

Quality Control and Improvement with MINITAB
Prof. Indrajit Mukherjee
Shailesh J. Mehta School of Management
Indian Institute of Technology, Bombay

Lecture - 36
Factorial Experiments (Contd.)

Hello and welcome to session 36 on our course on Quality Control and Improvement with MINITAB. I am Professor Indrajit Mukherjee from Shailesh J. Mehta School of Management, IIT Bombay. So, previous lecture what we are doing is that, we are studying factorial design and we have seen 2 square design that is generalization or 2^k design that we have used and 2 factors at 2 levels.

So, with replicates and without replicates examples we have taken over there, and now we will try to extend that when with more than 2 factors like that. So, we will try to do in this session some 2^k design which is having factors more than 2 like that ok.

(Refer Slide Time: 00:57)

Quality Control and Improvement using MINITAB

Effects in The 2^3 Factorial Design

The 2^3 Factorial Design

Geometric Cube View

Design Matrix

Treatment	Factor		
	A	B	C
→ (1)	-	-	-
a	+	-	-
b	-	+	-
(ab)	+	+	-
c	-	-	+
(ac)	+	-	+
(bc)	-	+	+
(abc)	+	+	+

Prof. Indrajit Mukherjee, SJMSOM, IIT Bombay

So, let us take one example which is more than 2 factors and this is a 2 cube design; that means, 3 factors at 2 levels over here. So, we have 2 levels and 3 factors over here. And this is a cube design that I have already told last time that this is cube like design where we have a factor A and factor B and factor C, and you can see in 3 dimensions this is the space that it will cover.

So, this is the total space that it will cover. And treatment combination that is given over here there is 1 means all at minus level. So, this is the 1 treatment combination over here. Similarly, A at plus level over here and all other at 0 level. This is a A condition that we are getting, this is a second experimental trial points like that. Like this we can represent that in a cube structure 3 dimensional where we can see up to 3-x variables over here.

So, this is the cube view other design matrix what we are seeing over here. So, this is the design matrix that is used over here. So, there will be 2 factor interaction over here that is A multiplied by B that there is an interaction. So, these are the 2 factor interaction over here there, can be 3-factor interaction also present in the analysis when we are doing the analysis.

So, this is A multiplied by B multiplied by the C ; when the three acts together then it can have an impact on the Y variable over here. So, that is also possible. So, this is the design structure that we are using over here. So, there are please remember there are three factors over here.

(Refer Slide Time: 02:24)

Quality Control and Improvement using MINITAB									
An Example of a 2^3 Factorial Design (8)									
The Plasma Etch Experiment									
Run	Coded Factors			Etch Rate		Total	Factor Levels		
	A	B	C	Replicate 1	Replicate 2		Low(-1)	High(+1)	
1	-1	-1	-1	550	604	(1)=1154	A(Gap, cm) 0.80	1.2	
2	1	-1	-1	669	650	a=1319	B (flow) 125	200	
3	-1	1	-1	633	601	b=1234	C (Power) 275	325	
4	1	1	-1	642	635	ab=1277			
5	-1	-1	1	1037	1052	c=2089			
6	1	-1	1	749	868	ac=1617			
7	-1	1	1	1075	1063	bc=2138			
8	1	1	1	729	860	abc=1589			

A = gap, B = Flow, C = Power, Y (CTQ) = Etch Rate

So, let us take a real-life example that is taken from Montgomery's book, again from Montgomery's book and this is a plasma etching experimentation that is done over here. There are 2 replicates in the process in the experimentation over here. So, because this is a 2-cube design, so, minimum number of trial is requirement is 8 over here and it is replicated 2 times. So, 16 trials are done over here.

So, A is a factor which is known as gap variable, then B is flow and C is power and the CTQ that we are measuring over here is Etch rate or response characteristics that is considered over here is Etch rate. Let us assume we want to maximize the Etch rate or we can assume that one for sake of analyzing the data let us assume that one.

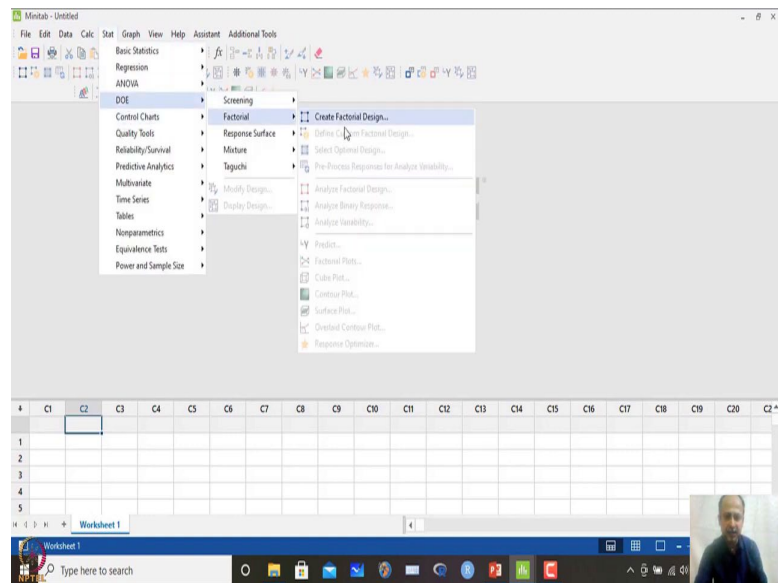
And A is having 2 levels over here; low level and high levels over here gap, these are the actual true variable that we have seen and we have seen how to code the variable if the true variable is known level is given. So, in that case it can be converted into minus 1 and plus 1. So, we can do the calculation with respect to minus 1 or plus 1.

Why we are converting into minus 1 and plus 1? Because, we want to see the effects with respect to the other effects or other factors like that. So, we can make a comparison because it will be unitless if we are coding the variables like that. So, in this case ABC, these are the variables of factors we can consider. So, in this case this is having true value of 125 and 200. These are the two levels low level and high level.

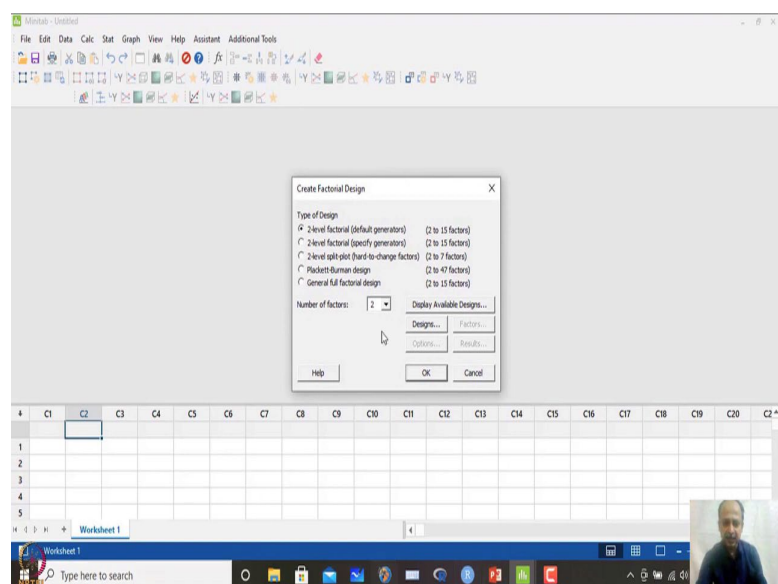
And for C factor this is the low level and high level that is. So, real life experimentation will happen with this variable, but while analyzing the data we can use the coded variable and based on that we can analyze the data, because we can always convert the coded variable into the actual values like that. So, that is possible. So, here we are using coded this design matrix over here and this is the design matrix.

And MINITAB can generate this one, MINITAB can automatically generate 2 cube factorial design, so in this case. So, this is the factor. So, how let me just recap how we have generated that one. So, I will just show you 2 cube design with 2 replicates like that how we are creating, then we will use this already created a design and the data is already created. So, we will analyze, use that for analysis like that.

(Refer Slide Time: 04:33)

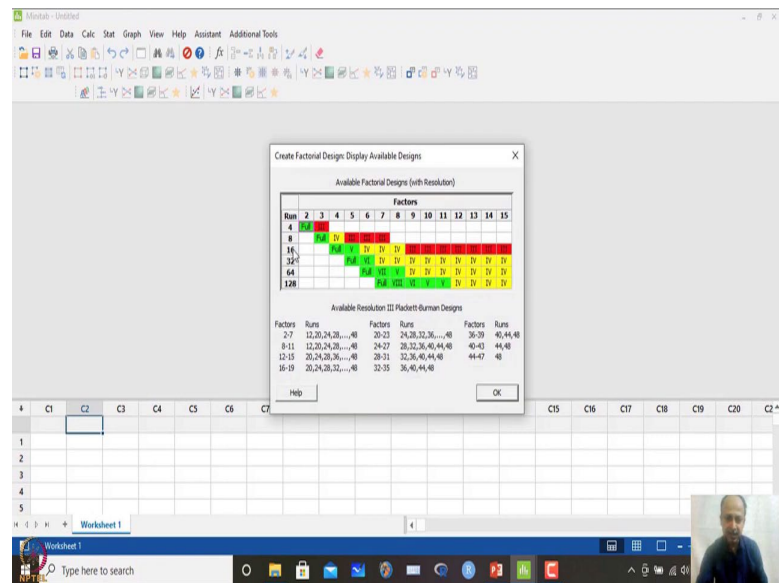


(Refer Slide Time: 04:36)



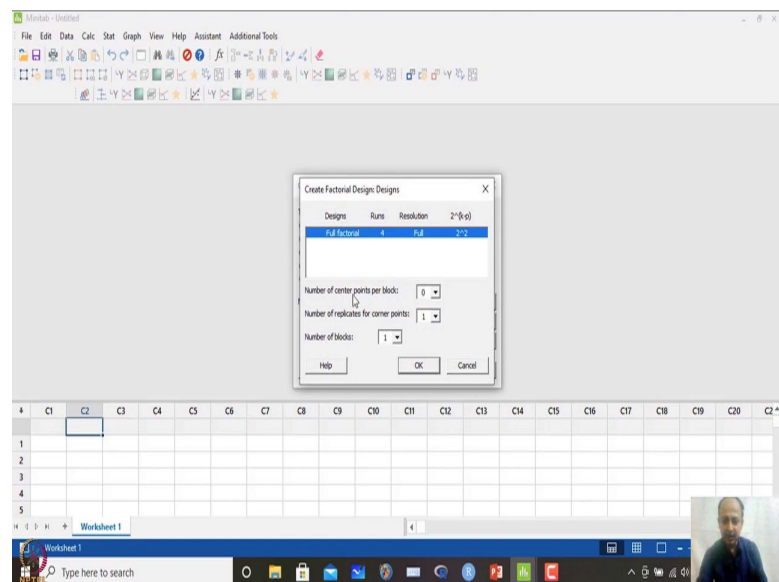
So, in this case what you have to do is that, you have to go to stat design of experiment factorial design, create factorial design. So, this will be 2 level factor. Last time also we discussed, number of factor is 2 available design.

(Refer Slide Time: 04:42)



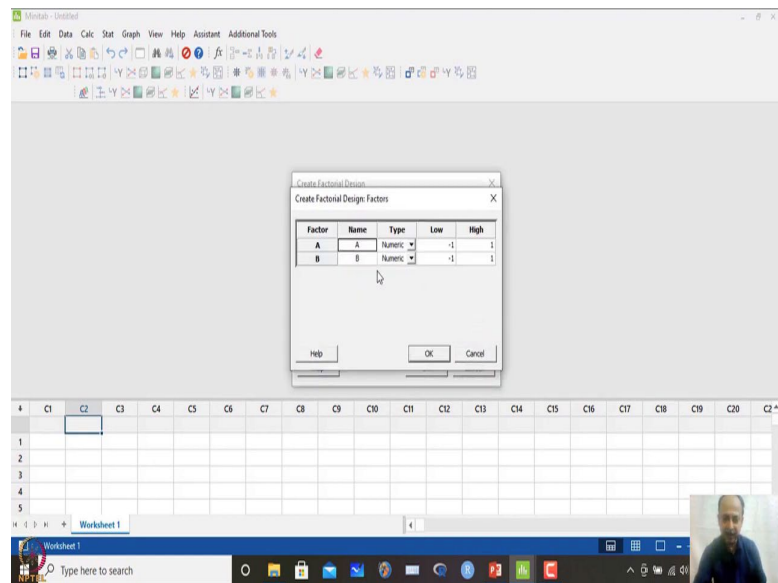
So, in this case we have factors number of factors as 3. So, it will be 8 trials, run will be 8 over here. So, minimum number of trials.

(Refer Slide Time: 04:51)

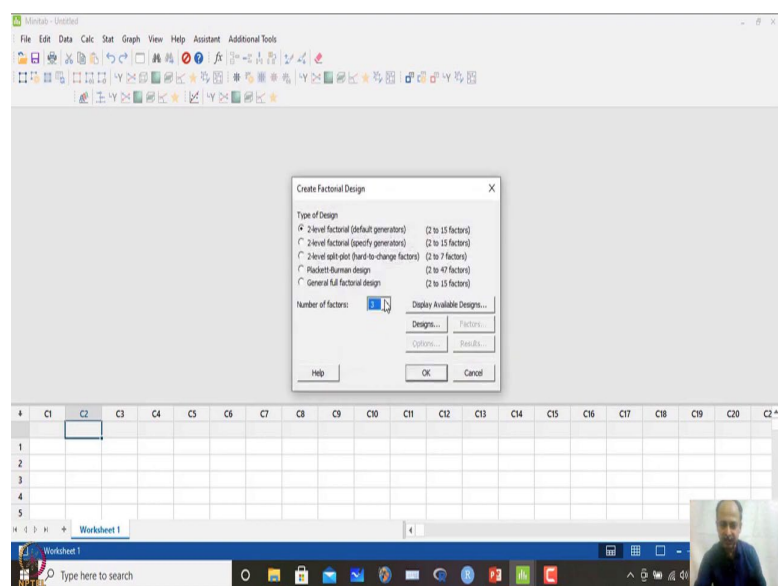


So, design will be over here. So, in this case number of center points we do not change this one, so let us keep as it is. So, a number of replicates we have 2 replicates over here in the corner points, and we assume that the default values of a number of blocks is taken over here. We will discuss about blocking up to just after this one. So, we are assuming that this is 1. So, by default we will treat this as 1 over here.

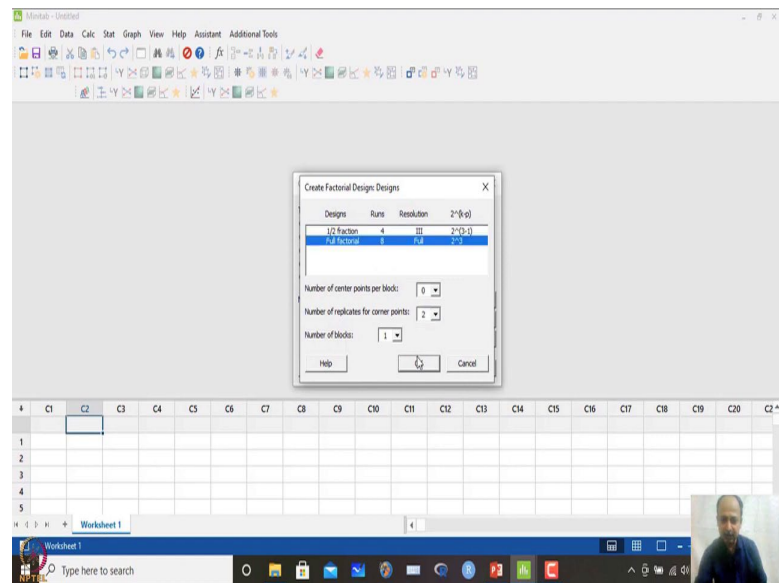
(Refer Slide Time: 05:15)



(Refer Slide Time: 05:21)

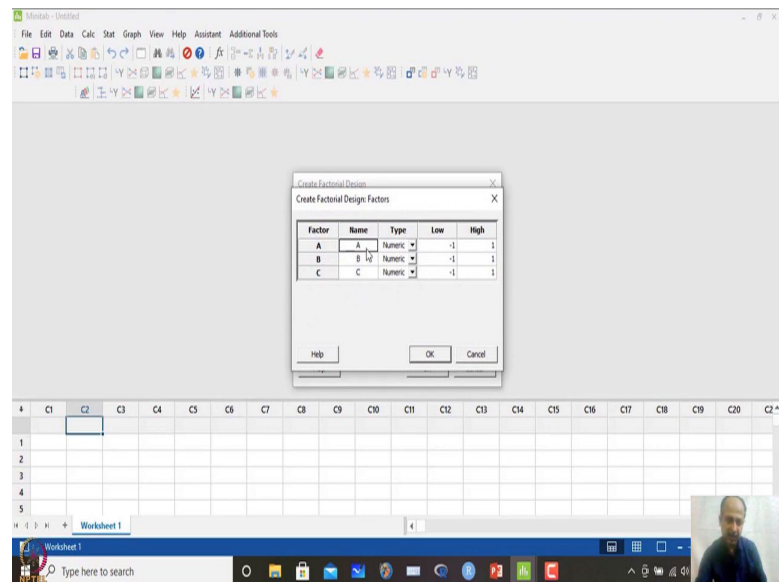


(Refer Slide Time: 05:24)



So, I click ok over here and then factors we can just name these factors over here. Sorry, this is number of factor is 3 over here. So, this we have this is 3 and number of replicates over here is 2 like that. So, in this case full factorial, we have to click on full factorial over here and then click ok.

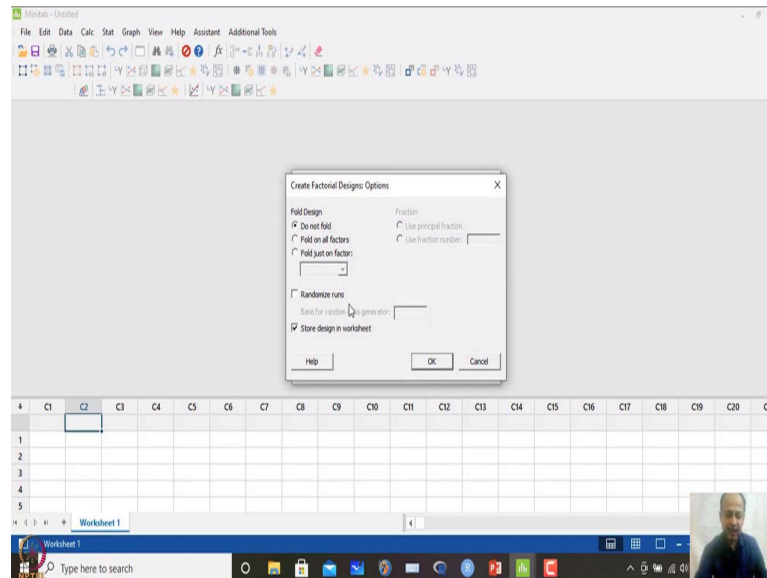
(Refer Slide Time: 05:31)



And then in factors we can assign the names over here, we can write names over here. And we can write the levels also. So, it is already coded, so I am using coded variables. All are numeric values that we are assuming over here and all are continuous variable

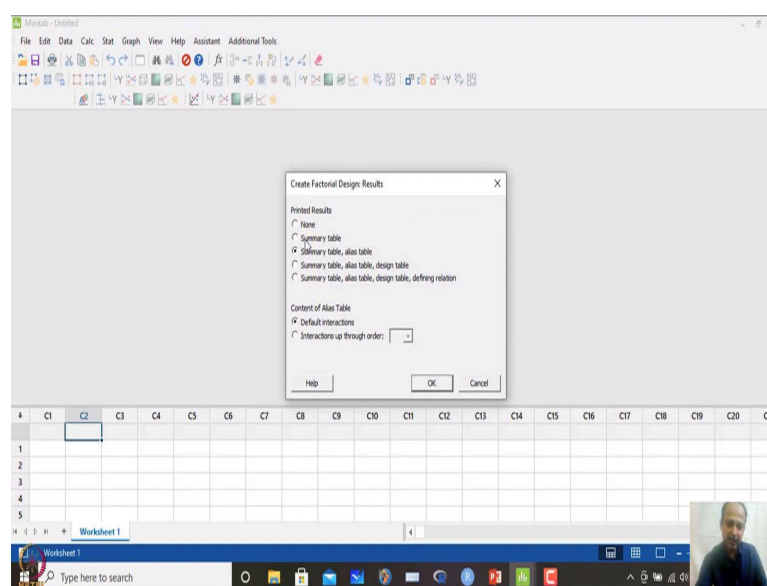
that is why this numeric variable we are assuming. So, this is already done and that factor already is mentioned.

(Refer Slide Time: 05:53)

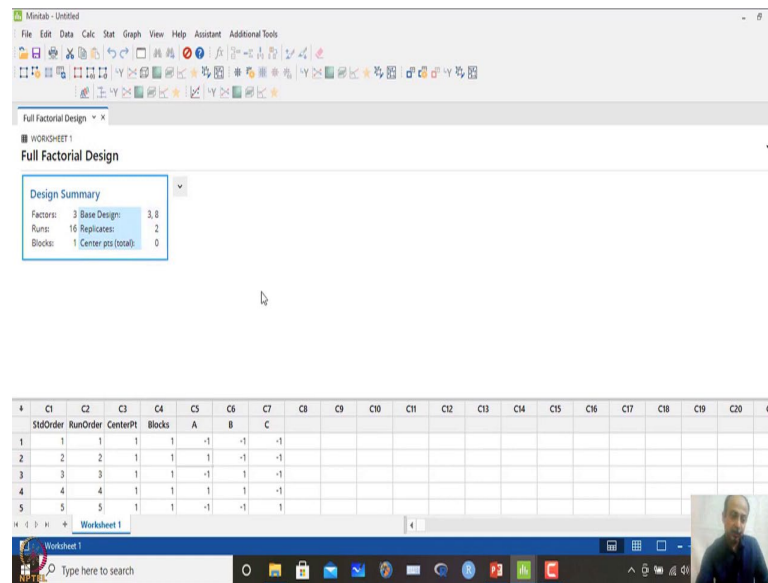


And we do not want to randomize, so we want to see the trial as per the as per the nomenclature that is followed in design of experiments. So, I am not randomizing this one. So, for your simplicity to understand. So, in this case what I will do is that I will click ok over here.

(Refer Slide Time: 06:07)



(Refer Slide Time: 06:19)



The screenshot shows the Minitab 'Full Factorial Design' dialog box with the following settings:

- Factors: 3 Base Designs: 3, 8
- Runs: 16 Replicates: 2
- Blocks: 1 Center pts (total): 0

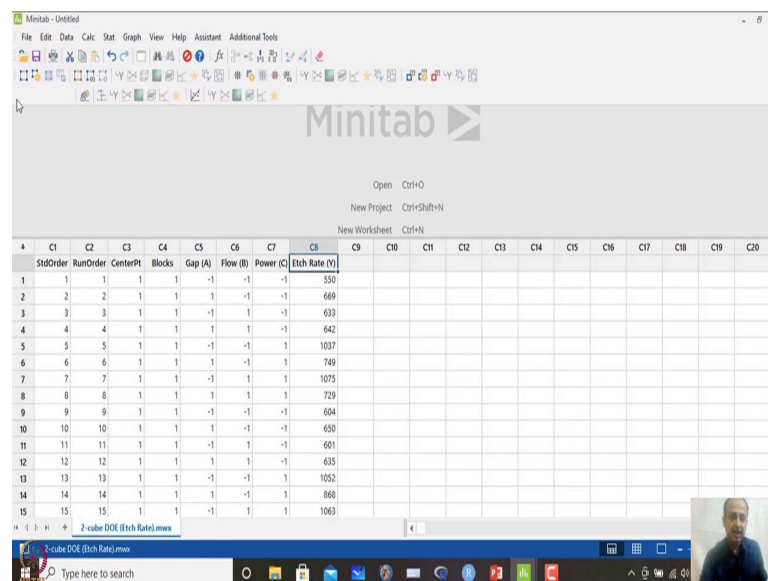
Below the dialog box, the design matrix is displayed in a worksheet. The matrix has 16 rows (trials) and 20 columns (factors and responses). The first 7 columns are labeled C1 through C7, and the last column is labeled G. The data is as follows:

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	G
StdOrder	RunOrder	CenterPt	Blocks	A	B	C															
1	1	1	1	1	-1	-1	-1														
2	2	2	1	1	1	-1	-1														
3	3	3	1	1	-1	1	-1														
4	4	4	1	1	1	1	-1														
5	5	5	1	1	-1	-1	1														

And then results what we can keep default, whatever is there. So, we can only summarize, because alias structure, alias table we have not studied. So, in that case it is not required. So, we will click ok over here and then click ok, so the design will be created over here and the design matrix is given over here.

So, factor 3 factors, total number of run is 16 over here, number of block is 1, center point 0, replicates 2 over here. So, 2 cube design basically 2 cube design over here. So, this is the matrix and you will have 16 trials that is shown over here.

(Refer Slide Time: 06:37)



The screenshot shows the Minitab '2-cube DOE (Etch Rate).mwx' worksheet. The data is as follows:

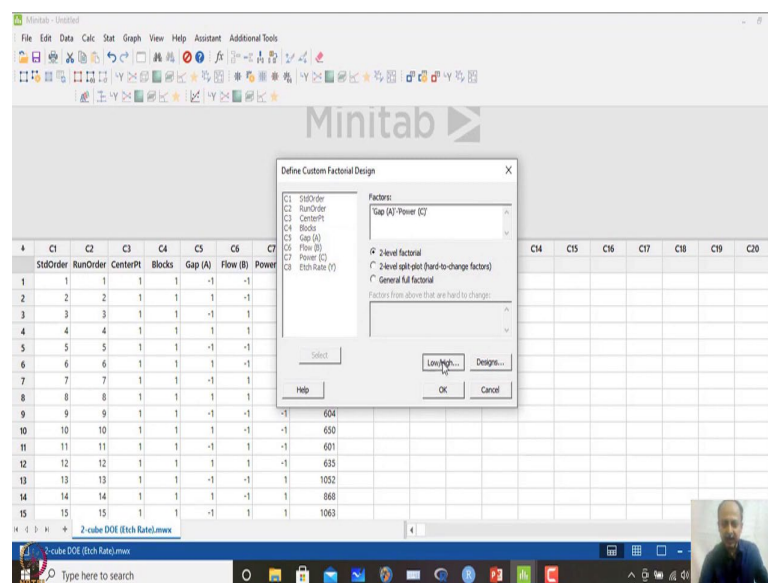
	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
StdOrder	RunOrder	CenterPt	Blocks	Gap (A)	Flow (B)	Power (C)	Etch Rate (Y)													
1	1	1	1	1	-1	-1	350													
2	2	2	1	1	1	-1	669													
3	3	3	1	1	-1	1	633													
4	4	4	1	1	1	1	642													
5	5	5	1	1	-1	-1	1037													
6	6	6	1	1	1	-1	749													
7	7	7	1	1	-1	1	1075													
8	8	8	1	1	1	1	729													
9	9	9	1	1	-1	-1	604													
10	10	10	1	1	1	-1	650													
11	11	11	1	1	-1	1	601													
12	12	12	1	1	1	-1	635													
13	13	13	1	1	-1	-1	1052													
14	14	14	1	1	1	-1	1	888												
15	15	15	1	1	-1	1	1063													

So, then what you do is that you just you have run the trials and this combination what was the value of Y that is already noted down. So, maybe this is Y1 we can mention and the second replicates we can be Y2 we can mention this one and note down the variable. So, the sorry this is not to be noted because we have already taken care of replicates.

So, one set of 8 experimental trial will be up to this point. So, one set will be like this and the next set will be placed over here. So, this is a complete experimental trials data can be placed over here. So, whenever you have place the data then we can analyze that one.

So, I am closing this one and already I have saved the data in MINITAB files like that. So, this is the MINITAB file where I have saved the data like that. And this is the gap that is given factor A, flow factor B and power factor C and in this is the Etch rate over here. So, in this case what we can do is that directly go to this design of experiment factorial design and we can analyze factorial design over here.

(Refer Slide Time: 07:35)



So, this it is showing that you have to mention what is the factors like that. So, I have to mention the factors over here. So, A, B and C these are the factors. I will select this factors over here 2 level factors over here.

(Refer Slide Time: 07:46)

Define Custom Factorial Design: Low/High

Low and High Values for Factors

Factor	Name	Type	Low	High
A	Gap (A)	Numeric	-1	1
B	Flow (B)	Numeric	-1	1
C	Power (C)	Numeric	-1	1

Worksheet Data Are

☐ Coded

☒ Uncoded

Help OK Cancel

(Refer Slide Time: 07:52)

Define Custom Factorial Design

Factors:

Gap (A) Power (C) Batch Rate (C)

☒ 2-level factorial

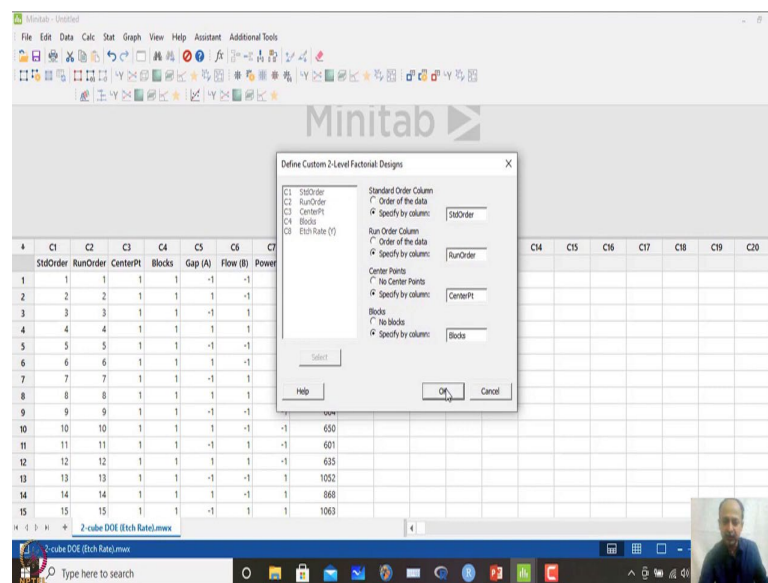
☐ 2-level split plot (hard-to-change factors)

☐ General full factorial

Factors from above that are hard to change

Help Low/High... Design... OK Cancel

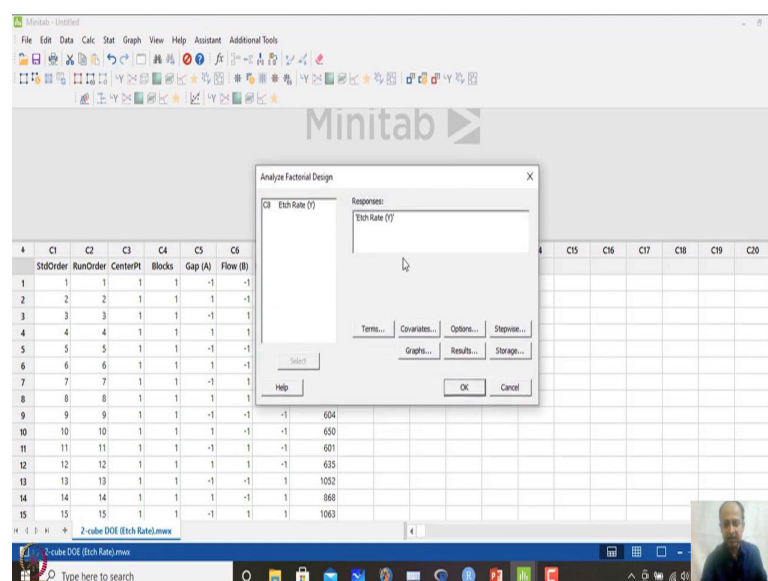
(Refer Slide Time: 07:53)



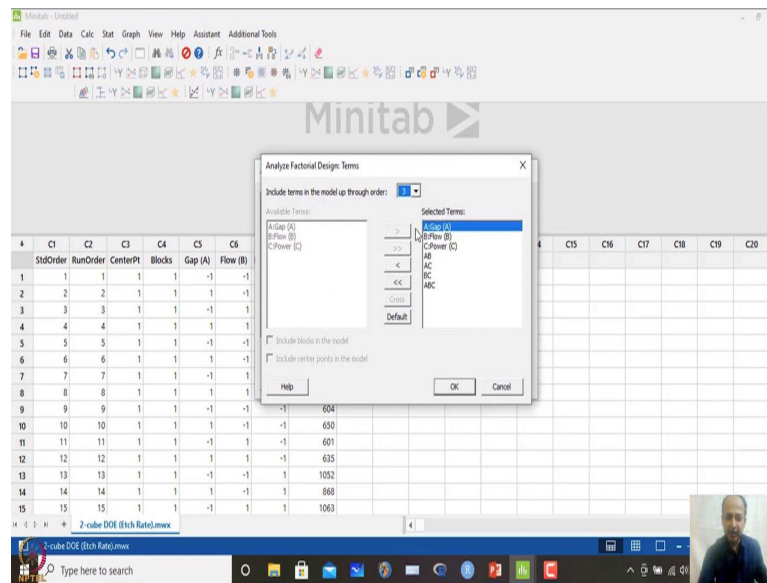
So, these this is the low level. So, this worksheet we can say that this is what, data is coded over here. So, I will place ok and design over here. So, specify the columns by. So, this is the standard order where it is mentioned. So, I have taken the standard order over here. Then run order what I will mention is that this is the run order over here.

And the last one is this over here, this is the center point where it is which column it is noted down. And blocks where the column is noted down over here. If you click this ok over here and then click ok.

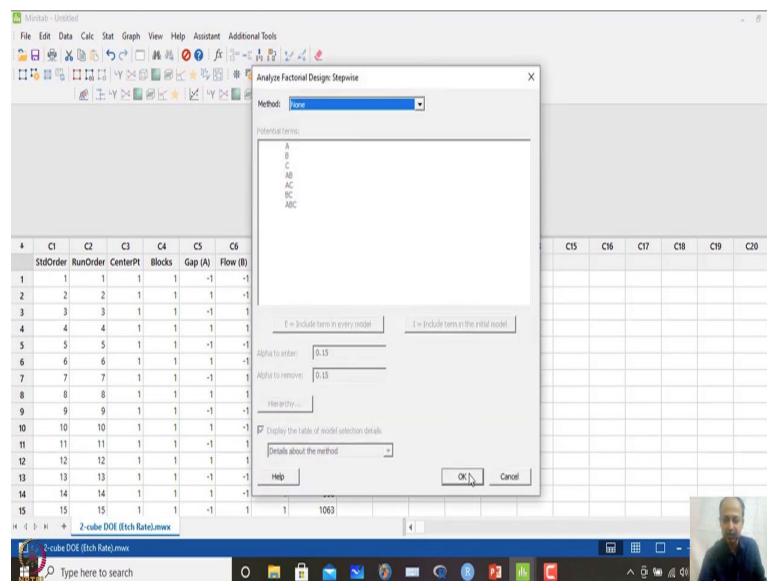
(Refer Slide Time: 08:17)



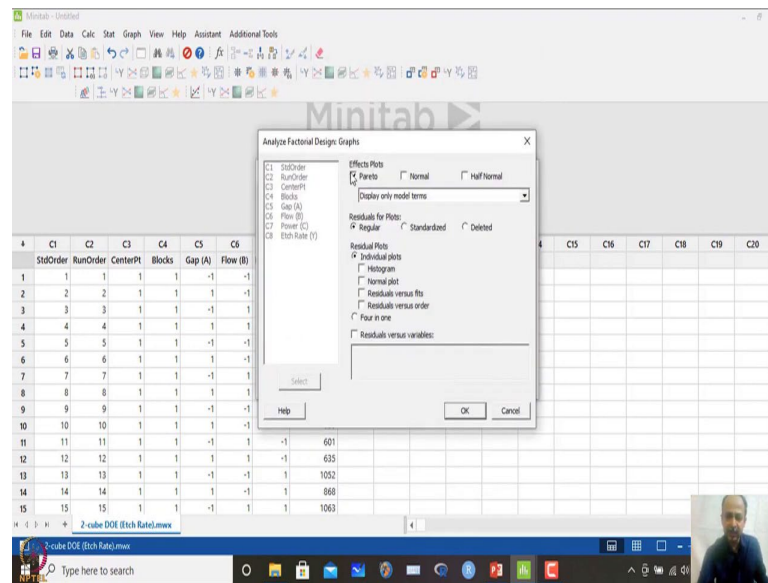
(Refer Slide Time: 08:21)



(Refer Slide Time: 08:43)



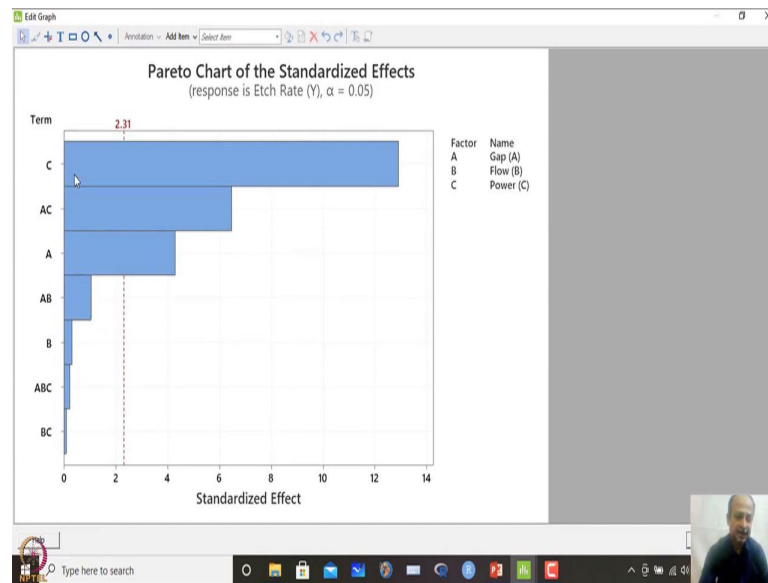
(Refer Slide Time: 08:46)



And then you mentioned what is response variable over here, then you mentioned which are the terms you want to see, so this is the include term. So, ABC all interaction up to third level we want to see over here. So, AB, AC and BC already there and ABC is the interaction effects that we want to study over here.

So, let us try to see that one, no covariates options over here, do not have to change because we are not transforming, stepwise we are not using regression over here. Stepwise regression graphically what we want to see maybe effect plots, we need a Pareto over here and normal plots over here let us try to see that one and other things we can ignore at this present moment. And then storage if you want to do later on also final model, we can store the residuals like that.

(Refer Slide Time: 09:05)

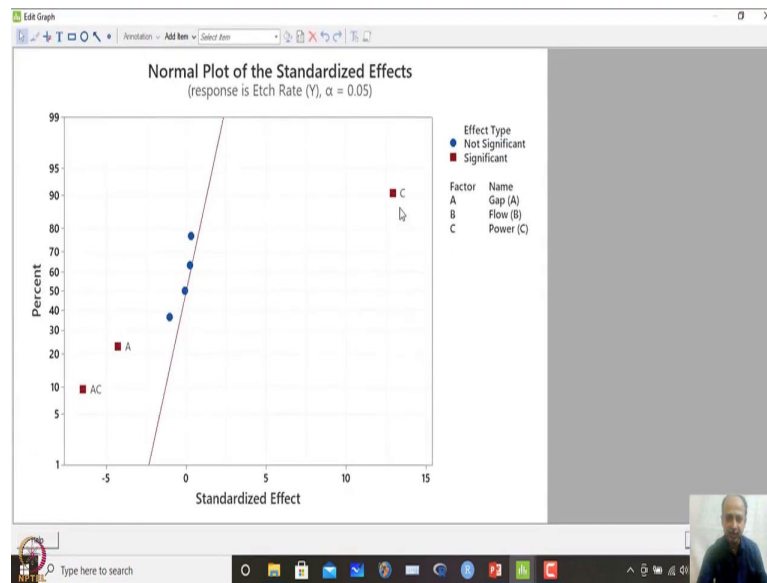


What we can do is that, we can click ok and try to see the effect plots over here and what happens let us try to see ok. So, when I have made the plot over here in the effect plots what we see is that, here A, C, and AC these are the factors what you can see over here. A, C and AC is primarily predominantly is having effects which is significant effects over here.

But other things are other interactions can be and main effects of B can be ignored over here. So, these things can be ignored over here AB, B, ABC and BC like this. Only information that we are getting from this is that A is important, C is important and AC interaction is significant that we have to consider in the subsequent while we are doing the modelling aspects of this.

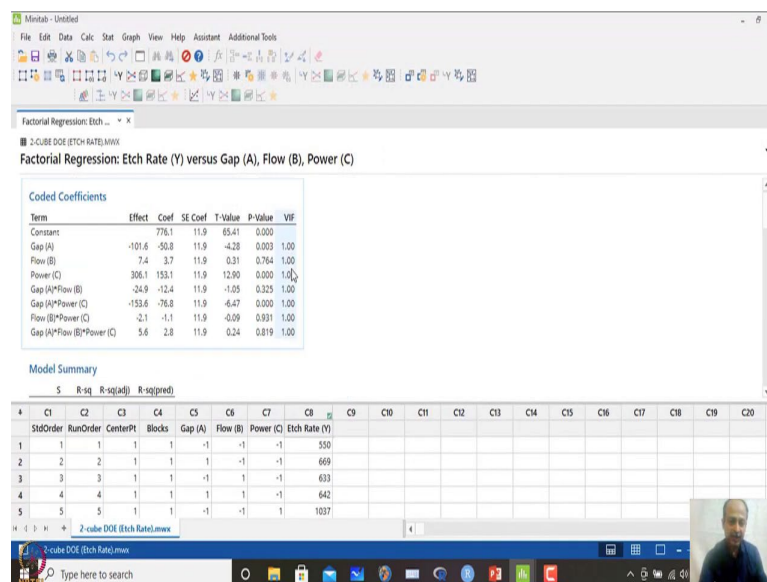
So, in this case AB can be ignored, B and ABC all higher order. So, third order terms can be ignored and this AC is the only term that needs to be considered over here.

(Refer Slide Time: 09:57)

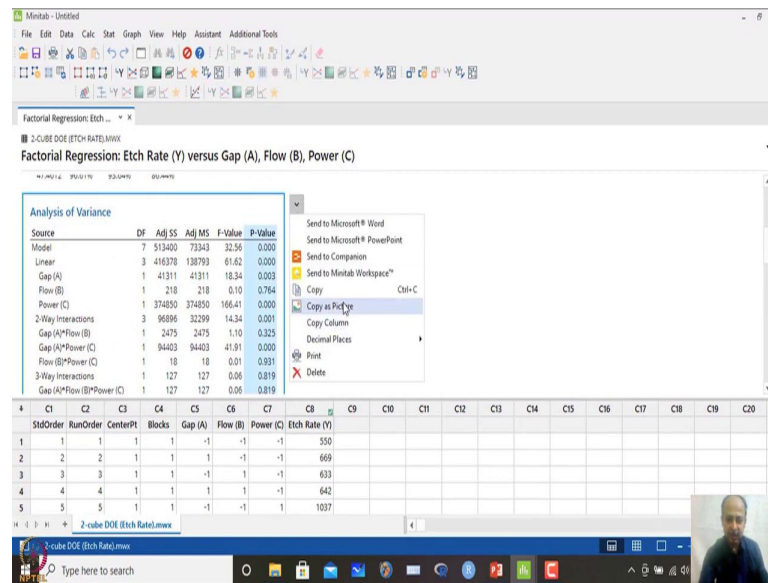


And also, this is prominent in the normal plot what you are seeing over here. So, in normal plot also you see that A is having a negative impact on the Etch rate and AC is having a negative impact, but C is having a positive impact on the Etch rate like that ok.

(Refer Slide Time: 10:18)



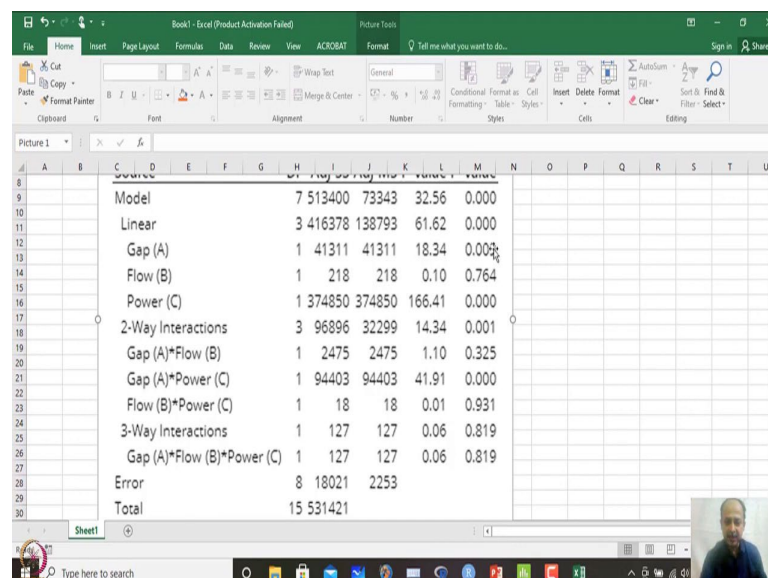
(Refer Slide Time: 10:20)



So, this is the effect that we are seeing normal plotting, normal plot we can see this one as we have explained earlier also. And what we see in this ANOVA analysis also we can just copy this as and we can paste it in excel like that. So, let us try to do that and try to see.

Enlarge this one and try to see which is significant which is not as per ANOVA analysis as per ANOVA analysis which is important which is not let us try to see that one. And in this case, we can paste this one over here.

(Refer Slide Time: 10:42)

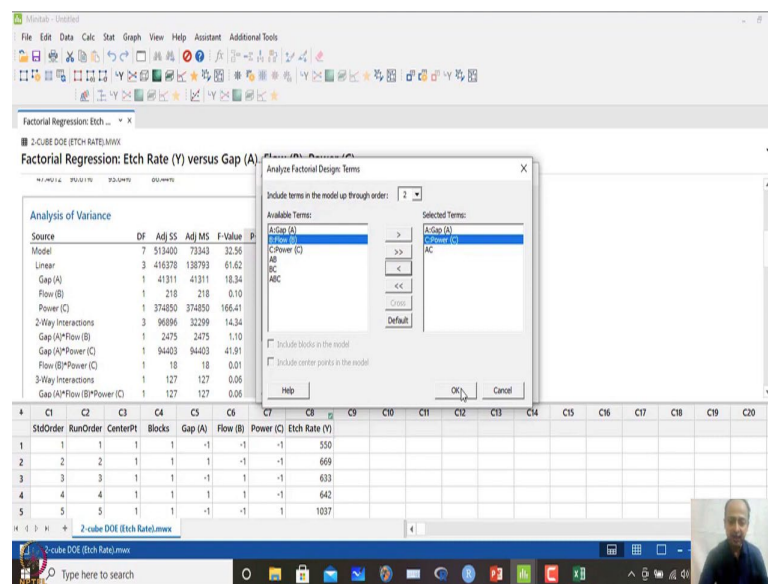


And then we can extend this one and just try to see. So, what you see over here is gap is 0.002. So, P value is 0.003 over here. So, A is significant B is 0.764. So, B is not significant, power C is significant over here 0.000. And here 2-way interactions; one is significant that is gap A and C.

This is significant over here 0.000. And third order interaction over here ABC is not significant 0.819. So, these same thing is revealed when what we have seen in the pareto plot pareto chart plots of the standardized effect, so that is also matching over here. So, in this case what we have to do is that and this is seen.

So, now we know that A, C and AC is the main important factors over here to consider. So, what we will do is that, we will just minimize the or eliminate the unnecessary terms that we are getting over here. So, analyze factorial design.

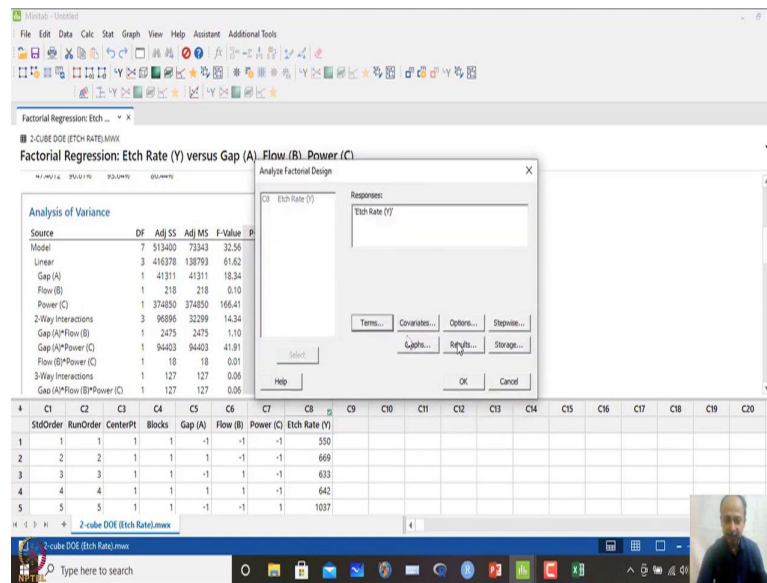
(Refer Slide Time: 11:41)



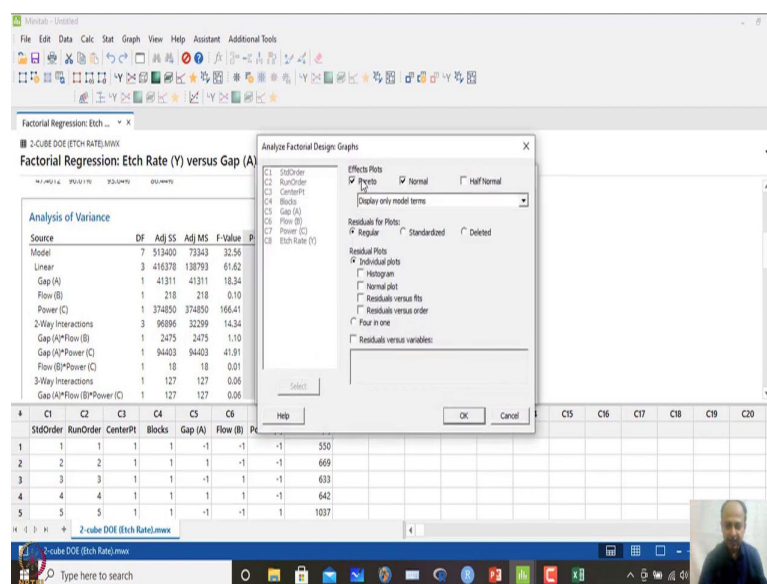
So, in this case in terms of terms what we will do is that, we will go to second order up to second order terms over here. So, here also AB is not significant we can remove this one. AC is significant. So, B can also be removed because B is not making significant impact on the overall CTQ or the response variability it is not impacting, we can remove this one.

AC and AC which is important which needs to be included in the model while prediction, while we are making the prediction out of the conditions are ok.

(Refer Slide Time: 12:11)

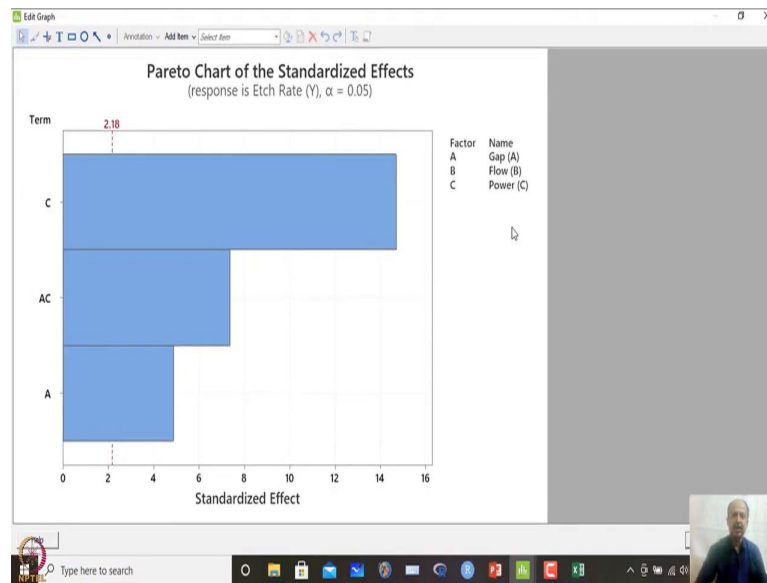


(Refer Slide Time: 12:15)

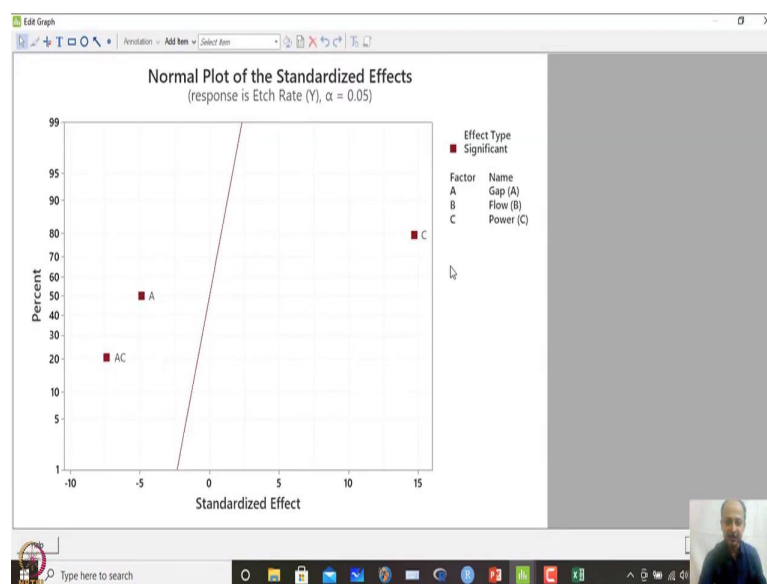


So, in this case I will place ok. So, we have done that one and again we can have the graph of pareto and normal plot over here and click this one and click ok over here.

(Refer Slide Time: 12:22)

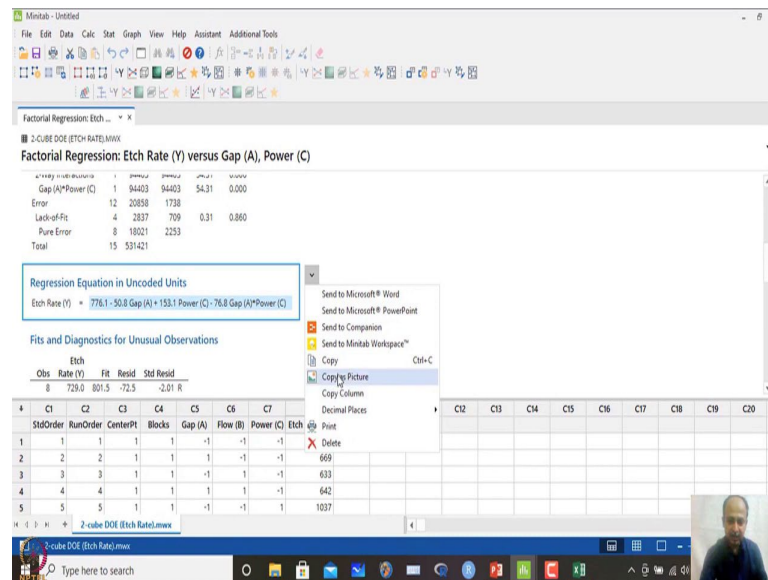


(Refer Slide Time: 12:32)



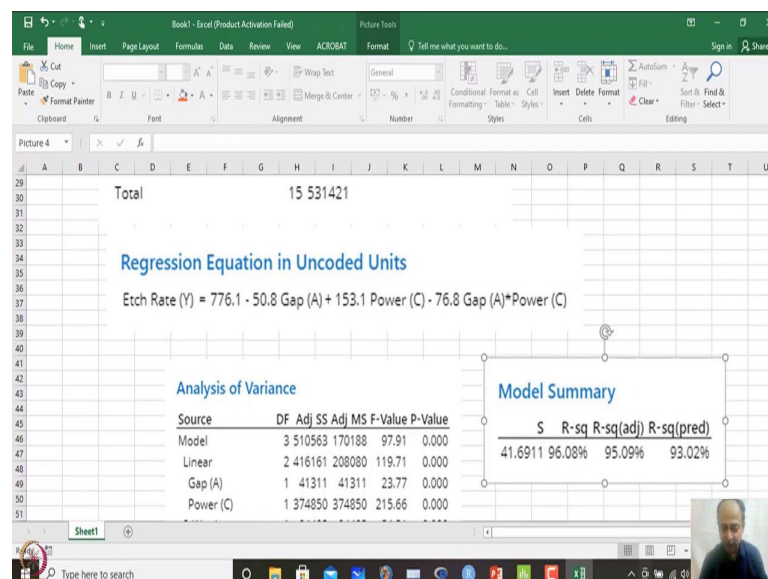
What do you observe is that, now what do you see in the pareto chart what you observe is that? All the factors are considered AC and AC interaction is considered over here and all are significant that it is showing. And also, the normal plots is also revealing the same information A, C and AC significant over here.

(Refer Slide Time: 12:40)



And when this is done you can we can see the regression equation that is developed over here. So, the regression equation copy this one and also, we can see what is also this we can paste it over here.

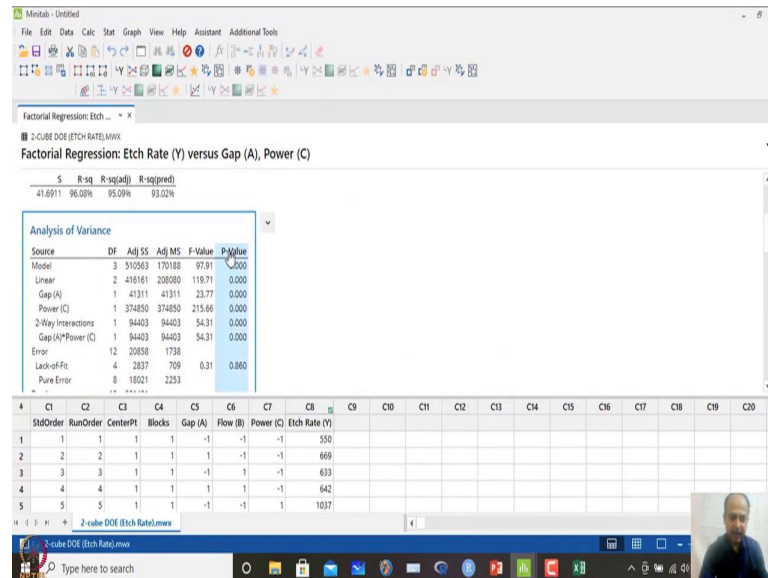
(Refer Slide Time: 12:48)



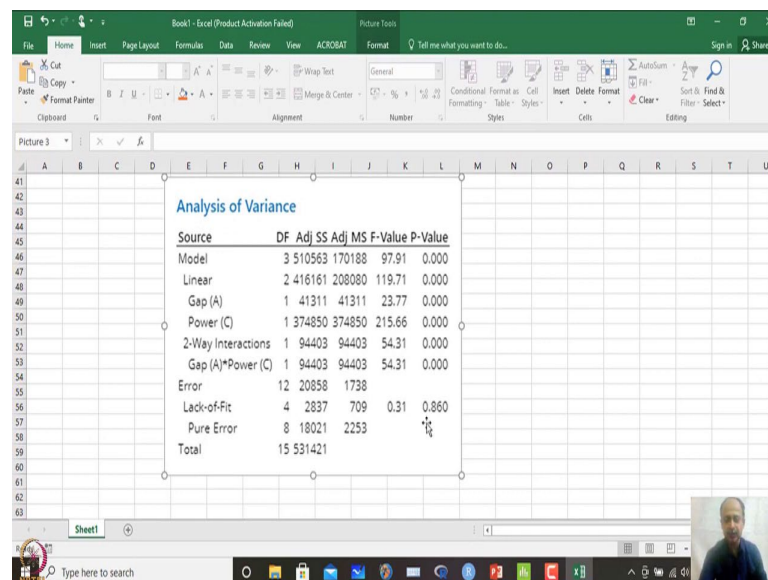
And let us try to see what is the equation. So, 7, this is a constant beta zero over here 50.8 is a positive impact A is making and the this C is having a positive impact. So, but when the interact it is having a negative impact on the Etch rate, it is having a negative impact on the Etch rate. So, gap A is having negative impact. Sorry I mentioned that one

earlier also, so this will have a negative impact. C will have a positive impact and, but AC will have a negative impact on the Etch rate like that ok.

(Refer Slide Time: 13:17)

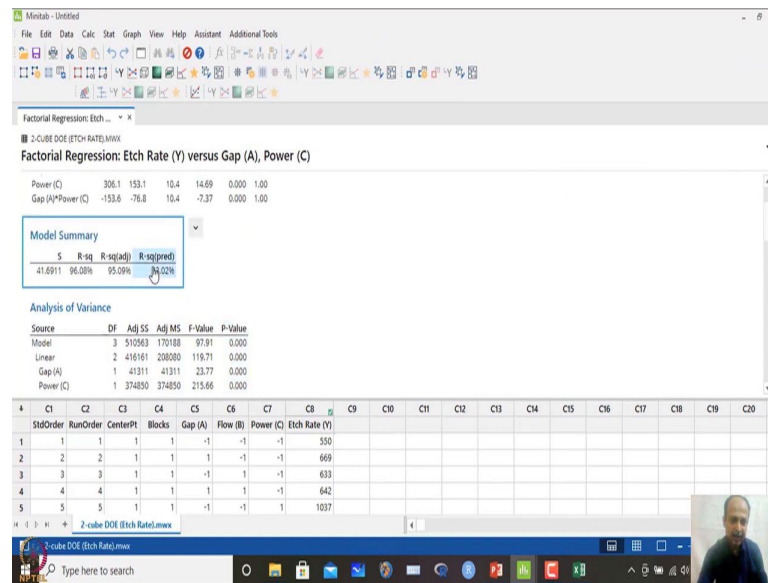


(Refer Slide Time: 13:28)



So, this is clear from this analysis what we have done and this is the after we have done reduced model. So, in this case what we can see is that, we can just place it over here and just enlarge this one and the model seems to be satisfactory, because lack of fit shows that 0.860 there is no lack of fit. So, this model seems to be adequate. And also the R square value we can check, what is the R square value of this model.

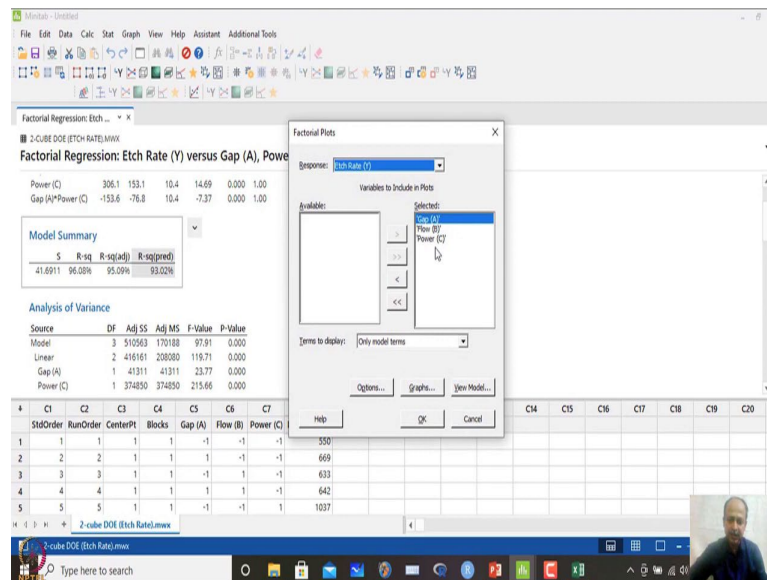
(Refer Slide Time: 13:40)



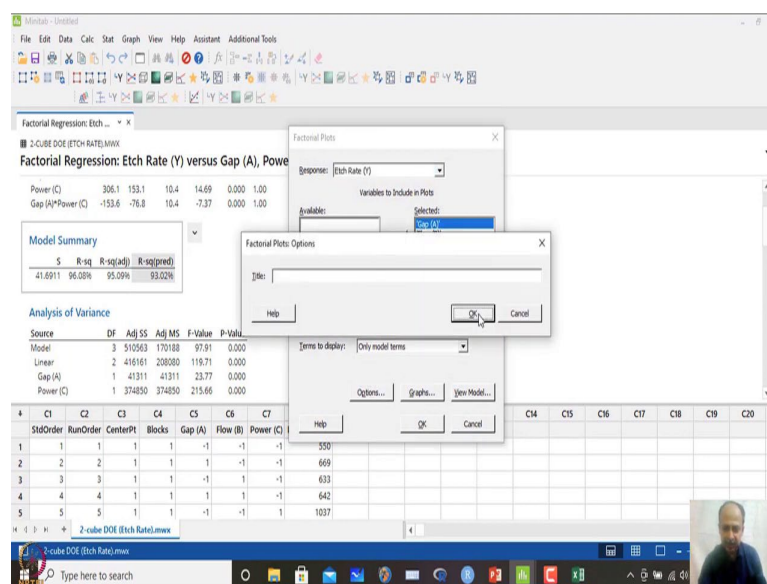
And R square predicted value is around 93 percent what we are seeing over here. So, I can copy this from here and we can paste it over here, and then enhance this one, so that it is visible to you. So, this is 93.02 which is which is very good basically, which is very good over here. So, now I have eliminated some of the factors which is unnecessary over here which is B over here.

So, prediction model is developed based on A, C and AC interactions like that ok. So, then what you can do is that you can just see the factor plots over here. So, I can have a factor plots over here.

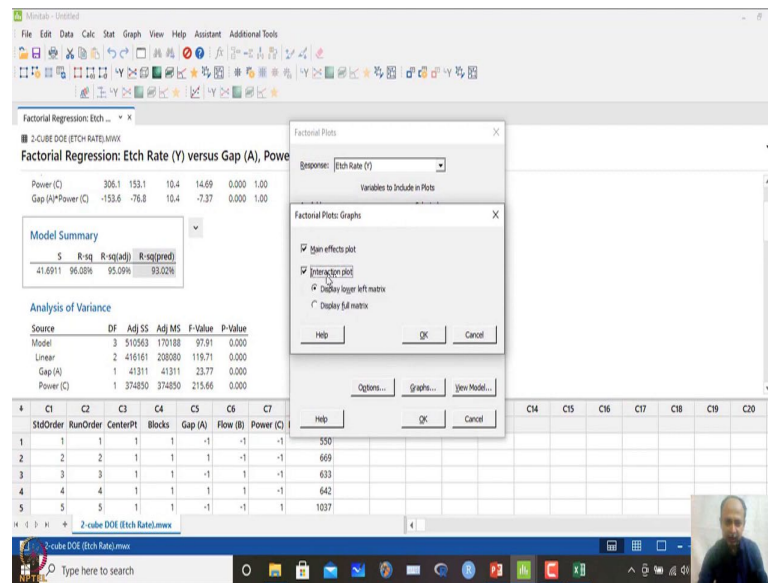
(Refer Slide Time: 14:14)



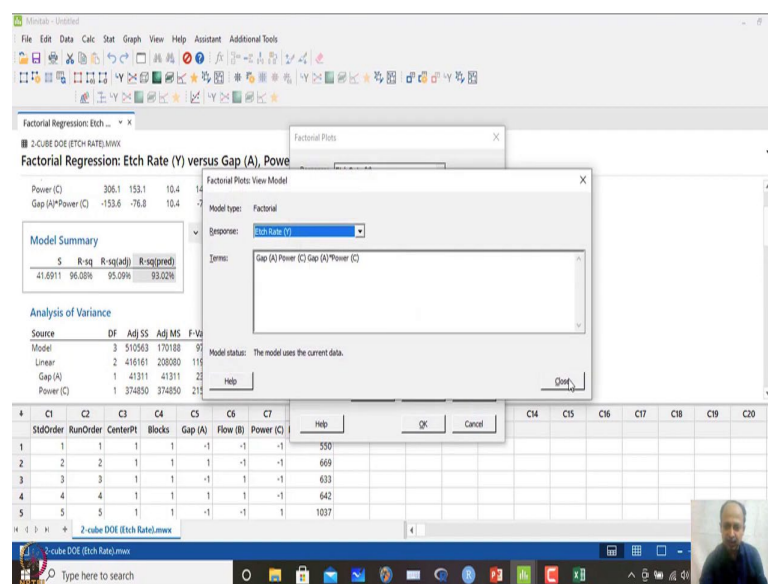
(Refer Slide Time: 14:17)



(Refer Slide Time: 14:20)

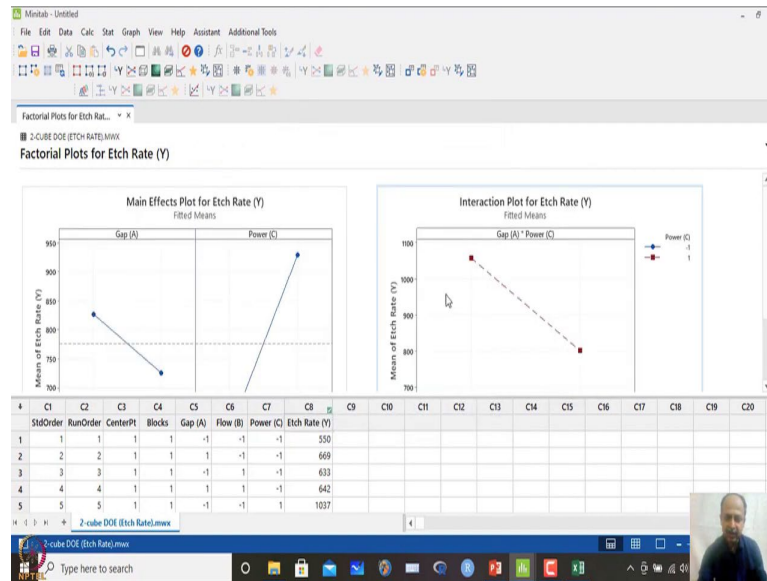


(Refer Slide Time: 14:25)

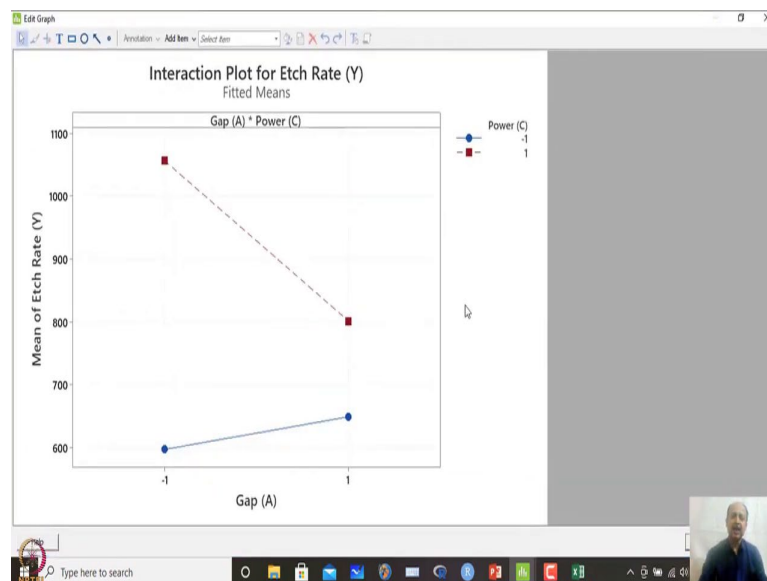


So, in this case we will include etch rates over here and options is that this is in this case. I want interaction plot to be visible over here, main effect plot and interaction plots. So, in this case view the model AC and AC, these are the models that is considered.

(Refer Slide Time: 14:29)



(Refer Slide Time: 14:31)



So, if we click ok over here then you will find that AC interaction is shown over here. So, interaction plot is given, and in this case, you can understand that to maximize the CTQ, let us see etch rate over here. This is the point I have to look it over here and this is corresponding to gap minus 1. So, A should be in negative level and C should be in positive levels.

So, minus 1 and C is plus 1 that is the that is the condition that we should be. For A and C that is the condition which will maximize the etch rate basically. Then somebody can

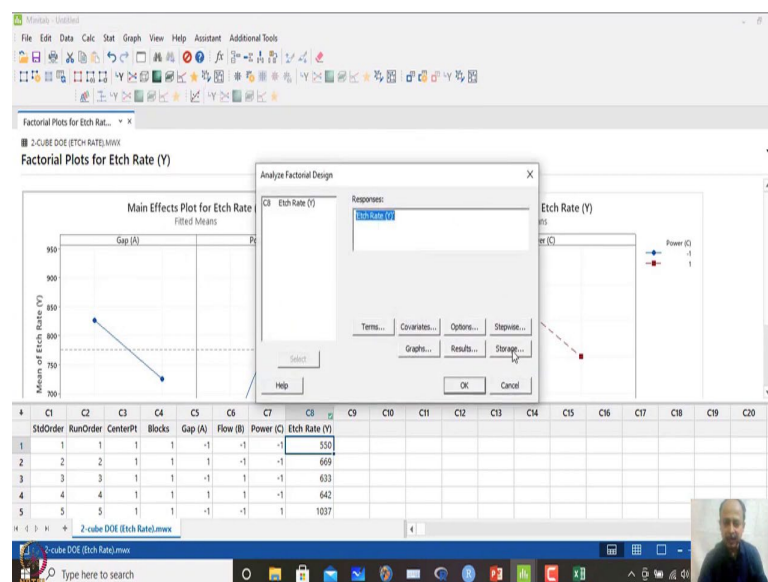
ask what should be the setting of B then what will be the setting of B then, because B is not included in the model. So, B can take any values and I told earlier also that it should be based on cost information that should be considered as a priority over here.

You can include B in the models also, but it will not have significant impacts like that. And if it does not increase the R square predicted value we can we do not need to consider that one. So, but we have to set the B B correct B B factor to certain levels over here.

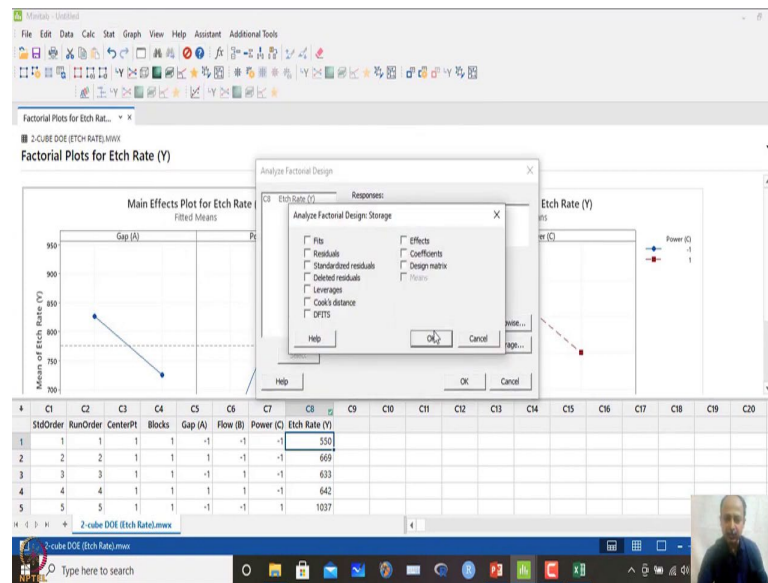
So, what we will do is that, we will set, will select the that level only which is having lower cost which is having lower cost like that. So, A will be set to the, A will be set at the negative level over here. So, A at low level and C will be set at plus level over here, B can be selected as minus or plus based on the cost information.

So, that will be the optimal combination that is to be used while we are setting the process like that while we are setting the process like that. That will be the condition we have to we have to consider over here ok. And we can whenever we have done that one. So, this is the models that is there.

(Refer Slide Time: 16:14)

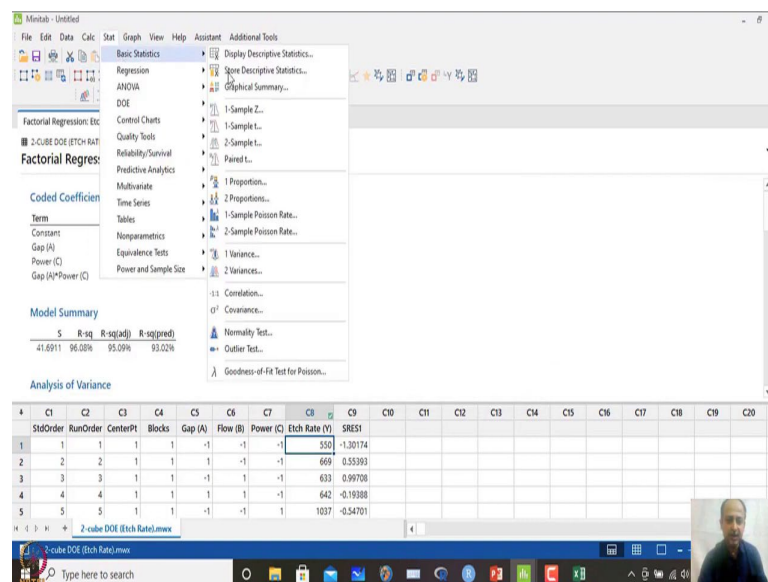


(Refer Slide Time: 16:15)

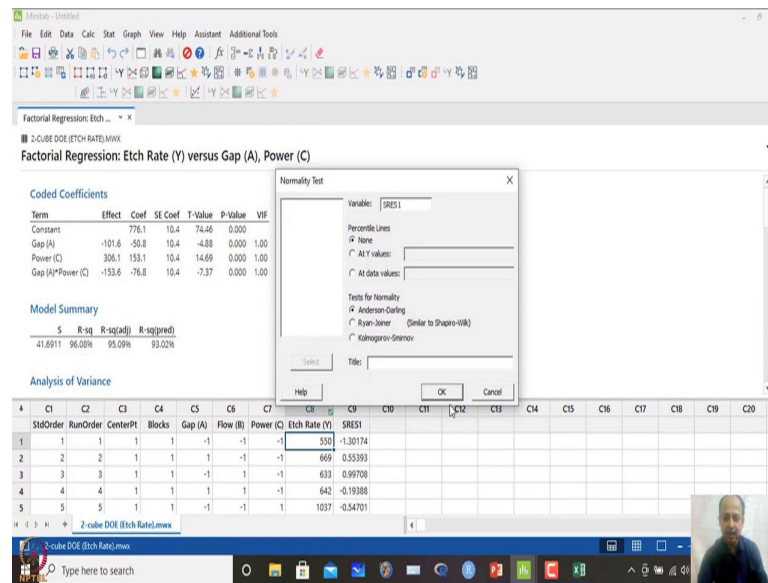


So, immediately what we can do is that design of experiment factorial, we can also save the residuals over here. And in case of storage what we will do standardized residual, just to make a check that everything is fine.

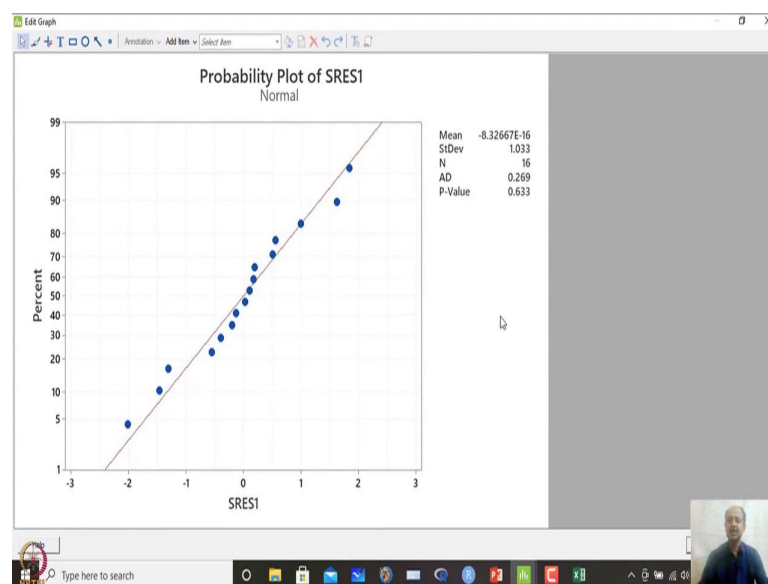
(Refer Slide Time: 16:20)



(Refer Slide Time: 16:23)



(Refer Slide Time: 16:26)



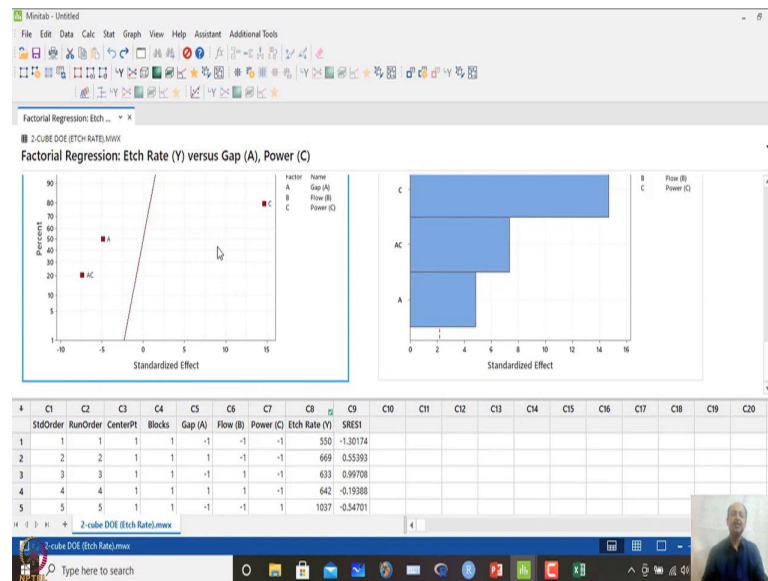
So, last column will be basic standard residual plots over here. So, I can use the residual, and I can just check Anderson Darling test and what we are observing over here in Anderson Darling test, is that the P value is more than 0.05. So, in this case normal normality assumptions is not validated at least.

So, these things and other checks also we have mentioned that we can see ok. Those things needs to be clarified before we implement the models and before we control the process like that ok. But this is preliminary experiments we are trying to assess which are

the factors to be considered like that, this is not complete optimization that we are doing over here.

But if you think that linear model is sufficient, we and this is the this is the working operating range of the process like that. So, within this what is optimal? This is the optimal condition that A at minus 1, C at plus 1 and B can be any level based on cost. So, that is a combination that we should look for ok.

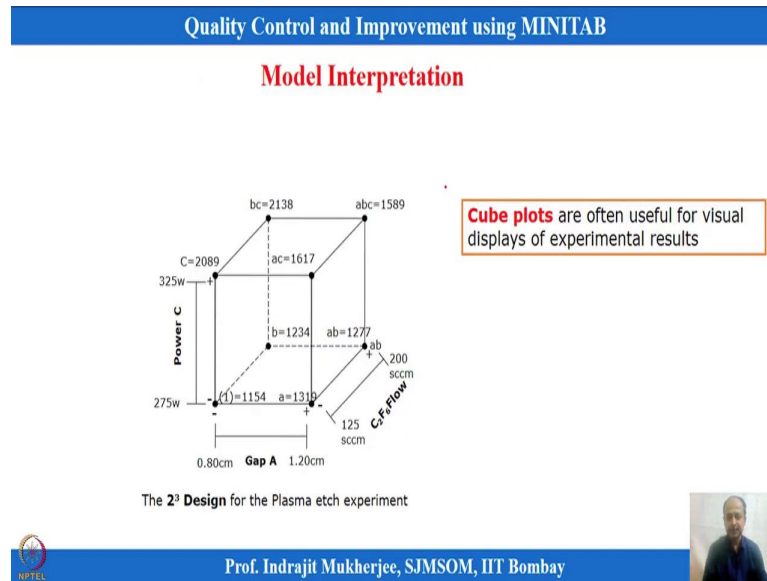
(Refer Slide Time: 17:18)



So, important aspects is that we have to we have to consider this pareto diagram and the normal plot which will give me indication that which are the factors to be considered, which are the factors to be removed like that and based on which we can take a decision about the setting conditions of the process like that ok.

So, this is replicated, this is replicated design over here and there are 3 factors 2 cube design that we have discussed over here. And let us go beyond this 2 cube model that we are having. So, this is the this is the experimentation that we have shown. So, this is the replicates 1 and 2 this data set we are using over here, these are the data sets that we are using. This is replication 1 and replication 2.

(Refer Slide Time: 17:52)



And these are the experimentation block design that we have told cubic view of the design like that and the values of experimentation is given over here.

(Refer Slide Time: 18:06)

Quality Control and Improvement using MINITAB

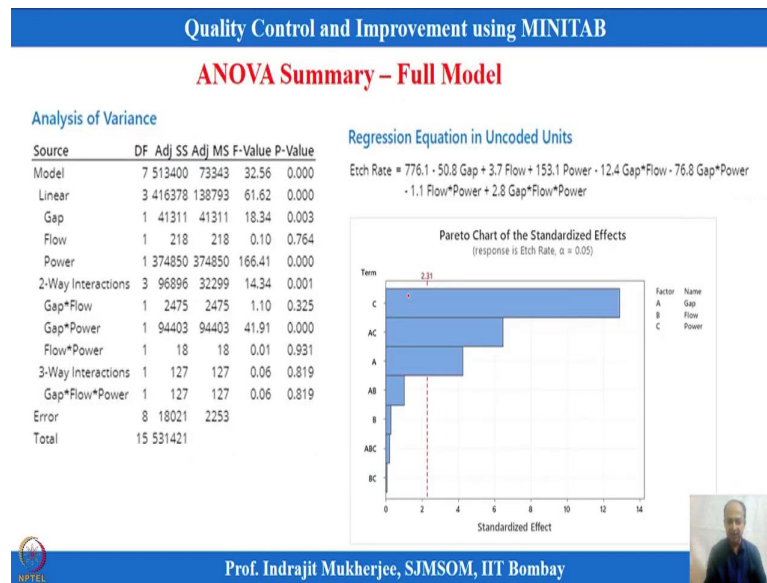
Estimation of Factor Effects

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		776.1	11.9	65.41	0.000	
Gap	-101.6	-50.8	11.9	-4.28	0.003	1.00
Flow	7.4	3.7	11.9	0.31	0.764	1.00
Power	306.1	153.1	11.9	12.90	0.000	1.00
Gap*Flow	-24.9	-12.4	11.9	-1.05	0.325	1.00
Gap*Power	-153.6	-76.8	11.9	-6.47	0.000	1.00
Flow*Power	-2.1	-1.1	11.9	-0.09	0.931	1.00
Gap*Flow*Power	5.6	2.8	11.9	0.24	0.819	1.00

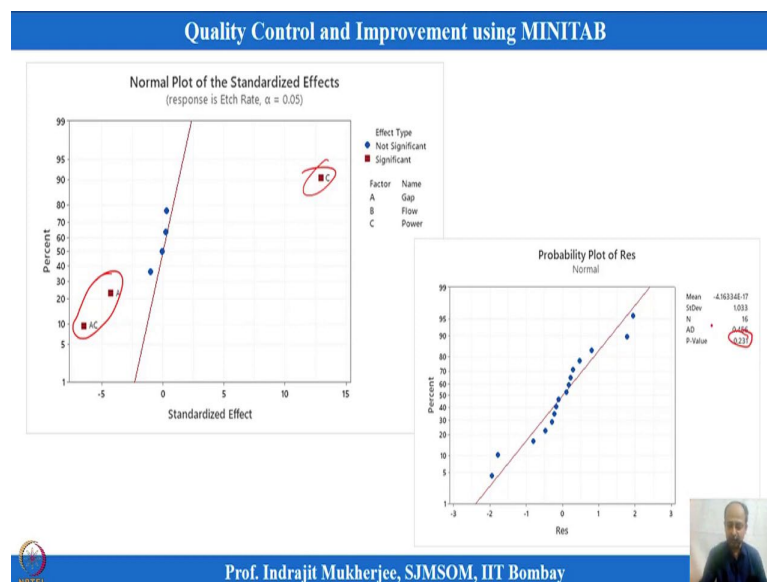
Prof. Indrajit Mukherjee, SJMSOM, IIT Bombay

(Refer Slide Time: 18:10)



So, these are the values that is experimentation that is done, ABC this is a cube plot. So, this analysis we have shown a coded variable, this is we have shown like that. So, A, C and AC this is significant over here.

(Refer Slide Time: 18:15)



(Refer Slide Time: 18:32)

Quality Control and Improvement using MINITAB

Model Coefficients – Reduced Model

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		776.1	10.4	74.46	0.000	
Gap	-101.6	-50.8	10.4	-4.88	0.000	1.00
Power	306.1	153.1	10.4	14.69	0.000	1.00
Gap*Power	-153.6	-76.8	10.4	-7.37	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
41.6911	96.08%	95.09%	93.02%

Regression Equation in Uncoded Units

Etch Rate = 776.1 - 50.8 Gap + 153.1 Power - 76.8 Gap*Power

Analysis of Variance

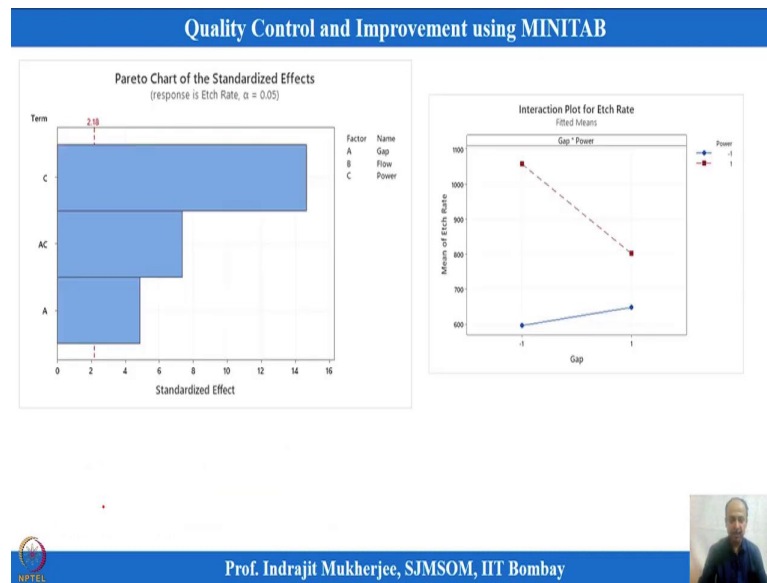
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	510563	170188	97.91	0.000
Linear	2	416161	208080	119.71	0.000
Gap	1	41311	41311	23.77	0.000
Power	1	374850	374850	215.66	0.000
2-Way Interactions	1	94403	94403	54.31	0.000
Gap*Power	1	94403	94403	54.31	0.000
Error	12	20858	1738		
Lack-of-Fit	4	2837	709	0.31	0.860
Pure Error	8	18021	2253		
Total	15	531421			

Prof. Indrajit Mukherjee, SJMSOM, IIT Bombay

So, then we have revised the models and based on that we see only A and AC is prominent over here and C is prominent over here. This is finally considered in the model and the P value of this is more than 0.05. So, it follows normal distribution, so there is no problem in that. So, this is the final model over here and lack of fit what we mentioned over here is more than.

And R square predictor is also quite good, so this model can be used and this is the regression model that can be used for prediction like that. So, this is the regression model over here that we are considering. So, final regression model which can be considered for prediction over here.

(Refer Slide Time: 18:47)



(Refer Slide Time: 18:52)

Quality Control and Improvement using MINTAB

2⁵ Full Factorial Design (without replicates)

- Number of Factors : 5
- Number of Levels : 2
- Possible Runs : 2⁵
- Min number of experiments : (32)✓

$2^5 = 32$ ✓

Prof. Indrajit Mukherjee, SJMSOM, IIT Bombay

So, this is the interaction plot that also I have shown to you and let us go to more complications in the factorial design. So, now, we are dealing with a scenario where we have 5 factors more than 3 factors. Now we have gone to 5 factors over here and each at 2 levels like that.

So, number of factor is 5 number of level is 2. So, it will be 2 to the power 5 experimentation. So, in this case minimum number of trials that is required is 32. So, minimum number, as we increase factors and levels what will happen is that, we will

have more number of trials that is required for experimentation basically ok. Here it is 2 to the power 5 means it is 32; 32 number of trials minimum is required and if you replicate once it will be 32 into 2.

So, this will be two replications if we consider. 64 trials that is not that is quite large number that is quite large number over here. So, in this case sometimes what happens is that? I do not want to do 64 experimentation, I do not I have to confine my analysis with only one replicates that or no replicates basically.

So, I can do up to 32 trials, but I cannot do 64 trials, because a huge amount of cost is involved over here. So, this is a single replicate experimentation that I am showing you how to analyze single replicate design like that. So, this is 2 to the power 5 without replications like that. So, this is no replication basically. Single replicates means no replicate basically over here.

(Refer Slide Time: 20:13)

S.No.	A	B	C	D	E	Avg. Strength	S.No.	A	B	C	D	E	Avg. Strength
1	-1	-1	-1	-1	-1	680.45	17	-1	-1	-1	-1	1	607.34
2	1	-1	-1	-1	-1	722.48	18	1	-1	-1	-1	1	620.8
3	-1	1	-1	-1	-1	702.14	19	-1	1	-1	-1	1	610.55
4	1	1	-1	-1	-1	666.93	20	1	1	-1	-1	1	638.04
5	-1	-1	1	-1	-1	703.67	21	-1	-1	1	-1	1	585.19
6	1	-1	1	-1	-1	642.14	22	1	-1	1	-1	1	586.17
7	-1	1	1	-1	-1	692.98	23	-1	1	1	-1	1	601.67
8	1	1	1	-1	-1	669.26	24	1	1	1	-1	1	608.31
9	-1	-1	-1	1	-1	491.58	25	-1	-1	-1	1	1	442.9
10	1	-1	-1	1	-1	475.52	26	1	-1	-1	1	1	434.41
11	-1	1	-1	1	-1	478.76	27	-1	1	-1	1	1	417.66
12	1	1	-1	1	-1	568.23	28	1	1	-1	1	1	510.84
13	-1	-1	1	1	-1	444.72	29	-1	-1	1	1	1	392.11
14	1	-1	1	1	-1	410.37	30	1	-1	1	1	1	343.22
15	-1	1	1	1	-1	428.51	31	-1	1	1	1	1	385.52
16	1	1	1	1	-1	491.47	32	1	1	1	1	1	446.73

Example
An experiment to study the effect of machining factors on ceramic strength was conducted based on Factorial design. Five factors are considered at two levels each: A : Table Speed, B: Down Feed Rate, C: Wheel Grit, D: Direction, E: Batch. The noted response is the average of the ceramic strength over 15 replications. The following data can be considered as single replicate of a 2^5 factorial design.

Source: Montgomery, D. C. (2005). Applied statistics and probability for engineers. John Wiley & Sons.

Prof. Indrajit Mukherjee, SJMSOM, IIT Bombay

So, that experimental trial is given over here. So, there are factors A, B, C, D and E over here. And this is we are trying to see the machining factors which impact strength over here, ceramic strength over here and the factors are considered as speed feed, wheel, grit, this is a grit size over here.

So, in this case this is the direction movement over here. So, this is 2 levels we have considered and E is batch which is considerable. So, these are 5 factors that is

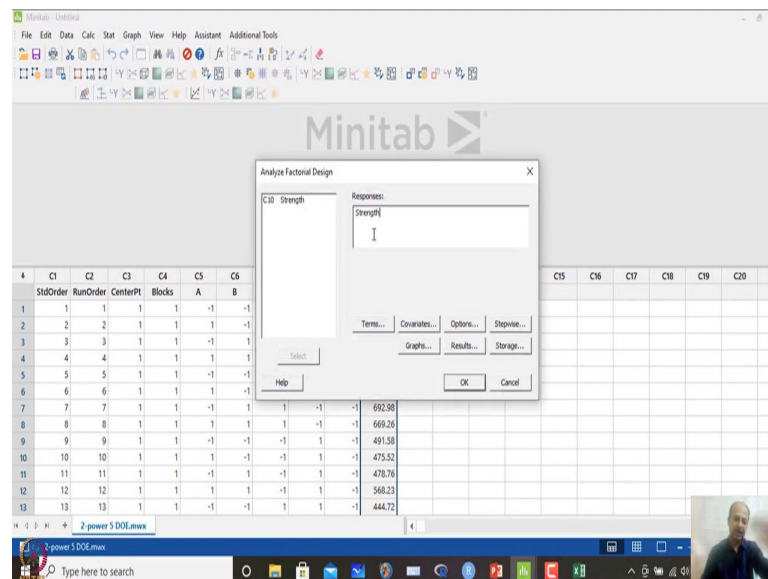
considered, some are continuous, some are discrete over here, some are categorical over here. So, in this case these are the factors that is considered and ceramic strength that the Y condition CTQ is continuous over here.

This is the average strength that is noted down over here and these are single observations that we have. So, this is 32 trials we have information over here, starting from 1 to 32 and the information of Y characteristics is also noted down. So, these are the Y characteristics which is only one observation we are having that is the average strength over here; 2 to the power 5 design.

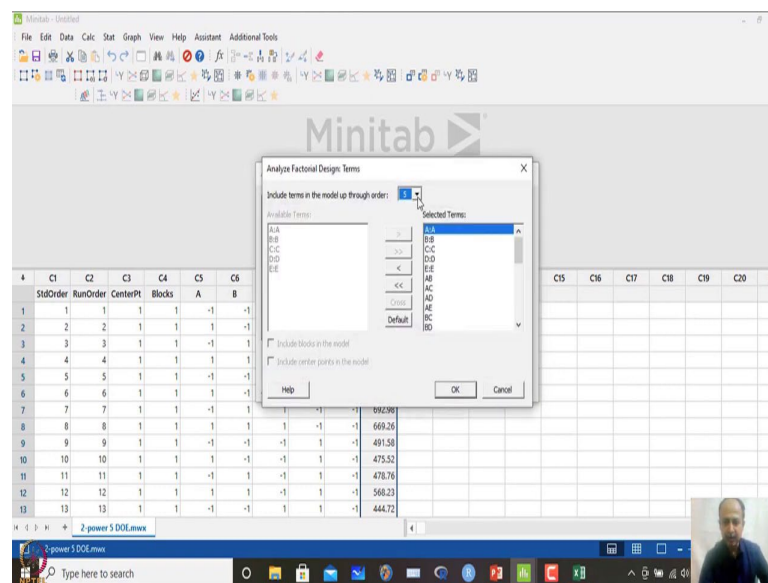
And we have we can create the design 2 to the power 5 design, using the same matrix see from a MINITAB we can generate that one. So, when we have 5 factors at 2 levels. So, in that case that is easy. So, this is taken from Montgomery's book example over here. So, how to analyze this one that is important for us.

So, in this case I will just go to the analysis part of this. So, what we can do is that, these are the 5 factors what we are seeing over here. This is the already created design matrix and based on that strength is the characteristics what we want to see over here.

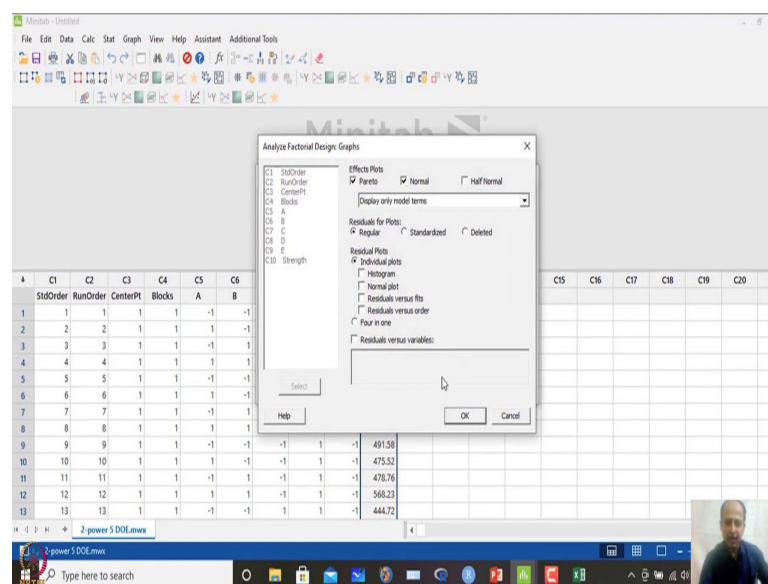
(Refer Slide Time: 21:58)



(Refer Slide Time: 22:04)



(Refer Slide Time: 22:12)



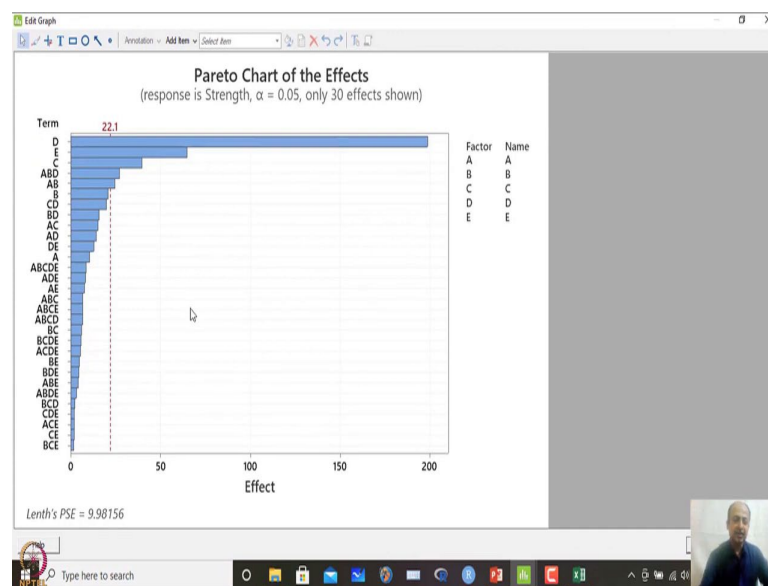
So, in this case what we will do is that STAT DOE factorial design and analyze factorial design what we will do is that, response is the strength. We want to maximize the strength let us assume that one. And in this case terms and we will include all five terms over here. So, in this case what happens we want to see.

So, first initially we are concerned about pareto and normal plot like that. So, in this case we will do that and we click ok, so to have an understanding and feeling which effect is

important now, when we are doing for basically 5 level interaction; that means, A, B, C, D and E.

And we do not have the degree of freedom that I told and that is why ANOVA analysis will not show results over here, because we do not have that much degree of freedom to. Because we have single replicates and we cannot do that, so in this case that is why it is star and we do not see any results. But my concern is I want to see the graphs basically how what the graph looks over here.

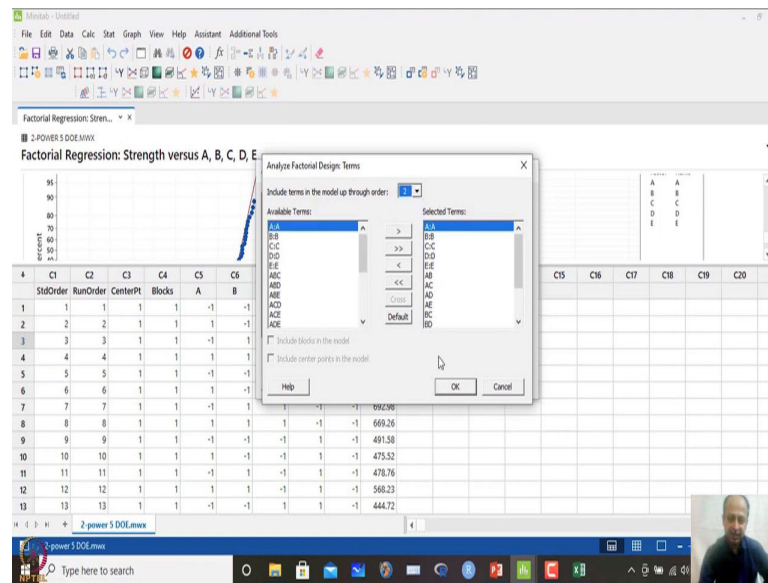
(Refer Slide Time: 22:45)



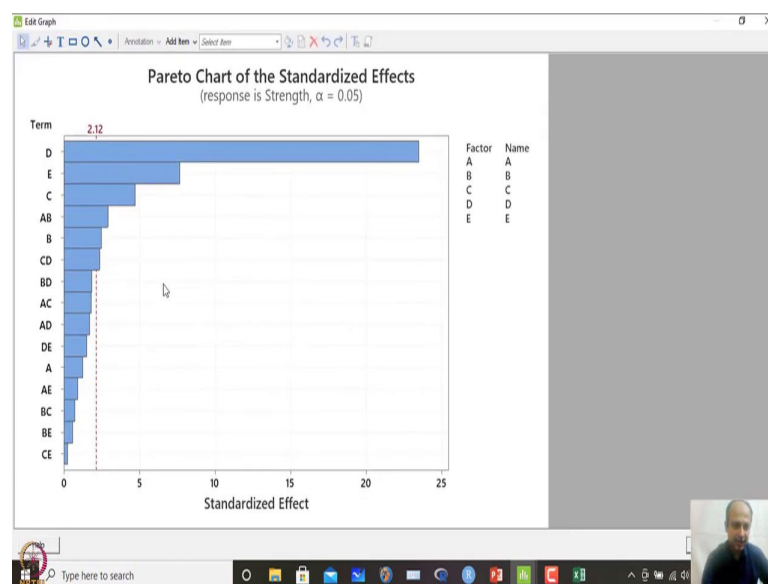
So, this is the pareto plot what we are seeing over here. So, if you see the pareto plots over here what you observe is that, one third level interaction is present, but otherwise it is only single factor A, B, C, D, E and maybe second degree equation is a second level interaction or C multiplied by DBC, this needs to be seen over here.

So, in this case what we will do is that? We will just include only second order interaction up to second order interactions like this. So, factorial analyze factorial design now.

(Refer Slide Time: 23:20)



(Refer Slide Time: 23:32)



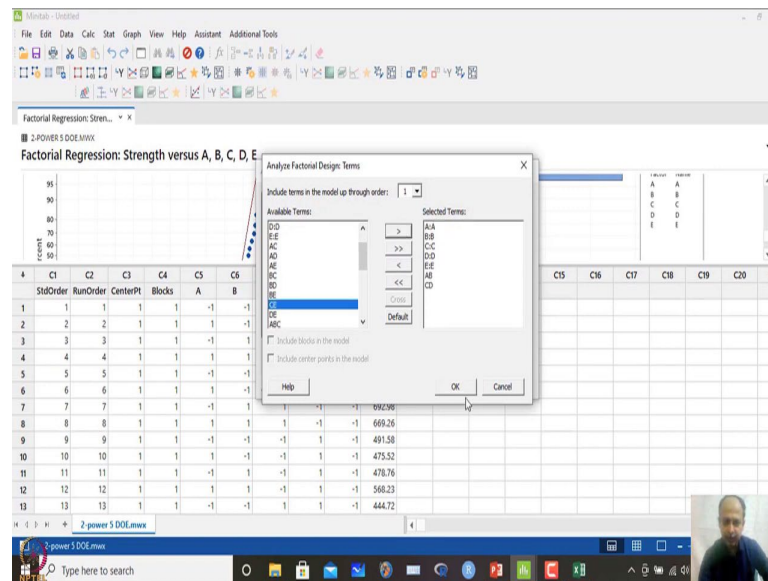
So, in terms what we will do is that, we will eliminate we will go for only 2 level interactions let us say. So, in this case if I do that and click ok and click ok I want to see the pareto plots how it looks like that ok. So, this is the pareto plot that we are seeing. And here we are seeing which is prominent.

So, all main effects we are seeing that the A, B, C, D, E, and only A is not visible over , A is not prominent, but AB interaction is prominent over here. So, we cannot ignore A over here. So, AB should be also included in the model so over here. And in this case

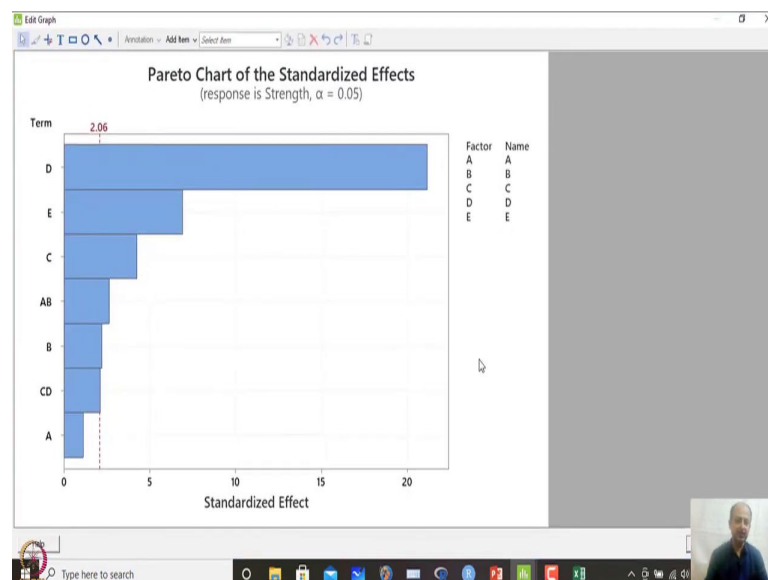
what we are observing is that? Not only AB interactions and CD interaction is prominent over here.

So, let us try to see that if we reduce this one again AB and CD will only be included, so what happens we will try to see.

(Refer Slide Time: 24:12)



(Refer Slide Time: 24:30)



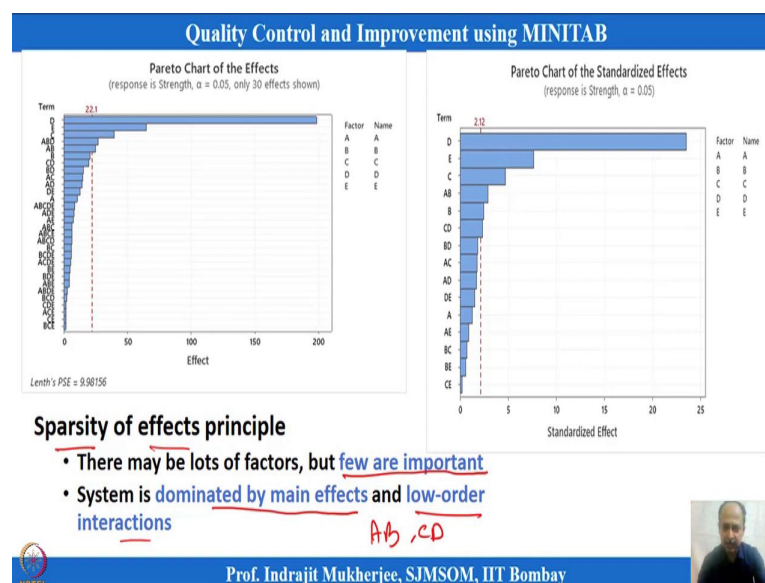
So, in this case then again, we will go to DOE factorial design, analyze factorial design. So, in the terms what we will do is that, we will only this one. So, in this case only AB

will we will include AB over here and maybe CD also we will include over here. So, which is also prominent like that.

So, let us click this one and let us try to see the pareto plot again over here what is observed. So, in this case A is not prominent, but CD, up to CD what we are seeing is that? This is the final combination that we are seeing. So, individual effects are prominent over here A B interaction is prominent, CD interaction is also prominent, but beyond that it is not so significant over here. So, that we are considering.

And this how we have done based on a property which is generally used extensively.

(Refer Slide Time: 24:56)



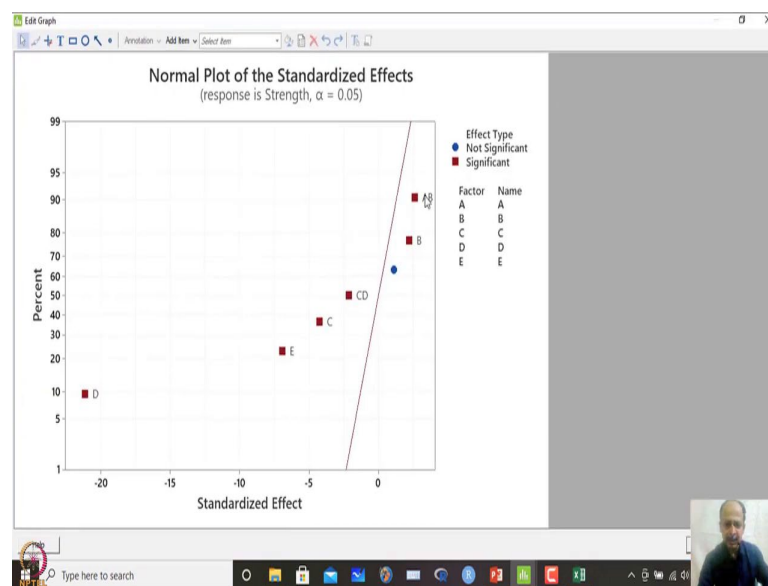
So, this is known as sparsity of effects principle over here, sparsity of effect principle over here. There can be lots of factor like 2 to the power 5. There are 5 factors over here, but few are basically important few are basically important which is controlling the behaviour of the process or CTQs like that ok.

System is dominated by the main effects; that means, A, B, C like that. So, those are the main effects and lower order interaction over here. We have considered that two-way interaction basically, we have considered AB or over here what we have considered is CD that we have considered over here. So, these are the lower order interactions over here. These are the.

So, it becomes easier for us to analyze the data like that. So, we are ignoring the other higher order interactions like that. So, we have ignored the higher order interaction. and based on that we are defining what should be the what should be the level of A, B, C and D and E like that ok. So, this is the condition that we are considering over here.

So, in this case what we will do is that. With this information we have built the model over here. So, we have developed. So, in this case. So, this is not the example the second one. So, in this case what we have done is that.

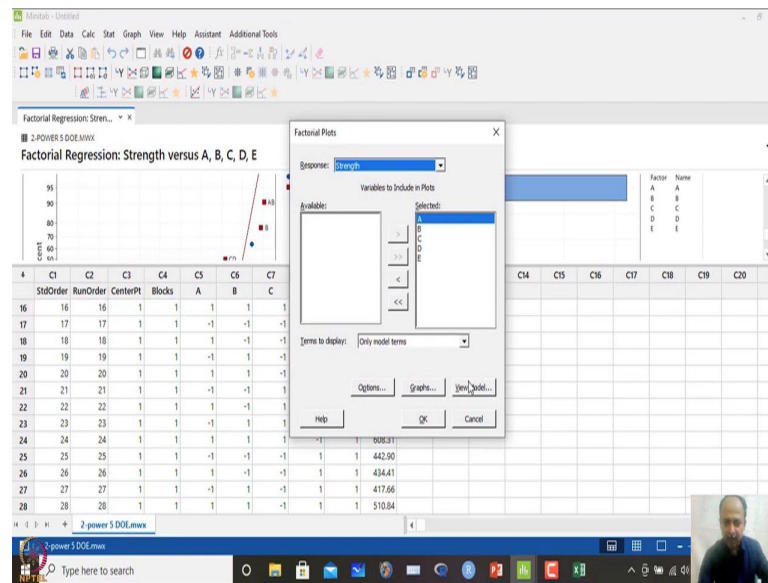
(Refer Slide Time: 26:11)



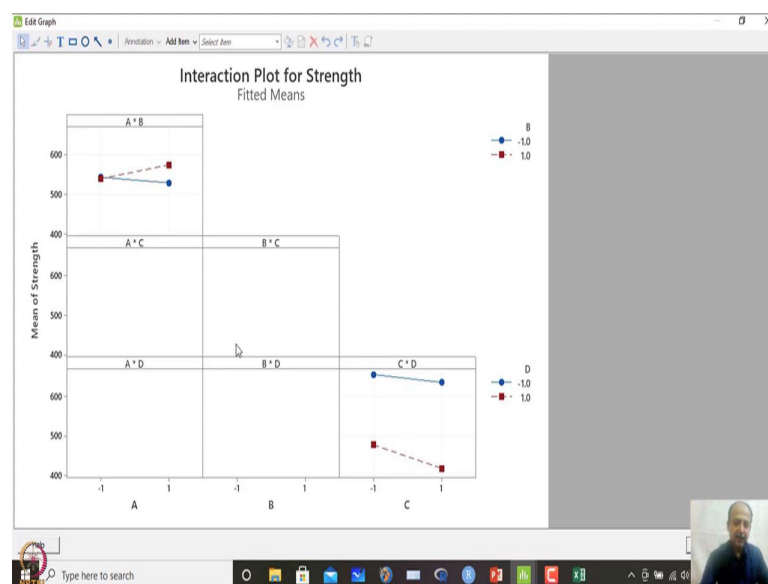
So, this is the this is the model, and also what is visible in the normal plot also we will find. So, AB and B have a positive impact, and C, E, CD, and D have a negative effect. So, AB and CD is important and other factors also levels. Only thing is that A will be defined which level based on AB interaction we will define what should be the level of A, and other things we can see it like CD interactions.

So, AB and CD will define what are the levels we should keep for AB and CD. Only thing E we have to check only the only one factor we have to check which is having making a significant impact. So, what we will do is that. We can make a graphical plot over here. So, in this case design of experiment factorial plots.

(Refer Slide Time: 26:52)



(Refer Slide Time: 27:01)



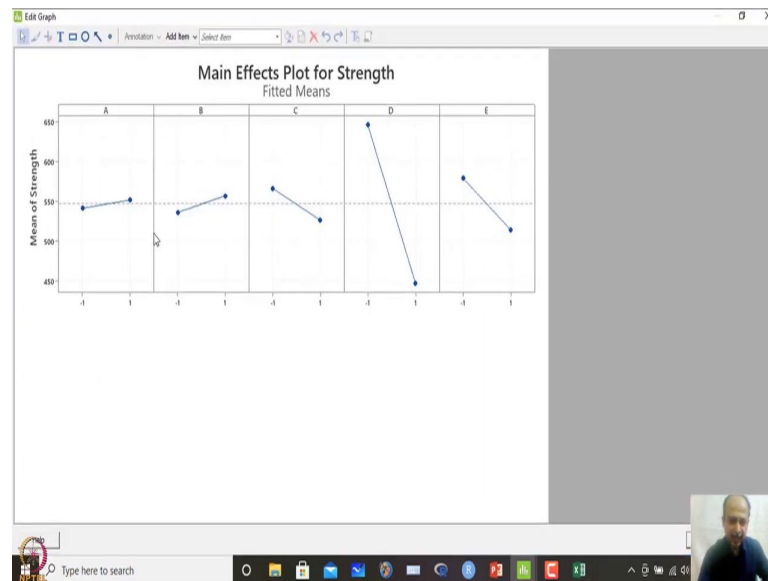
So, in this case we will use factorial plots over here. So, ABCDE and only a model terms that we will use, and in this case we will click ok. And then we will have this interaction plots we will have this interaction plots which will define what should be the level of A and B. We want to maximize the strength over here.

So, this is the for A and B. This is the point rate points that you are seeing over here. So, in this case B will be at plus 1 and A will be at plus 1. So, A and B is freezed at plus 1 plus 1. So, A plus 1 B plus 1 that is defined over here. And for C and D what we can see

is that this is the highest point over here. C will be at minus and D will be at D will also be at minus level over here; C minus and D minus.

So, A plus B plus and C minus D minus that is the level over here. So, that we have already defined from this graph. Only thing we have to see E ok.

(Refer Slide Time: 27:42)

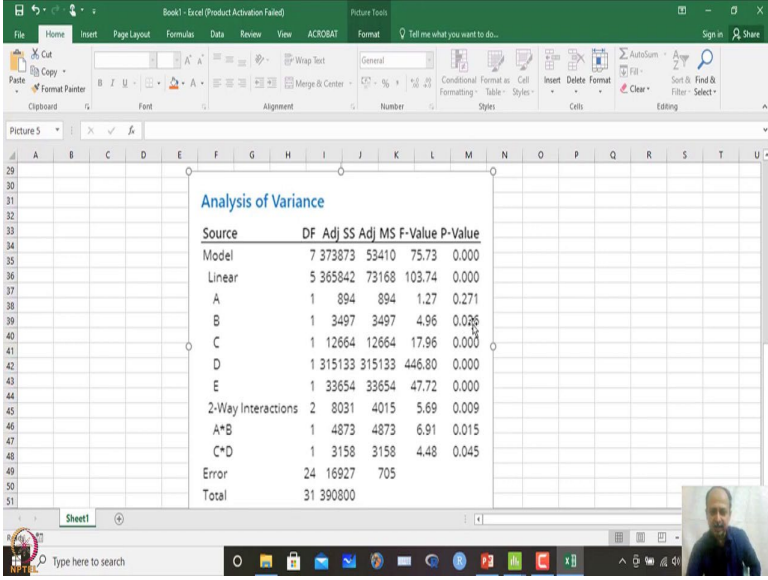


For E what we observe is that to maximize the strength over here and we are seeing the main effect plots over here. So, what we are seeing is that for E minus is the level that we have to define, because minus is giving me a higher mean strength over here. So, this is will be minus ok.

So, A at plus B at plus C at minus D at minus and E at minus that is the final combination that we have to use to optimize the CTQs like that. So, that is the combination that we should go for over here. So, whenever we have developed the final model so over here. And in this case regression equation also we can see. So, if you go up like that, so we can just see the earlier analysis over here.

And the how much variability is explained like that. So, this is the ANOVA analysis. So, I can copy this one and paste it over here.

(Refer Slide Time: 28:31)

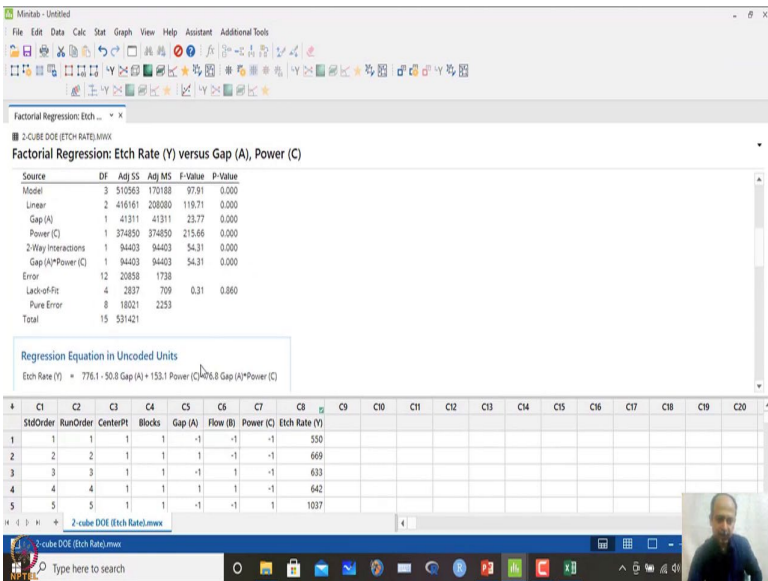


The screenshot shows an Excel spreadsheet with an ANOVA table titled "Analysis of Variance". The table is located in the center of the sheet, spanning from column F to column M and rows 33 to 50. The table has the following data:

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	7	373873	53410	75.73	0.000
Linear	5	365842	73168	103.74	0.000
A	1	894	894	1.27	0.271
B	1	3497	3497	4.96	0.036
C	1	12664	12664	17.96	0.000
D	1	315133	315133	446.80	0.000
E	1	33654	33654	47.72	0.000
2-Way Interactions	2	8031	4015	5.69	0.009
A*B	1	4873	4873	6.91	0.015
C*D	1	3158	3158	4.48	0.045
Error	24	16927	705		
Total	31	390800			

So, we can just see what is the ANOVA analysis. So, just enlarge this one. So, previous data and we moving from here and here we can see that what is happening. So, A is not prominent that is P value 0.271, B is just prominent, B 8 is less than 0.05, C is highly significant, D is also highly significant, E is also highly significant, A B interaction 0.015 that is significant, C multiplied by D there is also significant over here.

(Refer Slide Time: 29:01)



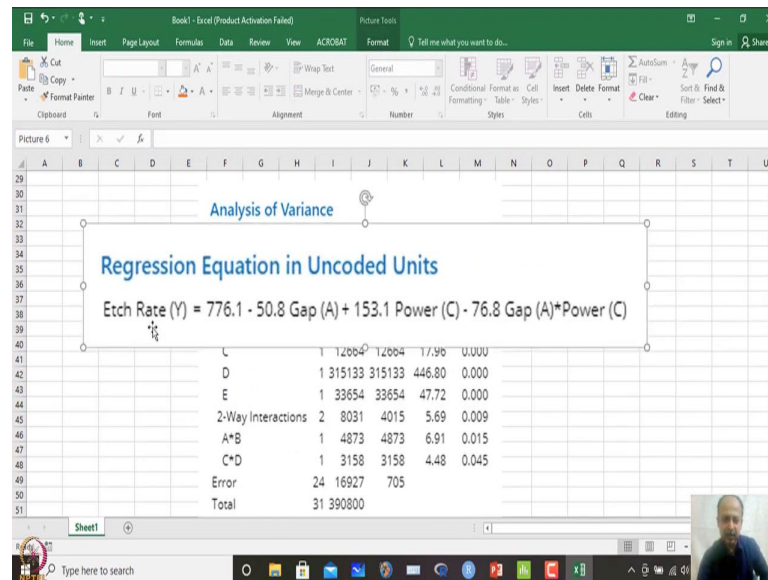
The screenshot shows the Minitab software interface with a Factorial Regression analysis. The analysis is titled "Factorial Regression: Etch Rate (Y) versus Gap (A), Power (C)". The table below shows the results of the analysis:

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	510563	170188	97.91	0.000
Linear	2	416161	208080	119.71	0.000
Gap (A)	1	41311	41311	23.77	0.000
Power (C)	1	374650	374650	215.66	0.000
2-Way Interactions	1	94403	94403	54.31	0.000
Gap (A)*Power (C)	1	94403	94403	54.31	0.000
Error	12	20958	1748		
Lack-of-Fit	4	2837	709	0.31	0.860
Pure Error	8	18021	2253		
Total	15	531421			

Below the table, the regression equation is displayed: $\text{Etch Rate (Y)} = 776.1 - 50.8 \text{ Gap (A)} + 153.1 \text{ Power (C)} + 76.8 \text{ Gap (A)*Power (C)}$.

And the regression equation is also given over here. So, regression equation is given over here that can be used for model controlling of the controlling of the process. So, what we can do is that we can just write it too, you can just paste it over here and enlarge this one.

(Refer Slide Time: 29:13)



Analysis of Variance

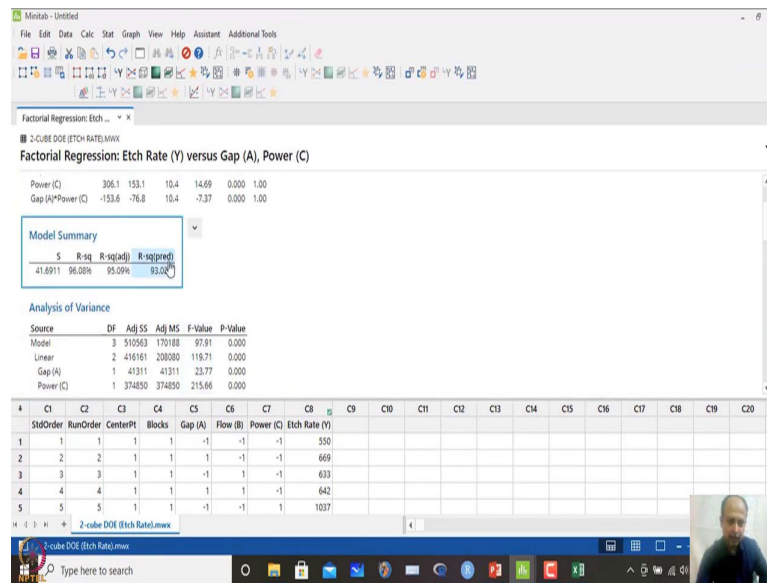
Regression Equation in Uncoded Units

$$\text{Etch Rate (Y)} = 776.1 - 50.8 \text{ Gap (A)} + 153.1 \text{ Power (C)} - 76.8 \text{ Gap (A)*Power (C)}$$

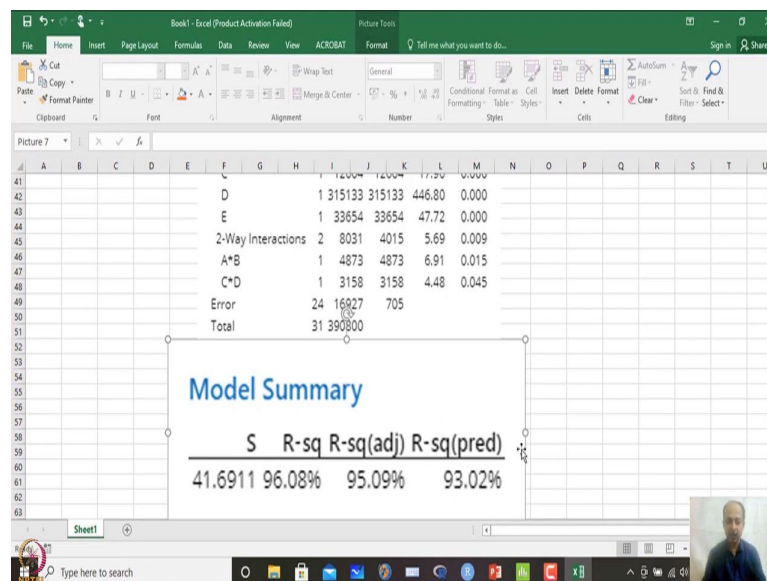
C	1	12664	12664	17.96	0.000
D	1	315133	315133	446.80	0.000
E	1	33654	33654	47.72	0.000
2-Way Interactions	2	8031	4015	5.69	0.009
A*B	1	4873	4873	6.91	0.015
C*D	1	3158	3158	4.48	0.045
Error	24	16927	705		
Total	31	390800			

So, this is the model that we are seeing over etch rate equals to 776.1 minus 50 that is A at minus level what we are seeing over here that the A has a negative impact. And this is the final equations that we have seen and setting condition we have already defined like that. So, in this case R square value that we are seeing over here. Let me see what is the R square value.

(Refer Slide Time: 29:34)

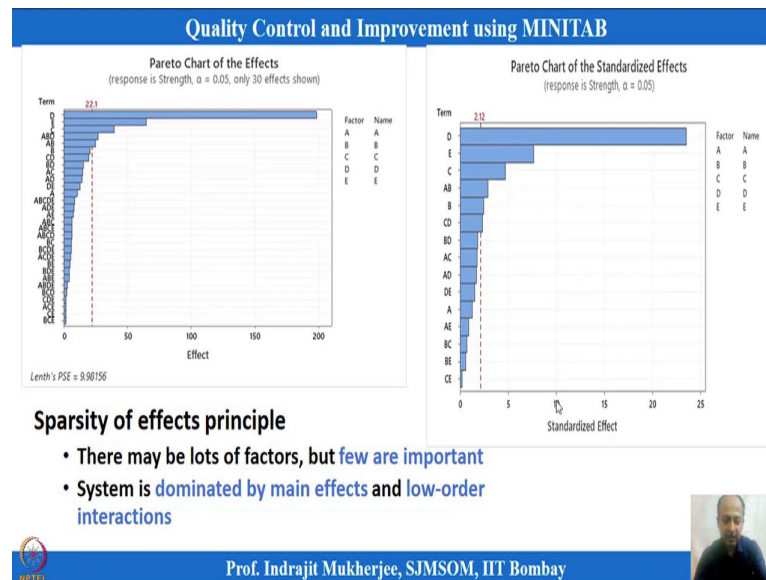


(Refer Slide Time: 29:44)



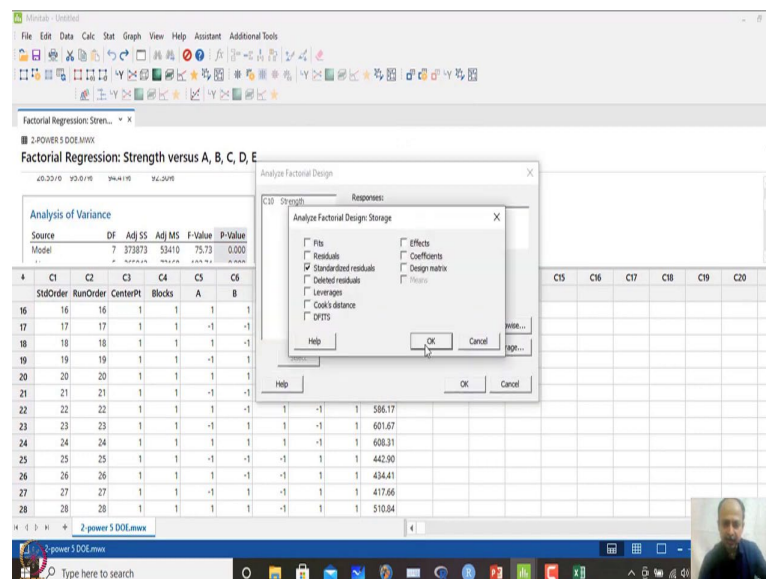
So, this is approximately 93 point that is very good. So, controlling the process is also with this model, what is happening is that. I have information R square predicted prediction or prediction of this models that we are using with all this factors into main effects and interaction effects of AB and CD considered that one. And R square adjusted value is 95.09 which is very good, and R square predicted value is 93.09 and 93.02 that is also quite good like that.

(Refer Slide Time: 30:09)

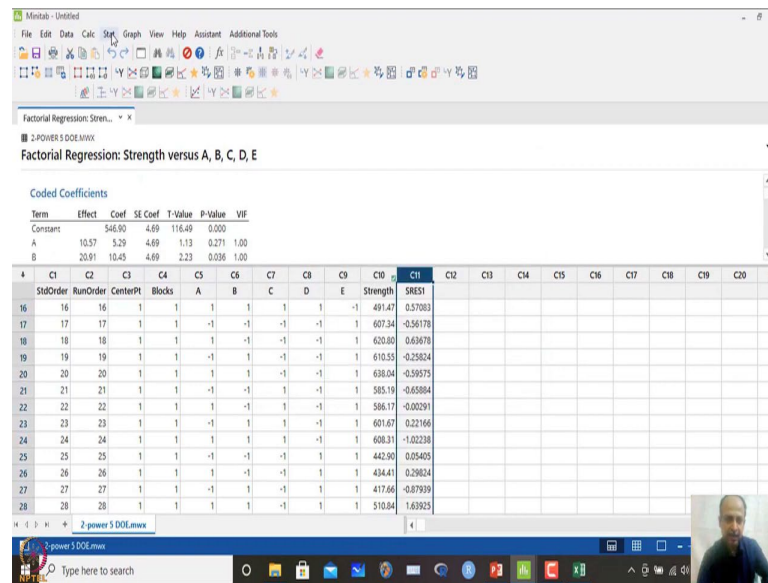


So, we have already find the best combination, and also we can check what is the distribution of the errors like that. So, we could have saved that one. So, we can just cross check whether the errors are normally distributed or not. So, in this case I can just save this one. So, in factorial what we can do is that.

(Refer Slide Time: 30:27)

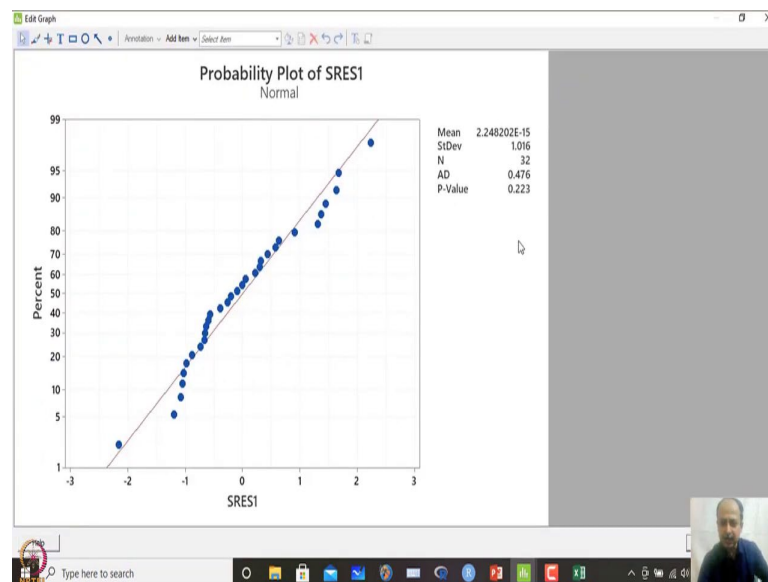


(Refer Slide Time: 30:34)



Analyze factorial only in storage we have not mentioned that standardized residual, we should store that one. And this is the residual that we have saved and we can see whether it is satisfying that conditions or not. So, we can just check the residual over here and click ok.

(Refer Slide Time: 30:45)



And we can just see what is happening over here. And what we see is that P value is more than 0.05, so there is no problem as such, the normality is concerned like that ok. And small deviation what I told is that in design of experiments, this is very robust

technique. And in that case small deviations can also be ignored in that case. Only if it is highly skewed like that then we have a trouble and we need to do some transformation on the data. So, that is also we have learned in our earlier lectures like that.

So, up to this point we have studied. So, now, we will also understand some more things about design of experiments and slowly and steadily a complexity will increase ok. So, thank you for listening we will start with a blocking principle in our next lecture ok.

Thank you.