Economic Growth and Development

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Lecture – 11
Strategies of Economic Development & Growth - VI

Hello and welcome to lecture 11 of the NPTEL MOOCs course on Economic Growth and Development. In today's lecture we will be studying the basic Solow model. And one of the characteristic features of a Solow model is that, it has made one of the biggest contributions to the neo classical theories of economic growth. And, to the understanding of why the rich countries are so and how the poor countries can catch up with richer countries.

Speaking at the annual meeting of the American economic association in 1989 economic historian David Lands choose as the title of his address the fundamental question of economic growth and development; that why are we so rich and they so poor. And this question has preoccupied the minds of economists for a very long period of time. The modern examination of the growth question by macroeconomists dates back to the 1950's and the publication of two famous papers: by Professor Robert Solow of the Massachusetts Institute of Technology. A Solow's theory is basically helped clarify the role of physical capital accumulation and the role of technology, as the ultimate driving force behind sustained economic growth.

Now, during the 1960's and to a lesser extent during the 1970's work on economic growth flourished. For methodological reasons however, important aspects of theoretical exploration of technological expansion or technology progress were postponed. In the early 1980s work at the University of Chicago by professors Paul Romer and Robert Lucas reignited the interest of a macroeconomics or macroeconomists in economic growth; emphasizing on the economics of ideas and human capital or human capital formation.

Taking advantage of the theory of imperfect competition Romer introduced the economics of technology to macroeconomists. And, following these theoretical advances various empirical exercises were carried out by economists such as Robert Barro of Harvard University; who quantified and tested theories of economic growth. And since, then both theoretical and empirical work on economic growth has increased, and has been sustained with a lot of professional interest.

Now, one of the things that we have been discussing in the last few classes is with respect to the different shapes and sizes of different economies of the world. So, world consists of economies of all shapes and sizes. And, we know that some countries are very rich some countries are very poor, and some countries are caught within these extremes of rich and poor. And, based on empirical studies of various countries, various some stylized facts with respect to growth and development have been worked on.

Before we move on with the basic Solow model, it makes sense to start with these certain stylized facts of growth and development. Why I call them stylized is that, or why economists call them stylized is that, these are all facts which are backed by empirical data which is supported by empirical data and on an average stands true for various countries across the world. Now, look at some of stylized facts, and these it is important that we keep in keep this in mind when studying growth models in general.

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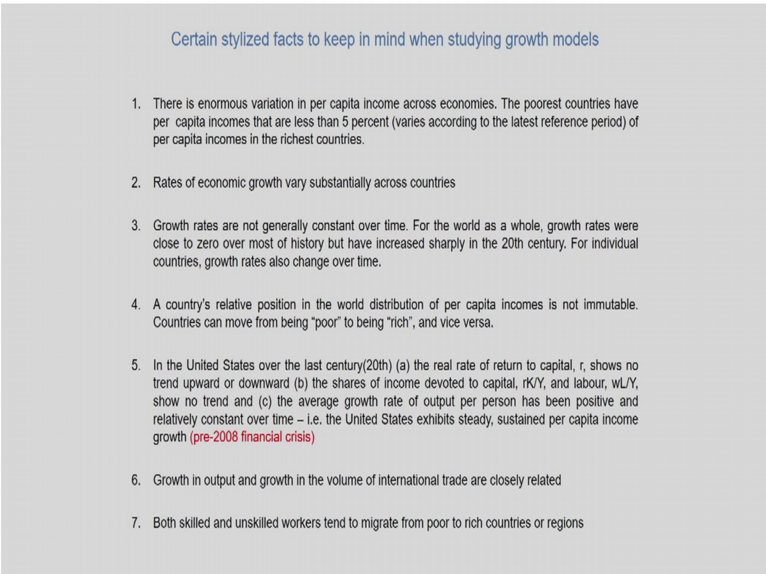

One of the first stylized fact is that there is enormous variation in per capita income across economies. The poorest countries have per capita incomes that are about less than 5 percent of per capita incomes in the richest countries. And the 5 percent estimate here refers largely to the period of the 20th century. And this variation in income has been seen largely in the 20th century. And, the stylized facts that we see with respect to the growth models are largely based up on the empirical data of the 20th century.

The second fact is that rates of economic growth vary substantially across countries. And this is something that is implicit in fact number 1. The third is that growth rates are generally not constant over time, for the world as a whole growth rates were close to 0 over most of history, but have increased sharply in the 20th century. For individual countries growth rates also change over time. And this is evident from the discussion that we have had in the earlier classes with respect to Simon Kuznets empirical studies on growth or growth of countries or income growth of countries over a period of time, which based upon historical data he concluded that the industrialized countries of the world seems to have grown at a very rapid pace largely in the 20th century compared to the historical growth rates.

The 4th stylized fact is that a country's relative position in the world distribution of per capita incomes is not immutable; which means the countries that are poor today may become rich tomorrow and vice a versa. So, there is nothing immutable about the distribution of per capita incomes and the relative position of the country's in world distribution of incomes.

The 5th stylized fact relates to the so called most powerful country of the world today the United States. And, the growth story of United States over the last 20th century has been highly celebrated, it has been highly cited. However, there are certain characteristic features of this growth story that has emerged based upon empirical analysis. And these characteristic features largely refer to the pre 2008 financial crisis period and they are as follows.

First that the real rate of return to capital are shows no trend upward or downward. In other words, the real rate of return to capital does not seem to be having any trend. The second is that the shares of income devoted to capital r into K by y, and labour w into L by Y show no trend. In other words, we do not know whether the share of income devoted to capital or labour is rising or declining. And the third the most important is that the average growth rate of output per person has been positive and relatively constant overtime; which means that United States exhibits steady sustained per capita income growth. And this is an important finding relating to the developed countries of the world.

The 6th stylized fact is that growth in output and growth in the volume of international trade are closely related. And, if you recall our discussion regarding the structural features characteristics of the developed and the underdeveloped countries, you would see that you would remember that the international the mix of commodities which are internationally traded is one of the important characteristic of developed countries, the developed countries seem to have a mix of commodities which is which has a very high incidence of manufactured goods.

And, one of a stylized facts based upon empirical data also tells us that, growth in output and growth in volume of international trade are closely related; which means that, higher the growth in international trade higher the growth of output, and higher the growth of income or economic growth of a country.

The 7th stylized fact is that both skilled and unskilled workers tend to migrate from poor to rich countries or regions. There is labour mobility between the countries or regions. And it is mostly the migration is usually seen from the poorer countries to the richer countries or richer regions of the world. So, these stylized facts that we have just seen generates a lot of interest in growth theories with respect to economic policies. And a surveying these kinds of evidence with respect to economic growth in the 1985 Marshall lecture at Cambridge University, Robert Lucas junior expressed the sentiment that well research on economic growth for the decade of a 1990’s.

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“ I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action government of India could take that would lead the Indian economy to grow like Indonesia’s or Egypt’s? If so, what exactly? If not, what is it about the “nature of India” that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else (Lucas, 1988, “On the Mechanics of Economic Development”, Journal of Monetary Economics 22 (July), p.5).

And he said something like this. He said that, I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so what exactly? And if not what is it about the nature of India that makes it so? The consequences for human welfare involved in questions like these are simply staggering once one starts to think about them it is hard to think about anything else.

So, basically he was trying to say that all of these empirical facts bad stylized facts tell us that, why the different countries across the world have been growing at different pace, what why the rates of growth have been different for different countries across the world. And if all of these growth rates could be mathematically determined based upon what variables could be increased by what proportions, then why is it that in spite of making attempts towards that some countries still lag behind the rest.

For example, the Asian miracle countries like South Korea have grown rapidly over the period of the 1980s whereas, the South Asian countries have lagged behind. Therefore, in this context it is important to understand what is it what are the structural characteristics of certain countries or is it the structural characteristics of certain countries that make them relatively backward then the rest of the countries.

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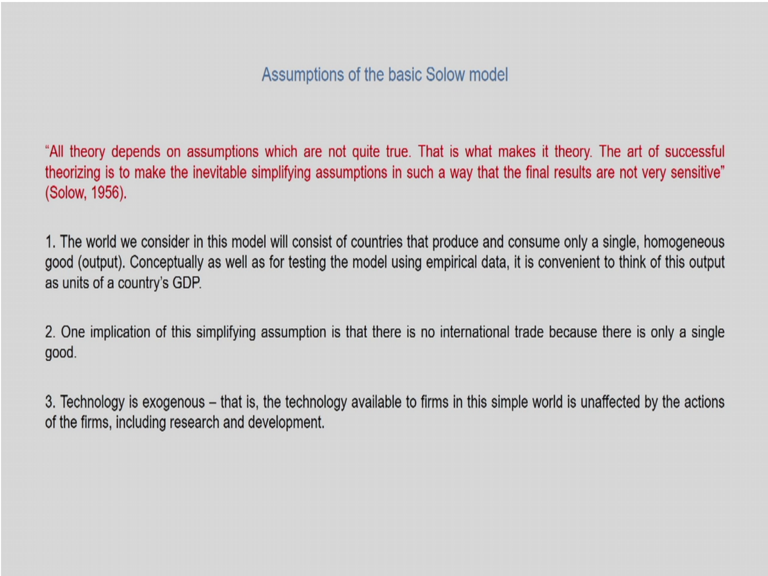

So, in 1956 Robert Solow he published seminal paper on economic growth and development titled a contribution to the theory of economic growth. And for this work and many of his subsequent work after this he was awarded the Nobel Prize in economics in 1987. And in this lecture let us develop the model proposed by Solow, and explore it is ability to explain some of these stylized facts that we have just discussed.

It has actually been claimed that Solow model provides an important cornerstone for understanding why some countries flourish while others are impoverished. Before we proceed, let us understand a few things about an economic model, and since we are dealing with models here there are every model has certain assumptions every economic model has certain heroic assumptions made.

Let us see what Solow himself talked about assumptions. Here he says that in 1956 he wrote that all theory depends on assumptions which are not quite true, and that is what makes it theory. And the art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive. So, a certain assumptions are bound to take place in any model so as to come up with some predictions or implications with respect to the solutions that we are attempting.

Now, a little more about economic models; in modern economics a model is basically a mathematical representation of some aspect of the economy. You can think of such an economy as a toy economies consisting constituting of various kinds of robos. And we specify exactly how these robos behave which is typically to maximize their utility. Now we also specify the constraints the robos faced to be able to maximize their utility. For example, the robos that populate our economy might want to consume as much output as possible, but they are limited by in how much output that they can produce, because of the given levels of technology within the economy or the techniques of production that are at their disposal.

And the best models often are very simple and they convey enormous insights into how the world works. So, for example, consider the supply and demand model of microeconomics frameworks of micro that we generally use in microeconomics. They are very simple frameworks yet they are highly effective in explaining to us, the changes in prices and quantities demanded of such varied goods such as the health healthcare sector or computers or nuclear technology ah; that respond to changes in the changes in the economic environment.

So, with this understanding of how and why economists developed models, let us highlight some of the important assumptions that we need to make with respect to the Solow model in our case. These are some of the important assumptions of the basic Solow model. Let me also make it very clear here that in this lecture we are attempting a basic Solow model. And notice that I am using the term basic Solow model, because Solow in his original paper came up with a very basic understanding of how the economic growth takes place, or how steady state equilibrium takes place in the economy. What is the steady state value of output per worker? What is a steady state value of capital per worker and so on.

However, various extensions and improvements have been made to the basic Solow model to come up, with many sophisticated tools of economic growth models. For example, the Solow swan model which is an extension of the Solow model. Various parameters such as technology have been variables such as technology has been added on to the basic Solow model to tell us how a sustainable the economic growth that has been attempted based upon the basic Solow modelist.

However, in this lecture we are looking at only the basic Solow model, what are the basic equations that are required to formulate the basic Solow model, how does one arrive at the steady state equilibrium value, and what is the implication of the steady state value that we are talking about. So, that is the topic of this class's discussion.

So, let us look at some of the assumptions of the Solow model. The first resumption that we are considering here is that the, this in this model the economy is producing only a single homogeneous good or a composite good. Only one good is being produced. And conceptually if we have to test this model using empirical data, we can very conveniently think of this output as units of a country's GDP. Why because, GDP as we know is a composite indicator which takes which is constructed using data for all goods and services that is produced within the economy. So, and if GDP is a composite indicator, it can be this model can be tested using GDP as an indicator. So, it makes sense to make the assumption that the economy is producing only a single homogeneous good.

And, because the economy is producing only a single homogeneous good, it is implicit in the first assumption that this is a closed model; if there is no international trade, because there is only a single good. Technology is exogenous that is, the technology available to firms in this simple world is unaffected by the actions of the firms including research and a development. It is important that we mention here that much progress in economics has been made by creating a very simple world and then seeing how it behaves and misbehaves. And a common problem in economics is for an individual to decide how much to consume today, and how much to save for what fraction of the income needs to be saved for the future. Another is for individuals to decide how much time to spend going to school accumulating skills, and how much time to spend working in the labour market.

So, instead of writing these problems down formally, we will always assume that individuals save a constant fraction of their income as the savings and spend a constant fraction of their time in accumulating skills. So, these are extremely useful simplifications and without them the models that we use will be difficult to solve because there will be a need for more advanced mathematical techniques. And for various purposes these are very fine assumptions to make, and this will help us run through a very basic model of economic growth. So, these are the assumptions that we can considered as given.

Now, being armed with the assumptions let us now look at the basic Solow model. Now the Solow model is built around 2 equations. One is a production function equation, and another is a capital accumulation equation. We will come to the capital accumulation equation in sometime. Let us spend some time on the production function equation. So, this a function basically describes how inputs various inputs are combined to produce a given level of output.

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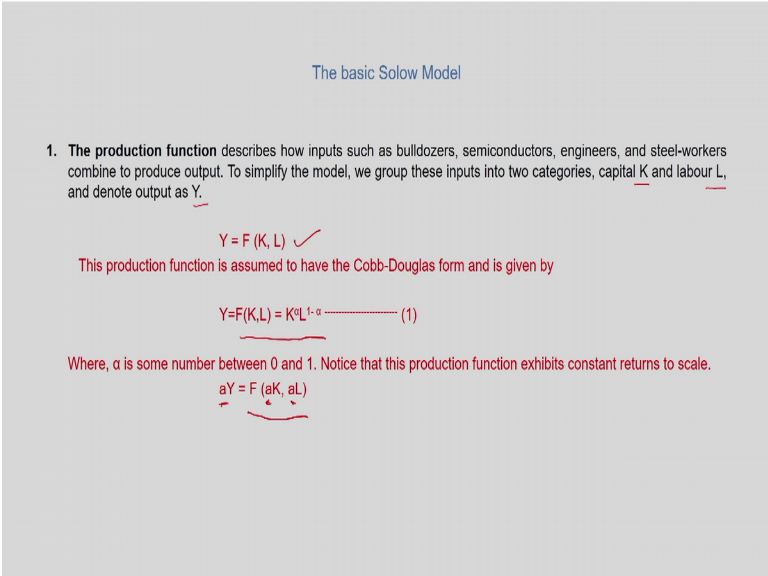

Various inputs say for example, how a bulldozers semiconductors engineers and steel workers are combined to produce output. We usually simplify the production function. So, to simplify the model we group these inputs into 2 categories. Capital denoted by K, labour denoted by L and output denoted as Y. So, here we are showing a function output as a function of capital and labour. And this production function is assumed to have the Cobb Douglas form. And is given by Y is equal to F of K L K alpha L 1 minus alpha. Where alpha is some number between 0 and 1. And you have to note here that this Cobb Douglas production function exhibits constant returns to scale; which basically means that if all the inputs are doubled output will exactly doubled.

So, in this example capital and labour are increased by some amount a and L, and output also changes by the amount a. So, the production function used in the Solow model is of the Cobb Douglas form, and it exhibits a constant returns to scale. Now to take this idea forward we have also assumed that the reduction in this economy is taking place in a perfectly competitive framework.

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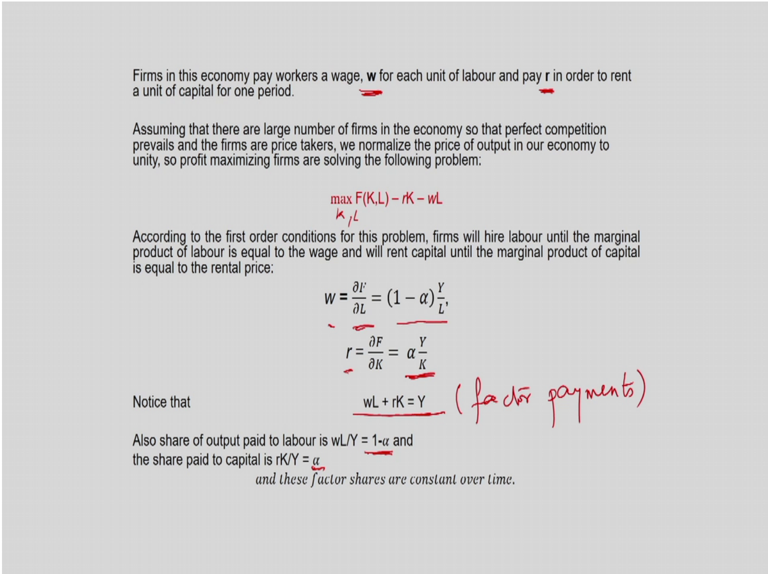

Firms in this economy pay workers a wage which is W for each unit of labour. And they pay the firms pay rent on a unit of capital for one period which is denoted by r. So, this is the rent that is paid on capital, and this is the wage that is w which is paid to the labourers.

Now, we have assumed that there are large numbers of firms in this economy and perfect competition prevails. And therefore, firms are price takers. So, it is safe to assume that we normalize the price of output in an in our economy to unity. And so, what are the firms doing? The firms are basically a solving a profit maximization function here. So, they are solving this following constraint here they are maximizing, how to maximize this equation F of K L minus r K w L given the endowments of capital and labour.

So, according to the first order conditions for this problem, firms will hire labour until the marginal product of labour is equal to wage. And they will rent capital until the marginal product of capital is equal to the rent of to the rental price. So, the partial derivatives of the with respect to labour gives us this wage rate is equal to 1 minus alpha Y L. And the first order derivative partial derivative with respect to capital gives us r which is equal to alpha Y K. This can be written in this form, w L plus r K. So, what is this? This is basically the total output Y of the economy been distributed as wages to labour and rent on capital.

So, which means that the payments to inputs or so you can also say that these are factor payments. So, the payments to inputs or factor payments completely exhaust the value of this output produced. So, that in this economy there are no economic profits to be earned. And this important result is a general property of production functions with constant returns to scale. You can also notice here that the share of output paid to labour is 1 minus alpha, and the share paid to capital is alpha. And these factor shares are constant over time.

And, if you recall the stylized facts that we considered in the beginning of this lecture; you will know that what we are typically interested in explaining is in terms of output per worker or per capita output. So, with this interest in mind we can write the production function in terms of output per worker.

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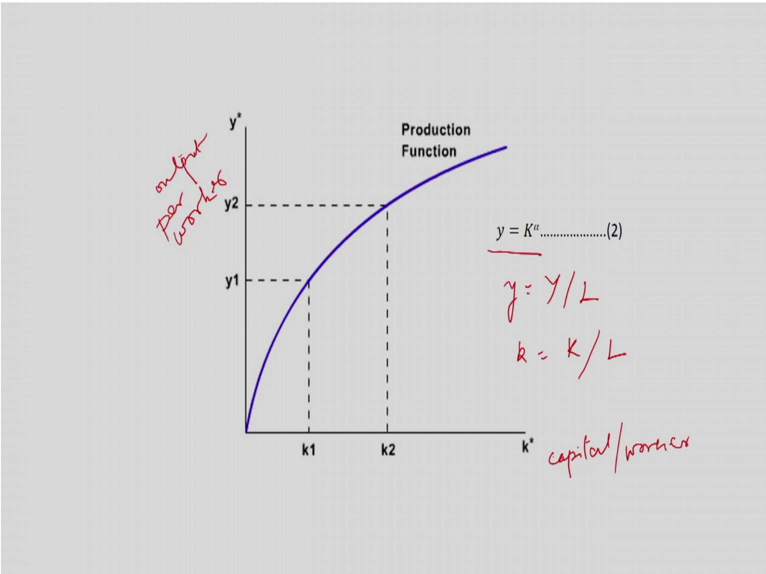

Where y is equal to Y by L, and capital per worker k is equal to K by L. So, this can be plotted in terms of a production function y is equal to K alpha. So, this is the production function this curve is a production function, the x axis shows us capital per worker. This shows capital per worker and the y axis shows us output per worker. And this production function you would see is rising upwards and then become convex to the x axis.

So, what does the shape of the production function tells us is, that with more capital per worker firms are producing more output per worker. However, there are diminishing returns to capital per worker. Because when we give each additional unit of capital we give to a single worker, it increases the output of that worker by less and less. So, which means that the production function exhibits diminishing returns of capital per worker; so, this is the first equation the production function equation of the Solow growth model.

Let us just summarize it once again. So, we began with this production function a simplified production function it takes the Cobb Douglas form exhibiting constant returns to scale. So, the firms are price takers. So, we have normalized price of output in our economy to unity. And the firms are maximizing profits so, they are solving this problem they are maximizing this function, given the endowments of capital and labour. And based on this, we also come to see that the total output in the economy is basically given out as factor payments to labour and capital. So, we get this equation w L plus r K is equal to Y. The share of output paid to labour is 1 minus alpha and the share of output paid to capital is alpha. And these factor shares given the Cobb Douglas production function are constant over time.

And based on that we also we can see what is the general shape of the production function. And the shape of the production function that we are we are considering is that which shows diminishing marginal productivity of capital per worker overtime. So, which means that as with time when a when firms produce more output per worker, there is diminishing returns to capital per worker. So, each additional unit of capital we give to a single worker increases the output of that worker by less and less. So, that is the production function equation.

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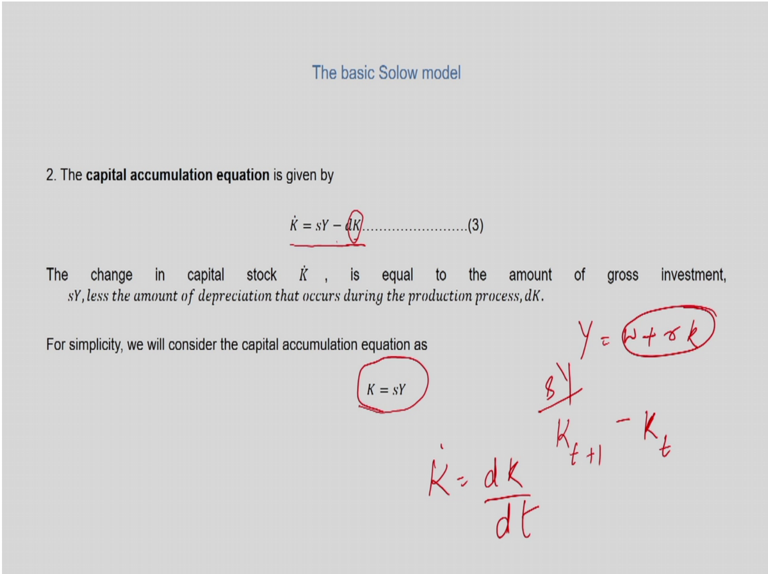

Next we come to the capital accumulation equation, which is a very important equation in the Solow model. And it will also be of interest to know that the Solow model emphasis on capital accumulation as one of the determining factors of economic growth in countries; has also given it the name of a capital accumulation model. The Solow model is also known as a capital accumulation model.

So, the second equation here the second K equation the Solow model is the capital accumulation equation. And this describes how capital accumulates. So, the capital accumulation equation is given by K dot is equal to s Y minus d K. Now so, this equation has 3 terms. The first term a K dot, second term s Y and the third term a d K. So, according to this equation, the change in capital stock K dot is equal to the amount of gross investment s Y minus the amount of depreciation that occurs during the production process which is d K. And this term K dot is basically a continuous time version of capital. So, it is basically saying how much capital has changed over a period of time.

So, let us say we begin with some capital in time t. So, it is the difference in of capital in time t plus 1 minus capital in time t. So, it is basically talking about the change in capital stock per period. And we usually use a dot notation to denote a derivative with respect to time. So, this K dot is telling us how much capital stock has changed over a period of time. And so, this K dot is basically d K by d t it is showing how much capital has changed over a period of time.

The second term s Y as I have already said represents gross investments. And following Solow we have assumed that the workers or the consumers save a fraction of a constant fraction s of their combined wage and rental income, Y is equal to W plus r K. So, we can also say so we know that Y is equal to W plus r K. And some fraction s of this is saved in the form of a is saved by the workers in Solow model. However, it is also important to know that in the Solow model all consumers are also workers. So, they are all working all the time all the and so, whatever is being saved is ultimately invested. So, that is the savings income identity savings investment identity, whatever is being saved is all being invested because all the workers are working all the time the consumers are working all the time. So, the consumers are basically workers they draw a wage.

And it is not hard to understand this assumption again if you recall the structural characteristics of the developed and the underdeveloped countries. One of the features was that the developed countries have a very high worker population or a large numbers of the large share of the population is in the workforce unlike the underdeveloped countries. So, in this model it is being assumed that all consumers are basically workers. So, there is no scope for increased consumption in this economy; whatever is being saved is invested and therefore, there is a lot of capital accumulation that is taking place.

So, this s Y basically shows that a fraction s of income is saved. And the economy is closed as I have just said. So, savings equals investment and the only use of investment in this economy is to accumulate capital. And the consumers then rent this capital to firms for use in production. So, that is the second term gross investments. The third term d K here reflects the depreciation of capital stock that occurs during production and the standard functional form here used here implies that a constant fraction d of the capital stock depreciates every period. Regardless of how much output is produced, often in macroeconomics we assume d is equal to 0.05; which means that 5 percent of the machine and factors used in a model depreciate every years. So, 5 percent is generally taken as an average of depreciation regardless of how much output is produced in an economy.

Now, first because we are developing the Solow model here for simplicity we will we can also rewrite this equation as K is equal to s Y by neglecting the role of for depreciation here just to keep the simple. However, we can continue with depreciation in the equation and then move ahead. Now, this is the second key equation of the Solow model, but we need to move ahead of this and understand how the evolution of output per person in the economy takes place. So, we rewrite the capital accumulation equation in terms of capital per person.

So, here we are talking about capital accumulation only in terms of the investment made. There is no scope for including population growth here. So, the capital accumulation will; obviously, change when we add population to this equation. So, this equation needs to be rewritten in terms of capital per person. What will happen then? Then what will happen is that the production function will tell us the amount of output per person produced for whatever capital stock per person is present in the economy. So, for that we first need to consider the growth rate of labour force in the economy and population growth is considered exogenously in this model.

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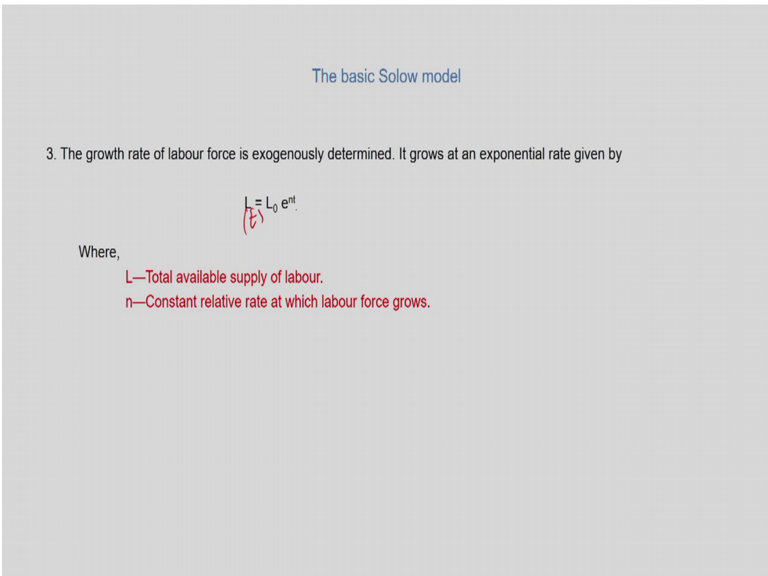

So, an important assumption here is that labour force participation rate is constant, and the population growth rate is given by the parameter n. So, if n is equal to a 0.01, then population and labour force are growing at one percent per year. And this is shown in the form of an exponential growth and this exponential growth can be seen in this a relationship. So, this is growth of population in time t given by L 0 e n t. So, population growing between the time period 0 to time t.

Now, so, this is the population growth equation. Now we need to combine all of these equations. So, we have seen a production function equation, we have seen a capital accumulation equation. And we have seen the population growth rate in the economy. Now we need to combine all of this equation to come up with the capital accumulation per worker in this economy.

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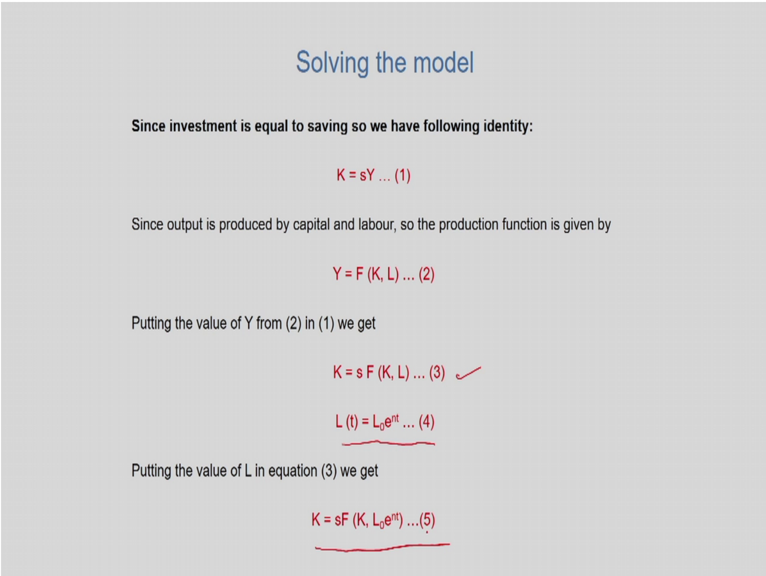

So, to be able to do that let us first bring all the equations on one slide. So, the first equation K is equal to s Y, we have removed depreciation from this equation; however, depreciation can very well be a part of this. This has been done just to keep this derivation simple here in this lecture. The second is the production function where output is a function of capital and labour.

So, if we substitute 2 in 1, we get this functional form where K is equal to s F of K L. We already know that the population growth rate takes this functional form. So, if we substitute the value of 4 in 3 we get the functional form which reads like this K is equal to s F K L 0 e n t.

So, this is capital accumulation which is accounting for gross investments as well as population growth rate. So, this is capital accumulation per worker; however, we have to now differentiate this equation with respect to time; to be able to see how overtime, how does the capital accumulation per worker equation reads like. So, for this we need to differentiate with respect to time.

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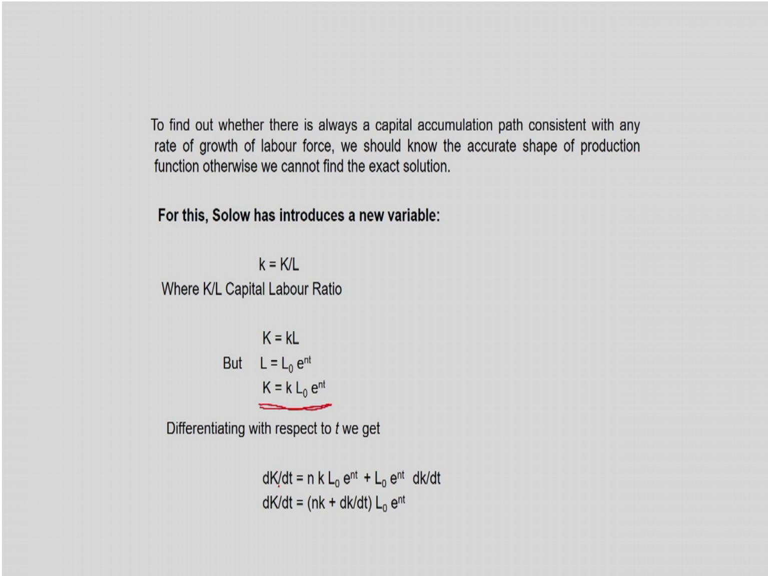

And here Solow introduces a new variable which is basically the capital labour ratio. K is equal to small k is equal to K by L. So, if we rewrite this equation, it would read like this K is equal to so, capital stock is equal to k into L 0 e n t. So, we differentiate this equation with respect to time.

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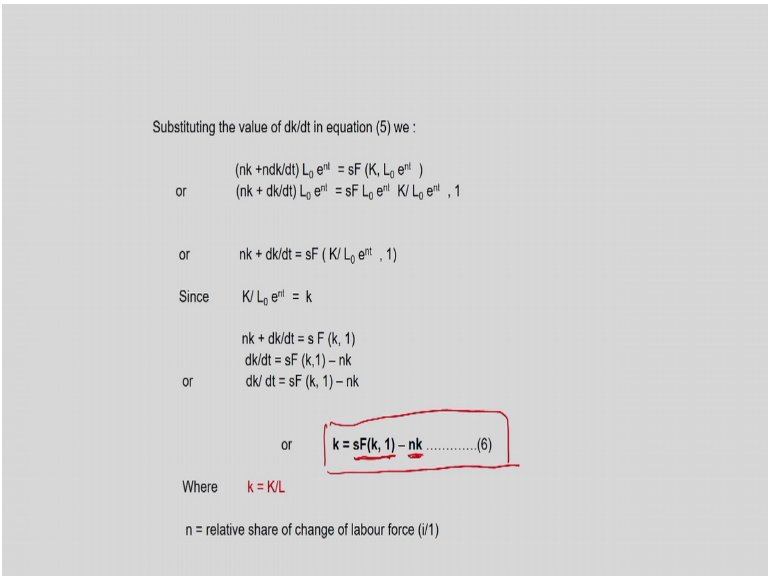

And after having done so, we come up with this most important function of the Solow model; where capital accumulation per workers. So, k the small k is capital accumulation per worker, and that is determined by 2 terms here one is investment per worker and the other is population growth rate here. So, s F k 1 shows investment per worker. What is the effect of investment per worker on capital accumulation per worker? And n K tells us: what is the effect of population growth on capital accumulation per worker. And based on this equation, the one of the first this relationship says that when investment increases when this term s of k 1 increases, then this investment per worker increases and capital accumulation per worker will also increase.

However, when population growth rate increases capital accumulation per worker will decline. If we had depreciation as a third term in this equation, we would have known that, if depreciation increases over a period of time, then capital accumulation per worker would also decline. So, investment per worker has a positive influence on capital accumulation per worker and population growth rate and depreciation has a negative impact on capital accumulation per worker.

Now, this equation this is the most important equation of the basic Solow model. And this can also be explained in the form of a diagram which is the basic Solow diagram. To be able to come to the diagram let us reframe the questions in a more coherent manner.

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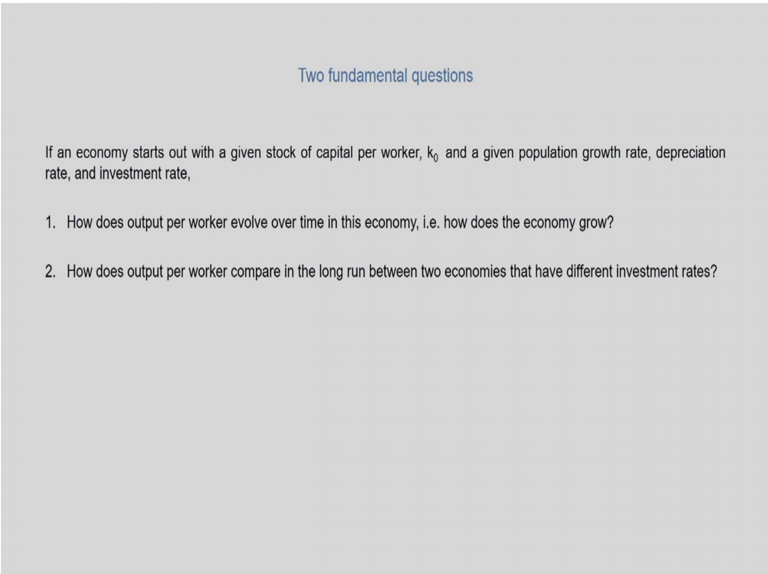

So, the basic problem that we are concerned with is, if an economy starts out with a given stock of capital per worker say k 0 and a given population growth rate depreciation rate and investment rate. There are two fundamental questions that come up. The first question is how does output per worker evolve overtime in this economy or how does the economy grow. And the second question is how does output per worker compares in the long run between 2 economies that have different investment rates. So, these questions are best answered in a Solow diagram.

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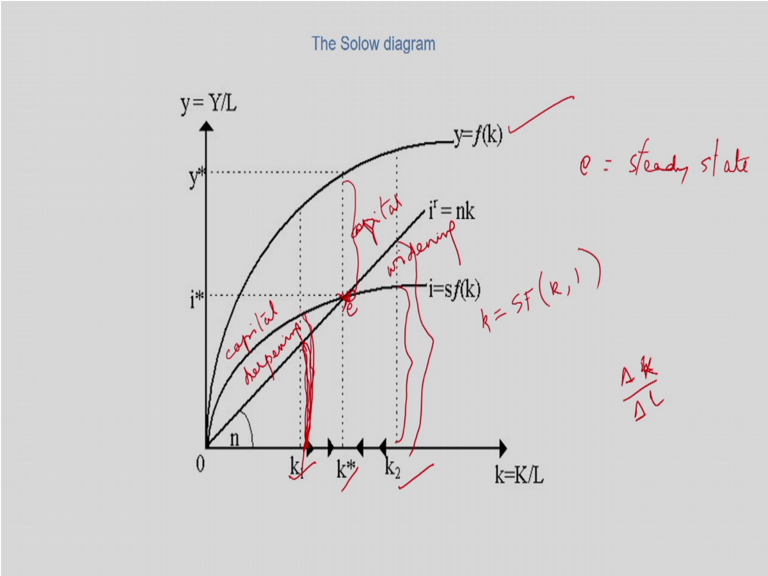

And this is a standard Solow diagram. For the time being I would ask you to ignore the production function curve, and concentrate only on these 2 curves. The gross investments curve and the constant capital labour ratio. This is the investment rate with when capital labour ratio is changing. And this is the investment rate that is required to maintain a constant capital labour ratio. So, this is the basic Solow diagram consisting of 2 curves, i equals s of k or this can also be written as we have derived as k is equal to S F k 1 and this is the n k curve.

So, the first curve, this curve i is equal to S, S F k is basically the amount of investment per person. Or, and this curve has the same shape as the production function that we have seen earlier. However, the only difference is that this curve is translated in terms of amount of investment taking place in the economy. The second curve is the line n k. And this represents the amount of new investment per person which is required to keep the amount of capital per worker constant.

Now, we have not included depreciation in this figure here; however, we have already understood that depreciation and a growing work force tend to reduce the amount of capital per person in the economy. Now, what we need to do to be able to evaluate this the steady state value in the Solow model; is to be able to calculate the difference between these 2 curves. The difference between i S F k and i r n k; so, this difference in these 2 curves what are the difference in these 2 curves tell us? It tells us: what is the change in the amount of capital per worker.

So, this difference basically tells us: what is the change in the amount of capital per worker. The intuition is that, when the difference in this change is positive and the economy is increasing it is capital per worker which is basically the phase below this point of equilibrium. Let us say this is the point e. So, below this point e, this is the point where the change is positive, the economy is increasing, and it is capital per worker is increasing. We say that capital deepening takes place.

So, this is the phase of capital deepening. This is the phase of capital deepening and when this per worker change is 0, but the actual capital stock k is growing. Because of population growth then we say that capital widening is taking place. So, to the right of this equilibrium point e, we say that capital widening take place. So, consider this diagram again. Suppose an economy has capital equal to k 1 today. Now what happens over time? Consider only these 2 figures i equals S F k and the figure curve i r is equal to n k. And we have started with his position k 1 which is the amount of capital labour ratio that we have today.

So, this it is equal to k 1. So now, we need to know what happens over time. At k 1, the amount of investment per worker which is this the amount of investment per worker, the amount of investment per worker exceeds the amount needed to keep the capital per worker constant. So, this is the amount of investment needed to keep capital per worker constant. And this is the actual amount of investment per worker which is taking place in the economy.

So, in other words the amount of investment per worker exceeds the amount needed to keep capital per worker constant. So, capital deepening is taking place; which means that k will increase over time, k will continue to increase over time. And this capital deepening will continue until k is equal to k star; at which these 2 curves are equal to each other so that k is equal to 0. And at this point the amount of capital per worker remains constant. And we call this point e as the steady state. So, this is the steady state point in the Solow economy.

Similarly, the question arises what would happen if instead the economy began with a capital stock per worker larger than k star, and let us say k 2; so, at points to the right of k star. So, all of these points the right to the right of k start the amount of investment per worker provided by the economy is less. So, this is the investment per worker provided by the economy it is less than the investment per worker required to keep the capital labour ratio constant. So, the term k is negative here and therefore, the amount of capital per worker begins to decline in this economy. And this decline the capital widening process this decline takes place until it falls back to k star.

So, the Solow diagram basically determines: what is the steady state value of capital per worker. And we can also juxtapose the production function equation in this model, which is y is equal to f of k if we bring the production function curve here we will basically be able to find out what is the output per labour given the amount of investment required to maintain the capital labour ratio. So, after juxtaposing the production function curve here, we you can note that the steady state consumption per worker is then given by this the; so, we are here we are being we are basically finding out the steady state consumption per worker when we are looking at the production function. And the steady state consumption per worker is given by the difference between steady state output per worker let us say o y star, which is the output per worker and the steady state investment per worker which is o i star.

So, this is the amount of consumption per worker taking place in the economy if we juxtapose the production function into the basic Solow model. So, the basic Solow model at the steady state equilibrium point, basically is able to give us k star y star i star and the difference between y star and i star. So, it is basically telling us what is the investment required per worker the capital required per worker, the output required per worker. And what is the steady state consumption per worker that can take place at the steady state equilibrium point.

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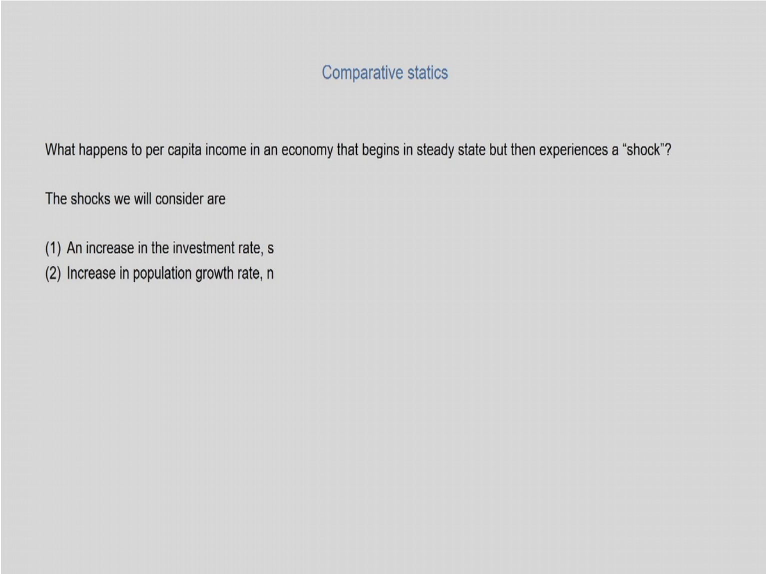

This is this in a nutshell is the basic Solow model in which the capital accumulation equation and the production function equation are brought together by giving a functional form to population growth rate. And the capital accumulation equation is differentiated with respect to time to be able to come up with the capital accumulation per worker; which is basically based upon how much investment per worker takes place to with the changing capital a labour ratio. And how much investment needs to take place to keep the capital labour ratio constant. And when these 2 curves meet we say that the steady state equilibrium is reached.

However, what happens to per capita income in an economy when there are sudden shocks in the economy. So, that can also be considered by bringing about certain changes to the basic Solow model. And the shocks that we can consider this is referred to as comparative statics; where the shocks that we will consider are increase in investment rate s and the increase in population growth rate.

So, that is how the steady state equilibrium can change, either through the capital deepening process or the capital widening process. So, we can either change the investment rate of the economy or we can change the population growth rate in the economy. So, let us first consider an increase in investment rate of the economy; is the same model that we have considered in the last diagram.

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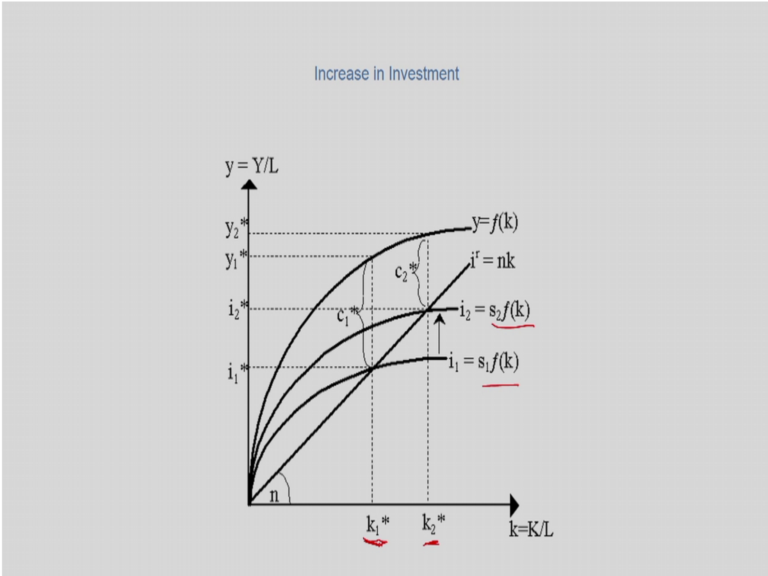

So, this is the basic Solow model where I one is the investment rate per worker, given the change in capital labour ratio. And i r is the investment rate required to keep the capital worker ratio constant. And k 1 star is the initial point of equilibrium the steady state point of equilibrium. So, suppose that the consumers in this economy have decided to increase the investment rate permanently from s 1 to s 2 here. So, what happens to a K and Y in this economy? The increase in investment rate then shifts the s curve to s 1 curve to s 1.

And the current value of capital stock which is k 1-star investment per worker will now exceed the amount required to capital to keep capital per worker constant. And therefore, the economy begins capital deepening process again. And this capital deepening will continue, until the new investment per worker curve coincides with the investment required curve at k 2 star. And from the production function we know that this higher level of capital per worker will be associated with higher per capita output.

So, in other words it means so, this at this at k 1 star the economy was producing output at y 1 start, but because of the capital deepening process. And the output shift of the investment curve. The k 2 star per worker is required to produce output at y 2 star. In other words, what has happened in this economy is that, the output has increased and the economy is now richer than it was before.

This is the effect of increase in investment when there is increase in investment per worker; capital deepening process takes place and therefore, there is an increase in the steady state value of k as well as y.

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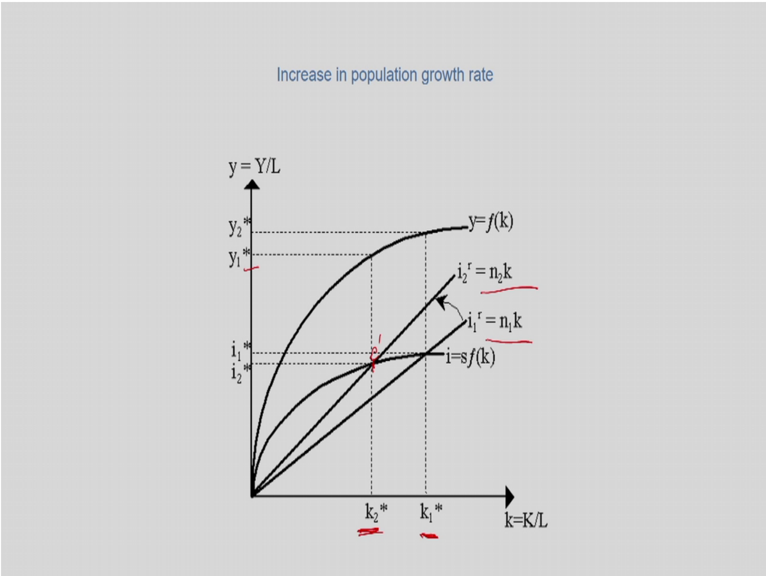

A second case is when there is an increase in population growth rate. When there is an increase in population growth rate, the line i r shifts inward from i r 1 to i r 2. So, suppose an economy has reached it is so, the exercise goes like this that. Suppose, an economy has reached it is steady state, but then because of immigration for example, the population growth rate of the economy rises from n 1 k to n 2 k, it rises from n 1 k to n 2 k here.

So, then what happens to K and Y in this economy that is the question. At the current value of capital stock k 1-star investment per worker is now no longer high enough to keep the capital labour ratio constant in the phase of rising population. Therefore, the capital labour ratio begins to fall, and it continues to fall until the point at which the investment per worker curve is equal to the n 2 k curve at this point let us say e bar at k 2 star. And at this point the economy has less capital per worker then it began with, and is it is therefore, poorer and the per capita output is ultimately lower after the increase in population growth the per capita output has reduced from y 2 star to y 1 star.

So, it is ultimately lower after the increase in population growth rate. That completes our discussion of the basic Solow model. To conclude this lecture let us go through some of the how to read literally the implications of the steady state growth model. In the beginning of this lecture, I was referring to this lecture by professor lands in the American economic association of in which he titled his lecture as why are we so rich and they so poor. So, this is basically the developed countries side of the story as to why are we rich why are they so rich and we so poor or why are we so rich and they so poor.

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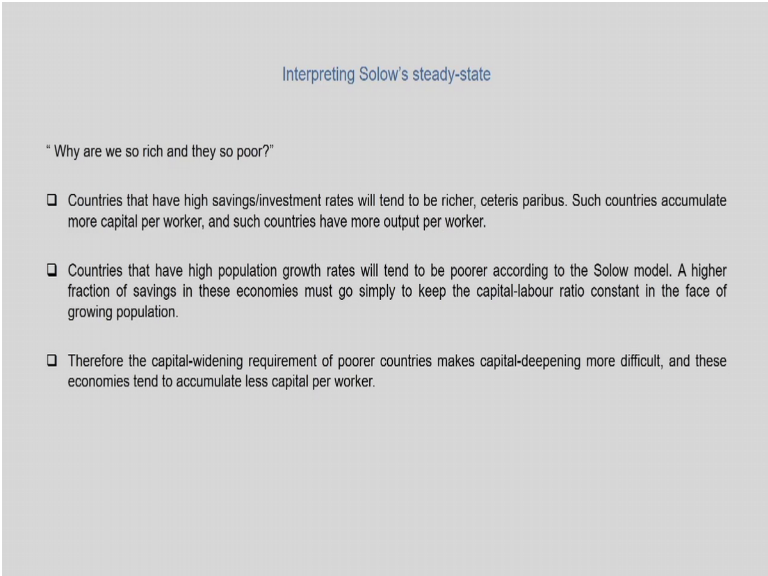

So, this the steady state value of Solow model is basically trying to answer this question. And these are the 3 answers to this question first is that countries and this is also how you can interpret the Solow's steady state value. First is that countries that have high savings investment rates will tend to be richer other things remaining equal.

So, such countries accumulate more capital per worker. And therefore, these countries have more output per worker. Secondly, countries that have high population growth rates will tend to be poorer according to the Solow model. A higher fraction of savings in this economy is must go simply to keep the capital per labour ratio per worker ratio constant in the phase of growing population. Third is there that therefore, the capital widening requirement of poorer countries makes capital deepening more difficult. And these economies tend to accumulate less capital per worker.

And a various the exercises have shown that the Solow model holds up empirically. For example, based on GDP estimates of various countries between 1980 and 1997 if we plots of GDP per worker against gross investment as a share of GDP, and against population growth rates show that countries with very high investment rates tend to be richer on average than countries with low investment rates. And similarly countries with high population growth rates tend to be poorer on average.

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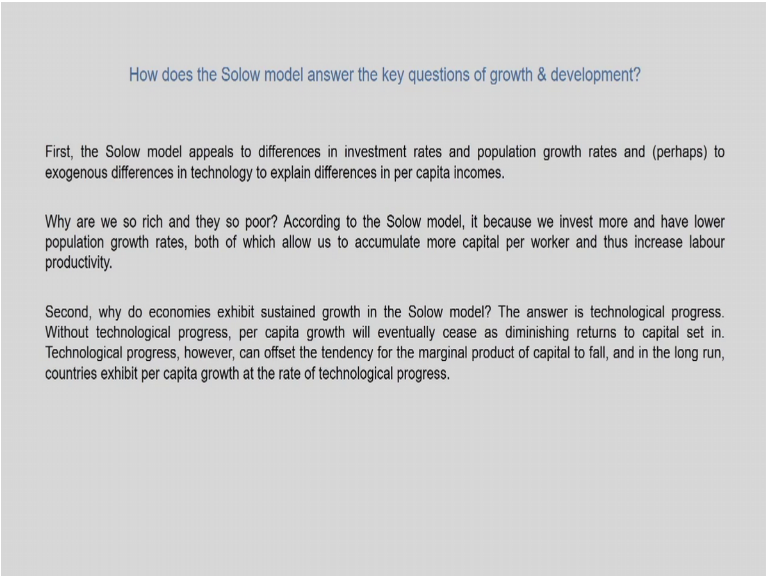

So, at this level then the general prediction of the Solow model seems to be supported by data. And if we have to answer the question how does the Solow model answer the key questions of growth and development. There are it answers the key questions in 2 ways. First is that the Solow model appeals to differences in investment rates and population growth rates. And perhaps to exogenous differences in technology to explain differences in per capita incomes.

So, to the question: why are we so rich and they so poor according to Solow model; it is because the rich countries invest more and have lower population growth rates. Both of which allow the rich countries to accumulate more capital per worker and therefore, increase labour productivity. And, the second question is how do how does the growth rate that is achieved become sustainable in the long run. So, why do economies exhibit sustained growth in the Solow model. For example, in the stylized fact number 5 which we were referring to in the context of USA which has shown steady growth rates.

The question is why do such economies exhibit a sustained growth. The answer the key answer provided by Solow is that because of technological progress. So, without technological progress per capita growth will eventually cease as diminishing returns to capital set in. And technological progress have the can offset the tendency for marginal product of capital to fall, and in the long run countries exhibit per capita growth at the rate of technological progress.

So, this is in a nutshell is the basic Solow model. And with the discussion of the basic Solow model, we complete our discussion on a growth and development models or the strategies of economic development and growth for those interested in studying the Solow model in detail, you may refer to these references here.

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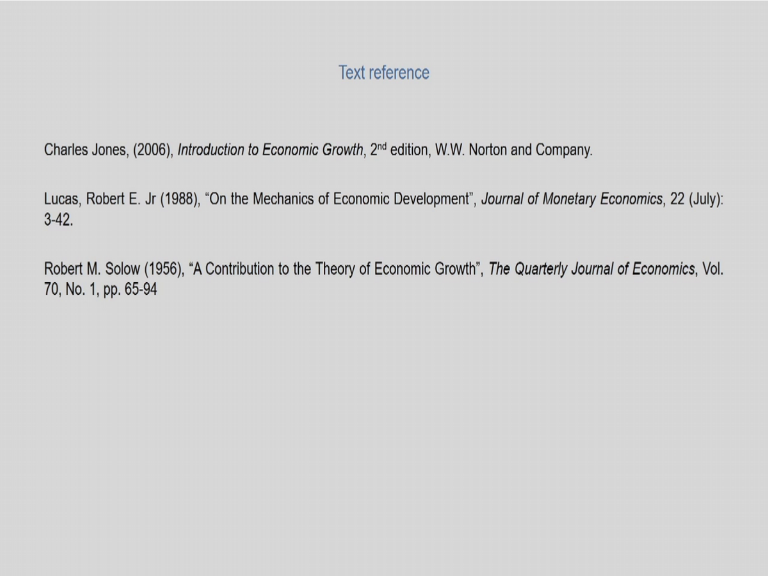

This lecture is largely drawn from Charles Jones, introduction to economic growth second edition. This gives a very good description of the Solow model as well as extensions to the Solow model in very and this skate us to a very wide audience.

So, those interested may look up this book. You may also try to read the original papers of Robert Solow titled the contribution to the theory of economic growth. And also Robert Lucas paper of 1988 on the mechanics of economic development which appeared in the journal of monetary economics. I will see you in the next class.

Thank you.