

Quality Control and Improvement with MINITAB
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Lecture - 38
Multiple response Optimization and RSM

Hello and welcome to session 38 on our course on Quality Control and Improvement with MINITAB. I am Professor Indrajit Mukherjee from Shailesh J. Mehta School of Management IIT, Bombay.

So, the last session we are discussing about blocking and then we talked about centre points like that. We will continue with the centre points and then move forward with some of the topics which is relevant ok.

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Quality Control and Improvement using MINITAB

Addition Centre Points in a 2^k Design

- Based on the idea of **replicating some of the runs** in a factorial design
- Runs at the center** (assuming X continuous) provide an **estimate of error** and **allow the experimenter to distinguish between two possible models**

First-order model (interaction) $y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i < j}^k \beta_{ij} x_i x_j + \varepsilon$

Second-order model $y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i < j}^k \beta_{ij} x_i x_j + \sum_{i=1}^k \beta_{ii} (x_i^2) + \varepsilon$

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So, let me just go through what we have discussed on centre points. So, in these case what we are trying to say is that we are having we are doing experimentation on these corner points over here. And when we are near to optimal scenarios what happens is that there may be curvature in the models there can be curvature in the models, if we are maximizing the model so, there will be.

So, maybe, we started with some region of experimentation and now, we are in this region and where the curvature is present over here.

So, first-order model may not work, first-order model may not work which is not suitable model at this phase for prediction. Maybe, second-order model is more appropriate, where we will have x_i square terms that needs to be incorporated in the model in the regression models like that ok. This is up to interactions. These are main effects and these are the interaction effects and this is the quadratic effects that we want to incorporate over here.

And so, factorial design is not sufficient for modeling this one. So, for specific designs has to be used for that ok. So, to make sure that we are near to the region of curvature, we are near to the region of curvature where in the response surface that we are generating. In this case, what we do is that we add a corner points over here, we add corner points in the design in experimentation.

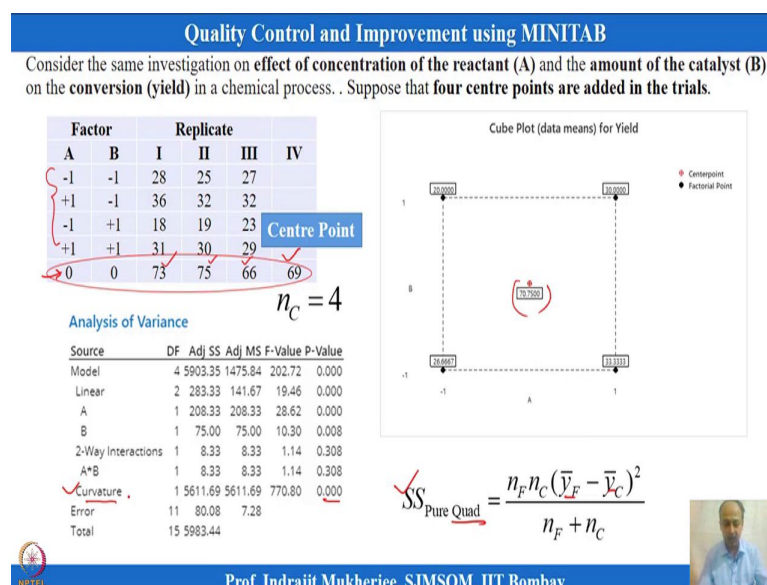
So, in that case what happens is that generally this 0-point is the this you can consider as the as is condition; that means, current scenario is the 0 condition like that; that setting is the current 0 conditions like that for a variable factor A and factor B.

So, 0 level is decided initially and then, you will have a plus level on one side and minus level on side for factor A and B like that ok. So, these are the corner points plus 1 and minus 1 like that and 0 is the centre point which is the which is the current operating condition you can think of.

And at this current operating condition, what is taken is that 3 or 5 runs or trial runs are made over here, 3 to 5; this is the suggested number of centre points that we replication that we do in the centre point setting that is 0 0 setting basically, is 0 0 setting. What we are doing is that we are running the experimental trial and 3 to 5 trials are optimal generally found to be quite suitable.

And then, what is expected is that the this extreme points that is the factorial points over here, what we are having these corner points. The average at the corner points and average at the centre points are compared basically, the average are compared.

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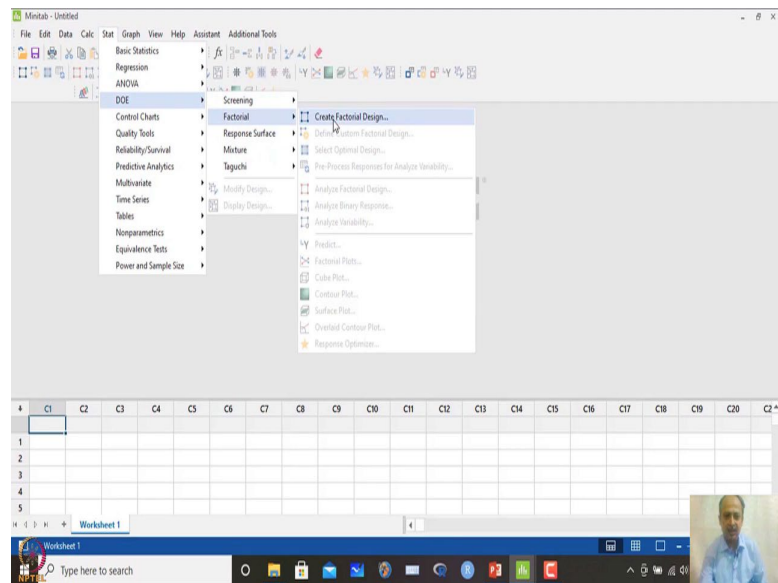
And based on that what happens is that we try to estimate what is the average value at the centre points and what is the average in the corner points like that. And that difference is taken over here, and statistically compared whether it is significant or not. And MINITAB will give you significance of this S S value of pure quadratic terms in the form of curvature that will be mentioned over here.

That means; you are experimenting in this zone and there is a centre point you have taken and the corner points you have completed the design. So, at centre points what you see over here is that at 0 points. I have run the trial and there are four replications done over here. So, four replicates are done over here. And this is the basic two square design that we are having. So, four trials are done with three replicates like that.

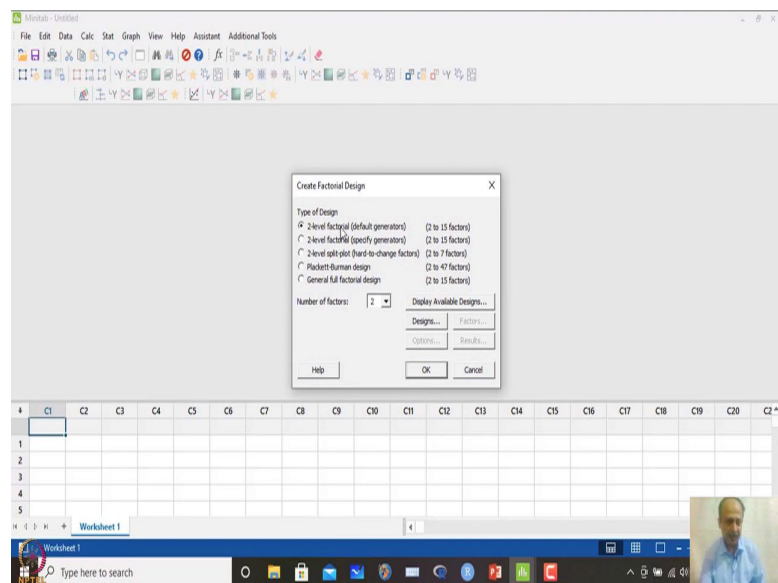
So, 4 into 3, 12 experimentation and then, we have another 4 experimentation, 16 experimentation completes the centre point design basically ok. So, when we do that, in that case, we can see what is the whether the curvature is significant or not. If the curvature is significant; that means, we need to consider second-order models in that case ok.

Quadratic terms that to be incorporated and that has to be that has to be done by using some other design that is suggested to determine the curvature determine the quadratic equation basically ok.

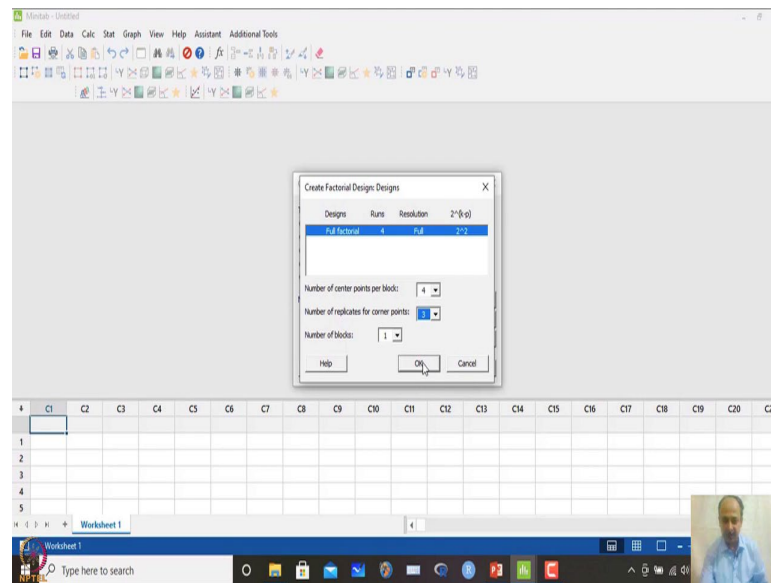
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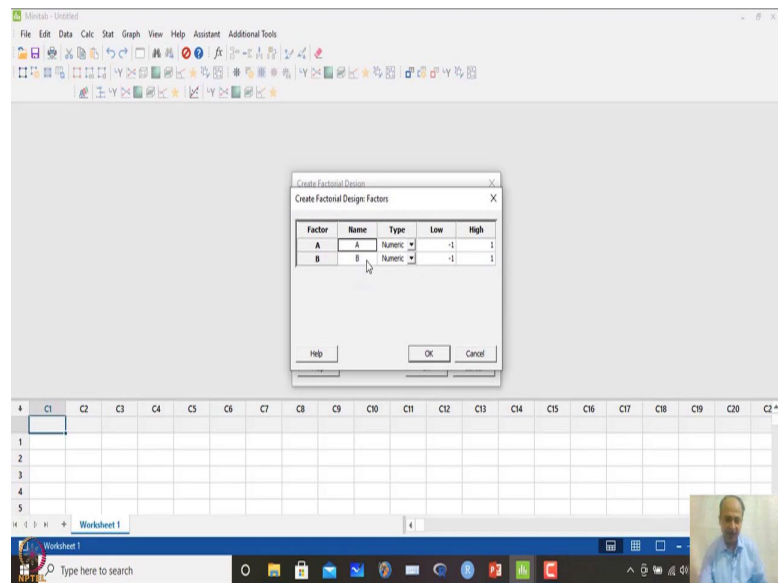


So, what we will do is that we will try to the centre points are added in MINITAB and the design is completed like that. So, we will go to a MINITAB file like that. So, in this case. So, what we will do is that stat go to design of experiments factorial design create factorial design maybe, 2 factors are 2-levels we can think of. So, number of factor is 2; that we have selected and then, we go to design.

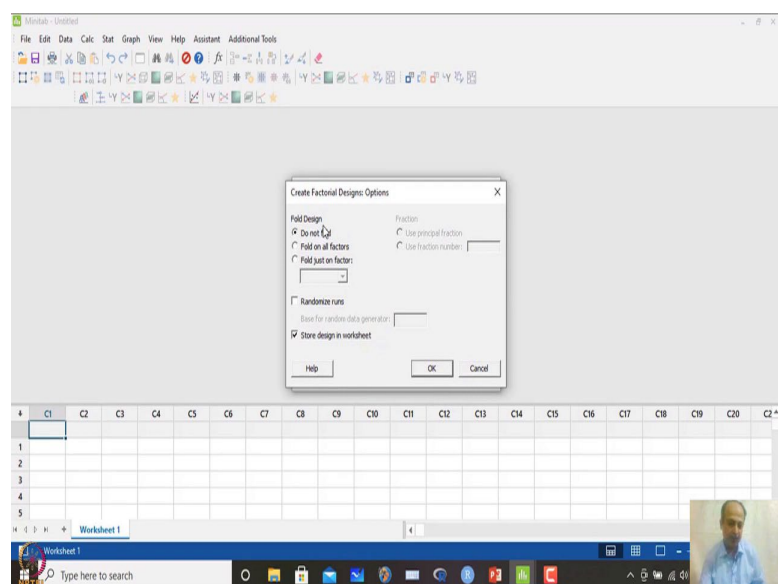
Full factorial number of centre points over here may be 4 centre points we have added let us say, at a centre point we want to replicate it 4 times like that and no blocking. So, block will be one over here. So, in this case we are not.

Number of replicates and maybe, we can assume that we have only 1 points we are not replicating that one. So, 2 square 4 observations. So, sorry number of replicates 3 times we have replicated maybe. So, 3 replicates; what we have seen. So, this is 3 replicates over here, and this completes the work and when we click ok.

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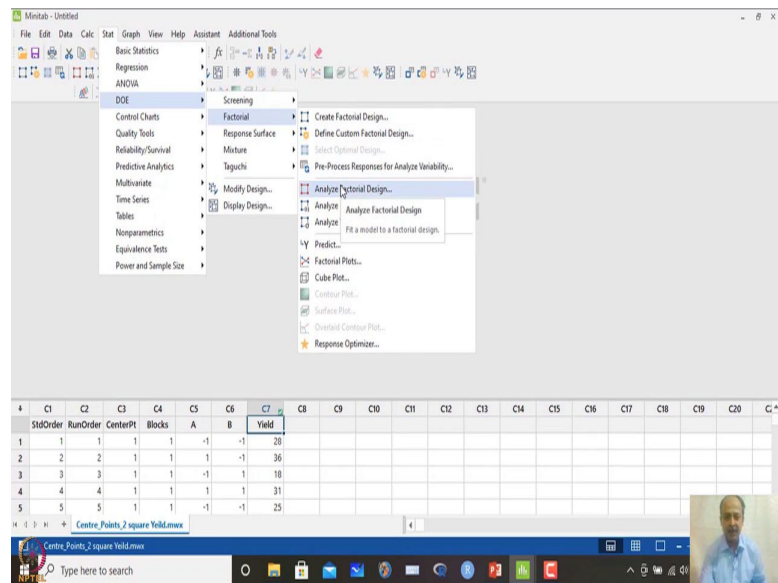


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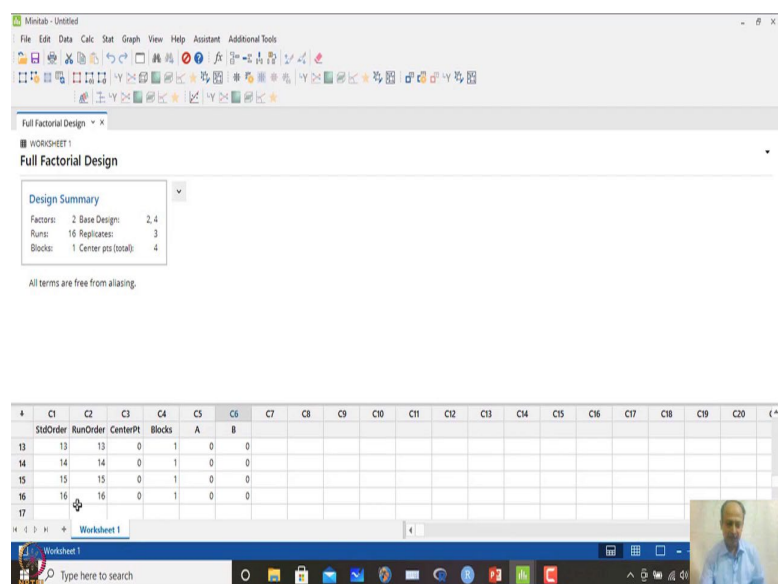


Then, we can name factor A and B. So, this is numeric we are assuming numeric over here. So, this will be ok. And lower and upper is minus 1 plus 1. We do not randomize this one we do not fold also over here. So, this is by default we will click.

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And then, we will click ok and you will get the design and design summary is given over here, and you will find that total 16 trials what I told is that. So, four corner points and two square design replicated 3 times like that. So, this is same as what we are seeing over here. So, A, B is the factor and we have 3 replicates and corner points has four points are replicated over here.

Now, when we have done this and we run the experimental trial with this you get the results that is given over here like 28, 25. These are the outcomes of CTQ which is over

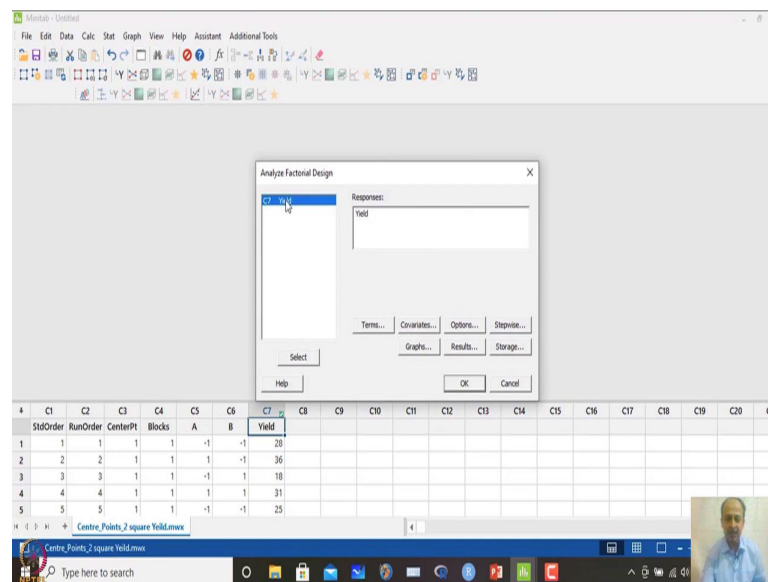
here, yield that is considered over here. And factor is concentration of the reaction and amount of catalyst is the factor B.

Now, we can analyze the data now, we can analyze the data and data is already saved somewhere in MINITAB. So, in this case we will just see that file where it is saved and we will use that file. So, this is with centre point. So, this is the data set that we are having.

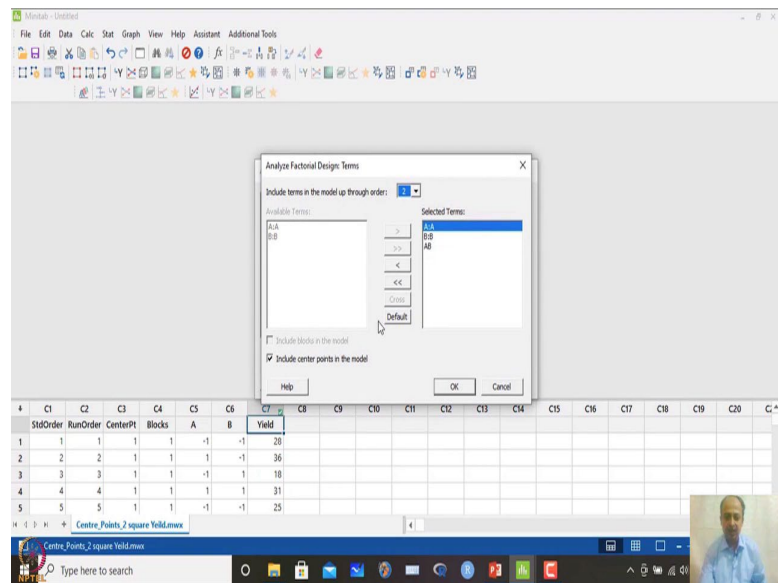
So, typical observation what you see is that; at the centre points, the values are quite high you see the average will be always high as compared to the average at other points like that at the corner points like that. So, this indicates that these values are significantly different from this, but MINITAB will confirm whether curvature is present or not.

So, in this case, but seeing the data it seems it should be. So, what we will do is that with centre point. So, this is the design and we have incorporated the yield data over here and this is the data set that we are having then, what we will do is that we can go to stat and go to design of experiments and then, go to factorial design analyze factorial design.

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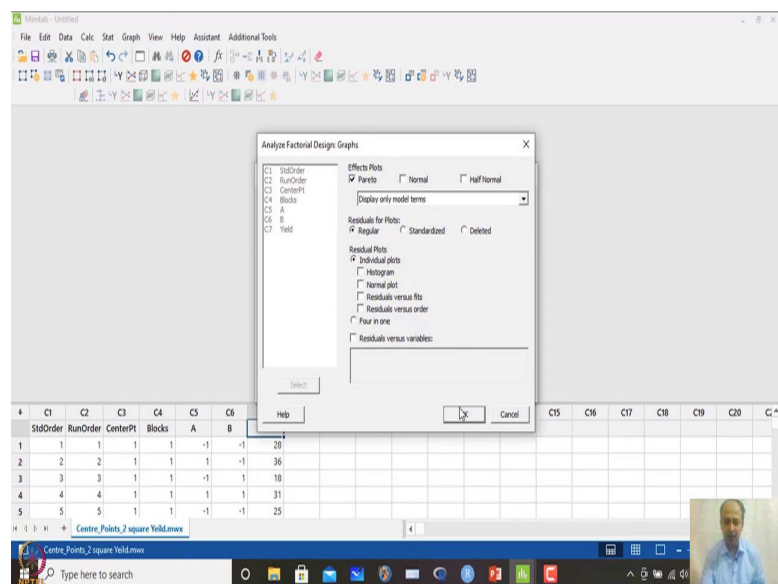


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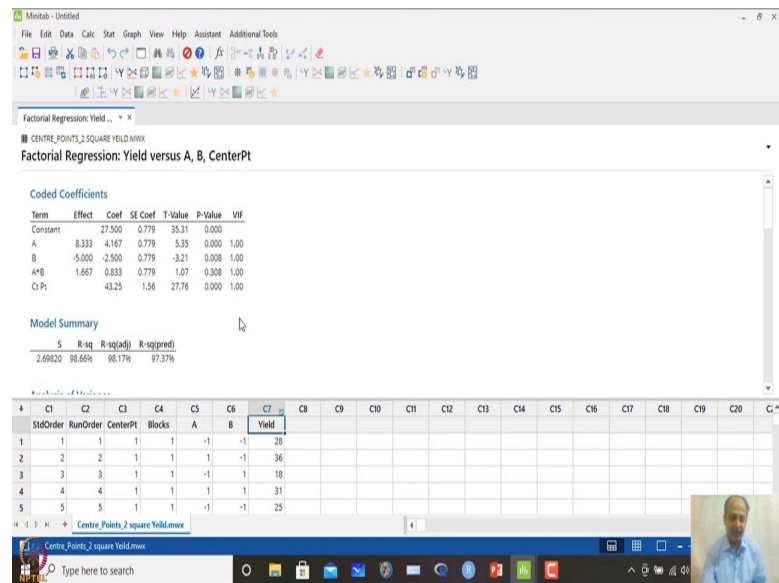


And in this case, we have to mention that yield is the response that we are looking for in terms of A, B and AB interactions we have an include centre points in the model. So, we will include this one. So, that SS pure quadratic can be calculated like that.

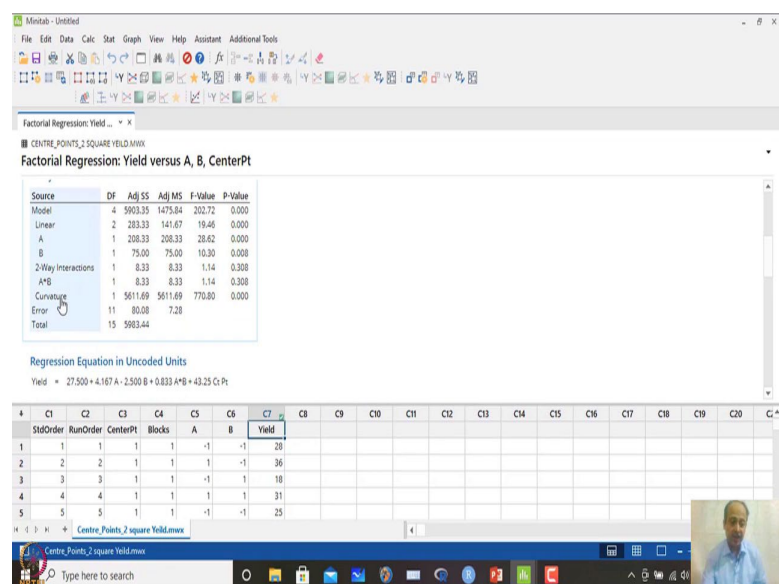
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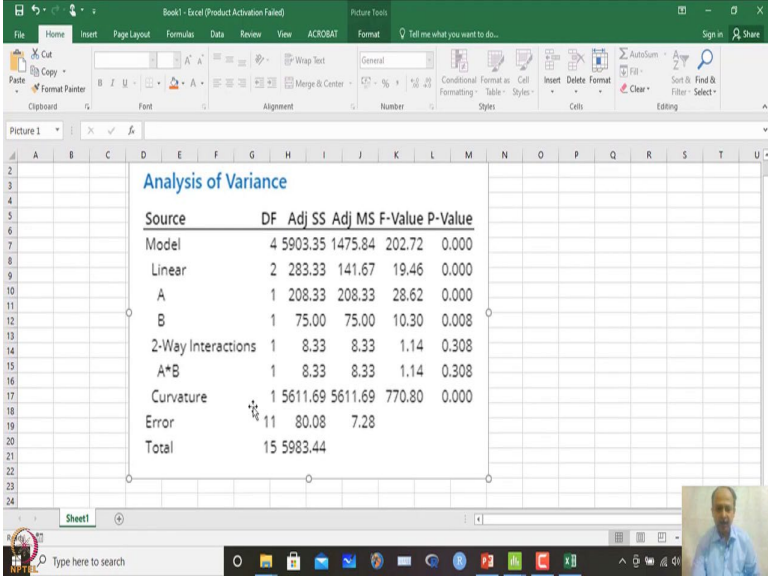
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So, in graph also, you can see pareto plots like that that we have discussed earlier also. And then, you will get the analysis over here.

And about R square value is R square adjusted is 98 seems to be quite good model, but we what we see is that curvature is present over here. So, if I paste this one. I copy this one and paste it in excel. So, we can see that one what is happening. And just enlarge the image what we are getting over here. So, that we can understand that what is the P-value significance or not like that. So, we can see that.

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Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	4	5903.35	1475.84	202.72	0.000
Linear	2	283.33	141.67	19.46	0.000
A	1	208.33	208.33	28.62	0.000
B	1	75.00	75.00	10.30	0.008
2-Way Interactions	1	8.33	8.33	1.14	0.308
A*B	1	8.33	8.33	1.14	0.308
Curvature	1	5611.69	5611.69	770.80	0.000
Error	11	80.08	7.28		
Total	15	5983.44			

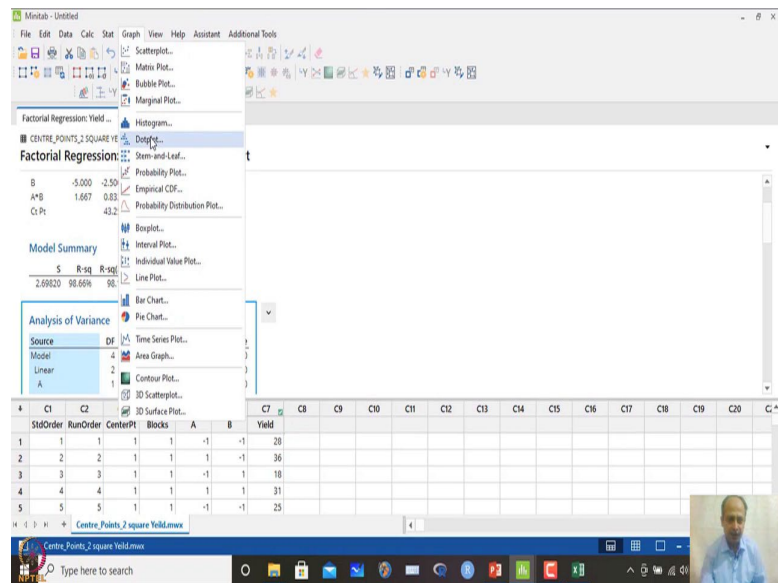
So, we will just open a excel file and then, we will paste this one. So, I have opened this one I will control-p over here. So, if I enlarge this one, what you observe is that you see for curvature, this is SS curvature term and this is significant over here. A is significant, B is significant, but AB interaction is not significant what we are seeing over here, but the curvature is significant.

Curvature is significant; that means, this model tells, that there is a curvature in the model which needs to be considered and while when we are developing the regression equation. So, in that case, some other way we have to find out the quadratic equation.

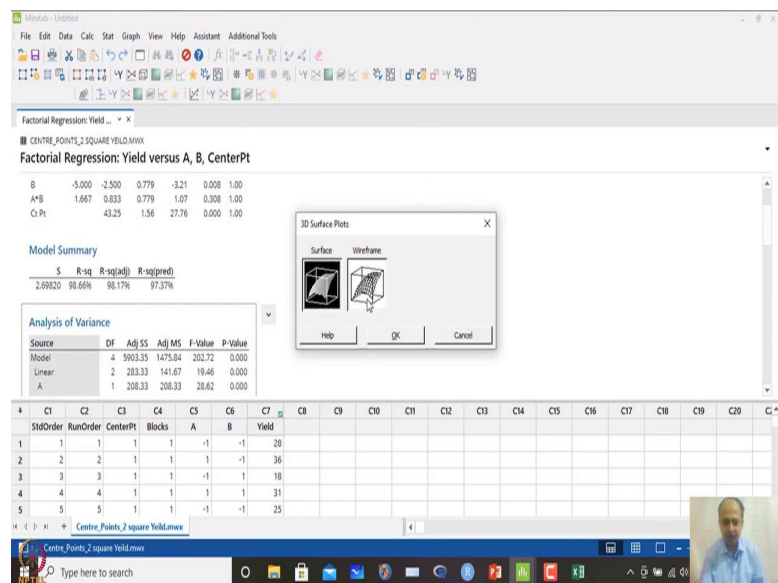
Now, this experimentation is not enough to calculate the β_0 estimation of the regression equation. If we incorporate the square terms like that. So, this design is not sufficient over here.

Now, what you can do is that what we can do is that we can just see that when we are doing this when we are doing this experimentation, we can just have a plot and see what is happening.

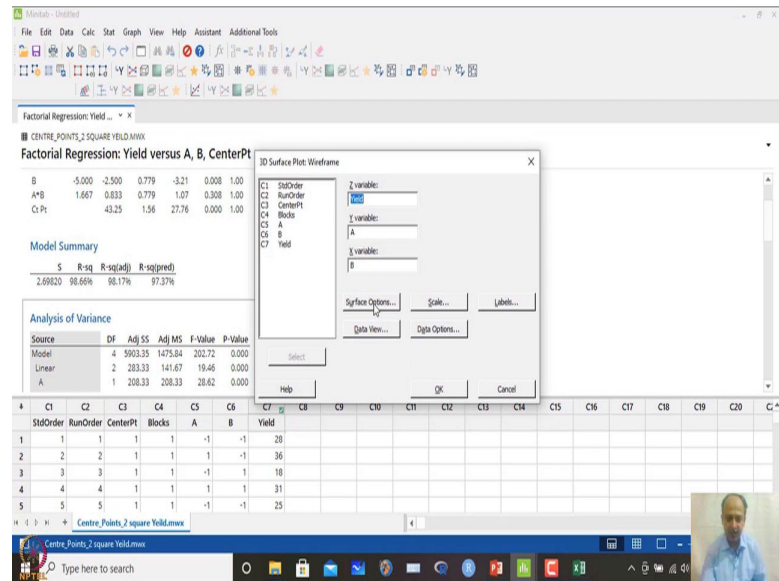
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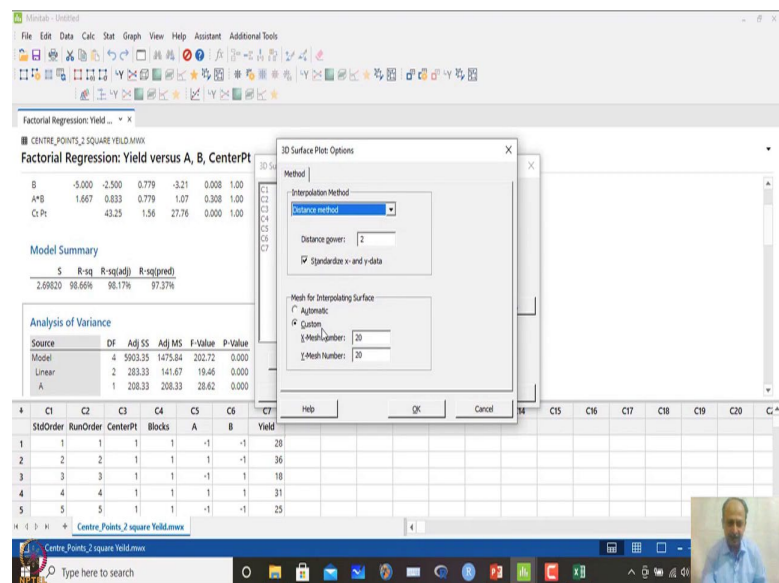
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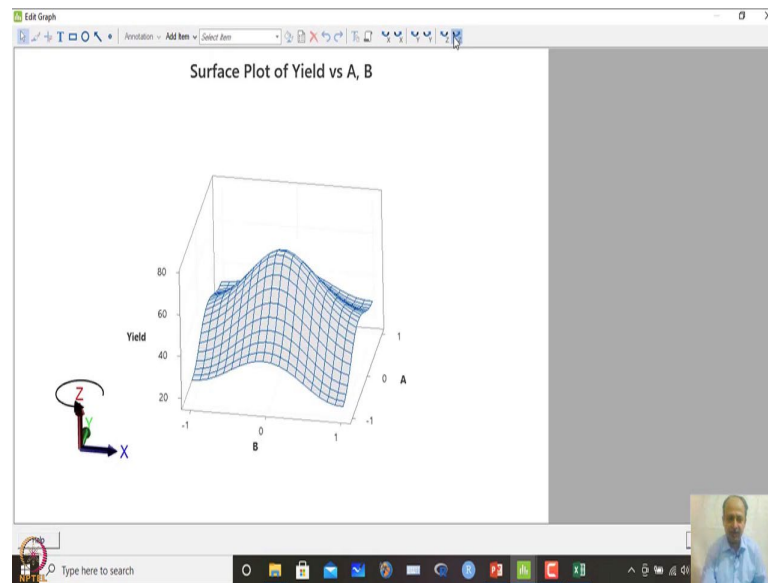
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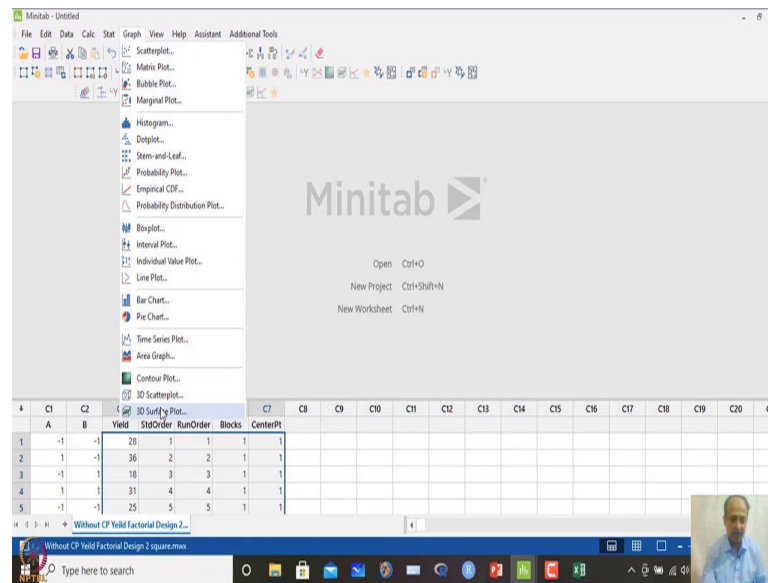


So, I can go to graph and show you what is happening. So, in this case 3D surface plot and when I use wire plots over here, and I use that Z-axis is the yield and A and B are the factors over here and surface options we can give as mesh over here 20 20 like that.

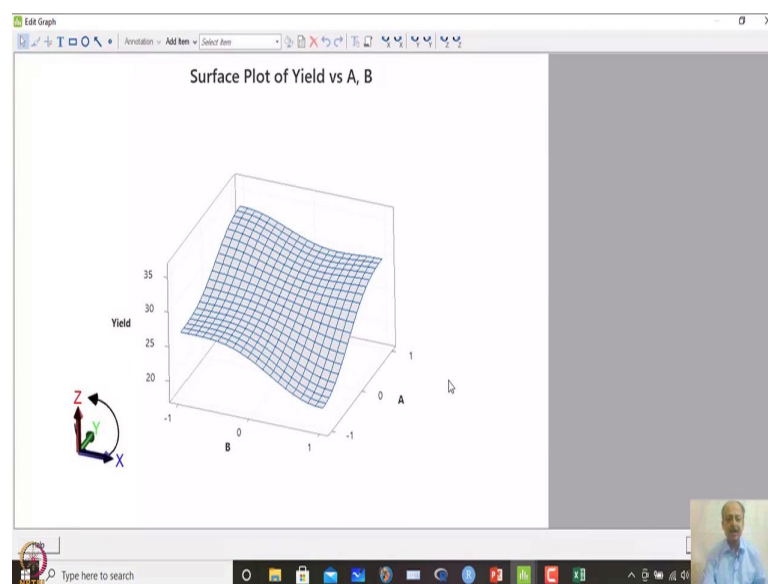
We can click this one and when you click ok what will happen is that you will see that this is the this is the curvature is visible over here. So, you can see this one and curvature is quite visible and you can just rotate the axis over here, and see what is happening.

So, here you see that there is a huge amount of curvature that is observed in the model. So, if this indicates that we need to add quantity terms in the model. So, we can also see that without centre points what will be the diagram.

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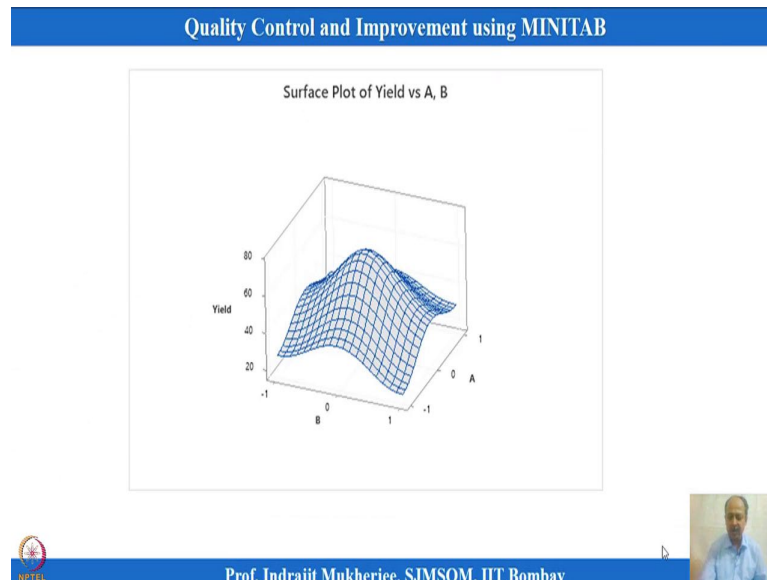


So, I have just removed the centre points over here, and this is the design that we have we have. In hand, I wanted to show you that how graphically it will look if we have not added the centre point. So, in this case 3-D surface plot again, we can use square plots over here, and these are the same things we have taken over here. And I click ok and the surface is not visible in this case.

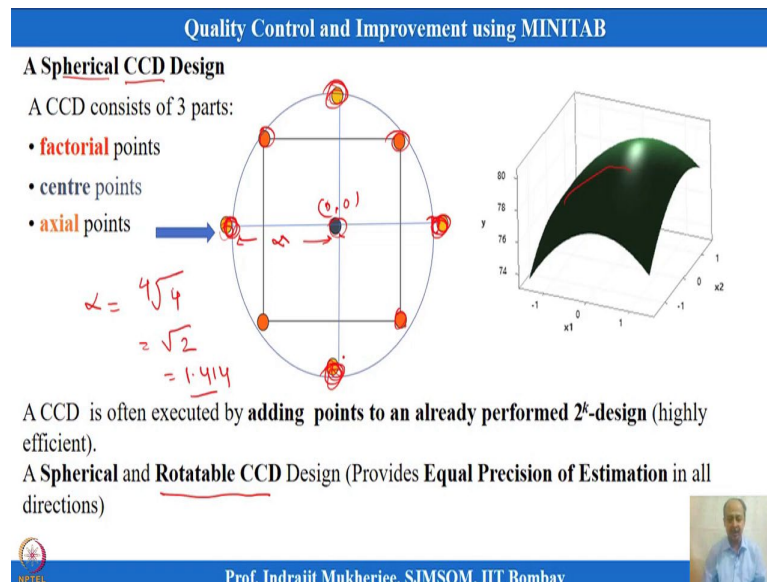
So, what we are seeing is that initial 2-square design. It is not showing it is quite flat over here. So, in this case this is not reflected, but whenever I have added a centre point. So,

over here and centre point experimentation was done what has happened is that I have seen that there are the mean is quite significantly higher than the corner points like that. So, that indicates that we need to add a we need to go for quadratic models over here we need to go for quadratic models over here.

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So, then how do you go for quadratic models how do you go for quadratic models? A specific design is available which is helpful for developing the quadratic design

quadratic equations over here; this is known as central composite design this is known as central composite design or CCD design like that.

So, this is one of the models that I am showing over here, this is known as spherical CCD design over here. There will be corner points over here, there is a factorial-points over here, there will be centre points over here, that is shown over here, but there will be something called which is known as axial points over here. This axial-points will be added in the design like that.

So, I will finish the experimentation with corner points that we have done earlier also. Now, we will add some axial points over here, so that we can estimate the quadratic beta values of the quadratic terms basically.

So, in this case what we are going to do is that we will add these axial points over here. So, these are the axial points four axial points we will add in the design and this is known as CCD design.

This is spherical and rotatable what it says is that equal precision of estimation is in all direction when we develop the response surface like this what you are seeing on the right-hand side of the screen response surface drawn and MINITAB.

So, in this case what happens is that estimation at any corner of this is quite precise. So, that is why we use CCD design. And in this case, we say that it is a rotatable CCD design this is a rotatable center composite design and for that.

Then, what will be the axial point. this is 0 0 point over here and these are the corner points minus 1 plus 1 this these points what we are seeing over here then, what will be the value of axial points ok.

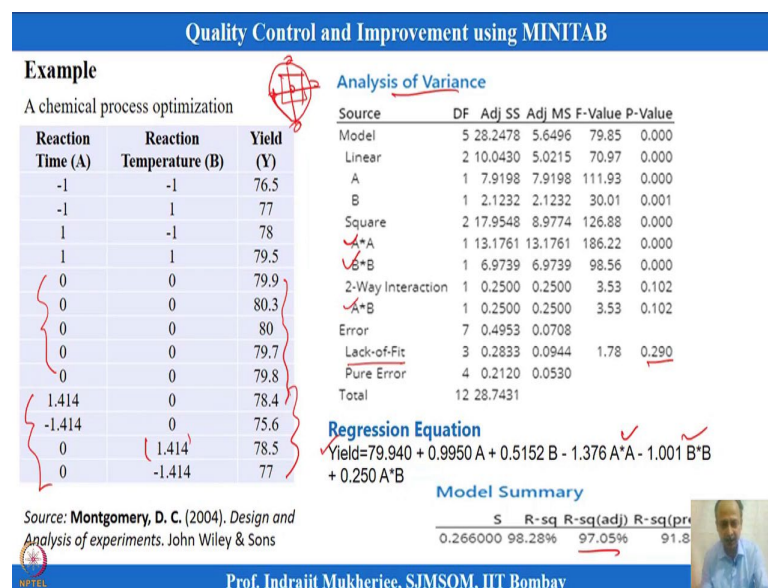
So, this is known as the distance from the centre point this will be noted by an α indicator that this distance from here to the centre points is around α over here and what should be the α that is the question over here.

So, generally α is taken as number of corner points over here. So, there are 4 corner points over here. So, 4 th root of corner points like that. So, 4 corner points over here.

So, this will be square root of 2 and this will be 1.414 that is a α value that is selected over here. So, this is the trend that is this is a formulation that is generally used. So, that it is rotatable CCD design like that. So, α is taken as 1.414.

So, one four experimentation has to be completed at this distance from the centre point and these are the four points if you experiment that one. And then, you can just generate the response surface which is in quadratic term we can generate the response surface like that ok.

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And this was done this experiment was completed. So, earlier we have done for the centre points let us say. And now, these points are different what we are seeing is that yield values are different over here. So, this is minus 1 and plus condition are different over here. So, in this case we have reached to about the optimal scenarios global optimal setting conditions like that near to that.

So, what we are doing is that; we are we have we are incorporating centre points over here. We have a minus 1 plus 1 condition like that. We are experimenting in some region and this is the centre points over here, then we are adding these axial points which is square root of 1, square root of 2 that is 1.414. So, this is added over here.

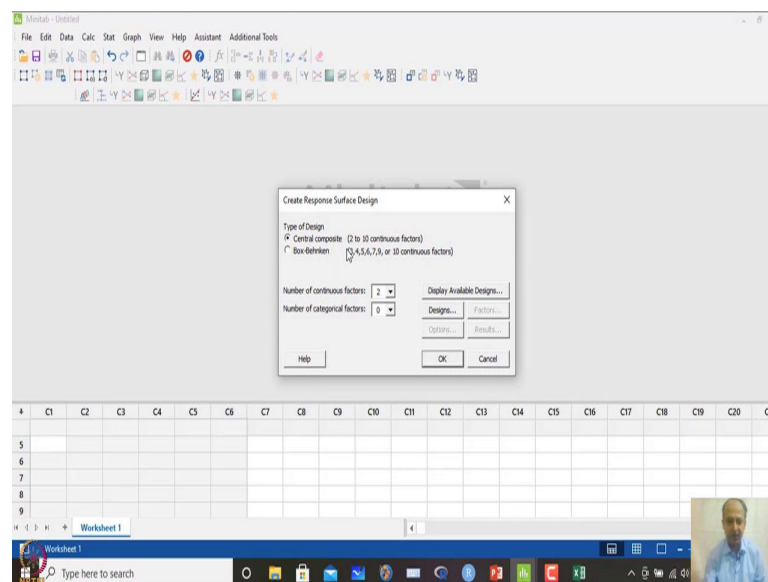
And then, when we add this one over here that is 1.414. So, these four trials are completed over here and these are the results that we are getting over here and then, we

can analyze and we can. When we do analysis of variance what you will find is that A square term will be incorporated B square term will be incorporated along with AB interaction.

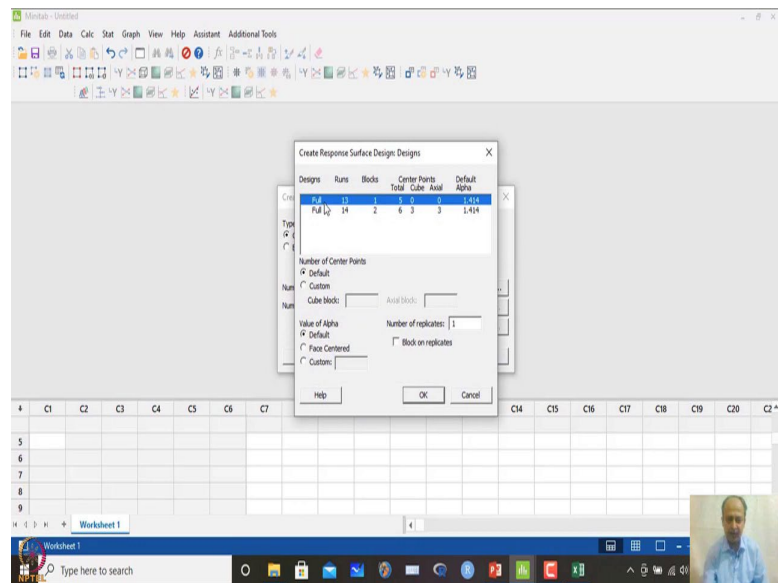
And you can also see the lack of fit there is no lack of fit. Model seems to be adequate and the regression equation we can get what is the what is the model. So, this is A square times B square terms and R square is coming out to be quite ok that is 97.05 that is quite fine. So, in this case. So, now, how to create the design CCD design. So, that I can get quadratic terms like that. In that case, we have to go to MINITAB and then, I will show you how it is generated like that.

So, this is this we have discussed this we have discussed. So, in this case we will go to a CCD design like that. So, and we can just see the last one. So, I will delete this one a blank sheet like that and then I will create a CCD design. So, what I will do is that I will go to stat and then, go to DOE. I have to go to response surface over here. We cannot go to factorial design to create this. I have to go to response surface and then, create response surface design.

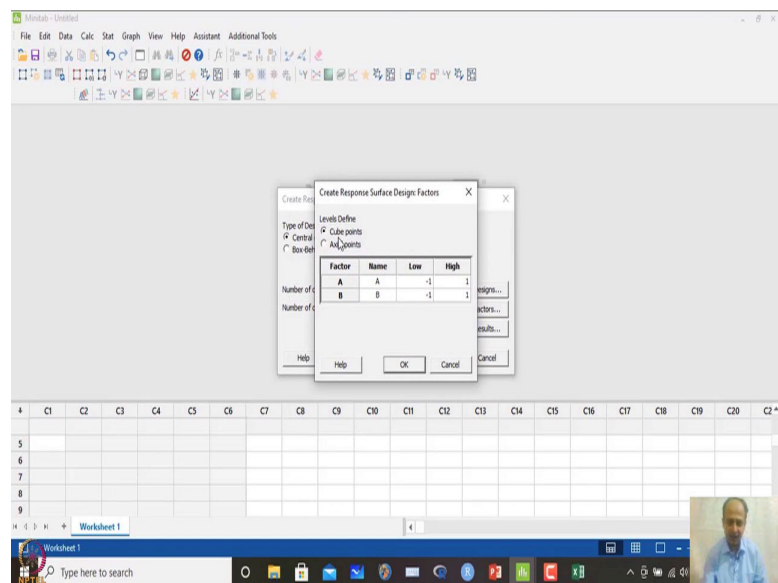
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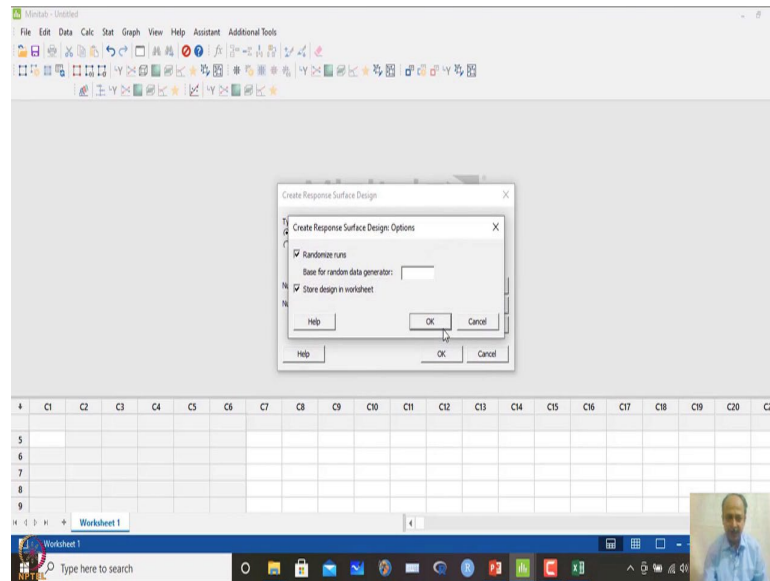
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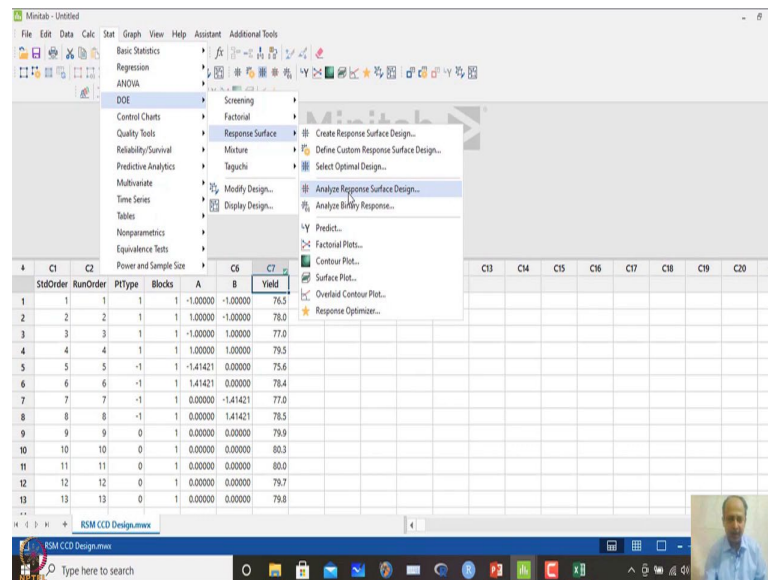


Here, you will find central composite design. So, in this case, I will say central composite design I want to create. Number of continuous factor is basically 2; we are having A and B that is the 1.

And in design matrix what we will do is that we will use the full 13 runs with block one and centre point 5. This is by default he will take like that. And then value of α is also default it will be square root of and that this is default α is 1.414; that is given over here.

So, when you click ok and then, factors you can write what is the factors. So, this is you do not have to do anything over here. And in options you do not randomize the run let us create the design. So, if you click ok.

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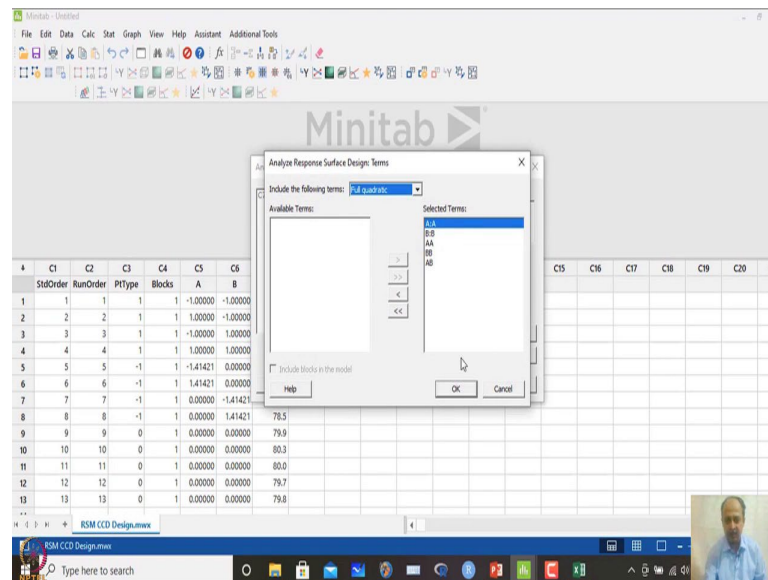


What will happen this matrix this is already created these 13 runs are created over here. Four corner points and then we have centre points five centre points over here and then we have axial points that is given over here four axial points that is given over here. Then, what you have to do is that CTQ measures if you put over here and then, analyze the data nothing more we can do over here yeah.

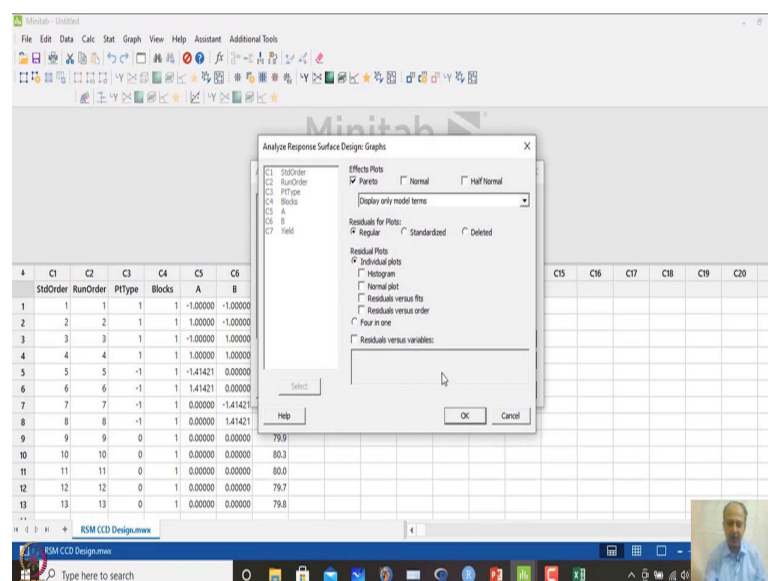
So, in this case we have already incorporated this data. So, we do not want to we do not want to we do not want to waste our time over here. So, we will go to directly to that and this is the response of specificity design that is there over here.

So, this is the data set is completed and placed over here in the yield column over here. Then, what you have to do is that to create the to analyze this, I will go to DOE then go to response surface over here. And then, go to analyze response surface design over here I will go to analyze response surface then I will click yield over here.

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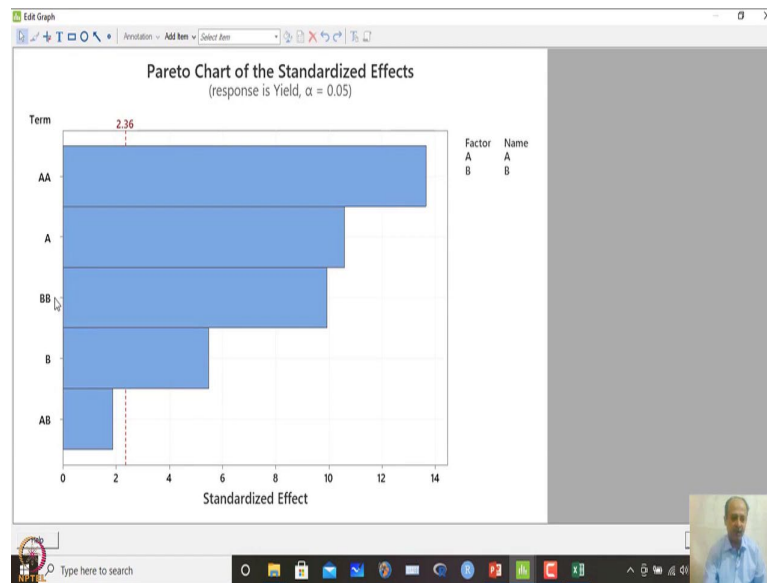


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And the terms that we will add is full quadratic terms that we will add over here. When we add this one and click ok, other things remain same and we can see pareto plots of this. And then, what we can do is that we can click ok over here and then, what happens is that pareto plot will show you which is significant which is not.

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So, in this case what you see is that A square is important A is important BB is B square is important B is important, but AB is not so much significant over here interaction between A and B is not significant, but quadratic term is significant over here. So, we can retain those terms, because we have gone to high and second-order model over here. So, we can we can keep interaction or we can drop also interaction over here.

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Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	28.2478	5.6496	79.85	0.000
Linear	2	10.0430	5.0215	70.97	0.000
A	1	7.9198	7.9198	111.93	0.000
B	1	2.1232	2.1232	30.01	0.001
Square	2	17.9548	8.9774	126.88	0.000
A*A	1	13.1761	13.1761	186.22	0.000
B*B	1	6.9739	6.9739	98.56	0.000
2-Way Interaction	1	0.2500	0.2500	3.53	0.102
A*B	1	0.2500	0.2500	3.53	0.102
Error	7	0.4953	0.0708		
Lack-of-Fit	3	0.2833	0.0944	1.78	0.290
Pure Error	4	0.2120	0.0530		
Total	12	28.7431			

So, when we do this you will get the ANOVA analysis what is shown over here. So, I will click this and I will paste it in excel and that will give me the equation idea of the equations what we are using.

So, in this case this is the. So, what we see is the linear model A is significant B is also significant. According to P-value, A square is significant B square is significant also, interaction is not significant. But lack of fit also does not show anything that shows that there is lack of fit. So, model seems to be adequate over here. So, in this case this quadratic equation can be used like that.

So, this we can adopt this CCD design to develop equations where quadratic terms are significant. So, whenever you are near to the global optimal solution generally, what happens is that; we try to use CCD design and then develop the quadratic equation and then, try to optimize the response surface like that we will try to optimize the response surface.

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Response Surface Methodology

- Primary focus of previous topics is **factor screening**
 - Two-level factorial Design, fractional factorial, Taguchi Method are widely used
- Objective of **RSM** is optimization
- RSM dates from the 1950s; early applications in chemical industry
- Modern applications of RSM span many industrial and business settings
- Collection of **mathematical and statistical techniques** useful for the modeling and analysis of problems in which a response of interest is influenced by several variables

- Factor screening
- Finding the **region of the optimum**
- Modeling & Optimization** of the response

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And that is the objective of this response surface methodology. What generally happens is that; we start experimentation from certain region over here. And then, we slowly move towards the optimal global optimal points and then we use CCD design over here and finally, reach the optimal global optimal solution or setting conditions like that; that is the overall idea of response surface methodology.

And it can use a specific optimization algorithm. So, to reach from this point to this point like that. But what I am saying is that generally linear model works over here in the initial stages and when we go to this point then, we need a quadratic equation over here. So, for that the design is CCD design that is generally used and then the optimal solution.

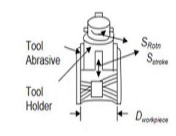
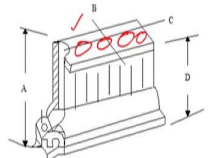
So, this is a sequential experimentation that happens in response surface methodology. So, we try to model and optimize we try to model and optimize. So, that is a brief overview of response surface methodology, but there are rules how we move from initial condition to the final condition that will be that we can see in any books like that. So, this is what we have in response surface methodologies and.

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Multiple Response Optimization (MRO)


- **Multiple response:** More than one response characteristic (Dependent variable).
- **Optimization:** Simultaneously maximize (or minimize, as desired) all the responses.


Consider an automobile engine cylinder liner bore grinding (honing) process.

Response variables: surface finish, cross hatch angle, ovality and taper. afg 1 2

Controllable variables: Speed, feed, depth of cut, grinding wheel type.



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Now, an important topic which we I thought we should address like that which is known as Multiple Response Optimization which is known as multiple response optimization or a MRO Multiple Response Optimization problems like that.

So, this is the general situation what is generally encountered in industries like that, when we have multiple number of responses which is important like that for the from the customer standpoint basically. So, it is not only yield, it may be many other characteristics which are important like that ok.

So, over here like one of the example that is taken over here is that; engine cylinder liner bore grinding operation is going on in a engine manufacturing unit let us say or assembly; assembly line it is happening like that and machine; machining line basically it is happening and then, it goes to assembly line basically.

So, in the machining what is happening is that there will be this is the cylinder block that you see over here. And that will be seeing the bores over here and we need to we need to machine this one and then, we need to generate the surface what is required by the customer.

And generally, what characteristics they SES or CTQ's that they observe over here, is basically surface finish whether it is ok or not maybe cross such angle is another important characteristic, which is observed ovality of the bore that is there that is also observed and taper of the bore can also be one of the characteristics.

So, there are a number of characteristics over here is 1 2 3 and 4 that you are seeing over here. So, there are four characteristics. Simultaneously, it should adhere to the specification. So, that is important over here.



There is no one single characteristic. So, I cannot optimize one variable over here. I have to optimize all four variables over here and determine which is the setting that will give me this optimal scenarios like that for all four CTQ's like that ok. See, that is the scenario what happens is that technology the problem is known as multiple response optimization problem.

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MRO Solution Approaches

- Overlaying contour plot
- Dual response approach
- Mathematical Programming
- **Desirability function approach**
- Generalized distance and Taguchi Loss function approach

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How it can be solved. There are many ways of solution of this. So, we will talk about one method that is known as desirability function approach which is available in MINITAB interface like that, it is available in MINITAB interface like that.

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Mathematical Programming



A popular approach is to formulate the problem as constraint optimization problem and solve using NLP techniques (e.g. **Direct Search**).

Maximize y_1

Subject to,

$62 \leq y_2 \leq 68$

$3200 \leq y_3 \leq 3400$

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So, the problem is basically we can think of a non-linear programming optimization problem like that. So, we have to maximize one variable let us say, this is one way of optimization over here, where one of the variables can be considered as to be maximized which is the priority variable.

So, primary objective can be selected as maximize y_1 and then, we can put constraints to the other response function over here. So, that is other CTQ's over here.

So, this should be subjected to this y_2 should be within this and this and y_3 should be within this and this. And if I can solve this problem maybe, one of the feasible solution we can get. And that may be suitable for implementation like that.

So, this is mathematical programming and we can use some of the algorithms like that search for is used most of the time many of the scenarios. You can use evolutionary algorithms also. So, there are many ways of doing this mathematical programming we can do that ok.

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Example

A chemical engineer is interested in determining the operating conditions that **maximize** the yield (y_1) [LTB], **on-target** viscosity (y_2) [NTB], and **minimize** molecular weight (y_3) [STB] of a process. Two controllable variables that influence process are: reaction time (x_1) and reaction temperature (x_2).

$70 \leq y_1 \leq 80; T_i = 80$
 $62 \leq y_2 \leq 68; T_i = 65$
 $3200 \leq y_3 \leq 3400; T_i = 3200$

Reaction Time (x_1)	Reaction Temperature (x_2)	Yield (y_1)	Viscosity (y_2)	Molecular Weight (y_3)
-1	-1	76.5	62	2940
-1	1	77	60	3470
1	-1	78	66	3680
1	1	79.5	59	3890
0	0	79.9	72	3480
0	0	80.3	69	3200
0	0	80	68	3410
0	0	79.7	70	3290
0	0	79.8	71	3500
1.414	0	78.4	68	3360
-1.414	0	75.6	71	3020
0	1.414	78.5	58	3630
0	-1.414	77	57	3150

Source: Montgomery, D. C. (2004). *Design and analysis of experiments*. John Wiley & Sons

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So, this is one way and another way what we have is that we can also use one desirability function approach one which is known as desirability function approach to. So, to apply desirability function approach what is required is that; I need to understand what is to be maximized what is to be minimized or something like that.

So, this is one of the experimentation; like last time what we have seen is that yield and reaction time and reaction temperature that experiment we have done and CCD design was used over here.

Now, instead of one response that is over here, we have three response over here which is to be optimized over here. Earlier case we can determine the response surface and then we can find out the optimal condition.

Now, we have three general desirability function approach is used when we have more than one response scenario. So, in this case, I have three response scenarios like that. So, in this case a problem becomes difficult because tradeoff solutions is required over here and there may be interrelationship between these y is the y characteristics that you are seeing over here.

So, this yield and this are interrelated they are correlated like that and in that case, we have to find out the setting for x_1 and x_2 , which will optimize all the variables. And it is not easy it is difficult like that a difficult problem which is known as multiple response optimization problem.

So, CCD design was used and we have the response over here. Now, we want to maximize all together maximize yield. We want to keep the viscosity at a certain level on targets like that and we may want to minimize the molecular weight that is y_3 . So, one is maximization, one is on target, one is minimization problem that we are trying to track it over here ok.

And this problem is taken from Montgomery's book that is analysis of experiments over here. You can get all the data information see whatever data I am using I am referring the book. So, you can just refer the books and see that chapter and you will find the data.

So, if you go to the Montgomery's book like apply probability and statistics for engineers or it is statistical quality control or it is design analysis of experiments. So, this is you can see in any of the any of the books and you can just enter the data and then you can follow the procedure what is shown in the video and you will get all the results that we have discussed like that.

So, this problem is maximization of yield on target is the viscosity and minimization of this. So, here the information that is given in Montgomery's book is that y_1 should be within 70 and 80 over here target value is 80 it has to be maximized. So, this is I have to reach this value over here.

Then second one is y_2 should be on target. So, target is 65. So, the upper range and lower range is given as 62 and 68 over here ok. And the operating reaction temperature over here. These are the x variables so; it varies from minus 1.414 to plus 1 over here.

So, in this case plus 1.414 that is a positive side we are seeing. And this is similar to x 2 also minus 1.414 to plus 1.414 like that ok. And the third one is to be minimized. So, in this case target value it should be less than 3400 and the target value is defined as 3200 let us if we can reach this one that is ok for us.

So, this is the problem statement that we are having on hand and this data can be. You can create the CCD design and then, enter the data that is given yield viscosity and molecular weights like that we can create that. And how to analyze that in MINITAB that is important.

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Quality Control and Improvement using MINITAB

Desirability function:
The desirability function transforms each response variable to a corresponding **scale free desirability value** (say d_j), which lies between zero and one ($0 \leq d_j \leq 1$). The value of increases as the “desirability” of the corresponding response increases.

Composite Desirability:
It (say, D) is defined as **geometric mean** of the individual desirability values and mathematically it is defined as.

$$D = \left(\prod_{j=1}^n d_j \right)^{1/n}$$

The composite desirability is also lies between 0 and 1.



0.93

$0.93 \leq 1$

$\sqrt[3]{d_1 \cdot d_2 \cdot d_3} < 1$

$\sqrt[3]{x_1 \cdot x_2 \cdot x_3}$

$y_1 \ y_2 \ y_3$

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And for this we MINITAB uses a desirability function approach we MINITAB uses desirability function approach over here. So, what it will do is that the response that you are getting over here. So, these values will be converted these values that you are getting over here will be converted into a desirability values like that this.

All these values that you are seeing over here will be converted into a value desirability which lies between zero to one which will lie between 0 to 1 and it will depend on what type of whether we want to maximize.

So, if you want to maximize a variable and there is a desirability function for that maximization any types of y characteristics which is to be maximized.

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Alternative Desirability Function Approach

For STB Response

$0 \leq d_j \leq 1$

$d_j = 1$ (let us)

$$d(y_j(X)) = \begin{cases} 0 & \text{if } y_j(X) \leq y_j^{\min} \\ \left[\frac{y_j^{\max} - y_j(X)}{y_j^{\max} - y_j^{\min}} \right]^{\tau_j} & \text{if } y_j^{\min} < y_j(X) < y_j^{\max} \\ 1 & \text{if } y_j(X) \geq y_j^{\max} \end{cases}$$

For NTB Response

$d_j = 1$

$$d(y_j(X)) = \begin{cases} 0 & \text{if } y_j(X) < y_j^{\min} \text{ or } y_j(X) > y_j^{\max} \\ \left[\frac{y_j(X) - y_j^{\min}}{\tau_j - y_j^{\min}} \right]^{\tau_j} & \text{if } y_j^{\min} \leq y_j(X) \leq \tau_j \\ \left[\frac{y_j^{\max} - y_j(X)}{y_j^{\max} - \tau_j} \right]^{\tau_j} & \text{if } \tau_j < y_j(X) \leq y_j^{\max} \end{cases}$$

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So, these functions will be used. There are alternative desirability functions that is available for. If it is smaller the better type of function, I can calculate the value of d_j which lies between 0 and 1 which will lie between 0 and 1 over here.

So, if you tell me the values of y_j which is the actual value; I can convert into a value between 0 to 0 and 1. And for a specific types of characteristics if smaller the better type of characteristics we want so, in this case this is the function that is given.

So, this is a function that is given by Deringer and Switch. So, in this case what is required this is simplified one ok, Harrington first proposed this one. So, this is desirability function approach you can see in any research papers you will find this one.

And MINITAB uses this one. So, it will convert this variable this response variable into 0 and 1 variables over here. So, this will be converted. Similarly, for nominal the best or on target values d_j values can be calculated like that this will also vary between 0 and 1 like that.

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Quality Control and Improvement using MINITAB

Desirability Function Approach

For LTB Response

$$d_j = \begin{cases} 0 & \text{if } \hat{y}_j(\mathbf{X}) \leq y_j^{\min} \\ \left[\frac{\hat{y}_j(\mathbf{X}) - y_j^{\min}}{y_j^{\max} - y_j^{\min}} \right]^q & \text{if } y_j^{\min} < \hat{y}_j(\mathbf{X}) < y_j^{\max} \\ 1 & \text{if } \hat{y}_j(\mathbf{X}) \geq y_j^{\max} \end{cases}$$



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And similarly, larger the better also we can get the value of desirability which will lie between 0 and 1 like that. So, you can think of that nominal the best if I want to explain this one nominal the best over here. So, in this case if the value of y falls outside the specification which is defined like that there is definition over here.

So, this is defined as y_2 should be between 62 and 68. So, if the value is below 62 if the value is below 62 then, in that case, if this is below 62 this is the minimum value let us say, this is 62 over here then, desirability value will be equal to 0. If the value is greater than this one. So, in this case the value will be also equals to 0. So, if you are outside the specification, the reasonability value will be 0 like this.

This is the target value over here this is the upper specification this is the lower specification limit. So, if you do not hit the target if you do not hit the target and move away like this up to a certain point there will be some desirability d_j value, but if you are outside the specification, d_j value will be equal to 0.

But if you are hitting the target exactly the d_j value will be equals to 1. So, 1 is desirable in any case. So, one will be desirable. So, we want to maximize the value of d_j and the maximum value is given as 1 like that.

So, similarly over here using this function when you put this one. So, smaller the better. If it is near to 0 if it is smaller the better than let us say, the target value is 0. So, when

you reach 0, the desirability value will be equals to 1. Similarly, when we are using this larger the better type of function. So, if it has to be more than certain values if it is more than certain values what will happen is that; desirability will be converted as 1. Well, we will be considered as 1 over here.

So, this is the function that is given which converts the response into the relative measures for and there are three types of functions over here. One is for larger the better, one is for nominal the best and one is for lower the better or smaller the better like that. So, this functions will be used by MINITAB and that will convert the value.

So, when I am searching for a optimal scenario of x_1 and x_2 ; it will convert for that what is the predicted value of y and for that predicted value of y . It will calculate what is the desirability value and all the desirability value for y_1 , y_2 and y_3 for that setting condition will be calculated

And then, using this d_1 , d_2 and d_3 value, it will calculate a composite desirability value which is given over here, which is known as composite desirability and it is taken as geometric mean.

If there are three variables, this will be $\sqrt[3]{d_1 \cdot d_2 \cdot d_3}$, that you have gone for a given setting and d_3 value that you have gone. So, cube root of this will be used over here. So, 1 by R this is the formulation over here, 1 by r over here; that is known as composite desirability. And composite desirability will also lie between 0 and 1. So, if it is near to 1; that means, you are reaching the tradeoff condition which is the best one.

So, if you hit 1; that means, all the target values are completed like that you have reached all the target values like that ok. If this is equals to 1 this is equals to 1 that is and the third one is equals 1; that is the most favorable scenario, but this does not happen in real life, because their inter coordination and getting the global optimal solution is not easy.

You need to have a tradeoff solution like that you can be near to 1, but getting exactly 1 is rarest of rare occasions that you will find ok. So, I have to get the setting of x_1 , x_2 and x_3 that will optimize y_1 , y_2 and y_3 and y_1 , y_2 , y_3 will be having a desirability value d_1 , d_2 and d_3 and multiplication of this if it is equals to 1 and cube root of 1 is means 1.

So, if this is the ideal scenario, but you will never reach this ideal scenario. You will get some values that is desirability capital D that will be equals to that will lie within 0 and 1 like that. So, maybe 9.3 maybe, the values of composite desirability that we will get, when the solution is achieved by MINITAB.

So, MINITAB will give you what is the composite desirability value that MINITAB has reached and what is the setting condition of x_1 , x_2 , x_3 that we mentioned. So, we have done x_1 and x_2 , it can be more number of variables like that. So, that is not a constraint ok.

So, we will discuss about implementation of this in our next session and then we will move forward with some other topics like that. So, this is an important topic. So, we will move from here. So, that we can address other things like that.

So, this is a typical scenario is most manufacturing organization and there are different ways to solve this multiple response optimization problem. One of the methods I am showing which is in MINITAB how it is done like that I will demonstrate that one. But you can see literature where there are many other approaches to solve multiple response optimization problem basically ok. So, we will continue from here, with examples on multiple response optimization problem.

Thank you for listening.