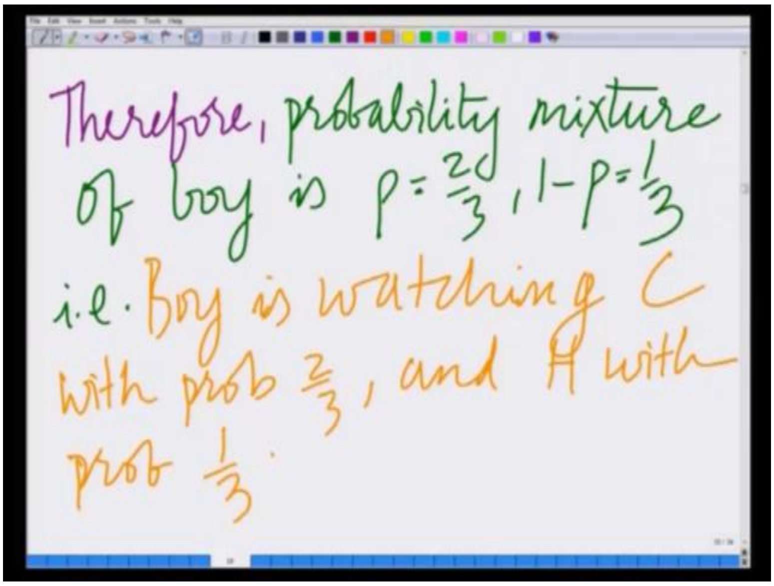

Therefore, which means that the boy is watching C with probability 2 by 3 and H with probability 1 by 3? That is fraction two third of the time he is choosing to watch the game of Cricket and fraction one third of the time he is compromising to watch the Harry Potter movie this is, the compromise strategy. That is not choosing one strategy always, but he is randomly choosing one or the other strategy. That is two third of the time with probability two by three he is choosing Cricket and the probability one by three is choosing to watch the Harry Potter movie and this is what we mean by a compromise strategy. Or, this is also a mixed strategy where is randomly choosing one or the other strategy.

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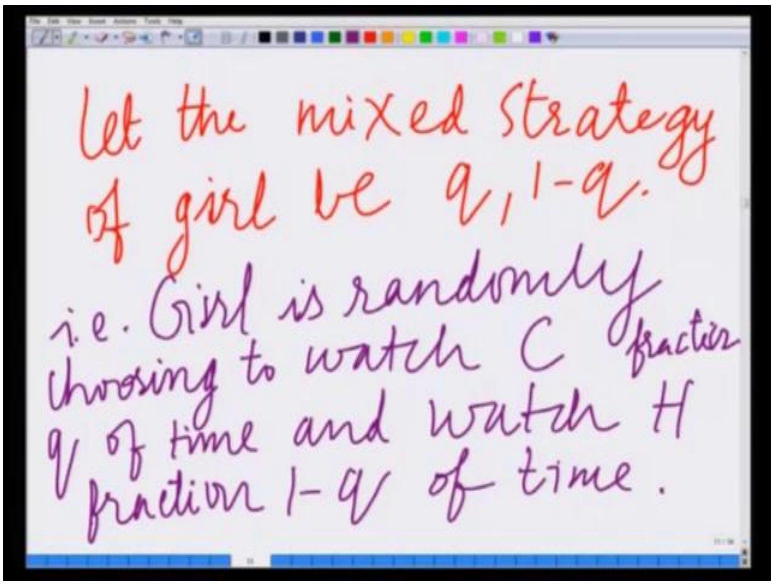

Now, similarly we also said that the girl is employing a mixed strategy. So, let us now try to find the mixture of the girl as similar to the match pennies game let us assume that the girl is mixing with probabilities q and 1 – q. Let the mixed strategy of girl that is q, 1 - q. Right that is girl is randomly choosing to watch Cricket fraction q of the time and watch the Harry Potter movie fraction 1 - q of time. Girl is randomly choosing to watch C fraction q of time and watch H fraction 1 - q of time right. So, the girl is using the mixed strategy q, 1 - q which means randomly fraction q of the time she is choosing to watch the game of Cricket and fraction 1 - q of times she is using to watch the Harry Potter movie.

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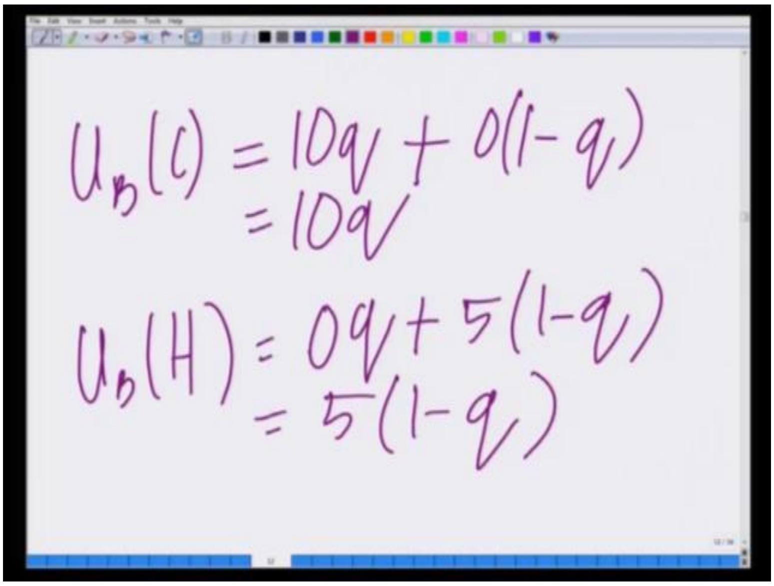

Now, let us look at the payoff at the boy again by now we should be very familiar with this technique. So, I will go a slightly faster if the boy always chooses to watch Cricket then the fraction q encounters the girl watching Cricket in which you get 10 with fraction 1 - q he gets zero because he encounters the girl watching the Harry potter movie. So, payoff of the boy to always watching Cricket can be obtained as 10 times q + 0 times 1 - q equals 10 times q. Similarly, if the boy is choosing to always watch the Harry potter movie if fraction q encounters the girl watching cricket in which case he gets 0. So, it is 0 times q and it fraction 1 - q encounters the girl watching Harry Potter movie in which case he gets 5. So, it is 5 times 1 - q therefore, the average payoff to always watching the Harry Potter movie equals 0 times q + 5 times 1 - q which is equal to 5 times 1 - q.

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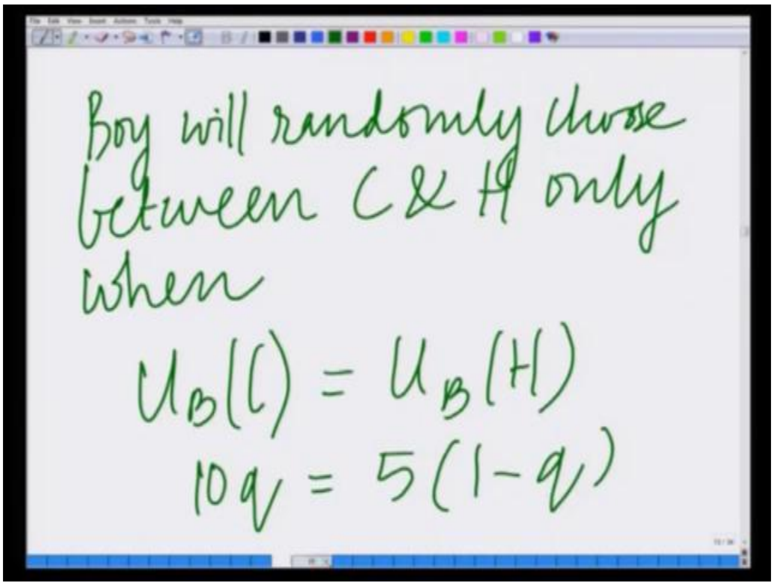

And again, he will randomly choose between C and H only when the payoffs are equal. Boy will randomly choose only when U B of C equals U B of H which means 10q equals 5 times 1- q.

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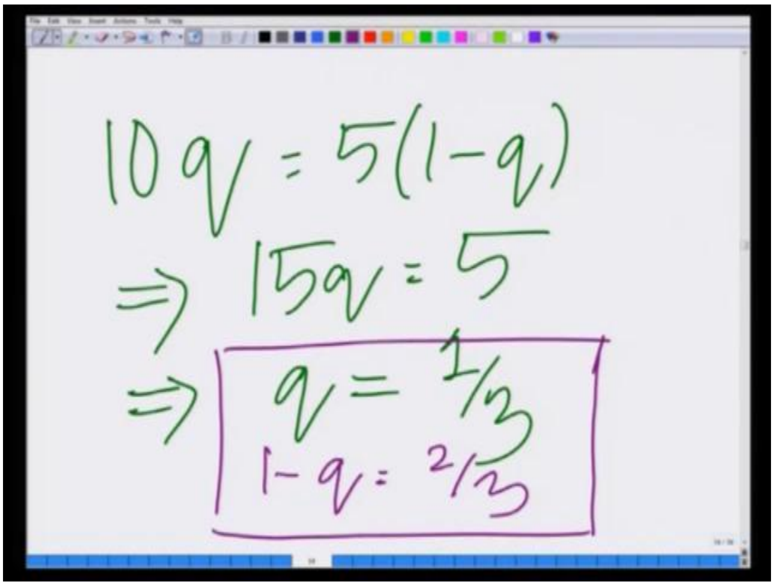

So, we have the condition 10 q equals 5 times 1 - q which implies that 15q is equal to 5 which imply that q is equal to 1 by 3 and 1 - q is equal to 2 by 3. Therefore, we have obtained the mixture of the girl. The, mixture of the girl is 3 equal to 1 by 3 1 - q equal to 2 by 3.

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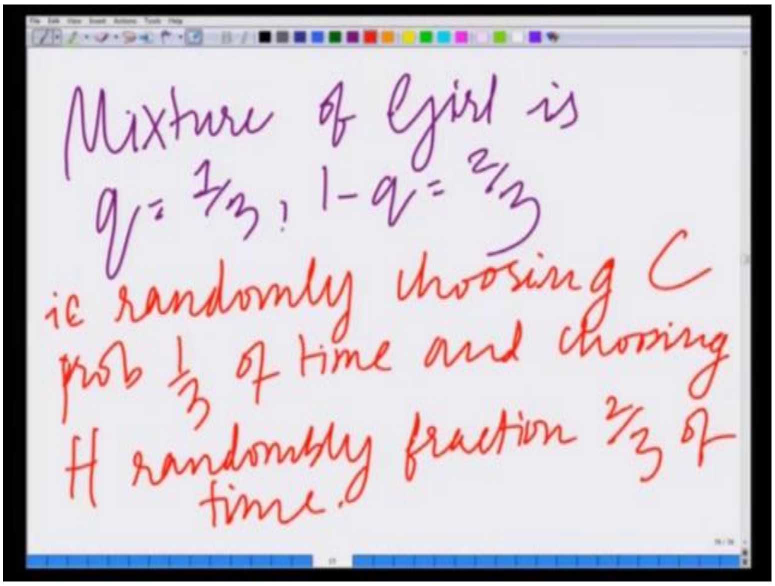

Mixture of girl where q equal to 1 by 3 that is randomly choosing C probability one third of time and choosing H randomly fraction two third of time therefore, we are saying that the mixture of the girl is to randomly choose to watch Cricket one third of the time at choose to watch the Harry potter movie two third of the time and therefore, we have derived mixed strategies of both the players at Nash equilibrium.

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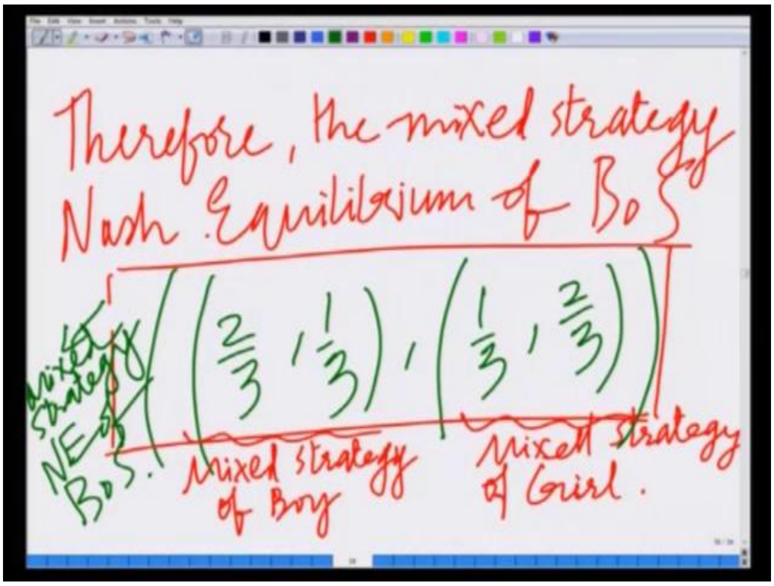

Therefore mixed strategy Nash equilibrium of BOS or the battle of the sexes game is the mixed strategy Nash equilibrium. Remember, we derived the mixture of the boy is 2 by 3, 1 by 3 and mixture of the girl is 1 by 3, 2 by 3. So, we have mixture 2 by 3, 1 by 3 mixed strategy 1 by 3, 2 by 3is the mixed strategy of the girl. This is the mixed strategy Nash equilibrium right. In this mixed strategy Nash equilibrium the boy is using the mixture 2 by 3 , 1 by 3 that is randomly fraction two third of the time he is watching Cricket and one third the time he is choosing the Horry Potter movie. And, on the other hand the girl is randomly choosing to watch the game of Cricket one third of the time and watch the Harry Potter movie two third of the time. And therefore, both of them are using randomize or mixed strategies and this is the mixed strategy Nash equilibrium of the battle of sexes game. So, let us stop this module here and let us continue with this in the next module.

Thank you.