Strategy: An Introduction to Game Theory

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Lecture - 10

Hello, welcome to another module in this online course Strategy, An Introduction to Game Theory. And today let us start looking at yet another different game, which is a game a market game between two different firms or two different companies producing two different goods in a market. This game is known as standard game and a very popular and a very established game known as the Cournot Duopoly.

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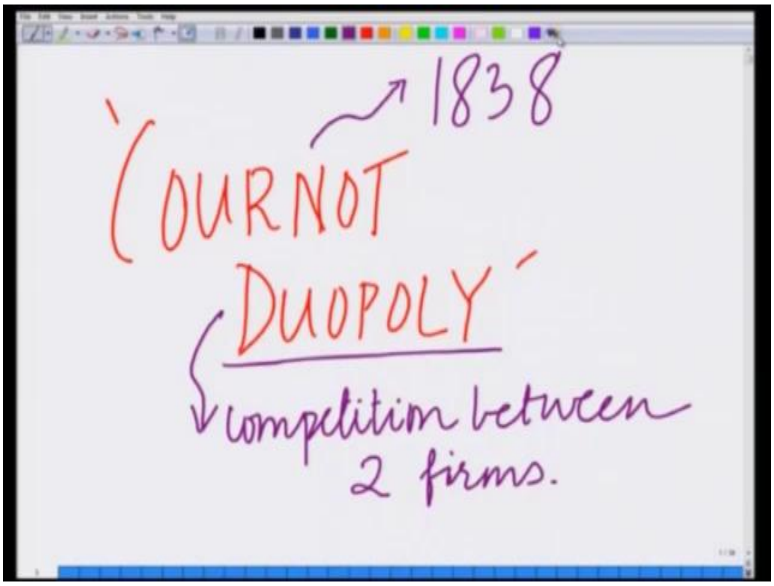

And as we have seen, we have seen various different examples of scenarios or real life scenarios that can be modeled as games. This game which is the Cournot duopoly, as the name implies it talks about a duopoly, as we are all familiar about a duopoly. It is a competition between market competitions between two firms or duopoly is a competition, where two firms are controlling an entire market by producing two different two kinds of goods which are different, but yet similar. In this sense, they have similar utility or similar applicability.

And cournot is the name of the economist, who first proposed this game in 1838. So, Cournot is an economist, who proposed this game in 1838 to study this interaction or this interaction in a duopoly when there are two firms competing to produce a good. So, the scenario that we are going to consider is a market game.

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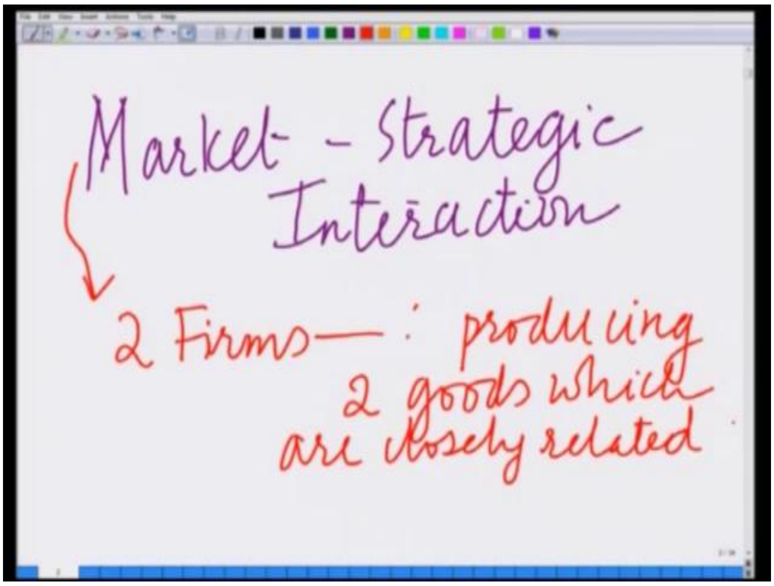

So, we are going to look at a market game or a strategic interaction market that is the strategic interaction of market based, a market based strategic interaction. A competition between two firms, which are producing two goods which are closely related they are producing, such that the consumers can substitute one good for the other that is the consumer's do not have the strong preference for one versus the other.

For instance, consider an example of two different soft drinks available in a market which are closely related or different soaps available in the market, which where consumer's do not have a strong preference of one soap for the other. And such goods which can be almost readily substituted for the other are also known as strategic substitutes.

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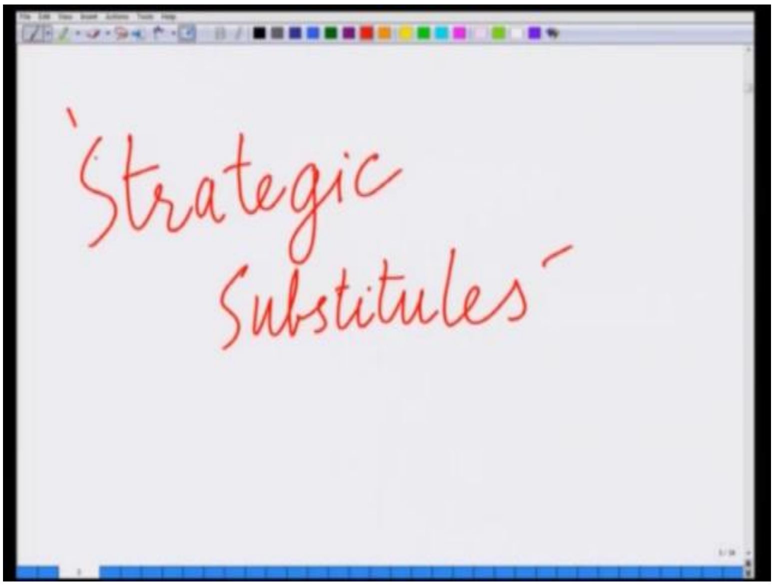

Such goods which are strategic substitutes, strategy substituted two goods, which are closely related to the other in a applicability and functionality. So, that the consumers do not have the strong preference for one versus the other, can more or less readily substitute one for the other, such that two popular soft drinks or popular soaps in a market and so on. So, these are often used to model the fast moving consumer goods, where typically people do not have a strong preference for one versus the other.

And in Cournot game, we are looking at a competition between two different firms in a market, this is duopoly. So, we are considering two different firms in the market which are producing such goods which are basically strategic substitute and what is the interaction between these two different firms, which are competing in the market place to sell the good or to capture the market and sell this goods to the consumers.

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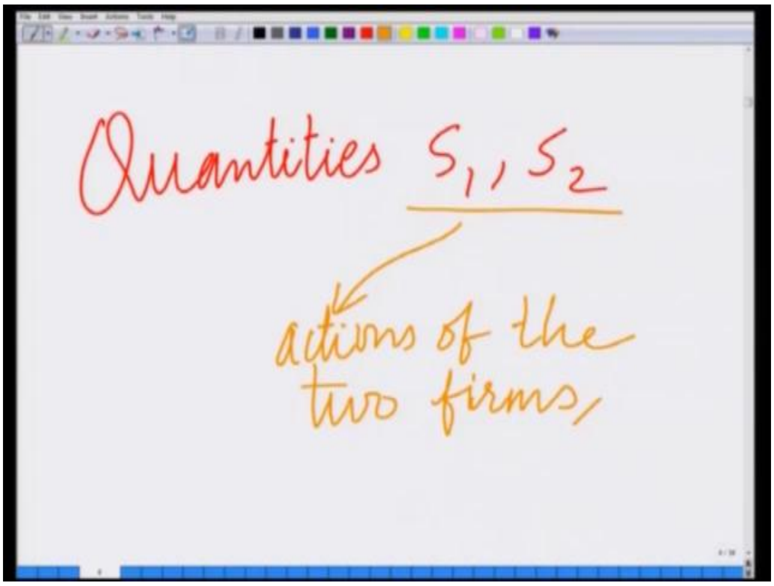

So, let us look at this game we have two firms F 1 and F 2 which are let say producing quantities, S 1 comma S 2 of these strategic substitutes. So, what are the actions? So, these S 1 and S 2 these are the actions or these are the strategies or what is, that is the strategy of each firm is to determine what is the quantity of the good produce, S 1 is the action of firm one, S 2 is action of firm two. So, these are the actions of the two firms that is, to determine how much quantity to produce of the good.

And also again, as we have seen previously in the strategy common example, again these quantities need not be restricted to a discrete set of quantities can possibly take an infinite set of values as we are going to see short. So, this is a duopoly and the competition between two firms and the two firms are competing by producing this different quantities of these goods from one is producing S 1, of the good firm two is producing S 2 of the good and these two goods are strategic substitution, in the sense one each can be substituted for the other by the consumer more or less readily.

So, this way there is an interaction, there is a competition between these two firms in the market place to capture more of the market. Not only to capture more of the market, but to increase the profit corresponding to the quantity produced. And, what are the utility functions? Well, the utility function for each firm can be modeled as follows. First, we are going to model the demand function for this good.

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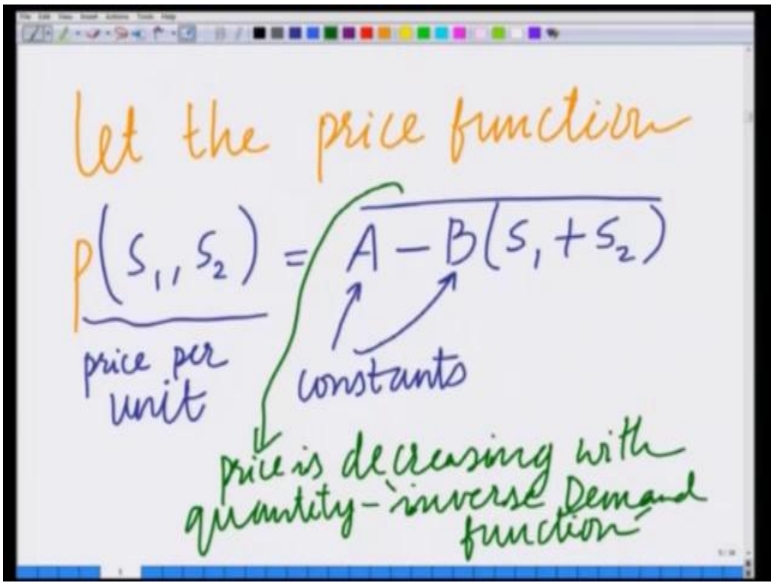

Let the demand function or the, let the price function, which is the price of the good as of function of S 1 comma S 2, this is the price per unit is given as A minus B times S 1 plus S 2. So, what are we saying, the price of the function is or the price per unit of good is A minus B times S 1 and S 2, where this A and B these are positive constants. These depend on the particular good and the market and also, we look at this, the price is decreasing with the quantity.

And this is also termed as an inverse demand function, which is basically the modeling of the price with the Cournot total quantity; that is as the total quantity of the good is increasing, then the price per unit is decreasing. Also, interestingly when the quantity, it only depends on the total quantity S 1 plus S 2 of the good produced; that is as S 1 it does not depend on S 1, I mean as the total quantity S 1 plus S 2 is increasing, the price is decreasing.

This shows that even if S 1 increases or S 2 increases, the price will decrease. Because, as we said these two goods are strategic substitutes, one can be more or less readily substituted for the other and that is reflected in the price function or the inverse demand price function, where the price is decreasing as the total quantity increasing. And more importantly, the price is decreasing is depending only on S 1 plus S 2; that is the total quantity of good 1 and good 2 produced by this two different firms as F 1 and F 2.

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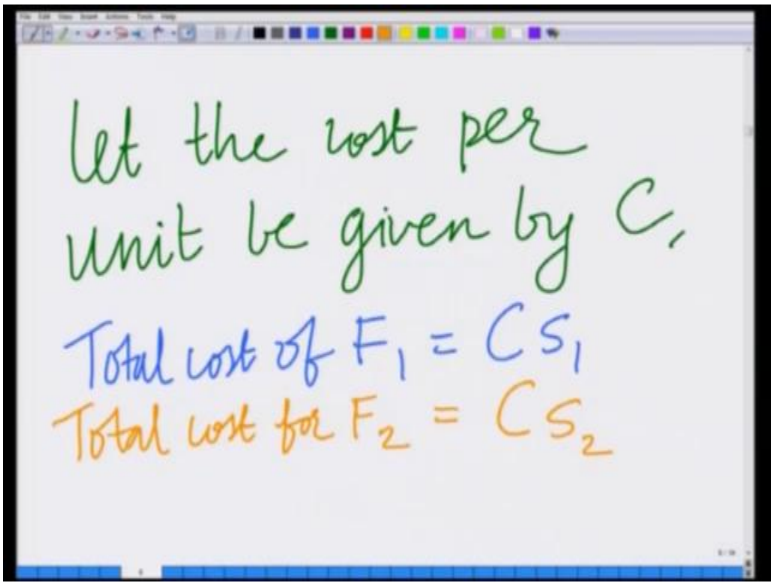

And further, let the cost per unit, let we given by C, which is another cost, let say the cost per unit in this game we given by C. So, the cost per unit produced is given by C. So, the total cost of F 1 and F 2, since F 1 is producing S 1 units, it is C times S 1 since from 2, it producing S 2 units, the total cost is C times S 2. So, cost total cost of F 1 is C times S 1, total cost for F 2 equals C times S 2. It is the cost per unit is C, total cost and for 1 is producing quantity S 1, total cost is C times S 1. For 2 it is producing quantity is 2. So, the total cost is C times S 2 and therefore, now the payoff for each firm can be obtained as follows.

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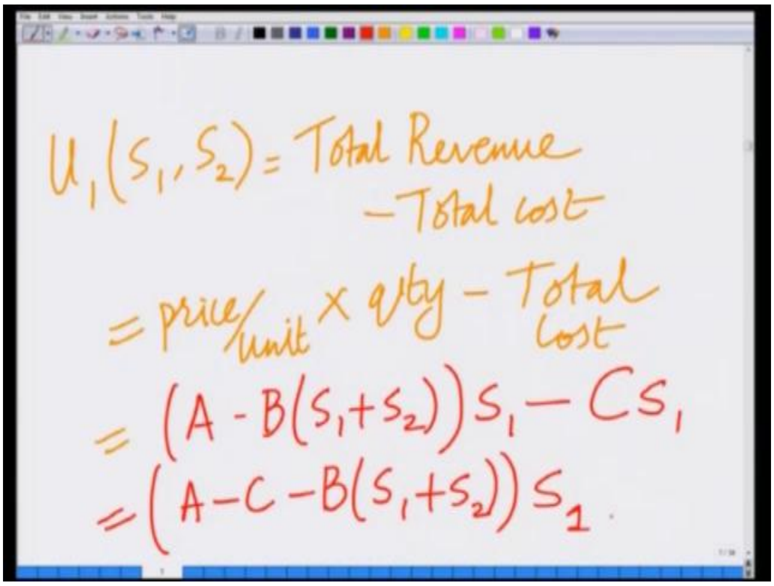

U 1 of S 1 comma S 2 equals total revenue minus total cost. Total revenue is the price per unit times quantity or the number of units minus the total cost. We have already seen ((Refer Time: 10:59)) that the price per unit is A minus B times S 1 plus S 2. So, this is A minus B times S 1 plus S 2 into the quantity S 1 minus the total cost, which is C times S 1. So, the total profit performed 1 is the price per unit times the quantity minus the total cost, which is equal to A minus B times S 1 plus S 2 times the quantity S 1 minus C S 1. This can be further simplified as A minus C minus B times S 1 plus S 2 times S 1.

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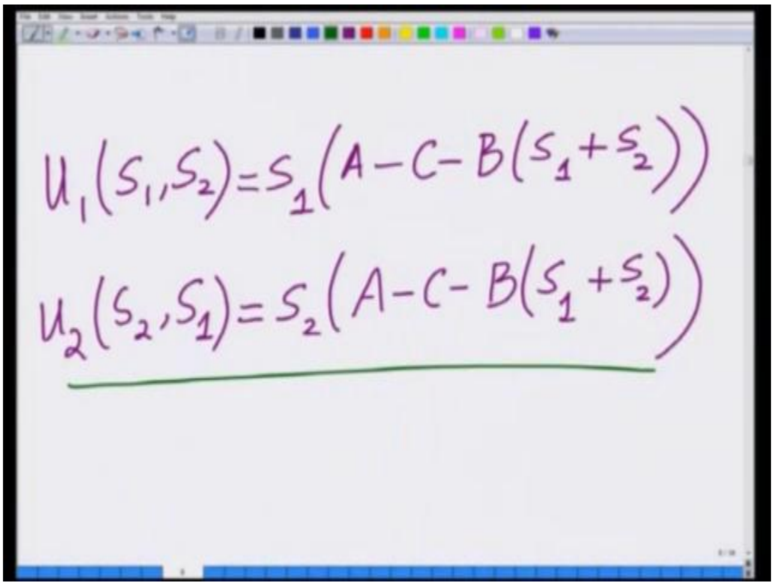

So, to again write this clearly, we have u 1 of S 1 comma S 2, which is a total profit of firm 1 equals S 1 into A minus C minus B times S 1 plus S 2. And similarly, now if we look at the payoff of firm 2, I can write similarly derive, since this is game is symmetric, this is S 2 times A minus C minus B times S 1 plus S 2. And therefore, now you can see this is the game, this is the strategic interaction between the firms F 1 and F 2.

Because, the profit depends not only on the quantity, for instance the profit of firm 1 depends not only on the quantity S 1 produced by it, but also depends on the quantity S 2 produced by firm 2. Similarly, the profit of firm 2 depends not only of the quantity S 2 produced by it, but also in the quantity S 1 produced by it is competitor.

Therefore, we have these two different utility functions and now, we have firm related, this in terms of the game for strategic interaction or in fact, this game is known as a duopoly at this as we have already said, this game was the Cournot duopoly, it was proposed by the French economist mathematician Cournot. And in this game, we are going to capture, what is the behavior of these two firms, what are the quantities produced by these two firms.

When, they are competing in this market by producing this good, which as been said is the strategic substitute, which are more or less different brand name, but more or less closely related in utility. Therefore, now how do we analyze this game, now we have the two different actions S 1 and S 2 and now, how do we analyze this game. Again, we have set the quantities produced can be continues, they need not be a finite set of quantities.

So, we are considering infinite possible set of quantities; that is the continuous that is set of quantity S 1 and S 2 can belong, can be real numbers belonging to a continuous interval. So, we cannot draw the game table and as we have seen previously in the example of the strategy of commons, we can now use differential calculus. The first find the best response and then find the Nash equilibrium.

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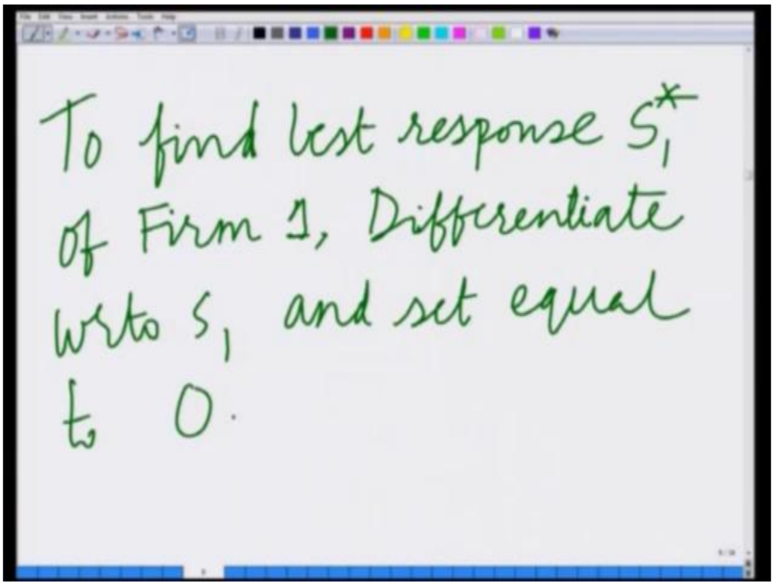

And therefore, to find the best response of firm 1, to find best response S 1 star of firm 1 differentiate with respect to S 1 and set equal to 0. That is how remember, we found the best response of each agent or each player in the strategy of common. Because, these are continuous functions differential functions with respect to S 1 and S 2. To find the best response S 1, I can differentiate it with respect to S 1 and set it equal to 0.

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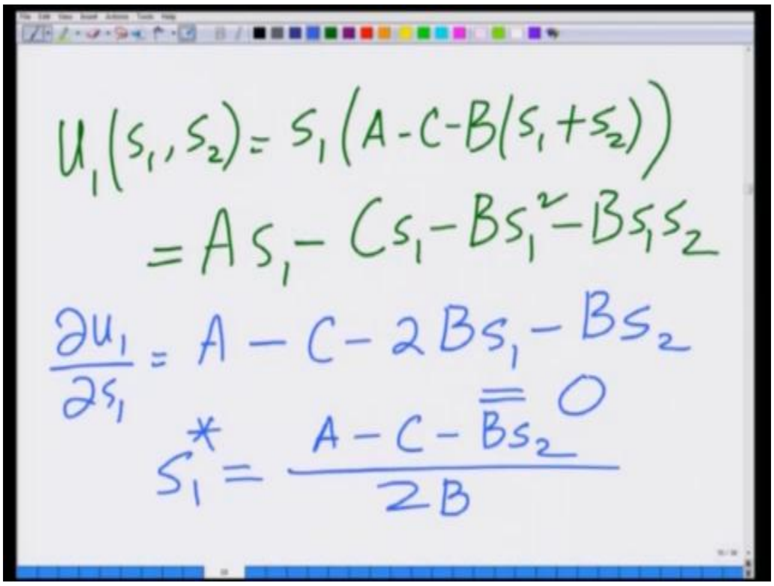

We have therefore, u 1 of S 1 comma S 2 equals, well S 1 into A minus B or A minus C minus B into S 1 plus S 2, which is equal to A times S 1 minus C times S 1 minus B times S 1 square minus B times S 1 S 2. Therefore, now if I differentiate this with respect to S 1 I have duo u 1 by duo S 1 equals A minus C minus 2 B S 1 minus B S 2 which I have equal to 0 and therefore, I obtained the best response S 1 star equals A minus C minus B S 2 divided by 2 B.

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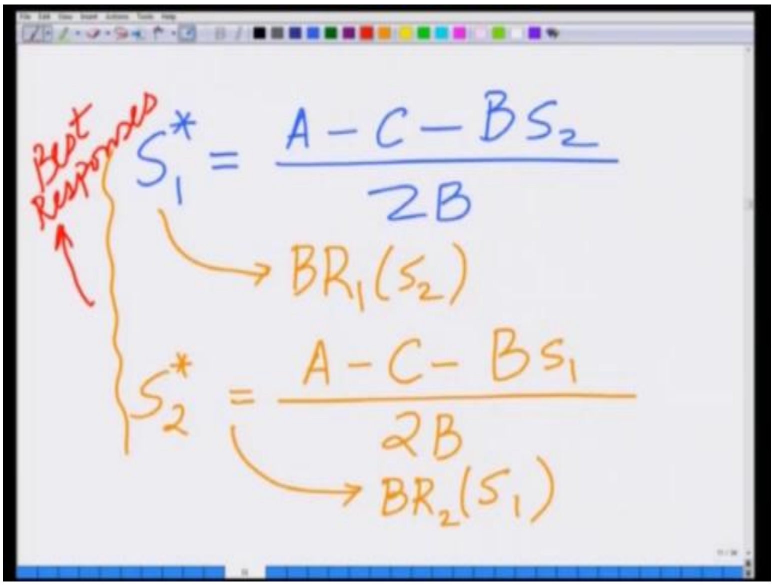

Therefore, I have the best response given as best response S 1 star given as S 1 star let me write this time again clearly S 1 star equals A minus C minus B S 2 divided by 2 B. And in fact, this is S 1 star equals B R 1 S 2 this is S 1 star which is the best response of firm one or which is the best response quantity of firm one to be produced in response to the quantity is S 2 produced by firm two and how did we find it, we find it found it by differentiating the payoff function and equality to 0 find that S 1 star were the payoff is maximized and therefore, this is indeed the best response.

Now, again using symmetry the best response S 2 star of firm two is given as S 2 star equals A minus B minus S 1 divided by 2 B, this is the best response 2 to the quantity S 1 produced by firm one. And therefore, now what I have be done, we have can successfully characterize the best responses for both these functions. And now once I have to characterize the best responses we can find the Nash equilibrium now, because we know that the Nash equilibrium is where these best responses intersect.

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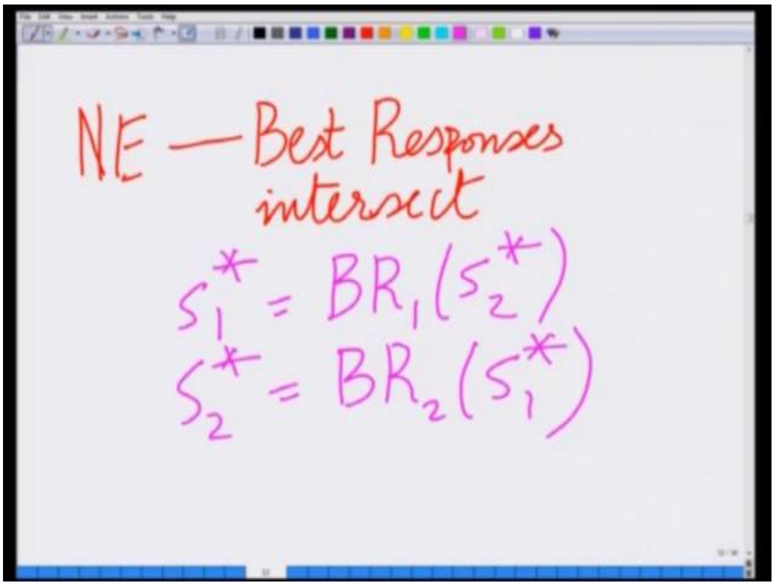

Therefore, the Nash equilibrium any is where the in best responses intersect that is S 1 star is best response one of S 2 star and S 2 star equals best response two of S 1 star therefore, they have each player playing his best response and that characterizes the Nash equilibrium.

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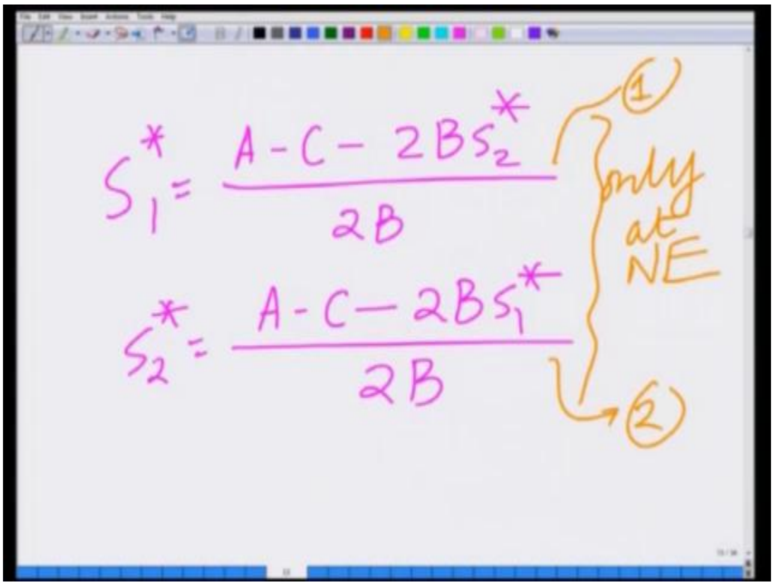

And therefore, now I can solve I can write down the system of equations to solve the Nash equilibrium as S 1 star equals A minus C minus 2 B S 2 star divided by 2 B and S 2 star equals A minus C minus 2 B S 1 star divided by 2 B this holds to only at any since each his playing the best response, since each is playing the best response. Now, again if I call this as equation number 2 and if I call this as question number 1 I can substitute the expression for S 2 star from question number 2 in equation number 1.

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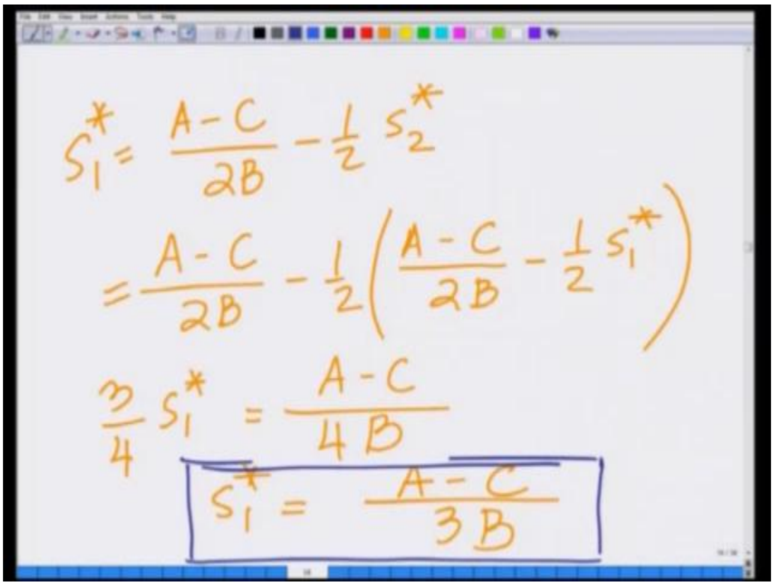

I have S 1 star equals A minus C divided by 2 B minus half S 2 star an substituting for S 2 star I have A minus C by 2 B minus half of A minus C divide 2 B minus half of S 1 star and solving this I have well 3 by 4 S 1 star equals A minus C divided by 4 B which implies S 1 star equals A minus C divide by 3 b. Therefore, I can the Nash equilibrium quantity S 1 star equals A minus C by 3 B. So, I am in this two equations I get S 1 star equals A minus C by 3 B.

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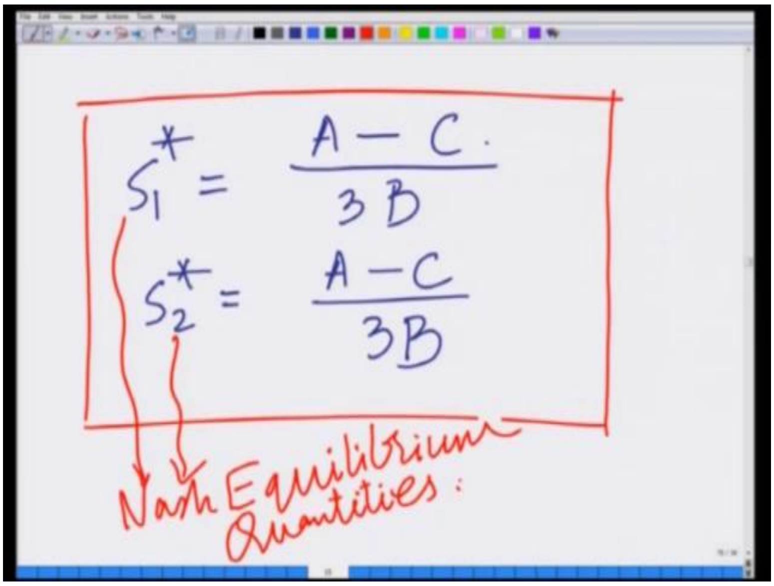

And also by symmetry, you can again see that is 2 star the quantities S 1 star and therefore, these are the Nash equilibrium quantities S 1 star S 2 star these are the Nash equilibrium quantity.

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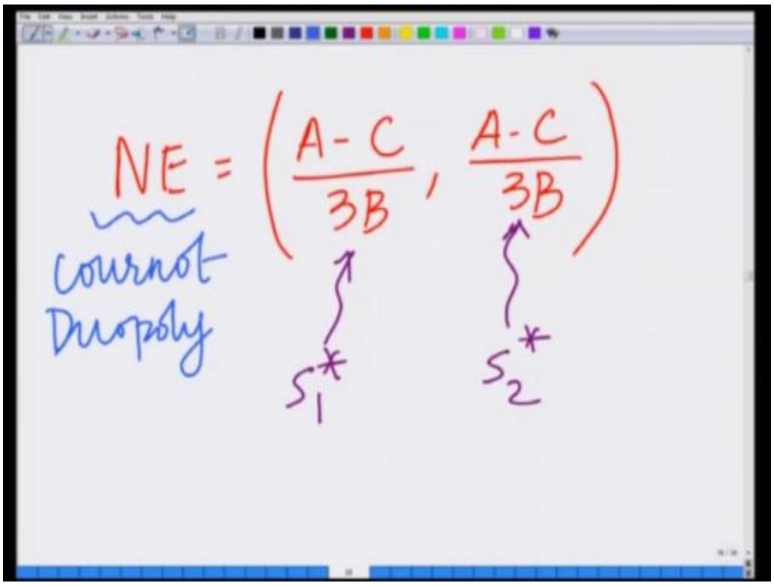

So, the Nash equilibrium is A minus C by 3 B comma A minus C by 3 B if this is the quantity S 1 star produced by firm one and this is the quantity S 2 star produced by firm two in equilibrium and this Cournot openly. So, this is the Nash equilibrium of the Cournot duopoly, now let us try to represent this graphically you to understand this better.

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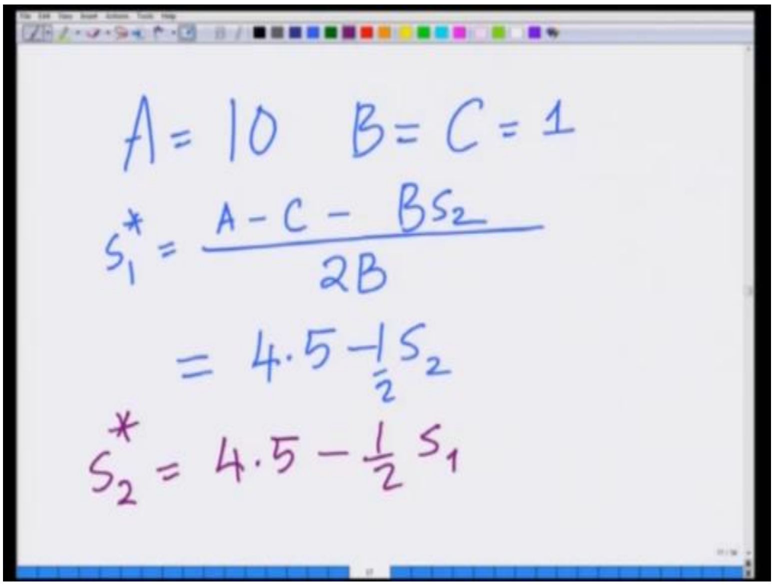

Let us take typical values let say A equal S 10, B equals C equals let say 1 then what we have is we have S 1 star equals A minus C minus B S 2 divided by 2 B which gives me basically 4.5 minus half S 2. So, I can write S 1 star equals 4.5 minus half S 2 taking these values equal to 10 B equal to C equal to 1. Similarly, S 2 star equals 4.5 minus half S 1.

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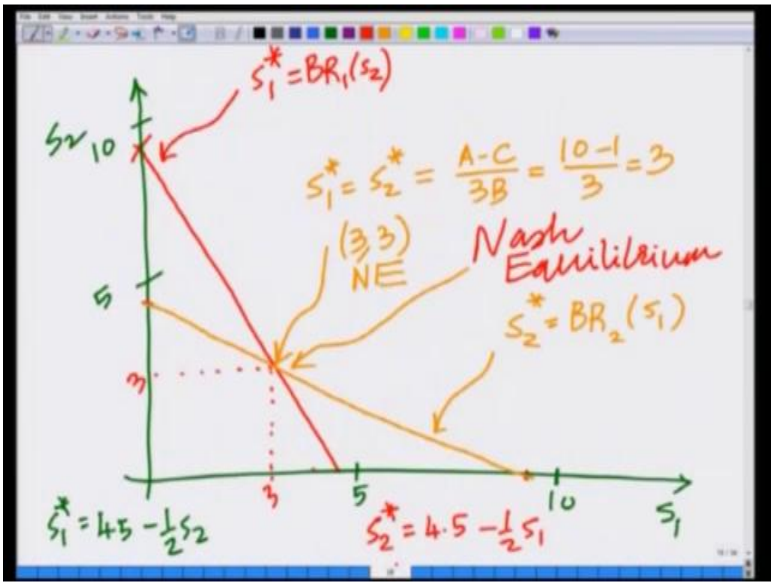

Let me try to draw it graphically on the x axis I am representing the quantity S 1 and the y axis I am representing the quantity S 2. So, this is 10 let say I am trying to the Nash accurate as possible of course, I am not using a scale. So, I cannot be very accurate in these measurements I am trying to approximately mark this things. Now, you can look at in the x axis I have S 1 and the y axis I have S 2 and S 1 star equals 4.5 minus half S 2.

So, S 1 star let me write it here S 1 star equals 4.5 minus half S 2, so if S 2 is equals to 9 somewhere over here if S 2 equals 9, then we have S 1 star is equal to 0. On the other hand if S 2 is equal to 0 then we have S 1 star is equal to 4.5 which is probably somewhere over the here and let me join these using a line. And therefore, now I have S 1 star this represents S 1 star which is equal to the best response one for any quantity S 2.

So, what I am drawn is the best response S 1 star as a function of S 2, if S 2 equal to 9 the best response S 1 star is equal to 0, if S 2 is equal to 4 point, if S 2 is equal to 0 the best response S 1 star is equal to 4.5. Similarly, we can draw it for S 2 star and that line which is S 2 star remember equals 4.5 minus half S 1, if S 1 is equal to 9 best response S 2 star is 0 and if S 1 is equal to 0 best responses S 2 star is equal to 4.5 that we look something like this.

So, this is the best response S 2 star equals best response 2 of S 1 and these are the best responses of both the firms this is the best response of firm 1 S 1 star is a best response of firm 1, S 2 star is the best response of firm 2 as a function of S 1 and the point where both of these intersect that is S 1 star equal to S 2 star in this case is equal to A minus C by 3 B which is equal to 10 minus 1 by 3 is equal to 3. So, this point which is 3 comma 3 we are saying is the Nash equilibrium.

So, this point is the Nash equilibrium of this game, this point is the Nash equilibrium of this Cournot duopoly games. So, this point we are saying is basically the point that is 3 comma 3 we are saying this is the Nash equilibrium is of this Cournot duopoly game. So, what we have done is we looked at another interesting yet another interesting example of a market competition between two firms, which are producing strategic substitutes.

And we said this is also termed as a duopoly and we are modeled the profit functions of both these firms and these profit functions of these utility functions, we have derived the best responses and finding the place or finding the best response the intersection of these best responses we found the Nash equilibrium, we found the Nash equilibrium quantities of both the Nash equilibrium quantities as we found are A minus C by 3 B A minus C by 3 B the both these forms and when A is equal to 10 and B equal to C equal to 1 these Nash equilibrium this 3 comma 3. So, at this point let us stop and we will continue our discussion of to study different properties of this Cournot duopoly and the next module.

Thank you.