Quality Control and Improvement with MINITAB
Prof. Indrajit Mukherjee
Shailesh J. Mehta School of Management
Indian Institute of Technology, Bombay

Lecture - 35
Factorial Experiments (Contd.)

Hello and welcome to session 35 of our course on Quality Control and Improvement with MINITAB. I am Professor Indrajit Mukherjee from Shailesh J Mehta School of Management, IIT Bombay. So, we are discussing about Factorial Design and we have taken one example where we have seen that if we do not get the replicates in that case what happens is that estimation of because of scarcity of the degree of freedom for the error.

What we faced is that we cannot do ANOVA analysis like that ok, but we can make an estimation of factor A, factor B, and interaction effects like that. And based on that, we can also see the regression equation because we are doing the factorial experimentation and for that regression equation, it is easy to develop. So, we have seen that regression is possible, but effect estimation and coefficient determination is not possible like that. So, one example that we are discussing; simple experimentation like that.

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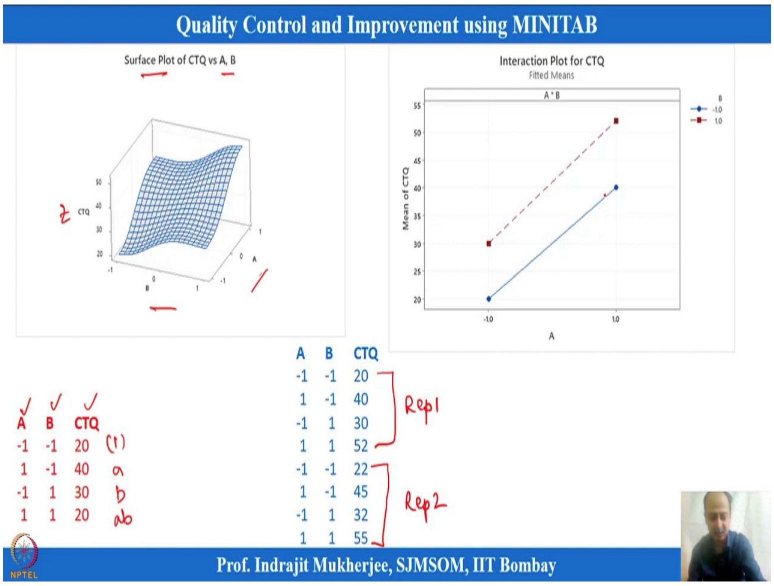

Quality Control and Improvement using MINITAB

✓ A	✓ B	✓ CTQ	
-1	-1	20	(1)
1	-1	40	a
-1	1	30	b
1	1	20	ab

A	B	CTQ	
-1	-1	20	Rep1
1	-1	40	
-1	1	30	
1	1	52	
-1	-1	22	Rep2
1	-1	45	
-1	1	32	
1	1	55	

Prof. Indrajit Mukherjee, SJMSOM, IIT Bombay

So, this is the experimental trials that we have considered factor A and factor B and this is the response variable which is the CTQ and these are the first experimentation that we this is the all at low level. Then A is at high level and then B is at high level, then A B is both A and B at high level that experimentation, we have seen how it is to be done ok and how the design is developed in MINITAB that also we have done ok.

So, now, I am just adding the some of the parts over here. So, I have added one replicates over here. So, one so, there are two replicates basically over here. So, this is replication first replicates that is kept as these replication one and this is kept as replication two over here. So, I have just changed the data sets like that. This is basically arbitrary or hypothetical scenarios that we are considering over here.

And just I have added the replicates to see whether the MINITAB can estimates a ANOVA analysis, it can do and what more we can do in MINITAB like that ok. This surface plot is also possible over here in MINITAB like if A B is continuous variable, we treat them as continuous variable and then Z dimension we can put at CTQ and then we can plot that one.

And interaction effects can be we can see it without replicates also we can do that and with replicates is always possible with replicates we can have the interaction plots also interaction plots is possible. And let us try to see this when I use this data set over here, what happens?

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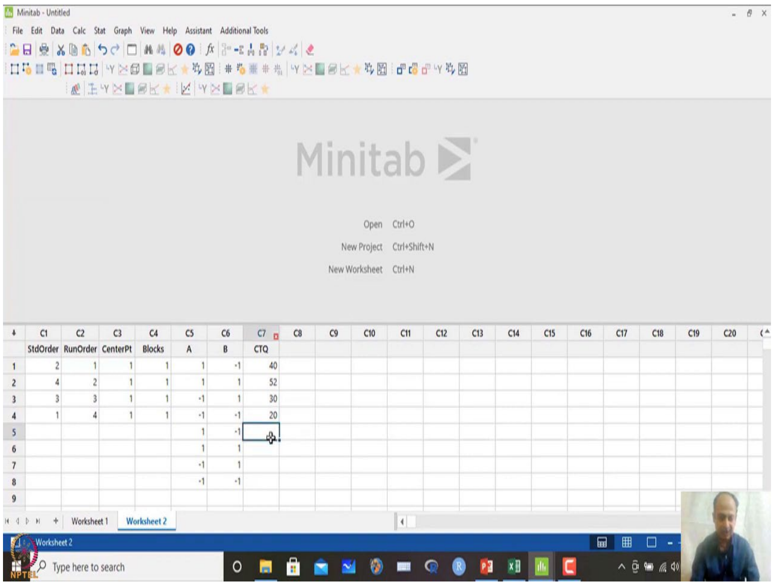

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	2	1	1	1	1	-1	40
2	4	2	1	1	1	1	52
3	3	3	1	1	-1	1	30
4	1	4	1	1	-1	-1	20
5					1	-1	
6					1	1	
7					-1	1	
8					-1	-1	
9							

So, what I will do is that how to create this with replicate. So, what we can do is that this was last time when we have one replication and this is the second replicates we have done let us say. And we can just eliminate this one with this analysis over here. Now with replicates, we want to make the design over here.

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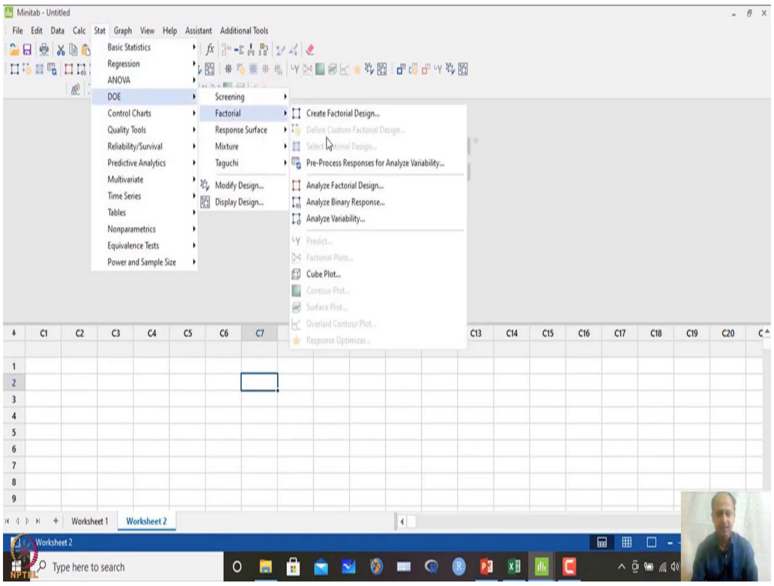

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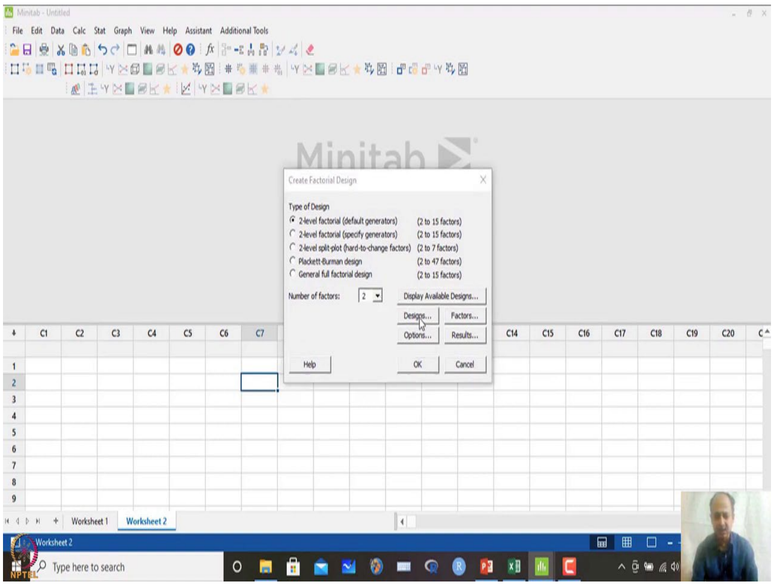
Create Factorial Design

Type of Design
- ● 2-level factorial (default generators) (2 to 15 factors)
- ○ 2-level factorial (specify generators) (2 to 15 factors)
- ○ 2-level split-plot (hard-to-change factors) (2 to 7 factors)
- ○ Plackett-Burman design (2 to 47 factors)
- ○ General full factorial design (2 to 15 factors)

Number of factors: 2

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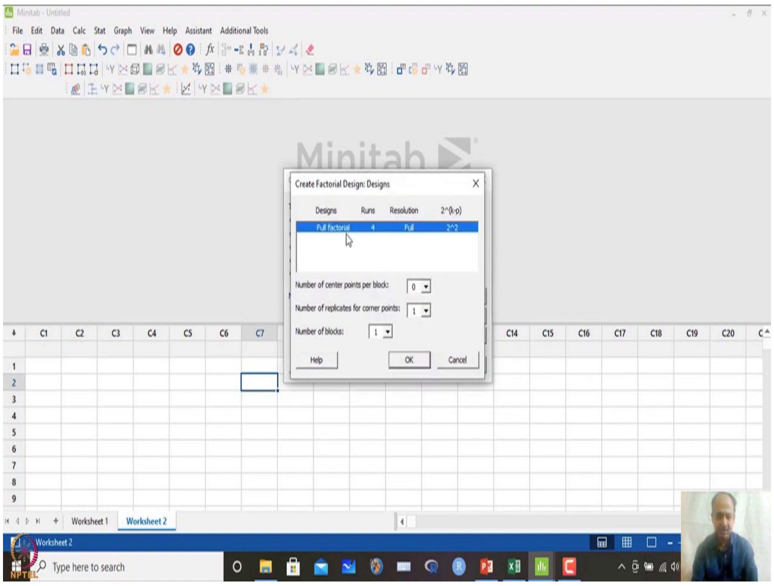

Create Factorial Design: Designs

Designs	Runs	Resolution	2^(k-p)
Full factorial	4	Full	2^2

Number of center points per block: 0

Number of replicates for corner points: 1

Number of blocks: 1

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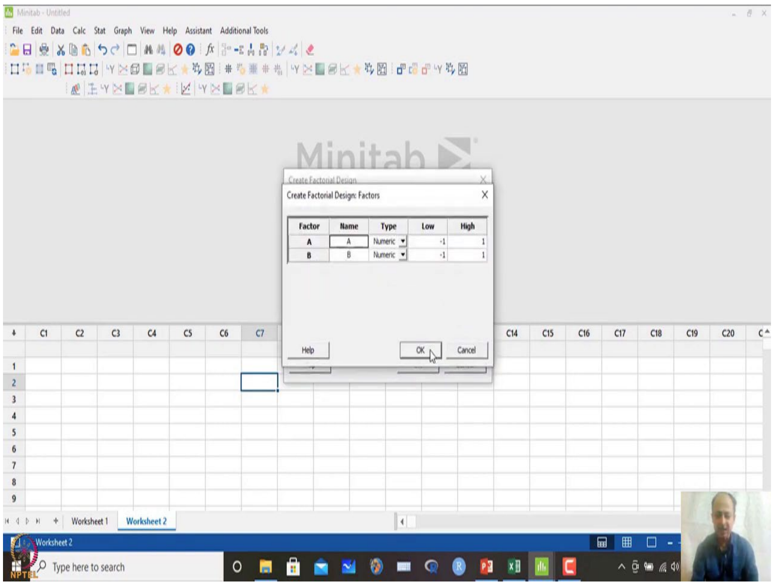

Create Factorial Design: Factors

Factor	Name	Type	Low	High
A	A	Numeric	-1	1
B	B	Numeric	-1	1

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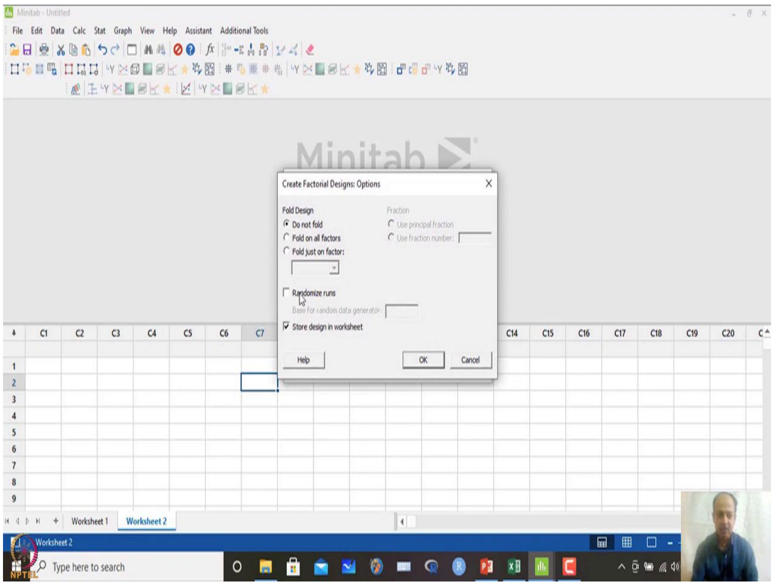

Create Factorial Designs: Options

Fold Design
● Do not fold
○ Fold on all factors
○ Fold just on factor:

Fraction
○ Use principal fraction
○ Use fraction number:

☐ Randomize runs
Base for random data generator:
☑ Store design in worksheet

So, in this case design of experiment factorial design, create factorial designs so, two level factors. Then we will mention 2, this is 2 over here in design matrix nothing is changing; only number of replicates I will make it 2. So, if you make number of replicates other things remain same. So, I am not changing anything so, I will click OK. So, I have just changed the number of number of replicates as 2 over here and factors remain same, we are not doing anything with the factors.

Then options over here we do not want to randomize like that you can randomize this one. When you randomize, it will generate random designs like that. So, that is during experimental trial, it is required actual experimentation. But for the time being, we are assuming that we are not doing this for sake of just illustration over here. So, in this case if you click OK and do not fold, we have to use these options of do not fold.

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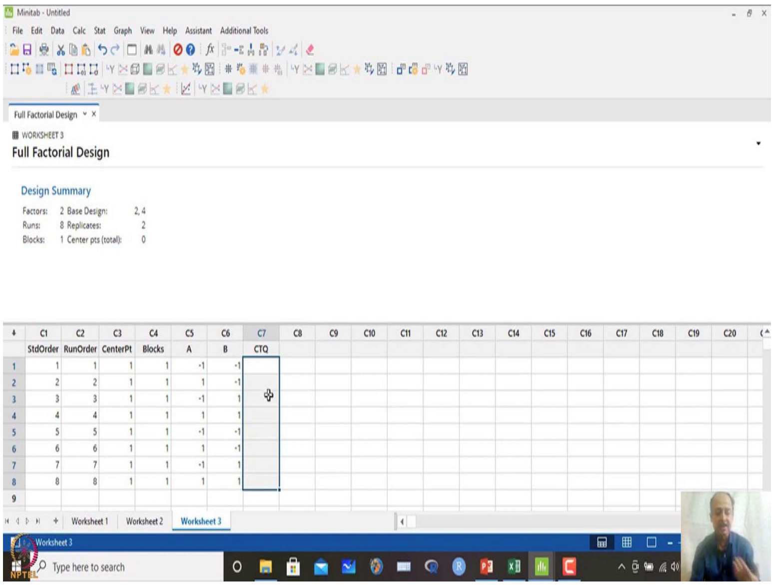

Full Factorial Design

Design Summary

Factors:	2	Base Design:	2, 4
Runs:	8	Replicates:	2
Blocks:	1	Center pts (total):	0

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	
2	2	2	1	1	1	-1	
3	3	3	1	1	-1	1	
4	4	4	1	1	1	1	
5	5	5	1	1	-1	-1	
6	6	6	1	1	1	-1	
7	7	7	1	1	-1	1	
8	8	8	1	1	1	1	
9							

If you click OK over here and click OK the design matrix will be created. So, this is the standard order, this is the run order center point by default is taken as 1, block is taken as 1. So, in this case this, we are not touching center point and block we are not defining over here. So, in this case this is by default what we are taking. So, A at low level and high level so, this CTQ measurements what we are doing over here; can be placed over here.

So, in this case, there is no problem design is created; then these 8 trials values we can put over here. So, I have already this file with me. So, in this case so, I do not want to replicate that one that doing it same time again. So, this data set is given over here, this is the data set that is given over here.

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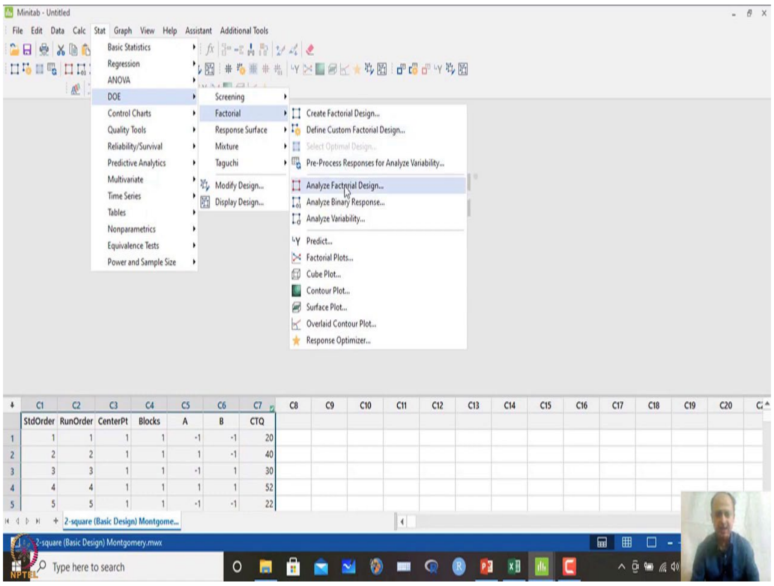

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

And it is already created over here. So, in this case what is required is that I want to do the analysis over here design of experiments. So, I want to see factorial design.

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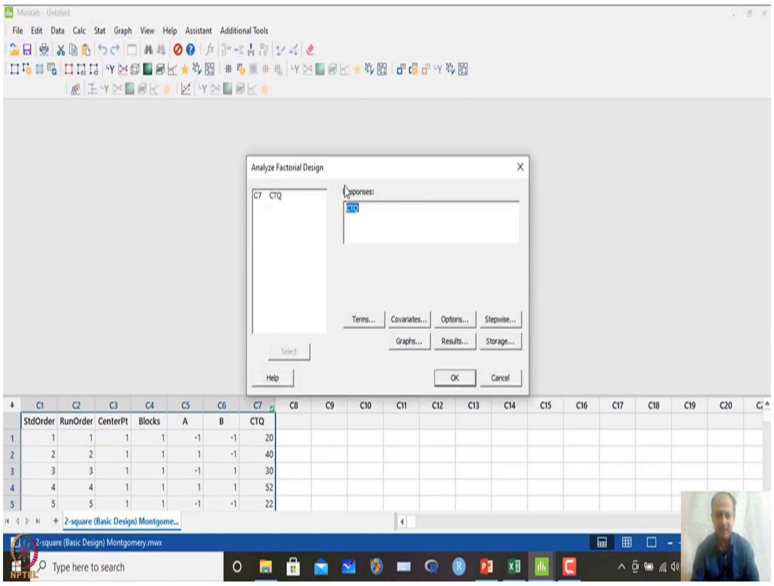

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

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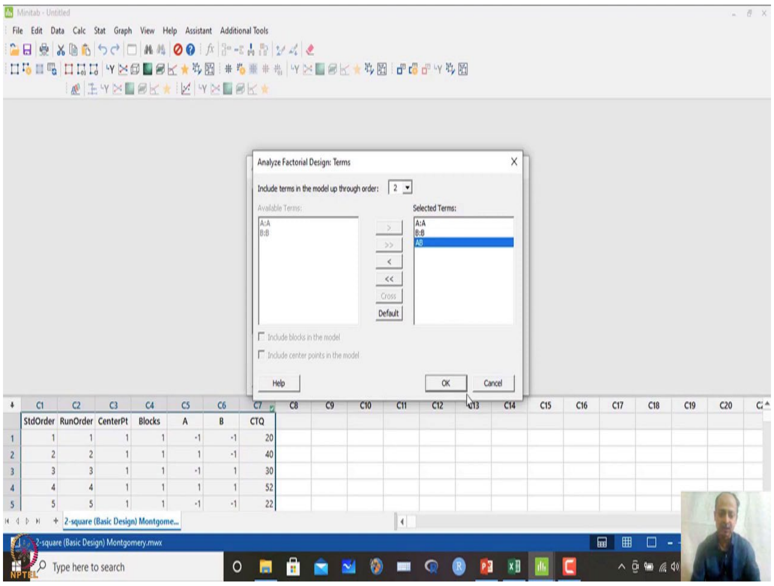

Analyze Factorial Design: Terms

Include terms in the model up through order: 2

Available Terms:
A:A
B:B

Selected Terms:
A:A
B:B
AB

☐ Include blocks in the model
☐ Include center points in the model

	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

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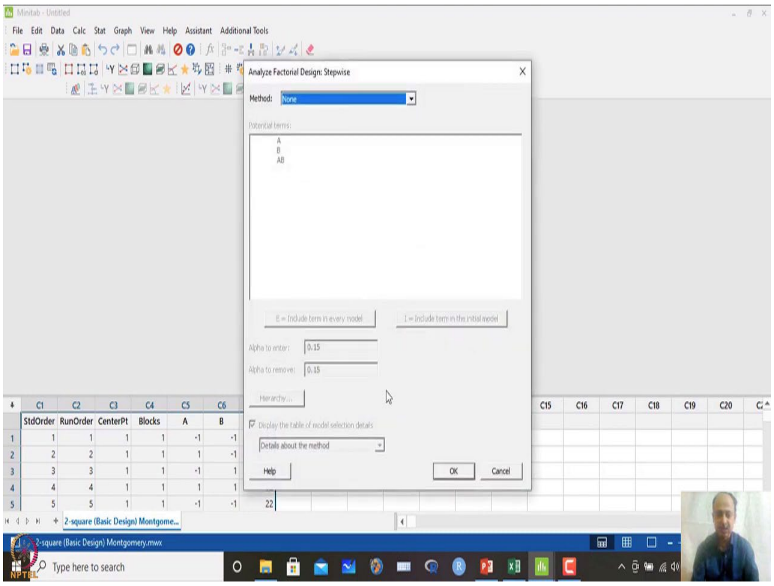

Analyze Factorial Design: Stepwise

Method: None

Potential terms:
A
B
AB

Alpha to enter: 0.15

Alpha to remove: 0.15

☑ Display the table of model selection details

Details about the method

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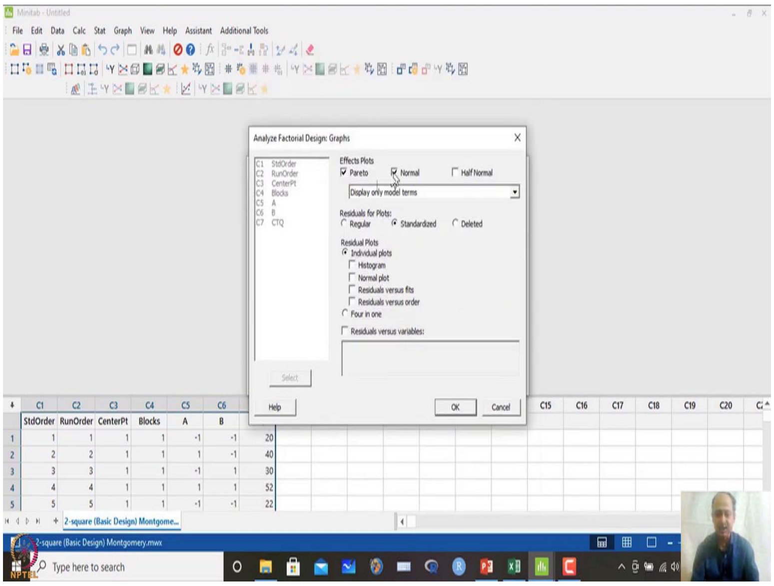


Analyze Factorial Design: Graphs

C1 StdOrder
C2 RunOrder
C3 CenterPt
C4 Blocks
C5 A
C6 B
C7 CTQ

Effects Plots
☑ Pareto ☑ Normal ☐ Half Normal
Display only model terms

Residuals for Plots:
○ Regular ● Standardized ○ Deleted

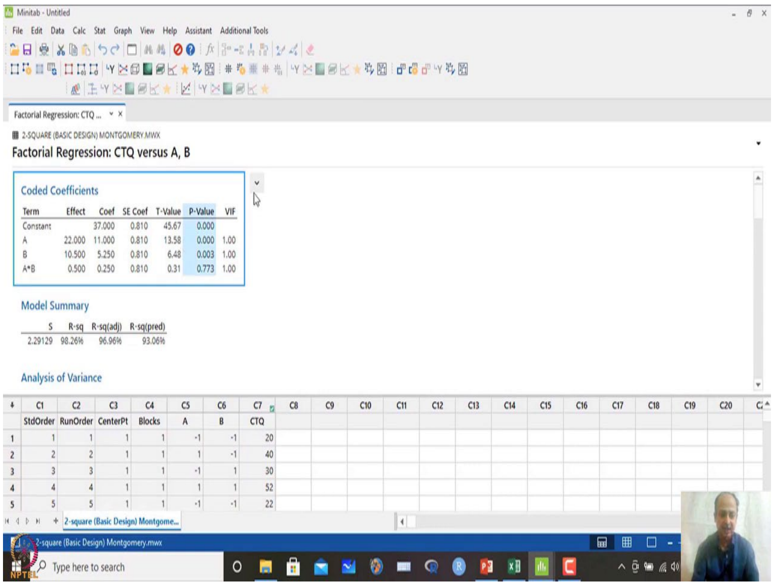
Residual Plots
● Individual plots
☐ Histogram
☐ Normal plot
☐ Residuals versus fits
☐ Residuals versus order
○ Four in one
☐ Residuals versus variables:

	C1	C2	C3	C4	C5	C6	
	StdOrder	RunOrder	CenterPt	Blocks	A	B	
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

So, analyze factorial design over here. So, then in this case what I have to mention is that which I have to analyze, then I will mention that A B interaction effects I want to estimate. So, I will click OK. So, and then covariance is not there other options. So, stepwise regression, we have not done anything over here. So, everything remains same.

So, graphical plots over here, normal plot and Pareto plot; these two plots, we want to see and then standardized residual normal plot of residual, we can see. And storage, at this moment, is not required. So, we can store it afterward when a final model is developed. So, in this case, I click OK.

(Refer Slide Time: 05:06)



Factorial Regression: CTQ versus A, B

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		37.000	0.810	45.67	0.000	
A	22.000	11.000	0.810	13.58	0.000	1.00
B	10.500	5.250	0.810	6.48	0.003	1.00
A*B	0.500	0.250	0.810	0.31	0.773	1.00

Model Summary

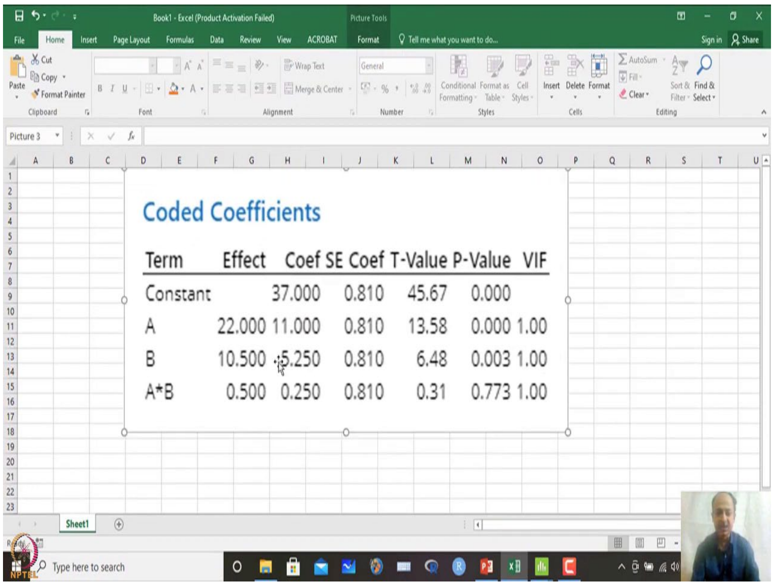
S	R-sq	R-sq(adj)	R-sq(pred)
2.29129	98.26%	96.96%	93.06%

Analysis of Variance

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

And what will happen is that you will get the ANOVA analysis, this is ANOVA analysis. So, I can copy this over here and I can paste that and try to see enlarge this what is happening.

(Refer Slide Time: 05:14)

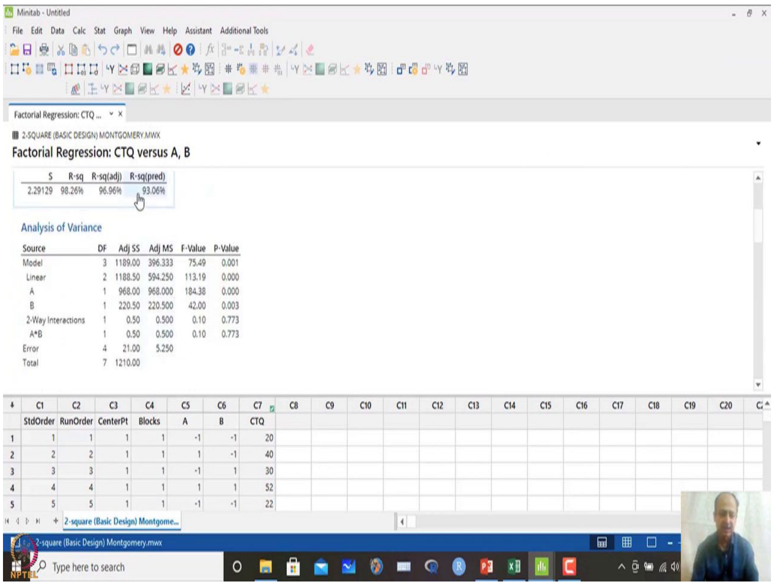


Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		37.000	0.810	45.67	0.000	
A	22.000	11.000	0.810	13.58	0.000	1.00
B	10.500	5.250	0.810	6.48	0.003	1.00
A*B	0.500	0.250	0.810	0.31	0.773	1.00

So, in this case factor A is significant, what we are observing over here and factor B is significant, but interaction effect is not significant that is 0.773. So, that is not significant over here. So, interaction effect is not significant; factor A and factor B is significant over here.

(Refer Slide Time: 05:33)



2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX

Factorial Regression: CTQ versus A, B

S	R-sq	R-sq(adj)	R-sq(pred)
2.29129	98.26%	96.96%	93.06%

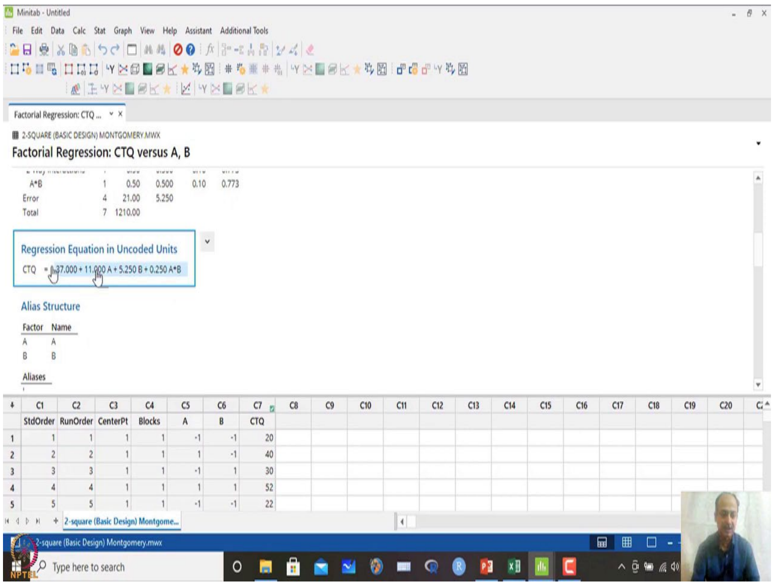
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	1189.00	396.333	75.49	0.001
Linear	2	1188.50	594.250	113.19	0.000
A	1	968.00	968.000	184.38	0.000
B	1	220.50	220.500	42.00	0.003
2-Way Interactions	1	0.50	0.500	0.10	0.773
A*B	1	0.50	0.500	0.10	0.773
Error	4	21.00	5.250		
Total	7	1210.00			

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

So, in this case and then R square values over here is very high, R square predicted is now about 93 percent, 93 percent like that very good models, we have developed. So, these two are the critical factors which we have; which we have screened over here, but interaction effect is not prominent they are independent and they influence the outcome like that ok.

(Refer Slide Time: 05:52)



2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX

Factorial Regression: CTQ versus A, B

Source	DF	Adj SS	Adj MS	F-Value	P-Value
A*B	1	0.50	0.500	0.10	0.773
Error	4	21.00	5.250		
Total	7	1210.00			

Regression Equation in Uncoded Units

CTQ = 37.000 + 11.000 A + 5.250 B + 0.250 A*B

Alias Structure

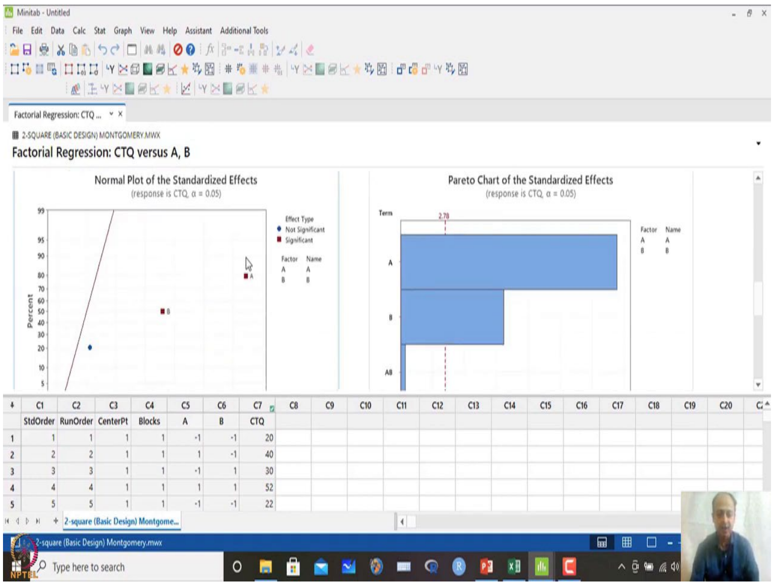
Factor	Name
A	A
B	B

Aliases

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

And then the equations for this is generated over here, regression equation will be generated the like the same way what we have shown the basic calculations, how the interaction coefficient and all these coefficient of mean effects and interaction effects can be calculated. A factor effects and the interaction effects can be calculated that from the experimental results, we can do that, that we have seen earlier also.

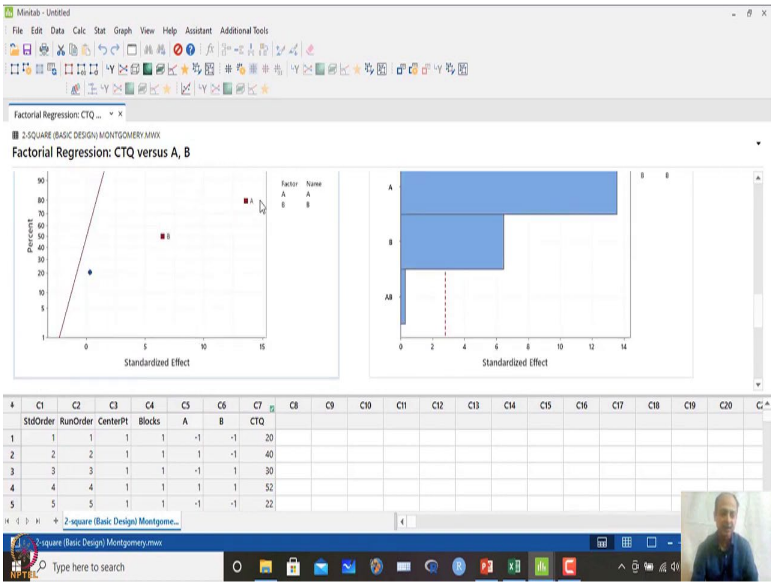
(Refer Slide Time: 06:12)



Factorial Regression: CTQ versus A, B

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

(Refer Slide Time: 06:16)



Factorial Regression: CTQ versus A, B

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

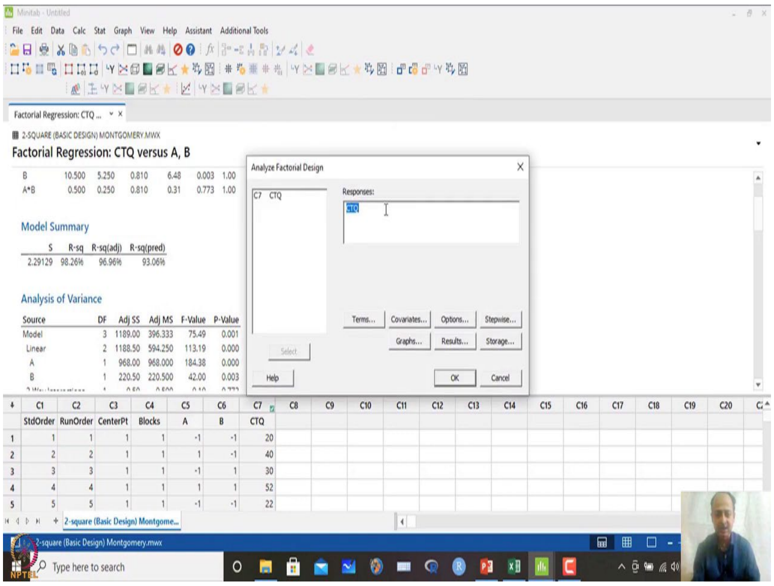
And the normal plot over here what it says is that A and B is on the right hand side of this graph over here and that means, it has a positive effect like that and coefficient will also tell you that it has a positive effect. So, coded coefficient what do you see over here, coefficient A and B is positive over here, C is also positive, this AB interaction is also positive over here; but in this case, it is not significant. So, we are concerned about AB.

So, in this case, what happens is that AB is only shown in the normal plot to be significant because this is red dotted over here. Anything that is red shows that this is significant and the side that it takes will indicate that whether it has a positive effect, whether it has a negative effect on the CTQ.

So, this has a positive effect over here and this Pareto chart also indicates that A and B is only significant. So, AB is not significant over here, AB is not significant. So, and the regression equation from the regression equation, we can drop the interaction effects like that because interaction is not significant.

So, why to include that one in the model, but if you want to retain that one for your prediction behaviour if it is more if it is giving R square predicted is quite high. So, in that case we can retain that one. So, but I am showing you in general how you can eliminate this one interaction effects if it is not so much making impact in the model.

(Refer Slide Time: 07:24)



2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX

Factorial Regression: CTQ versus A, B

```
B      10.500   5.250   0.810   6.48   0.003   1.00
A*B     0.500   0.250   0.810   0.31   0.773   1.00
```

Model Summary

```
      S     R-sq  R-sq(adj)  R-sq(pred)
2.29129   98.26%     96.96%      93.06%
```

Analysis of Variance

```
Source    DF   Adj SS   Adj MS  F-Value  P-Value
Model      3  1189.00  396.333    75.49    0.001
  Linear   2  1188.50  594.250   113.19    0.000
    A      1   968.00  968.000   184.38    0.000
    B      1   220.50  220.500    42.00    0.003
```

Responses: CTQ

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ													
1	1	1	1	1	-1	-1	20													
2	2	2	1	1	1	-1	40													
3	3	3	1	1	-1	1	30													
4	4	4	1	1	1	1	52													
5	5	5	1	1	-1	-1	22													

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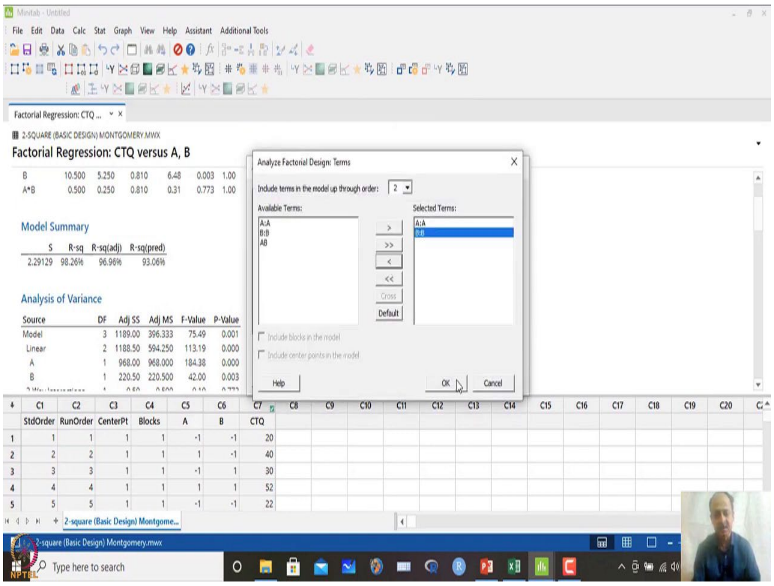
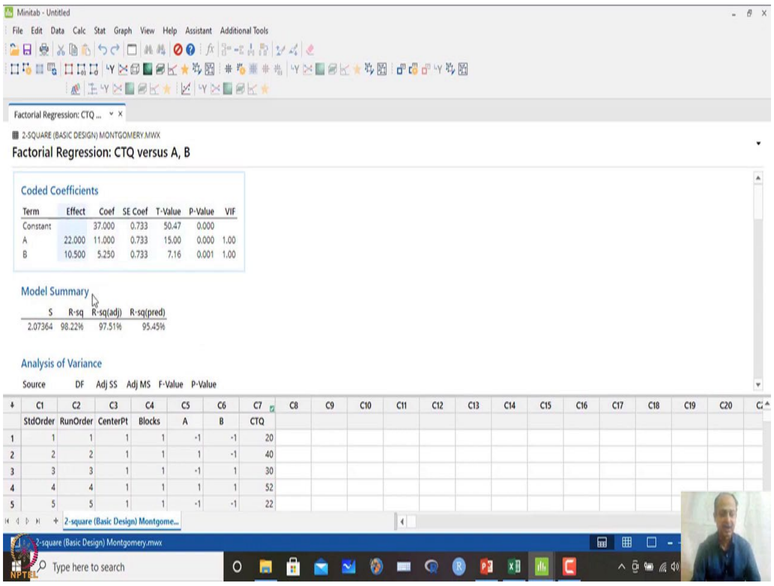


```
2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX
Factorial Regression: CTQ versus A, B

  B          10.500   5.250   0.810    6.48   0.003  1.00
  A*B         0.500   0.250   0.810    0.31   0.773  1.00

Model Summary

        S     R-sq  R-sq(adj)  R-sq(pred)
  2.29129   98.26%     96.96%      93.06%

Analysis of Variance

Source       DF   Adj SS   Adj MS  F-Value  P-Value
Model         3  1189.00  396.333    75.49    0.001
  Linear      2  1188.50  594.250   113.19    0.000
    A         1   968.00  968.000   184.38    0.000
    B         1   220.50  220.500    42.00    0.003
```

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 A	C6 B	C7 CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

So, what I will do is that analyze factorial design, I will go over here. In terms what I will do is that I will remove this term AB interaction because A B, I have seen already that A B is not making such amount of impact. So, in this case I will click OK over here and everything remains same, I will click OK over here.

(Refer Slide Time: 07:41)

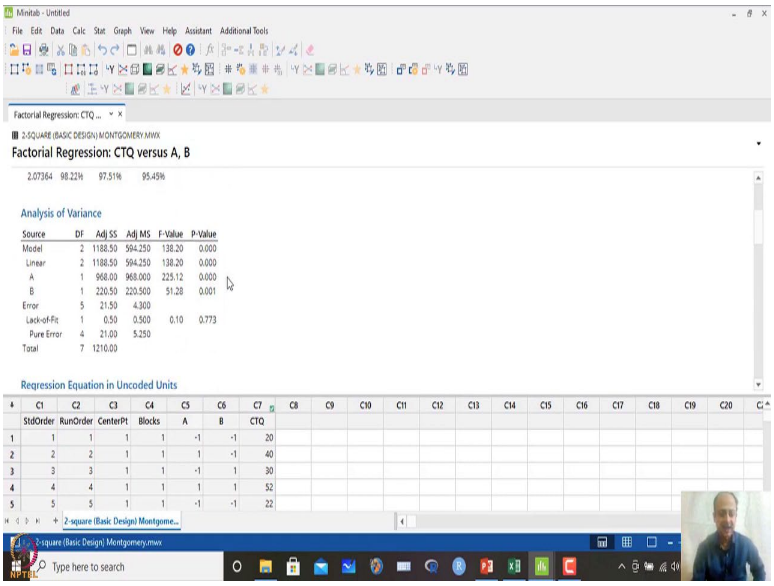


```
2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX
Factorial Regression: CTQ versus A, B

Coded Coefficients

Term       Effect    Coef  SE Coef  T-Value  P-Value   VIF
Constant           37.000    0.733    50.47    0.000
A          22.000  11.000    0.733    15.00    0.000  1.00
B          10.500   5.250    0.733     7.16    0.001  1.00

Model Summary

        S     R-sq  R-sq(adj)  R-sq(pred)
  2.07364   98.22%     97.51%      95.45%

Analysis of Variance

Source       DF   Adj SS   Adj MS  F-Value  P-Value
```

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 A	C6 B	C7 CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

So, when you do that coefficients are there calculated over here, only thing is that I lose something on the predictive behaviour over here. So, earlier it was 96, now it is 95 like that.

(Refer Slide Time: 07:50)



2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX

Factorial Regression: CTQ versus A, B

```
2.07364   98.22%    97.51%    95.45%
```

Analysis of Variance

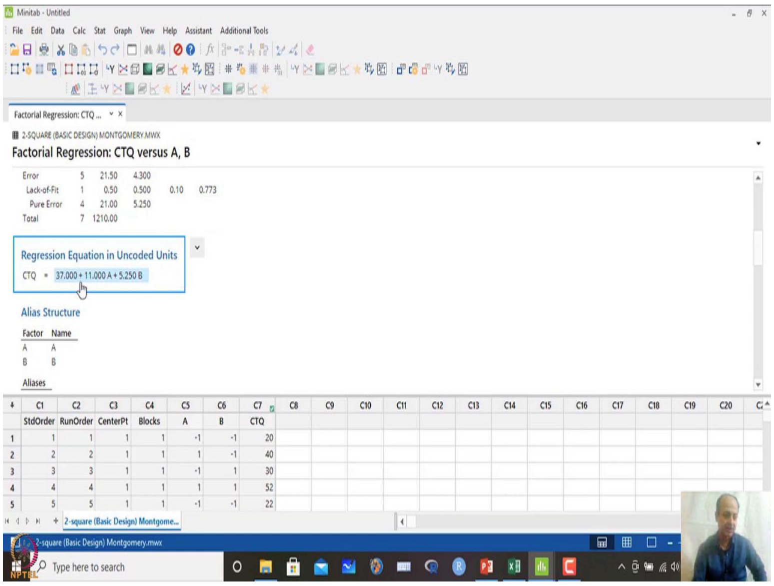
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	1188.50	594.250	138.20	0.000
Linear	2	1188.50	594.250	138.20	0.000
A	1	968.00	968.000	225.12	0.000
B	1	220.50	220.500	51.28	0.001
Error	5	21.50	4.300		
Lack-of-Fit	1	0.50	0.500	0.10	0.773
Pure Error	4	21.00	5.250		
Total	7	1210.00			

Regression Equation in Uncoded Units

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 A	C6 B	C7 CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

And the analysis of variance shows that there is no lack of fit over here. So, AB because we have multiple, we have replicated. So, we can calculate the lack of fit that we have mentioned in regression class; so, in this in regression session sorry.

(Refer Slide Time: 08:10)



2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX

Factorial Regression: CTQ versus A, B

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Error	5	21.50	4.300		
Lack-of-Fit	1	0.50	0.500	0.10	0.773
Pure Error	4	21.00	5.250		
Total	7	1210.00			

Regression Equation in Uncoded Units

CTQ = 37.000 + 11.000 A + 5.250 B

Alias Structure

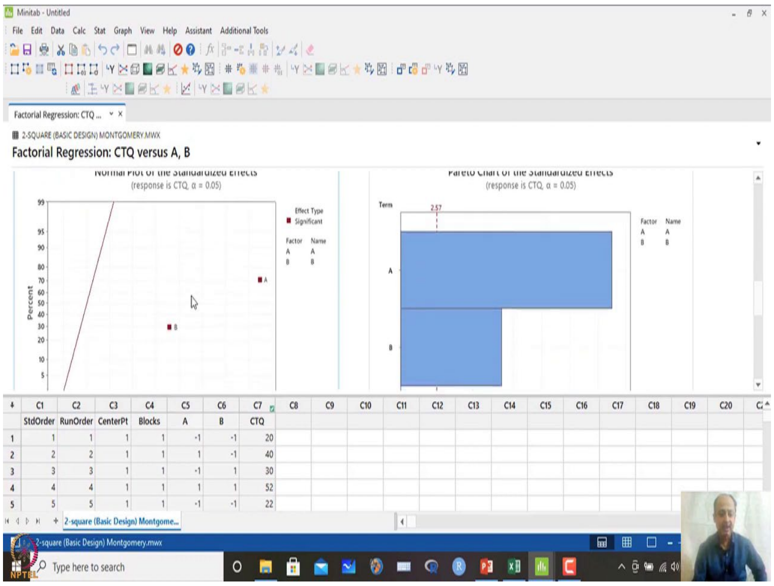
Factor	Name
A	A
B	B

Aliases

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 A	C6 B	C7 CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

So, in that case what we have told is that if lack of fit is not there so, the model linear model seems to be adequate and this is the model that we can use. So, this is the CTQ is y= 37 + 11*A + 5.25* B over here.

(Refer Slide Time: 08:18)



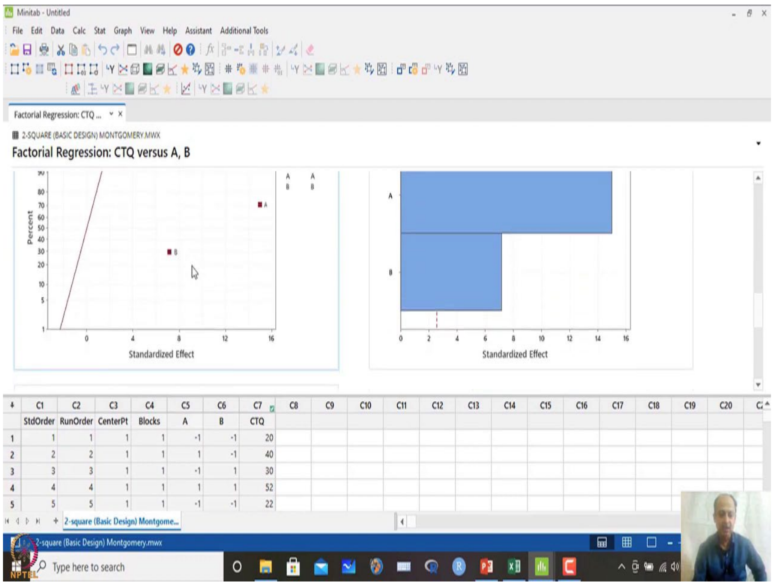
2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX

Factorial Regression: CTQ versus A, B

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

2-square (Basic Design) Montgomery.mwx

(Refer Slide Time: 08:18)



2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX

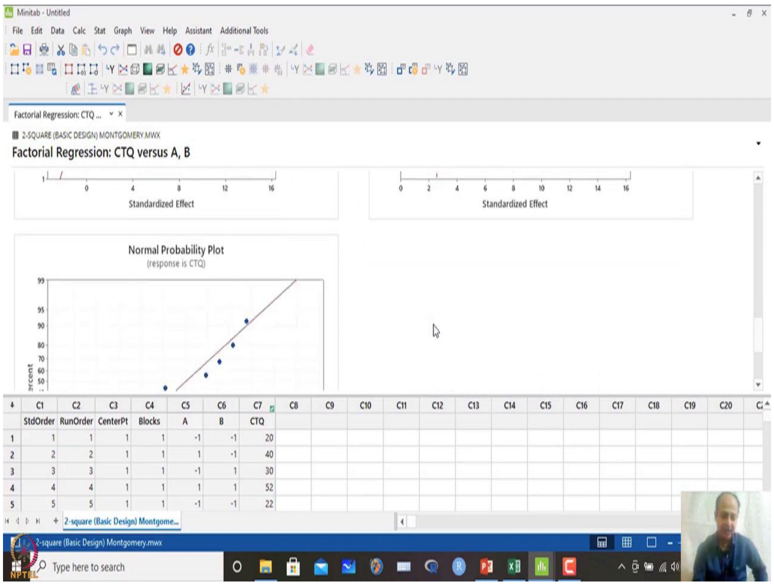
Factorial Regression: CTQ versus A, B

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

2-square (Basic Design) Montgomery.mwx

So, and normal plot over here because interaction, we have eliminated now. So, AB is quite significant that is also shown over here in Pareto chart like that. So, this is also evident over here. So, this is an evident over here.

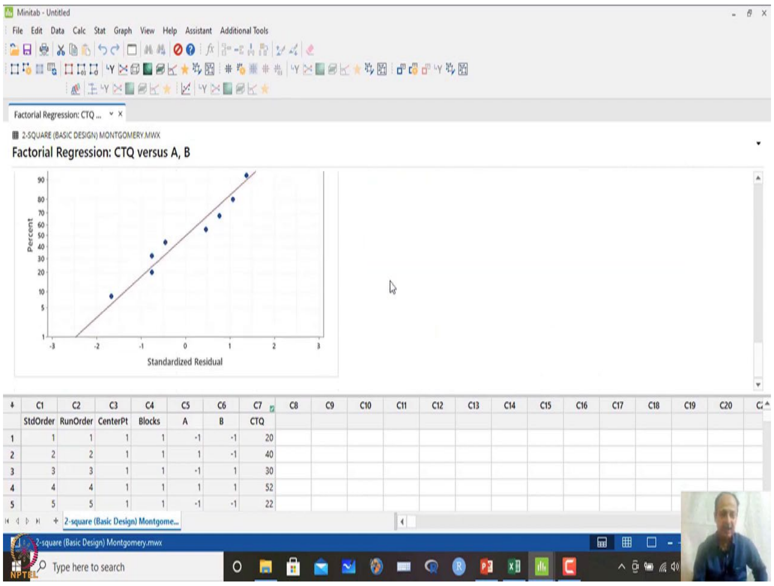
(Refer Slide Time: 08:29)



Factorial Regression: CTQ versus A, B

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

(Refer Slide Time: 08:29)

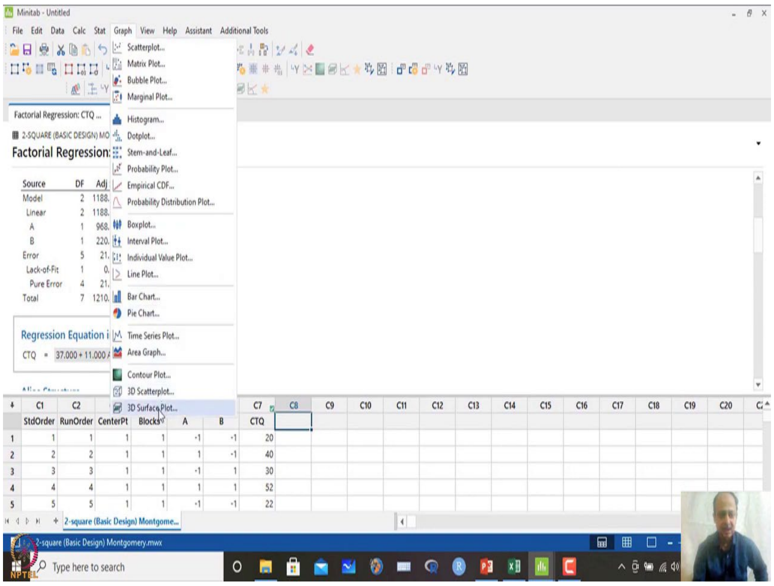


Factorial Regression: CTQ versus A, B

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

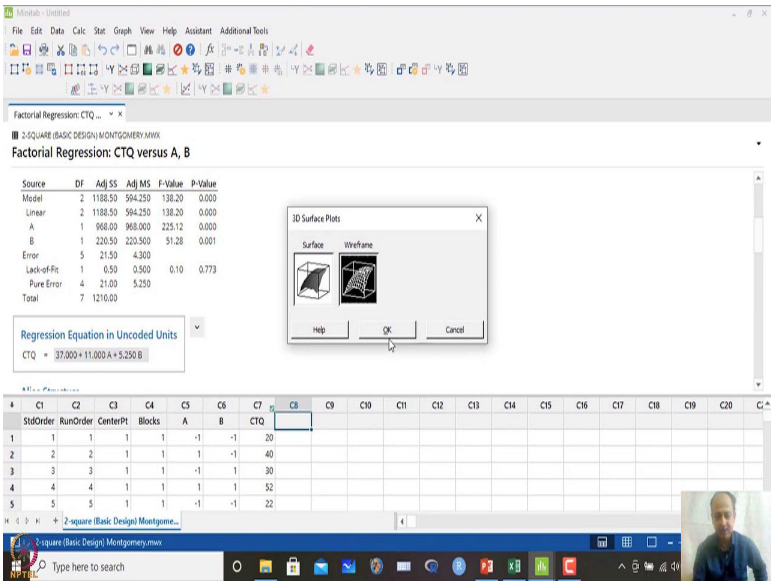
And we have saved this, this is standardized residual, residual plot is given over here; we have not saved the residuals. So, we can save the residual and check whether normal distribution assumption holds seems to be ok because all points are near to the line. So, there is no problem as such over here. So, this is the equation we can use and this is the final model that can be recommended over here.

(Refer Slide Time: 08:53)



	C1 StdOrder	C2 RunOrder	CenterPt	Blocks	A	B	C7 CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

(Refer Slide Time: 08:57)



2-SQUARE (BASIC DESIGN) MONTGOMERY.MWX

Factorial Regression: CTQ versus A, B

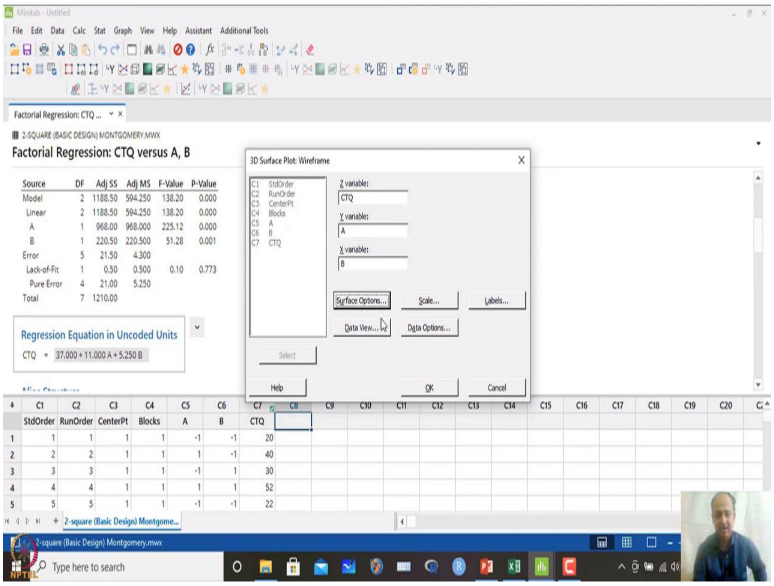
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	1188.50	594.250	138.20	0.000
Linear	2	1188.50	594.250	138.20	0.000
A	1	968.00	968.000	225.12	0.000
B	1	220.50	220.500	51.28	0.001
Error	5	21.50	4.300		
Lack-of-Fit	1	0.50	0.500	0.10	0.773
Pure Error	4	21.00	5.250		
Total	7	1210.00			

Regression Equation in Uncoded Units

CTQ = 37.000 + 11.000 A + 5.250 B

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 A	C6 B	C7 CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

(Refer Slide Time: 09:00)



Factorial Regression: CTQ versus A, B

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	1188.50	594.250	138.20	0.000
Linear	2	1188.50	594.250	138.20	0.000
A	1	968.00	968.000	225.12	0.000
B	1	220.50	220.500	51.28	0.001
Error	5	21.50	4.300		
Lack-of-Fit	1	0.50	0.500	0.10	0.773
Pure Error	4	21.00	5.250		
Total	7	1210.00			

Regression Equation in Uncoded Units

CTQ = 37.000 + 11.000 A + 5.250 B

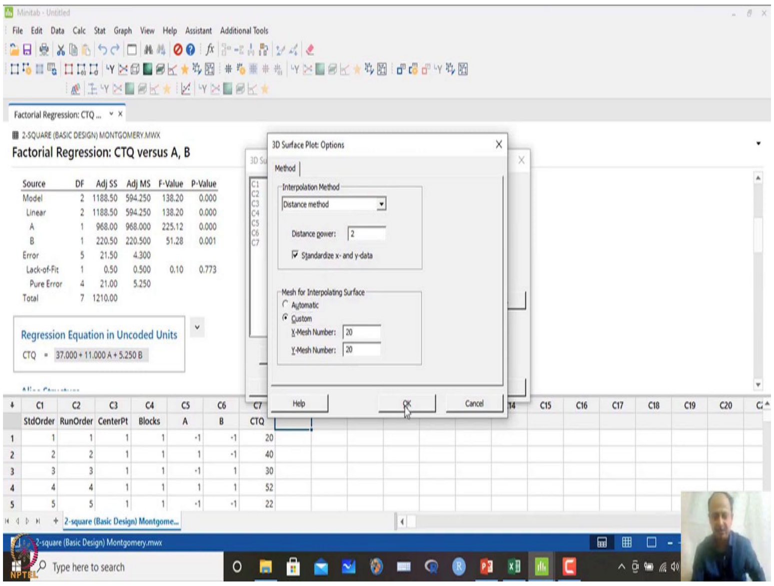
C1 StdOrder
C2 RunOrder
C3 CenterPt
C4 Blocks
C5 A
C6 B
C7 CTQ

Z variable: CTQ
Y variable: A
X variable: B

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

Now, if we want to draw graphically I want to see that surface plot let us say of this. So, I can use wireframe over here and I will click ok over here and then Z axis, I will say CTQ, let us take CTQ in the Z axis and let us take A in Y axis and B in this.

(Refer Slide Time: 09:11)



Factorial Regression: CTQ versus A, B

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	1188.50	594.250	138.20	0.000
Linear	2	1188.50	594.250	138.20	0.000
A	1	968.00	968.000	225.12	0.000
B	1	220.50	220.500	51.28	0.001
Error	5	21.50	4.300		
Lack-of-Fit	1	0.50	0.500	0.10	0.773
Pure Error	4	21.00	5.250		
Total	7	1210.00			

Regression Equation in Uncoded Units

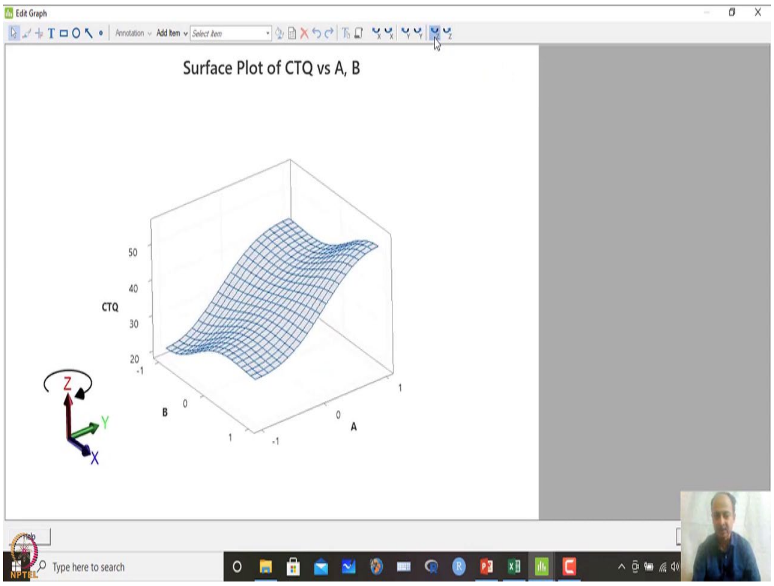
CTQ = 37.000 + 11.000 A + 5.250 B

Interpolation Method
Distance method
Distance power: 2
☑ Standardize x- and y-data

Mesh for Interpolating Surface
○ Automatic
● Custom
X-Mesh Number: 20
Y-Mesh Number: 20

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

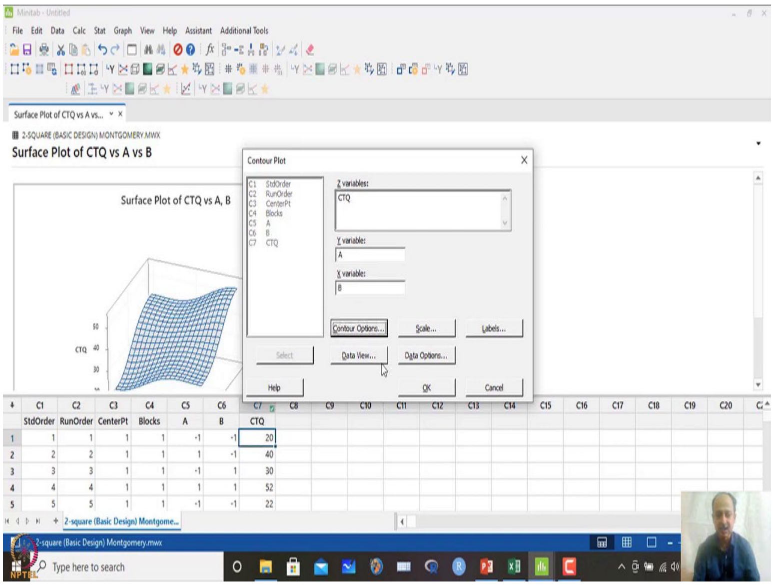
(Refer Slide Time: 09:23)



And then surface options in this case maybe I will customize this with 20 number of meshes over here; 20 number of meshes over here if I assume this one and then click OK; other things remain same as default, I will click OK. So, what will happen is that you will get the surface that is generated over here. So, this is the surface over here that is generated and this can be rotated also that I showed know.

So, last time we have seen that this can be rotated and we can see from any given direction what is happening. So, surface plot is possible over here. So, this gives you more or less flat surface because interaction is not so prominent. So, surface is flat. So, this is possible and this can be also seen like that. We can also see the contour plot over here so, because if I am assuming continuity over here, I can also see the contour plot over here.

(Refer Slide Time: 09:57)



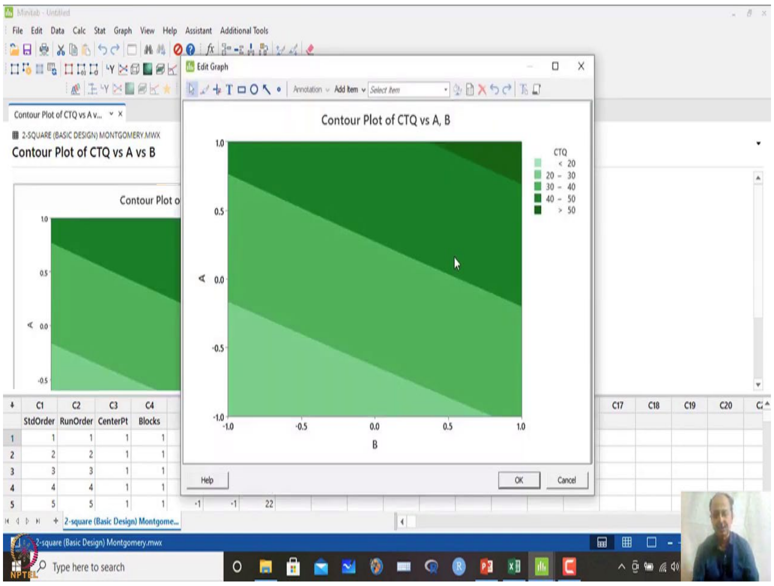
Z variables: CTQ

Y variable: A

X variable: B

	StdOrder	RunOrder	CenterPt	Blocks	A	B	CTQ
1	1	1	1	1	-1	-1	20
2	2	2	1	1	1	-1	40
3	3	3	1	1	-1	1	30
4	4	4	1	1	1	1	52
5	5	5	1	1	-1	-1	22

(Refer Slide Time: 10:03)

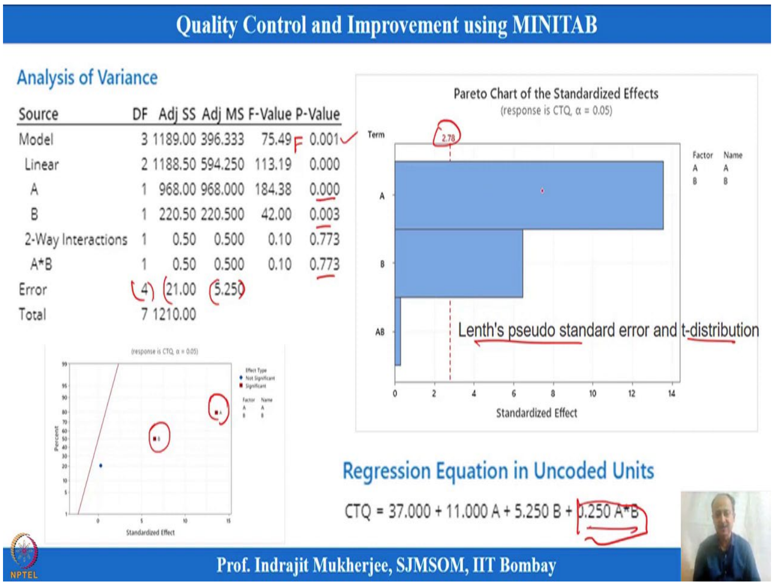


So, in this case Z axis will be CTQ and Y axis let us say, A and B like that and this contour plot is also possible. We can see that one and in this case, we will figure out that where it is maximized if I say that CTQ has to be maximized. So, in this case what I see is that B at plus level and A at plus level is giving me the because both have positive coefficients also what we have seen in regression.

So, as I increase A and B, it will increase the CTQ values like that. So, that is also prominent over here in the contour plot. So, contour plot, I told that the top view of the surface plot that you have seen. So, that will be given in contour plot like that. So, in this case I click OK so, I get the contour plot over here and we can also see some more optimization.

So, we will take some real example that will be easier for you to understand why we when we take a real case like that. So, let me take one more one or few case studies over few cases that we can analyze.

(Refer Slide Time: 10:58)



Quality Control and Improvement using MINITAB

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	1189.00	396.333	75.49 F	0.001 ✓
Linear	2	1188.50	594.250	113.19	0.000
A	1	968.00	968.000	184.38	0.000
B	1	220.50	220.500	42.00	0.003
2-Way Interactions	1	0.50	0.500	0.10	0.773
A*B	1	0.50	0.500	0.10	0.773
Error	4	21.00	5.250		
Total	7	1210.00			

Pareto Chart of the Standardized Effects
(response is CTQ, α = 0.05)

Lenth's pseudo standard error and t-distribution

Regression Equation in Uncoded Units

CTQ = 37.000 + 11.000 A + 5.250 B + 0.250 A*B

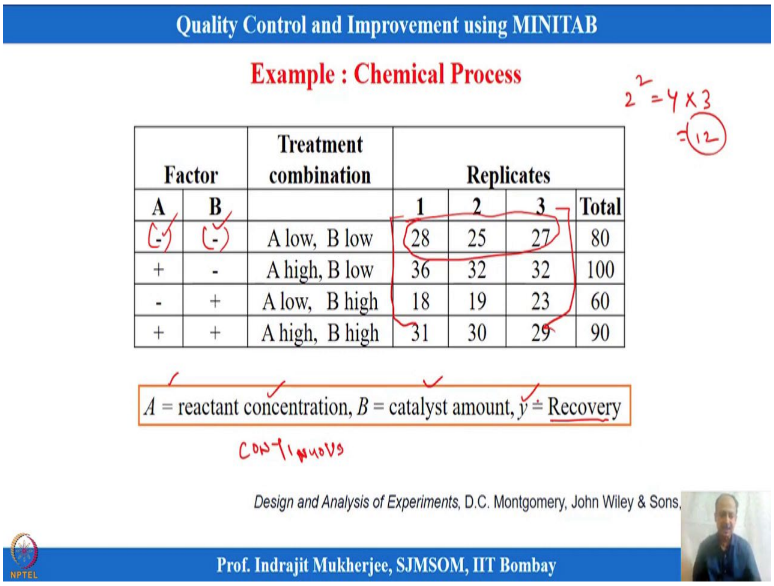
Prof. Indrajit Mukherjee, SJMSOM, IIT Bombay

So, I will go to the PPT's and try to figure out what is the next case that we have on these aspects like that. So, this is the graphical plot that that I have already generated like that. So, you see factor A is significant over here, factor B is significant AB interaction is not significant. So, in this case, this is not significant coefficient. So, we can drop this one and develop the regression equation again. So, that we have seen so, we have eliminated.

So, these two factors are only prominent in normal plots. So, that will give me indication which factor to drop which interaction to drop like that. So, that will give me some hints about that ok. And because of replications what we have seen the error degree of freedom is calculated over here and error degree of freedom, we are having over here and that is allowing us to estimate the mean square values. And then we can calculate the F values over here and the corresponding P values can be calculated over here ok.

So, Lenth's pseudo standard, I told that this is the thing that is used to find out these cut of values over here 2.78 and beyond that anything is there that effect has basically statistical significance that is the formulation says like that.

(Refer Slide Time: 11:59)



Quality Control and Improvement using MINITAB

Example : Chemical Process

$2^2 = 4 \times 3 = \boxed{12}$

Factor		Treatment combination	Replicates			
A	B		1	2	3	Total
-	-	A low, B low	28	25	27	80
+	-	A high, B low	36	32	32	100
-	+	A low, B high	18	19	23	60
+	+	A high, B high	31	30	29	90

A = reactant concentration, B = catalyst amount, y = Recovery

continuous

Design and Analysis of Experiments, D.C. Montgomery, John Wiley & Sons,

Prof. Indrajit Mukherjee, SJMSOM, IIT Bombay

And that we have seen. So, let us take one example from the chemical process over here. So, factors are coded over here what do you see coding values over here. So, actually they may have certain values which has two levels basically so, but I have coded this variable over here; what is the (Refer Time: 12:14).

When we code the variables like this, it becomes easier to compare the effects estimation and the coefficients that we get. So, effect which is larger effect which is smaller effect that is possible, but how people code that also we will the formulation for coding also can be seen from any books like Montgomery's book it is given, how it is done.

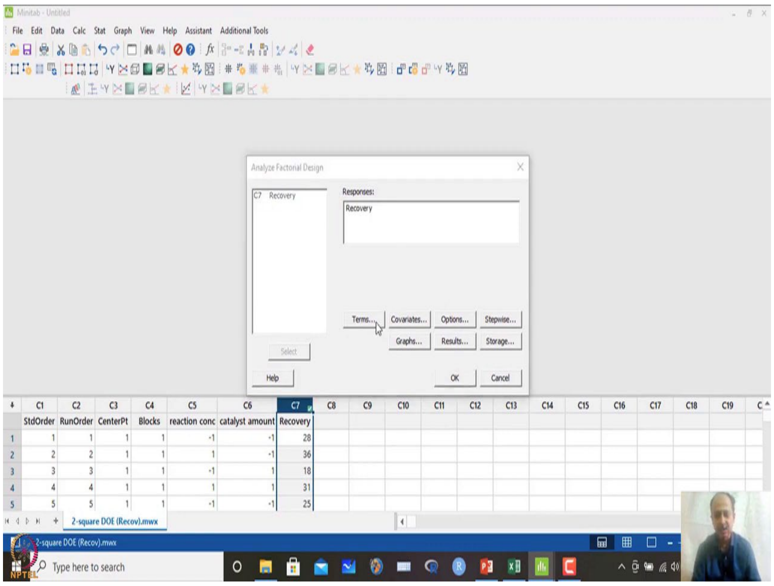
So, anyhow we have coded, let us assume the actual value is coded over here. So, this is minus 1 minus 1 and we have three replicates over here and this is chemical recovery why values of CTQ is taken as chemical recovery. So, it is recovery let us assume that we have to maximize the outcome and we have this A B factors only with us and A is changed in 2 levels, B is changed in 2 levels. So, this is basically a 2*2 square design over here.

So, minimum number of trials is 4, but I have taken 3 replicates over here so, 12 experimentation is done. So, total number of experimental trials is 12 over here. So, based on these data that is generated over here, we want to analyze what should be the level of A reaction concentration and what should be the level of B that is catalyst amount over here. So, both are assumed to be continuous over here. So, I am assuming continuity continuous.

So, in this case I want to analyze this experimentation and figure out that how I can maximize the recoveries over here. So, this is replications over here. So, now, we can create this design like what I have told like earlier lectures that we can just add the replicates and then based on that we can do the analysis like that.

So, I have this data set with me where already the data is the experiment, this MINITAB excel sheet was prepared like that and trial was done and these are the C5, C6 column of experimentation that we have with 3 replicates so 12 experimental trials. And these are the recovery values already, I have noted down over here. So, what I will do is that then I will run the trials. So, to figure out whether the factor A and B how it is; how it is like that. So, I will go to design of experiments factorial design, analyze factorial design.

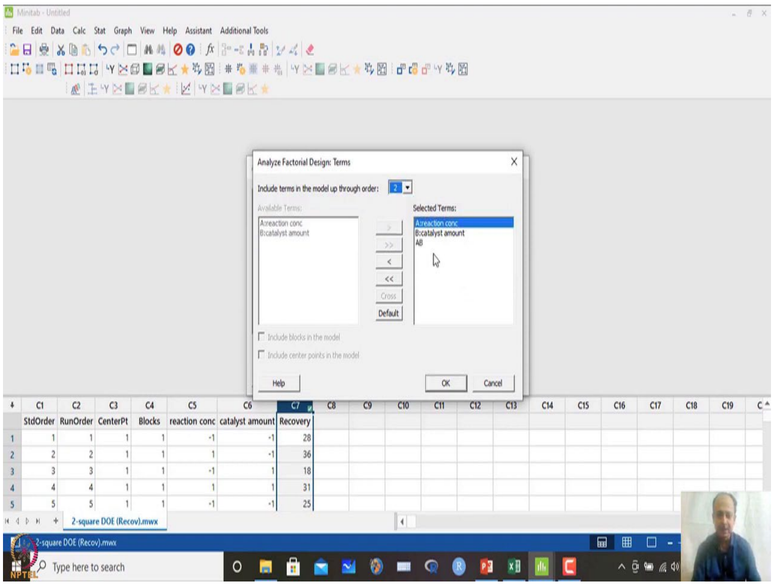
(Refer Slide Time: 14:17)



Responses: Recovery

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery
1	1	1	1	1	-1	-1	28
2	2	2	1	1	1	-1	36
3	3	3	1	1	-1	1	18
4	4	4	1	1	1	1	31
5	5	5	1	1	-1	-1	25

(Refer Slide Time: 14:22)



Analyze Factorial Design: Terms

Include terms in the model up through order: 2

Available Terms:
A:reaction conc
B:catalyst amount

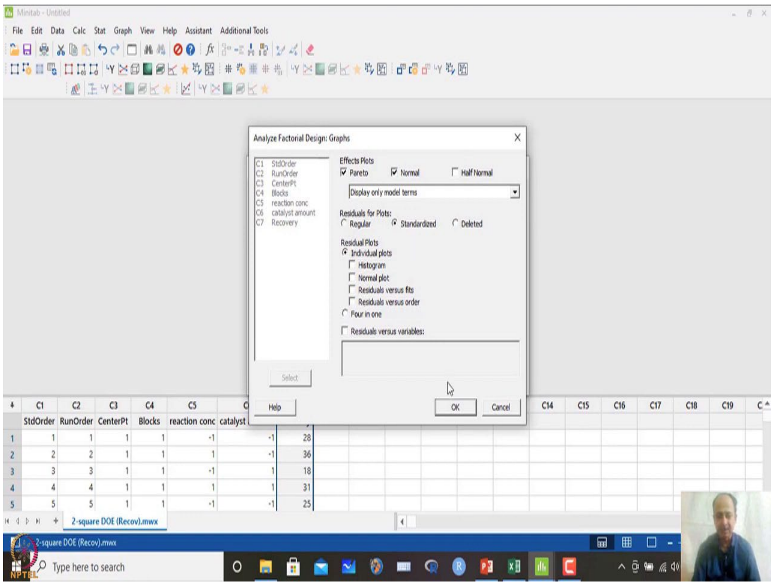
Selected Terms:
A:reaction conc
B:catalyst amount
AB

☐ Include blocks in the model
☐ Include center points in the model

	C1	C2	C3	C4	C5	C6	C7
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery
1	1	1	1	1	-1	-1	28
2	2	2	1	1	1	-1	36
3	3	3	1	1	-1	1	18
4	4	4	1	1	1	1	31
5	5	5	1	1	-1	-1	25

So, here I mentioned if it is not mentioned over here, you have to mention recoveries over here; many time I has taken it automatically. So, if you want to see interaction effects over here, you just include that into the model. So, I have this is 1. So, if you click 2, AB interactions will come over here. So, because we have three replicates so, in that case it is not difficult to calculate the mean square error. So, there will be no problem in my opinion.

(Refer Slide Time: 14:39)



Analyze Factorial Design: Graphs

C1 StdOrder
C2 RunOrder
C3 CenterPt
C4 Blocks
C5 reaction conc
C6 catalyst amount
C7 Recovery

Effects Plots
☑ Pareto ☑ Normal ☐ Half Normal
Display only model terms

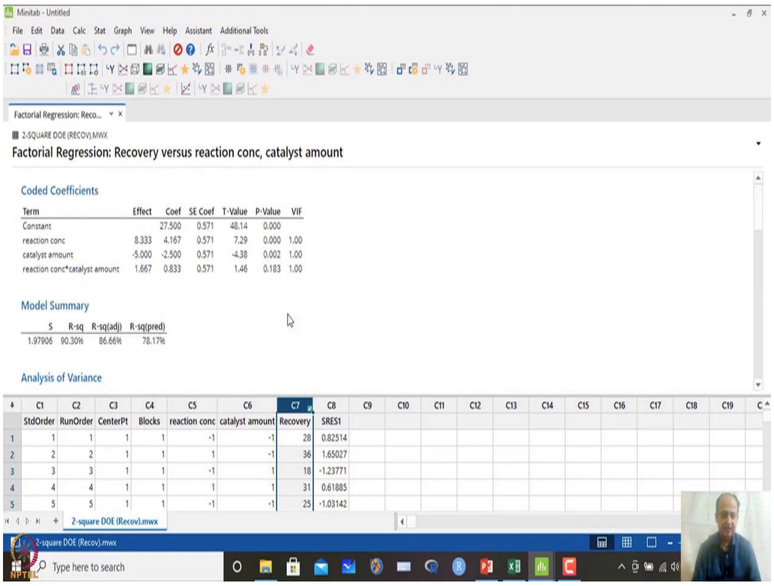
Residuals for Plots:
○ Regular ● Standardized ○ Deleted

Residual Plots
● Individual plots
☐ Histogram
☐ Normal plot
☐ Residuals versus fits
☐ Residuals versus order
○ Four in one

☐ Residuals versus variables:

So, then what we have done is that graph Pareto and normal plot, we can see what is the effect of that. Normal probability plot later on we can see so, standardized. So, this is not required at present. So, first we have to finalize the model, then we have to check all those things. So, then what we do? We just click OK, ok.

(Refer Slide Time: 14:57)



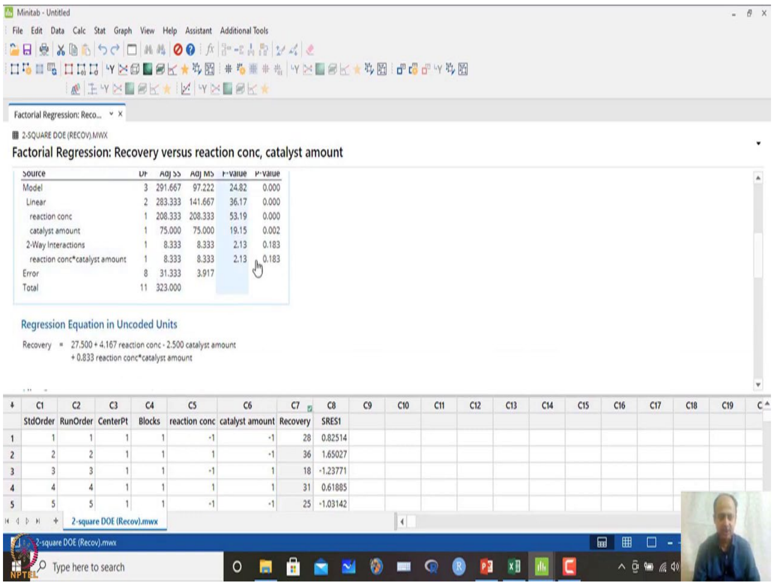
```
Factorial Regression: Recovery versus reaction conc, catalyst amount

Coded Coefficients

Term                              Effect    Coef  SE Coef  T-Value  P-Value   VIF
Constant                                  27.500    0.571    48.14    0.000
reaction conc                      8.333   4.167    0.571     7.29    0.000  1.00
catalyst amount                   -5.000  -2.500    0.571    -4.38    0.002  1.00
reaction conc*catalyst amount      1.667   0.833    0.571     1.46    0.183  1.00

Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
1.97906  90.30%     86.66%      78.17%

Analysis of Variance
```

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	0.82514
2	2	2	1	1	1	-1	36	1.65027
3	3	3	1	1	-1	1	18	-1.23771
4	4	4	1	1	1	1	31	0.61885
5	5	5	1	1	-1	-1	25	-1.03142

So, when we do OK over here, what we are observing is that A in the coefficient which is coded over here. So, in this case what we are seeing is that reaction concentration and catalyst they are basically significant over here, but interaction effect is not significant interaction effect is not significant over here.

So, and R square predicted value is about 78.17 if interaction is also included. So, 78.17, let us assume R square predicted more the R square predicted better is the model in our opinion. So, what is unknown observations can be predicted like that so with more accuracies. So, here it is 78.17 quite adequate.

(Refer Slide Time: 15:36)



Factorial Regression: Recovery versus reaction conc, catalyst amount

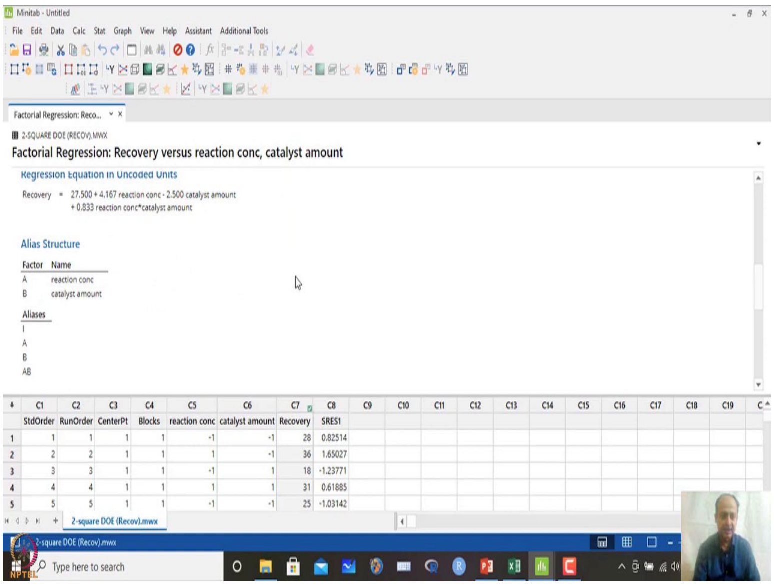
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	291.667	97.222	24.82	0.000
Linear	2	283.333	141.667	36.17	0.000
reaction conc	1	208.333	208.333	53.19	0.000
catalyst amount	1	75.000	75.000	19.15	0.002
2-Way Interactions	1	8.333	8.333	2.13	0.183
reaction conc*catalyst amount	1	8.333	8.333	2.13	0.183
Error	8	31.333	3.917		
Total	11	323.000			

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount + 0.833 reaction conc*catalyst amount

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	0.82514
2	2	2	1	1	1	-1	36	1.65027
3	3	3	1	1	-1	1	18	-1.23771
4	4	4	1	1	1	1	31	0.61885
5	5	5	1	1	-1	-1	25	-1.03142

(Refer Slide Time: 15:42)



Factorial Regression: Recovery versus reaction conc, catalyst amount

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount + 0.833 reaction conc*catalyst amount

Alias Structure

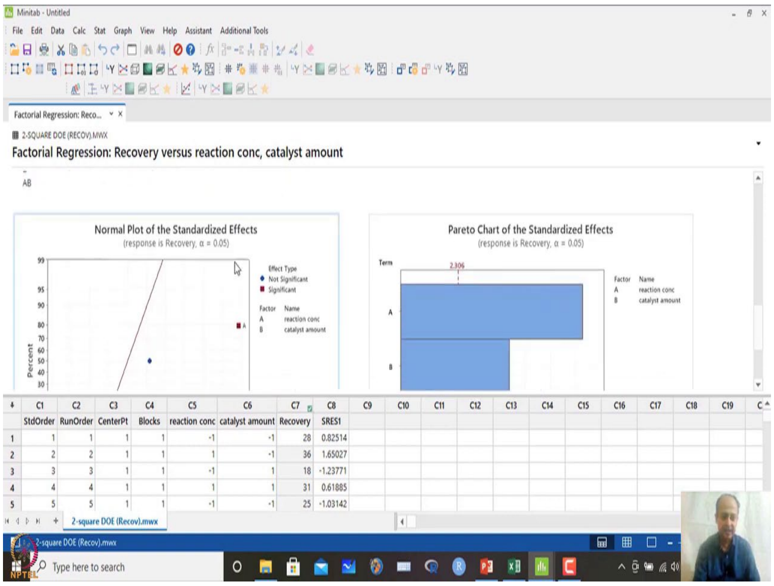
Factor	Name
A	reaction conc
B	catalyst amount

Aliases

I
A
B
AB

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	0.82514
2	2	2	1	1	1	-1	36	1.65027
3	3	3	1	1	-1	1	18	-1.23771
4	4	4	1	1	1	1	31	0.61885
5	5	5	1	1	-1	-1	25	-1.03142

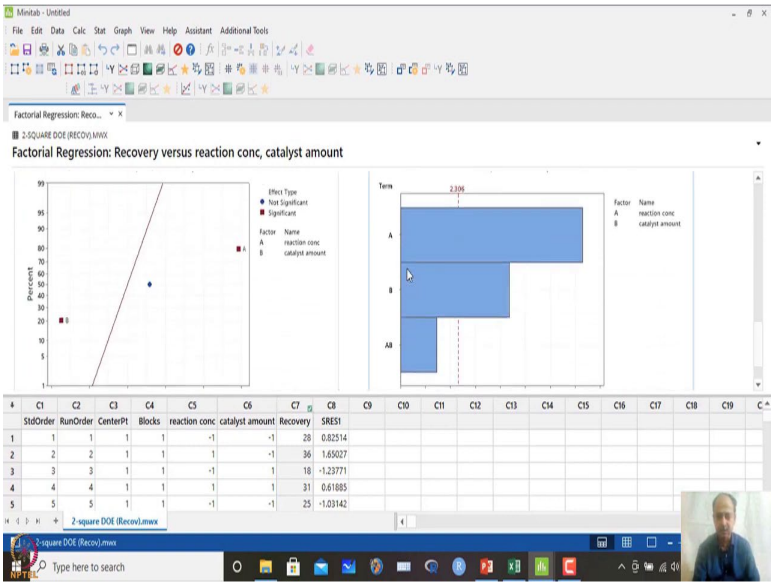
(Refer Slide Time: 15:44)



Factorial Regression: Recovery versus reaction conc, catalyst amount

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery	C8 SRES1
1	1	1	1	1	-1	-1	28	0.82514
2	2	2	1	1	1	-1	36	1.65027
3	3	3	1	1	-1	1	18	-1.23771
4	4	4	1	1	1	1	31	0.61885
5	5	5	1	1	-1	-1	25	-1.03142

(Refer Slide Time: 15:45)



Factorial Regression: Recovery versus reaction conc, catalyst amount

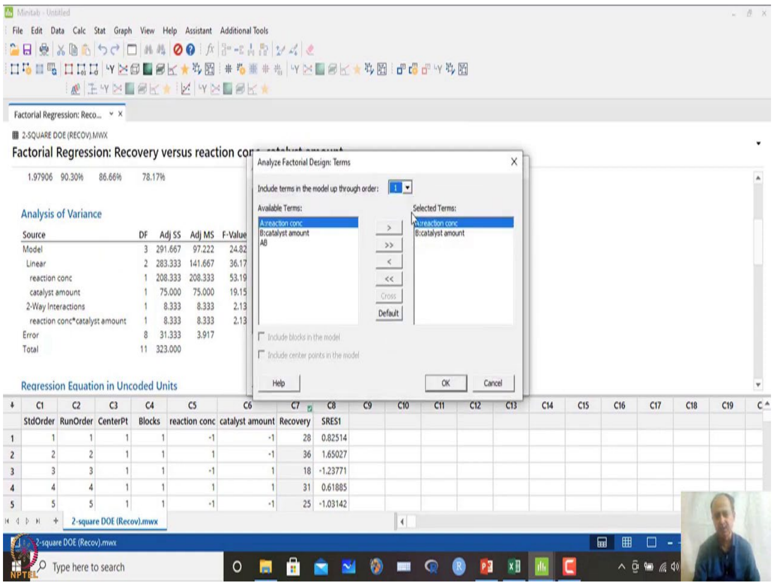
	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery	C8 SRES1
1	1	1	1	1	-1	-1	28	0.82514
2	2	2	1	1	1	-1	36	1.65027
3	3	3	1	1	-1	1	18	-1.23771
4	4	4	1	1	1	1	31	0.61885
5	5	5	1	1	-1	-1	25	-1.03142

And in this case two way interaction is not prominent, what we have seen. And so, and then we go to the Pareto effect plot standardized effect plot. So, in this case, also I see A B is significant, AB is not significant. So, in this case and here what you see is that A is having a positive effect and B is having a negative effect.

Although they are significant one sign convention will be different over here. So, A is having plus coefficient and B is having negative coefficient. So, if I increase catalyst amount, it will decrease the recoveries basically if I increase the reaction concentration, it will increase the recovery basically. So, this is contradicting with each other. So, we have to be careful when we are adding or reducing the amounts like that.

So, in this case what is possible is that because what we have seen is that interaction effect is not significant. So, let us finalize the model. So, in this case and we have to remember that the predicted R square value what we have got is approximately around 9, 78.17. So, we will now I am reducing the model. So, in this case what I will do is that factor factorial design analyze factorial.

(Refer Slide Time: 16:40)



Factorial Regression: Recovery versus reaction conc, catalyst amount

```
1.97906   90.30%   86.66%   78.17%

Analysis of Variance

Source                          DF   Adj SS   Adj MS  F-Value
Model                            3  291.667   97.222    24.82
  Linear                         2  283.333  141.667    36.17
    reaction conc                1  208.333  208.333    53.19
    catalyst amount              1   75.000   75.000    19.15
  2-Way Interactions             1    8.333    8.333     2.13
    reaction conc*catalyst amount 1   8.333    8.333     2.13
Error                            8   31.333    3.917
Total                           11  323.000

Regression Equation in Uncoded Units
```

Analyze Factorial Design: Terms

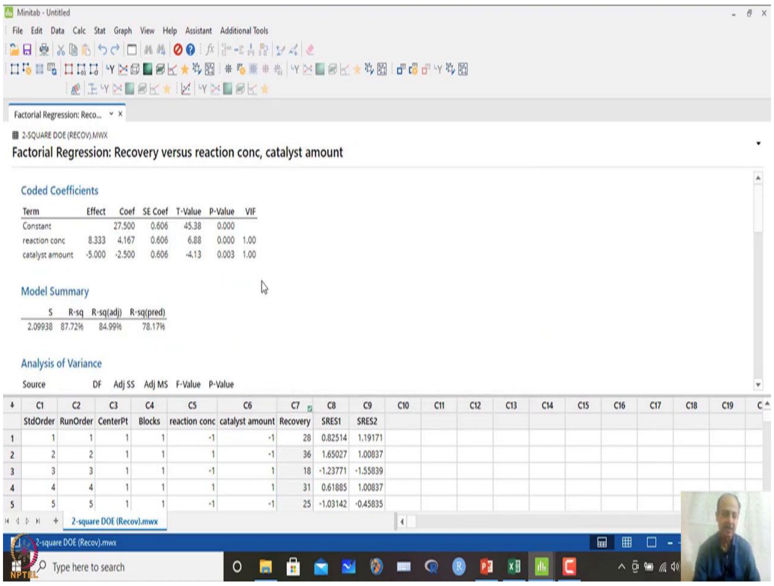
Include terms in the model up through order: 1

Available Terms:
A:reaction conc
B:catalyst amount
AB

Selected Terms:
A:reaction conc
B:catalyst amount

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery	C8 SRES1
1	1	1	1	1	-1	-1	28	0.82514
2	2	2	1	1	1	-1	36	1.65027
3	3	3	1	1	-1	1	18	-1.23771
4	4	4	1	1	1	1	31	0.61885
5	5	5	1	1	-1	-1	25	-1.03142

(Refer Slide Time: 16:49)

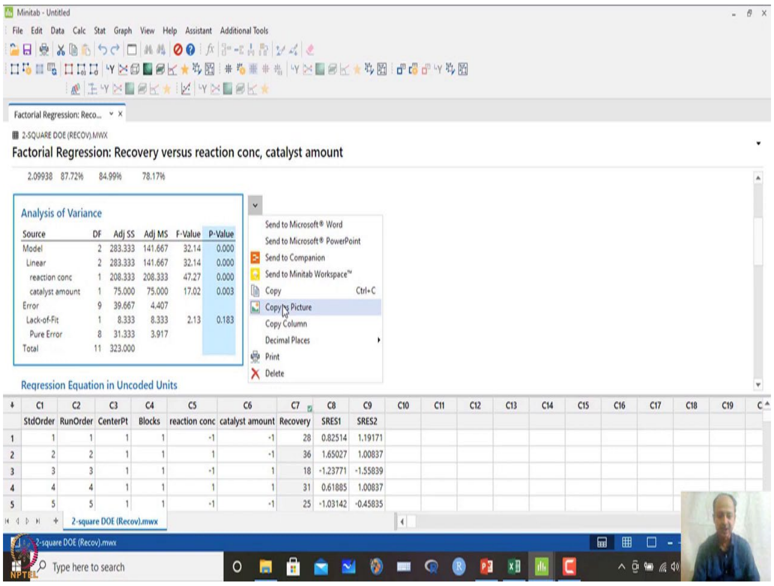


Factorial Regression: Recovery versus reaction conc, catalyst amount

```
Coded Coefficients

Term             Effect    Coef  SE Coef  T-Value  P-Value   VIF
Constant                 27.500    0.606    45.38    0.000
reaction conc     8.333   4.167    0.606     6.88    0.000  1.00
catalyst amount  -5.000  -2.500    0.606    -4.13    0.003  1.00

Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
2.09938  87.72%     84.99%      78.17%

Analysis of Variance

Source   DF   Adj SS   Adj MS  F-Value  P-Value
```

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery	C8 SRES1	C9 SRES2
1	1	1	1	1	-1	-1	28	0.82514	1.19171
2	2	2	1	1	1	-1	36	1.65027	1.00837
3	3	3	1	1	-1	1	18	-1.23771	-1.55839
4	4	4	1	1	1	1	31	0.61885	1.00837
5	5	5	1	1	-1	-1	25	-1.03142	-0.45835

(Refer Slide Time: 16:52)



Factorial Regression: Recovery versus reaction conc, catalyst amount

2.09938 87.72% 84.99% 78.17%

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	283.333	141.667	32.14	0.000
Linear	2	283.333	141.667	32.14	0.000
reaction conc	1	208.333	208.333	47.27	0.000
catalyst amount	1	75.000	75.000	17.02	0.003
Error	9	39.667	4.407		
Lack-of-Fit	1	8.333	8.333	2.13	0.183
Pure Error	8	31.333	3.917		
Total	11	323.000			

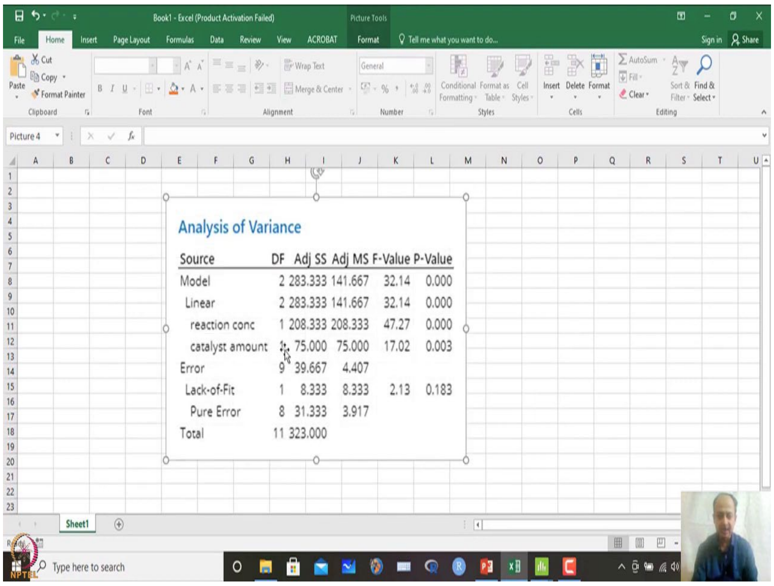
Regression Equation in Uncoded Units

	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1	SRES2
1	1	1	1	1	-1	-1	28	0.82514	1.19171
2	2	2	1	1	1	-1	36	1.65027	1.00837
3	3	3	1	1	-1	1	18	-1.23771	-1.55839
4	4	4	1	1	1	1	31	0.61885	1.00837
5	5	5	1	1	-1	-1	25	-1.03142	-0.45835

So, in case of term, I will not include the interaction. So, I have given include terms in the model up to order of 1. So, AB interaction is taken away over here. So, in this case, I will click OK and I will click OK what happens is that prediction behaviour does not change.

So, in this case I lose something on the R square adjusted value, but I do not miss lose much on the predicted value that is around same values what we are getting. So, this model seems to be adequate in this case reaction concentration and catalyst amount p-value is less.

(Refer Slide Time: 17:14)

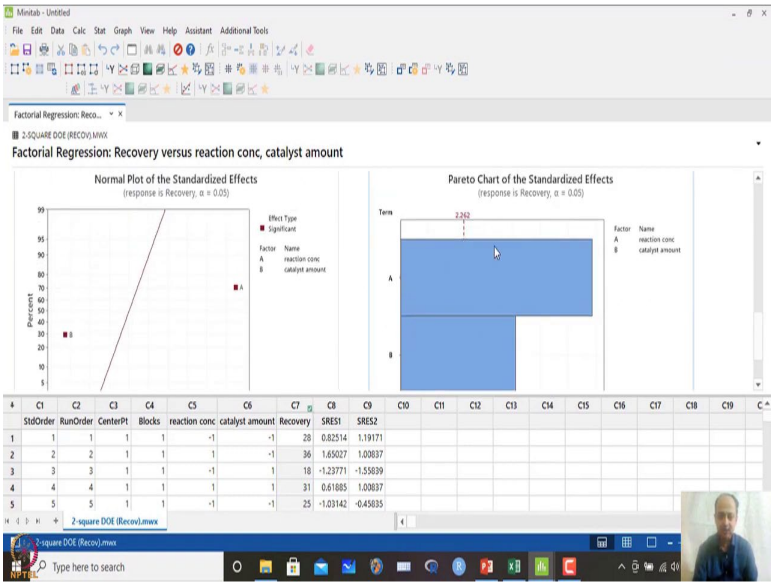


Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	283.333	141.667	32.14	0.000
Linear	2	283.333	141.667	32.14	0.000
reaction conc	1	208.333	208.333	47.27	0.000
catalyst amount	1	75.000	75.000	17.02	0.003
Error	9	39.667	4.407		
Lack-of-Fit	1	8.333	8.333	2.13	0.183
Pure Error	8	31.333	3.917		
Total	11	323.000			

So, if you want to see this final graphically when I place this one so, if I can place this one chemical recovery what I am getting is that I am getting a value; that means, reaction concentration is significant catalyst amount is significant. But there is no lack of fit in the model. So, 0.183; that means, linear model is quite adequate to express this one.

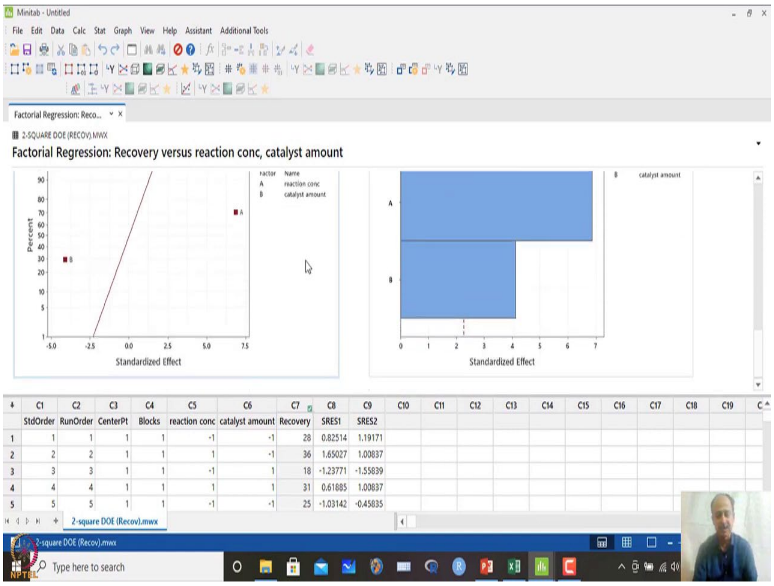
(Refer Slide Time: 17:29)



Factorial Regression: Recovery versus reaction conc, catalyst amount

	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1	SRES2
1	1	1	1	1	-1	-1	28	0.82514	1.19171
2	2	2	1	1	1	-1	36	1.65027	1.00837
3	3	3	1	1	-1	1	18	-1.23771	-1.55839
4	4	4	1	1	1	1	31	0.61885	1.00837
5	5	5	1	1	-1	-1	25	-1.03142	-0.45835

(Refer Slide Time: 17:35)

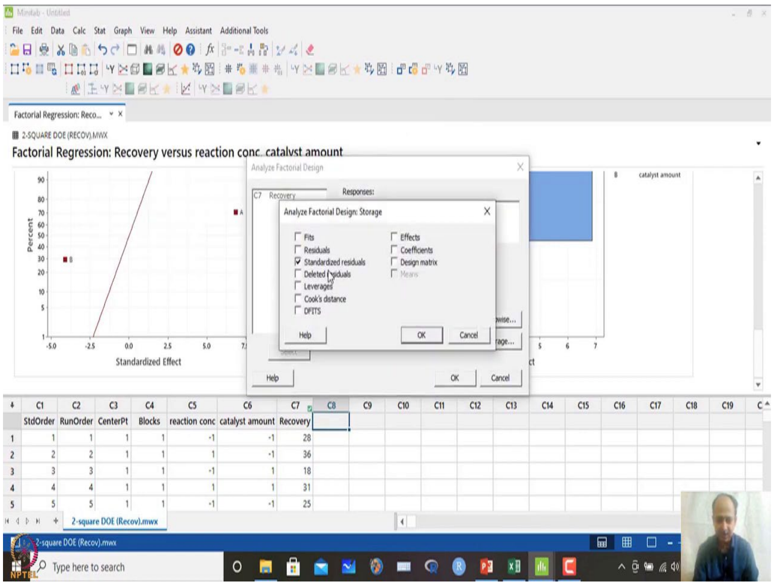


Factorial Regression: Recovery versus reaction conc, catalyst amount

	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1	SRES2
1	1	1	1	1	-1	-1	28	0.82514	1.19171
2	2	2	1	1	1	-1	36	1.65027	1.00837
3	3	3	1	1	-1	1	18	-1.23771	-1.55839
4	4	4	1	1	1	1	31	0.61885	1.00837
5	5	5	1	1	-1	-1	25	-1.03142	-0.45835

And then what I see is that equation is given over here and this standardized effect plot also shows that AB is significant over here and both are on the opposite side. So; that means, sign convention that I told one is effecting positively, one is effecting on the other side. So, when you run this, I do not know whether the residuals are saved or not. So, I can rerun these experimental trials and save the residual so, analyze factorial.

(Refer Slide Time: 17:53)



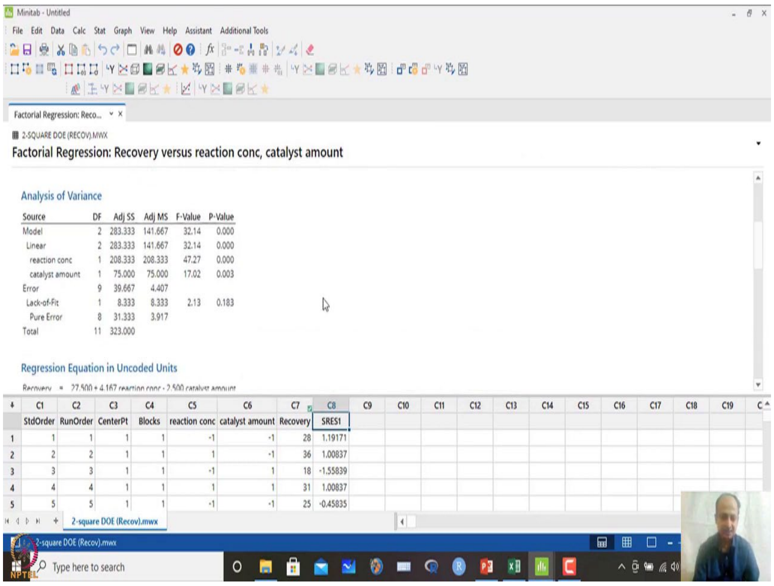
Factorial Regression: Recovery versus reaction conc, catalyst amount

Analyze Factorial Design: Storage

☐ Fits
☐ Residuals
☑ Standardized residuals
☐ Deleted residuals
☐ Leverages
☐ Cook's distance
☐ DFITS
☐ Effects
☐ Coefficients
☐ Design matrix
☐ Means

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	
1	1	1	1	1	-1	-1	28	
2	2	2	1	1	1	-1	36	
3	3	3	1	1	-1	1	18	
4	4	4	1	1	1	1	31	
5	5	5	1	1	-1	-1	25	

(Refer Slide Time: 17:55)



Factorial Regression: Recovery versus reaction conc, catalyst amount

Analysis of Variance

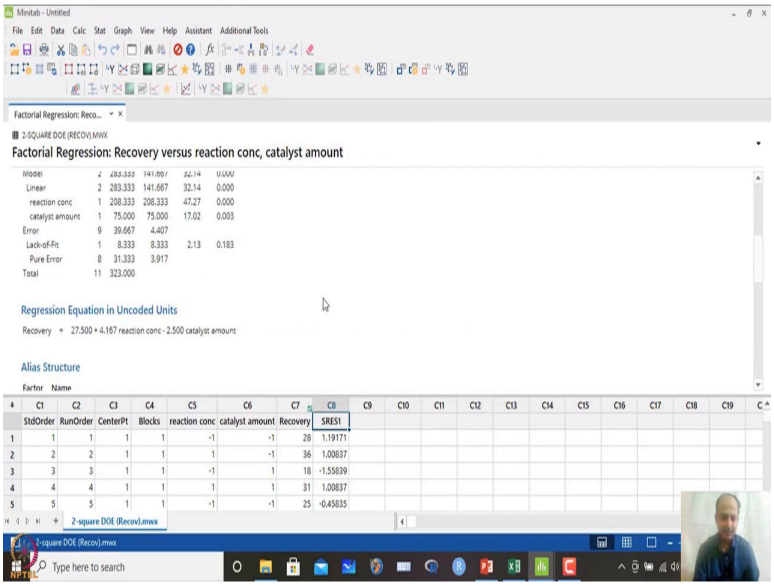
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	2	283.333	141.667	32.14	0.000
Linear	2	283.333	141.667	32.14	0.000
reaction conc	1	208.333	208.333	47.27	0.000
catalyst amount	1	75.000	75.000	17.02	0.003
Error	9	39.667	4.407		
Lack-of-Fit	1	8.333	8.333	2.13	0.183
Pure Error	8	31.333	3.917		
Total	11	323.000			

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	1.19171
2	2	2	1	1	1	-1	36	1.00837
3	3	3	1	1	-1	1	18	-1.55839
4	4	4	1	1	1	1	31	1.00837
5	5	5	1	1	-1	-1	25	-0.45835

(Refer Slide Time: 17:58)



Factorial Regression: Recovery versus reaction conc, catalyst amount

Model	2	283.333	141.667	32.14	0.000
Linear	2	283.333	141.667	32.14	0.000
reaction conc	1	208.333	208.333	47.27	0.000
catalyst amount	1	75.000	75.000	17.02	0.003
Error	9	39.667	4.407		
Lack-of-Fit	1	8.333	8.333	2.13	0.183
Pure Error	8	31.333	3.917		
Total	11	323.000			

Regression Equation in Uncoded Units

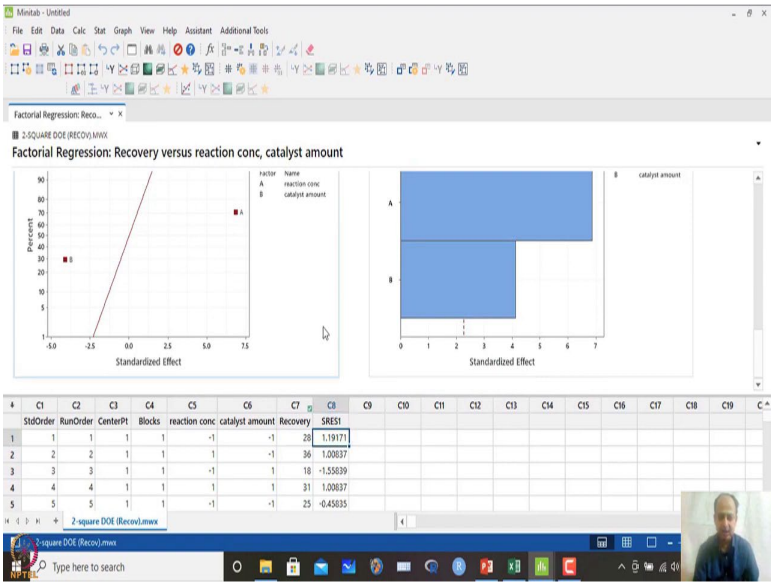
Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount

Alias Structure

Factor Name

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	1.19171
2	2	2	1	1	1	-1	36	1.00837
3	3	3	1	1	-1	1	18	-1.55839
4	4	4	1	1	1	1	31	1.00837
5	5	5	1	1	-1	-1	25	-0.45835

(Refer Slide Time: 18:01)

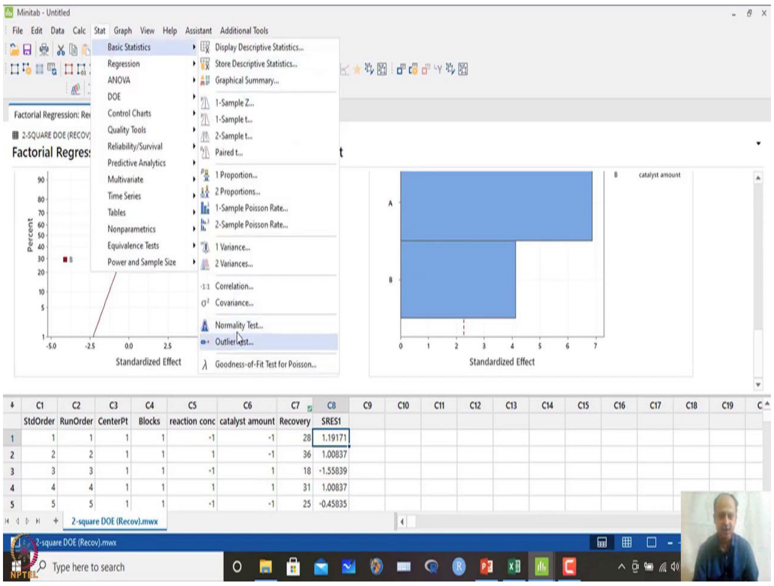


Factorial Regression: Recovery versus reaction conc, catalyst amount

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	1.19171
2	2	2	1	1	1	-1	36	1.00837
3	3	3	1	1	-1	1	18	-1.55839
4	4	4	1	1	1	1	31	1.00837
5	5	5	1	1	-1	-1	25	-0.45835

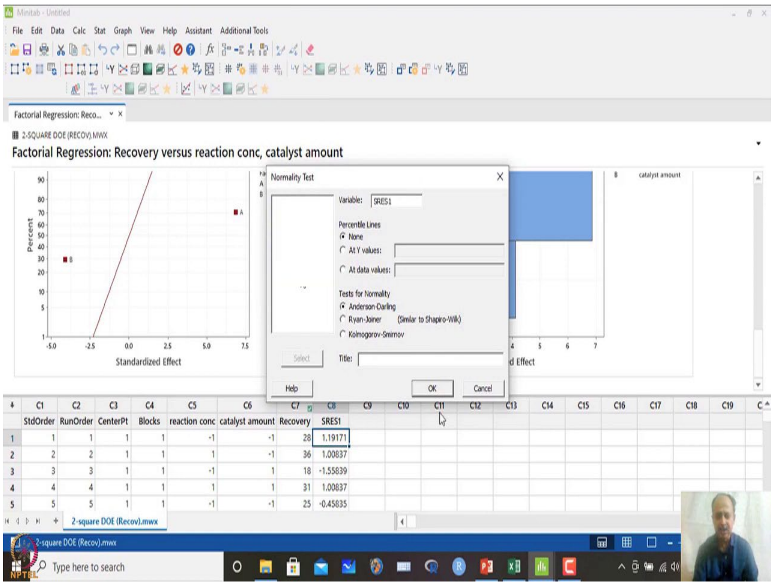
So, in this case storage, let me try to see yes standardized residual is done over here. So, in this case in this case so, I have saved the residual over here which is with 2 factors only A and B, no interaction over here.

(Refer Slide Time: 18:08)



	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	1.19171
2	2	2	1	1	1	-1	36	1.00837
3	3	3	1	1	-1	1	18	-1.55839
4	4	4	1	1	1	1	31	1.00837
5	5	5	1	1	-1	-1	25	-0.45835

(Refer Slide Time: 18:10)



Factorial Regression: Recovery versus reaction conc, catalyst amount

Normality Test

Variable: SRES1

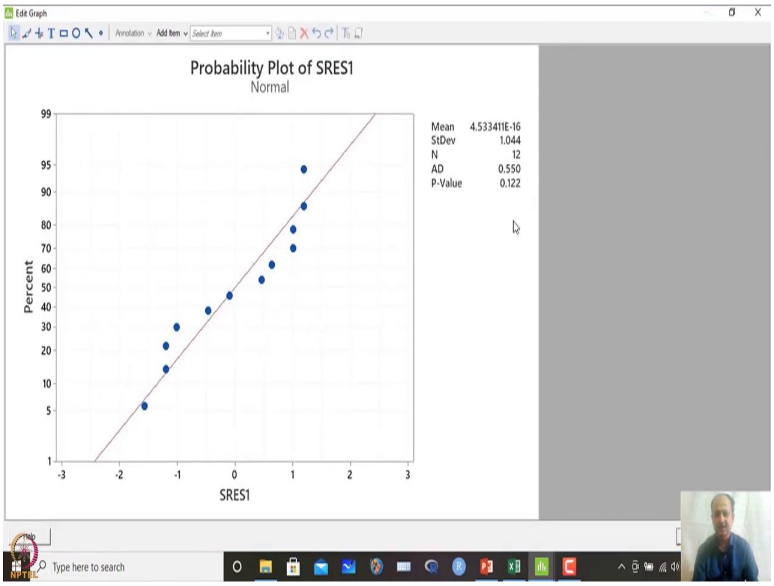
Percentile Lines
- ● None
- ○ At Y values:
- ○ At data values:

Tests for Normality
- ● Anderson-Darling
- ○ Ryan-Joiner (Similar to Shapiro-Wilk)
- ○ Kolmogorov-Smirnov

Title:

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	1.19171
2	2	2	1	1	1	-1	36	1.00837
3	3	3	1	1	-1	1	18	-1.55839
4	4	4	1	1	1	1	31	1.00837
5	5	5	1	1	-1	-1	25	-0.45835

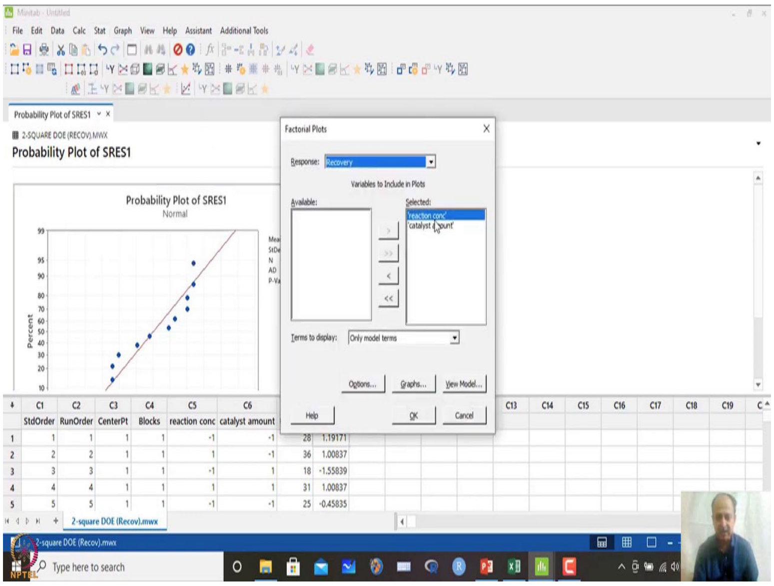
(Refer Slide Time: 18:14)



Mean 4.533411E-16
StDev 1.044
N 12
AD 0.550
P-Value 0.122

So, in this case I can check whether this is following the normal distribution assumption. So, I go to normal distribution, click the last residual over here and I do Anderson-Darling test and what we are observing is that Anderson-Darling test says that P value is more than 0.05. So, it is ok the model seems to be adequate over here and what we can do is that we can also see in the design of experiment factor plots like that.

(Refer Slide Time: 18:30)



Factorial Plots

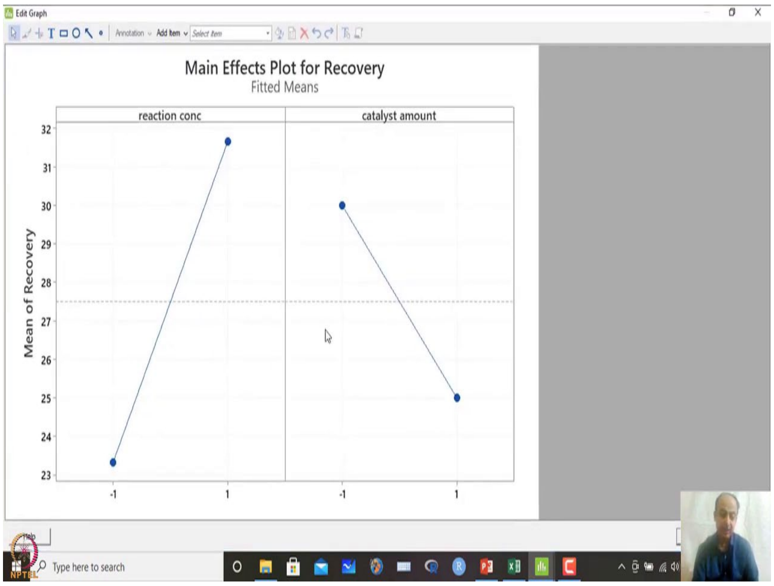
Response: Recovery

Selected: 'reaction conc' 'catalyst amount'

Terms to display: Only model terms

	C1	C2	C3	C4	C5	C6		
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount		
1	1	1	1	1	-1	-1	28	1.19171
2	2	2	1	1	1	-1	36	1.00837
3	3	3	1	1	-1	1	18	-1.55839
4	4	4	1	1	1	1	31	1.00837
5	5	5	1	1	-1	-1	25	-0.45835

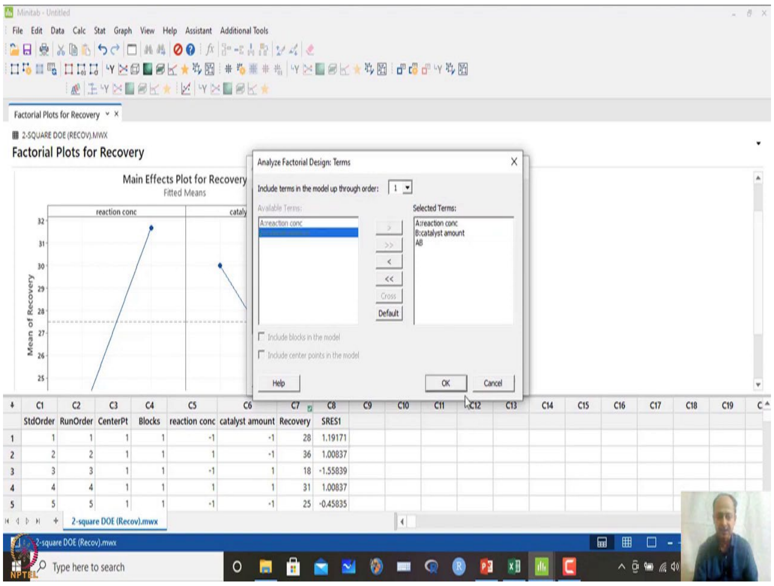
(Refer Slide Time: 18:40)



So, in this case factor plots can be seen over here. So, A B which is in our model say AB interaction is eliminated. So, we are not able to see that one, but if you click ok so, in this case what will happen is that, you will only see without interaction plot over here.

So, if you see these diagrammatically what is expected is that if you have to maximize the recoveries over here, then reaction concentration should be positive and catalyst amount should be negative over here. So, combination is A at positive level and B at negative level that is the combination that we are finding over here.

(Refer Slide Time: 19:05)



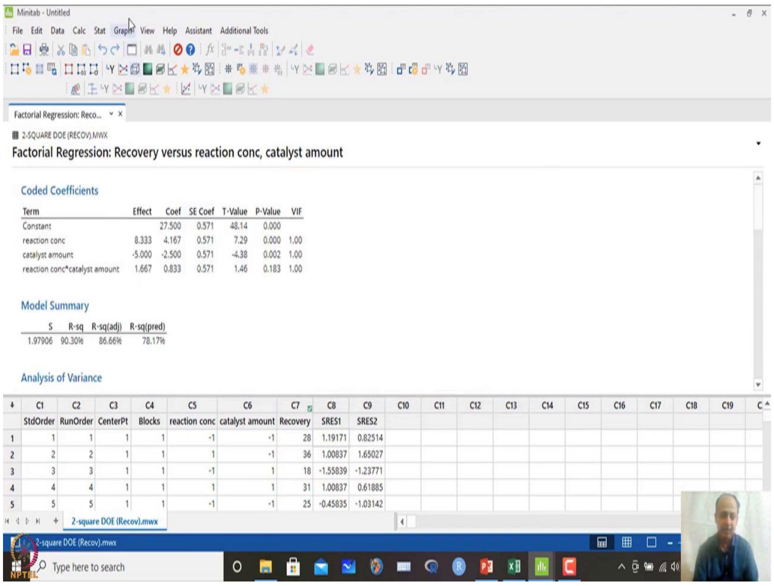
Include terms in the model up through order: 1

Selected Terms:
A:reaction conc
B:catalyst amount
AB

☐ Include blocks in the model
☐ Include center points in the model

	C1	C2	C3	C4	C5	C6	C7	C8
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1
1	1	1	1	1	-1	-1	28	1.19171
2	2	2	1	1	1	-1	36	1.00837
3	3	3	1	1	-1	1	18	-1.55839
4	4	4	1	1	1	1	31	1.00837
5	5	5	1	1	-1	-1	25	-0.45835

(Refer Slide Time: 19:10)



Factorial Regression: Recovery versus reaction conc, catalyst amount

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		27.500	0.571	48.14	0.000	
reaction conc	8.333	4.167	0.571	7.29	0.000	1.00
catalyst amount	-5.000	-2.500	0.571	-4.38	0.002	1.00
reaction conc*catalyst amount	1.667	0.833	0.571	1.46	0.183	1.00

Model Summary

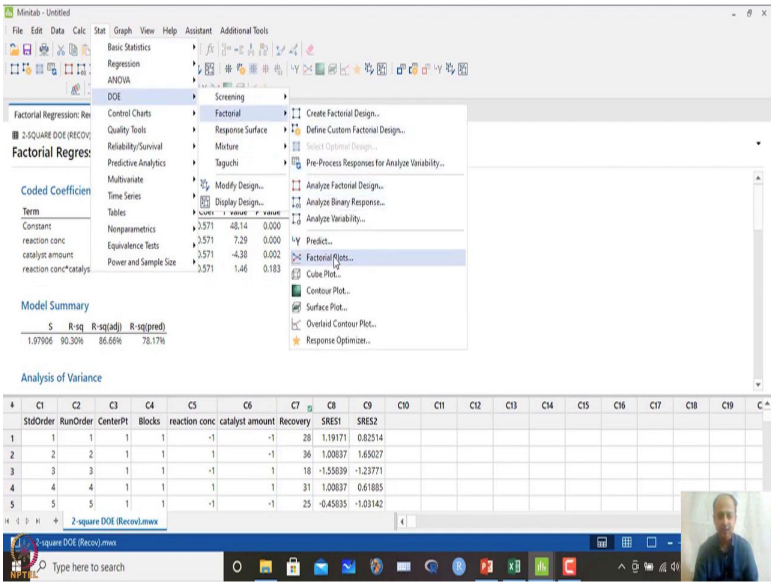
S	R-sq	R-sq(adj)	R-sq(pred)
1.97906	90.30%	86.66%	78.17%

Analysis of Variance

	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1	SRES2
1	1	1	1	1	-1	-1	28	1.19171	0.82514
2	2	2	1	1	1	-1	36	1.00837	1.65027
3	3	3	1	1	-1	1	18	-1.55839	-1.23771
4	4	4	1	1	1	1	31	1.00837	0.61885
5	5	5	1	1	-1	-1	25	-0.45835	-1.03142

So, if we want to see interaction effects like that. So, then you have to go to this again analyze this one and you have to include the A B interaction terms over here and then click OK and click OK over here.

(Refer Slide Time: 19:11)



Factorial Regres

Coded Coefficien

Term			
Constant	0.571	48.14	0.000
reaction conc	0.571	7.29	0.000
catalyst amount	0.571	-4.38	0.002
reaction conc*catalys	0.571	1.46	0.183

Model Summary

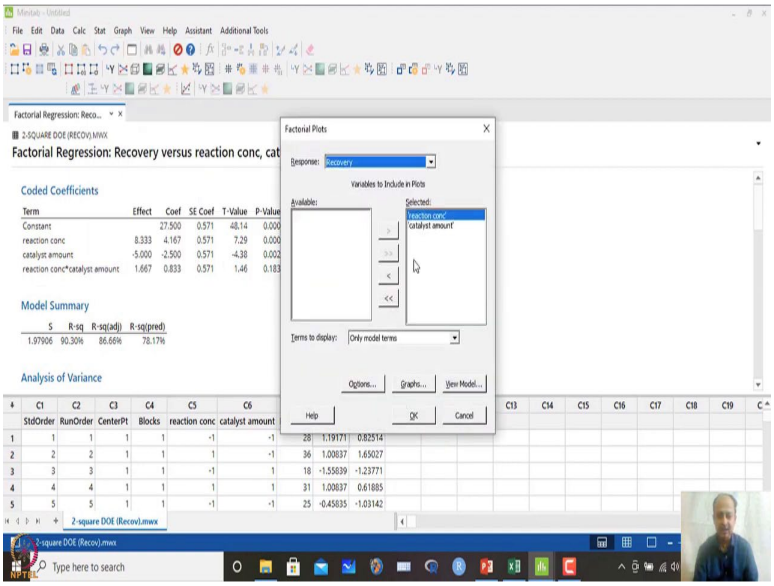
S	R-sq	R-sq(adj)	R-sq(pred)
1.97906	90.30%	86.66%	78.17%

Analysis of Variance

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery	C8 SRES1	C9 SRES2
1	1	1	1	1	-1	-1	28	1.19171	0.82514
2	2	2	1	1	1	-1	36	1.00837	1.65027
3	3	3	1	1	-1	1	18	-1.55839	-1.23771
4	4	4	1	1	1	1	31	1.00837	0.61885
5	5	5	1	1	-1	-1	25	-0.45835	-1.03142

2-square DOE (Recov).mwx

(Refer Slide Time: 19:15)



2-SQUARE DOE (RECOV).MWX

Factorial Regression: Recovery versus reaction conc, cat

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value
Constant		27.500	0.571	48.14	0.000
reaction conc	8.333	4.167	0.571	7.29	0.000
catalyst amount	-5.000	-2.500	0.571	-4.38	0.002
reaction conc*catalyst amount	1.667	0.833	0.571	1.46	0.183

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.97906	90.30%	86.66%	78.17%

Analysis of Variance

Response: Recovery

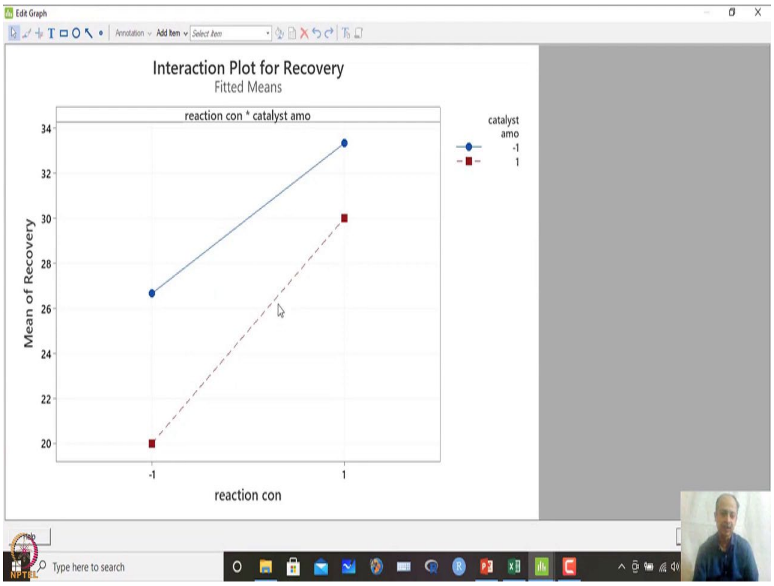
Selected: 'reaction conc' 'catalyst amount'

Terms to display: Only model terms

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount			
1	1	1	1	1	-1	-1	28	1.19171	0.82514
2	2	2	1	1	1	-1	36	1.00837	1.65027
3	3	3	1	1	-1	1	18	-1.55839	-1.23771
4	4	4	1	1	1	1	31	1.00837	0.61885
5	5	5	1	1	-1	-1	25	-0.45835	-1.03142

2-square DOE (Recov).mwx

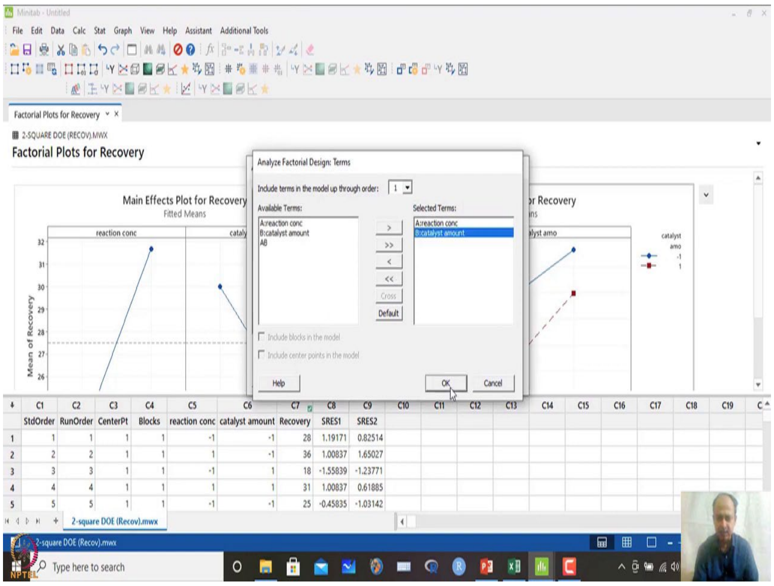
(Refer Slide Time: 19:18)



And then what I have to do is that design of experiments factorial, then we go to factorial plots and then when you mention this one this will be in the model, then interaction plot will come.

And then you can see the interaction plot its parallel lines you can see so, more or less interaction is not prominent that it disclosed that there is no interaction is not prominent over here ok. So, that is why we can eliminate that one from this. So, what we can do is that again rerun this one experimental trial so, over here analyze factorial design.

(Refer Slide Time: 19:41)



Analyze Factorial Design: Terms

Include terms in the model up through order: 1

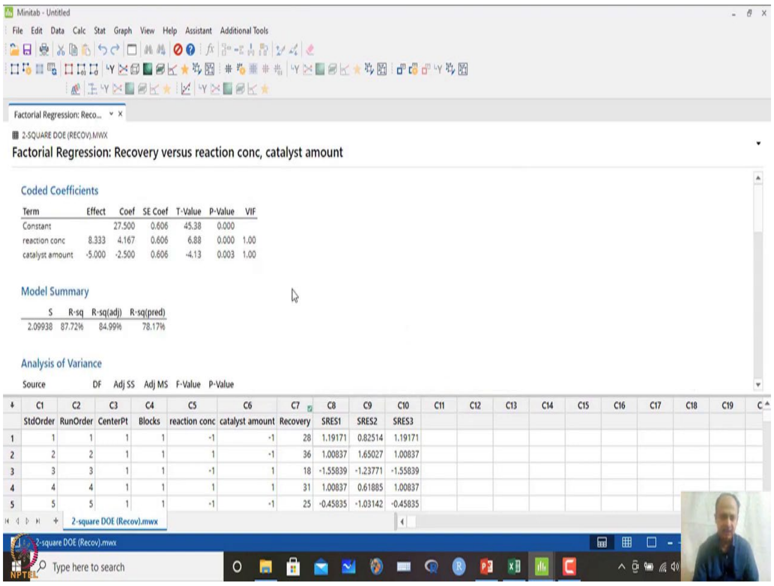
Available Terms:
A:reaction conc
B:catalyst amount
AB

Selected Terms:
A:reaction conc
B:catalyst amount

☐ Include blocks in the model
☐ Include center points in the model

	C1	C2	C3	C4	C5	C6	C7	C8	C9
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1	SRES2
1	1	1	1	1	-1	-1	28	1.19171	0.82514
2	2	2	1	1	1	-1	36	1.00837	1.65027
3	3	3	1	1	-1	1	18	-1.55839	-1.23771
4	4	4	1	1	1	1	31	1.00837	0.61885
5	5	5	1	1	-1	-1	25	-0.45835	-1.03142

(Refer Slide Time: 19:46)



Factorial Regression: Recovery versus reaction conc, catalyst amount

Coded Coefficients

```
Term             Effect    Coef  SE Coef  T-Value  P-Value   VIF
Constant                 27.500    0.606    45.38    0.000
reaction conc     8.333   4.167    0.606     6.88    0.000  1.00
catalyst amount  -5.000  -2.500    0.606    -4.13    0.003  1.00
```

Model Summary

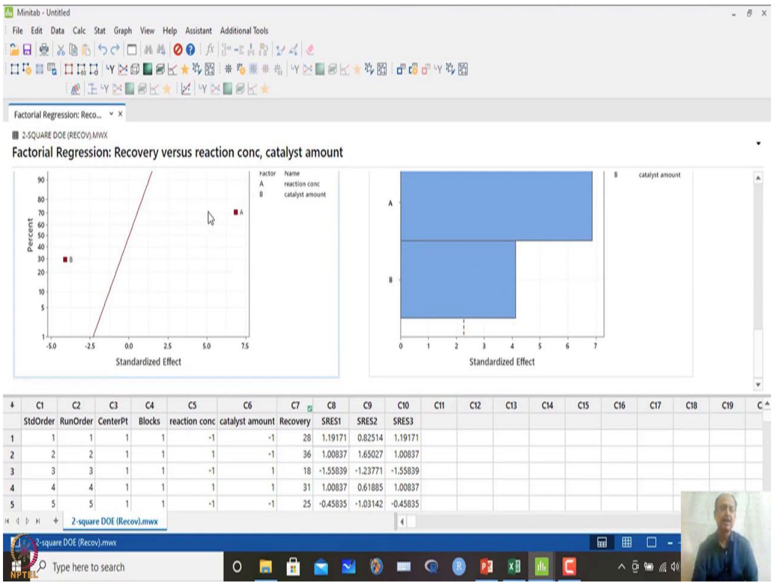
```
      S    R-sq  R-sq(adj)  R-sq(pred)
2.09938  87.72%     84.99%      78.17%
```

Analysis of Variance

```
Source          DF   Adj SS   Adj MS  F-Value  P-Value
```

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery	C8 SRES1	C9 SRES2	C10 SRES3
1	1	1	1	1	-1	-1	28	1.19171	0.82514	1.19171
2	2	2	1	1	1	-1	36	1.00837	1.65027	1.00837
3	3	3	1	1	-1	1	18	-1.55839	-1.23771	-1.55839
4	4	4	1	1	1	1	31	1.00837	0.61885	1.00837
5	5	5	1	1	-1	-1	25	-0.45835	-1.03142	-0.45835

(Refer Slide Time: 19:47)

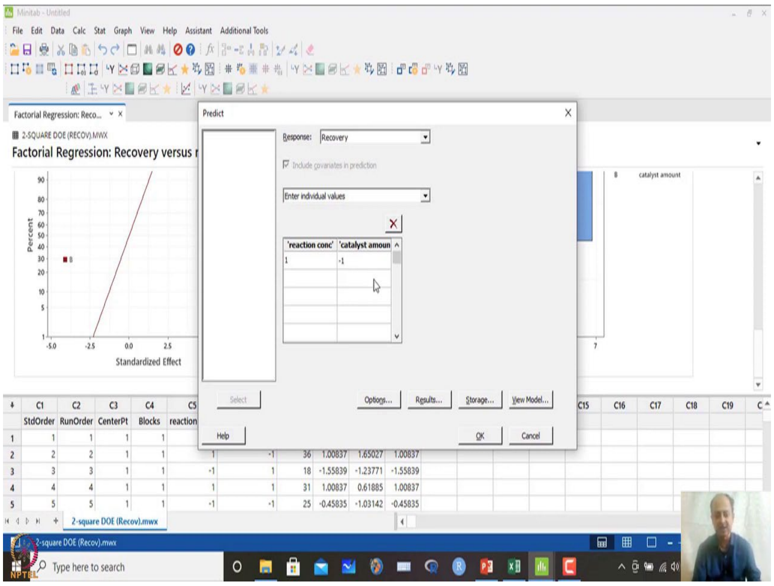


Factorial Regression: Recovery versus reaction conc, catalyst amount

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery	C8 SRES1	C9 SRES2	C10 SRES3
1	1	1	1	1	-1	-1	28	1.19171	0.82514	1.19171
2	2	2	1	1	1	-1	36	1.00837	1.65027	1.00837
3	3	3	1	1	-1	1	18	-1.55839	-1.23771	-1.55839
4	4	4	1	1	1	1	31	1.00837	0.61885	1.00837
5	5	5	1	1	-1	-1	25	-0.45835	-1.03142	-0.45835

And in this case terms what we have included, this we will eliminate over here and click OK and click OK like this. So, this is the final model that we are getting over here. And then what we can do is that we can make a design of experiments over here, factorial design.

(Refer Slide Time: 20:04)


Predict

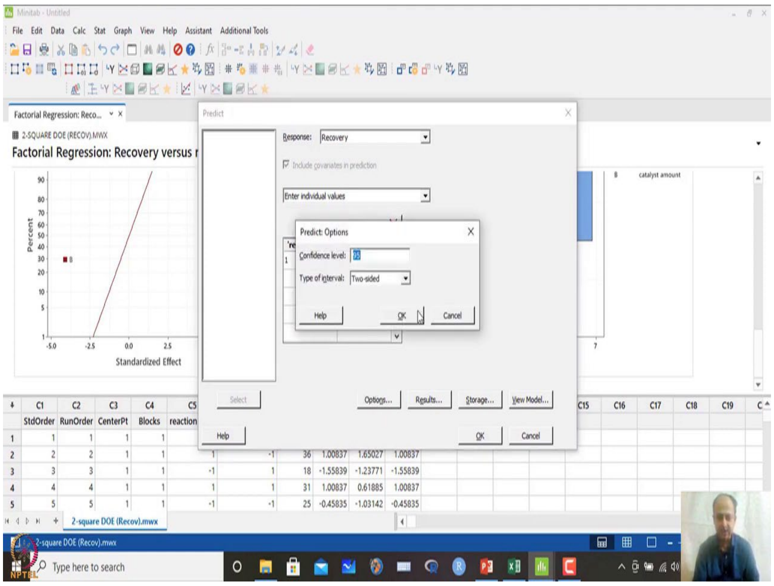
Response: Recovery

☑ Include covariates in prediction

Enter individual values

'reaction conc'	'catalyst amoun
1	-1

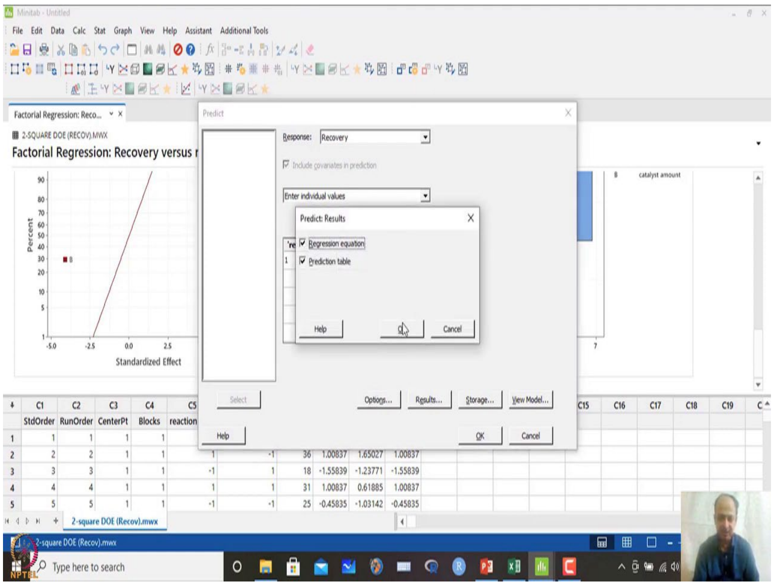
(Refer Slide Time: 20:21)


Predict: Options

Confidence level: 95

Type of interval: Two-sided

(Refer Slide Time: 20:22)



Response: Recovery

☑ Include covariates in prediction

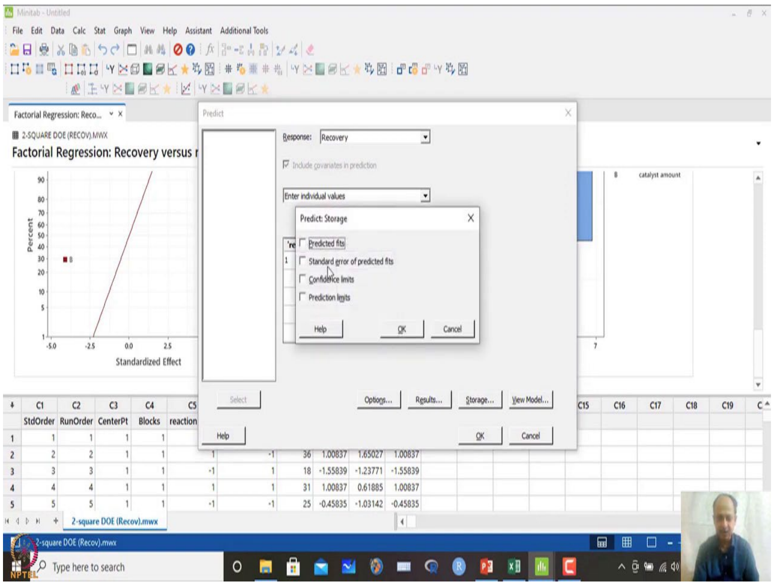
Enter individual values

Predict: Results

☑ Regression equation

☑ Prediction table

(Refer Slide Time: 20:25)



Response: Recovery

☑ Include covariates in prediction

Enter individual values

Predict: Storage

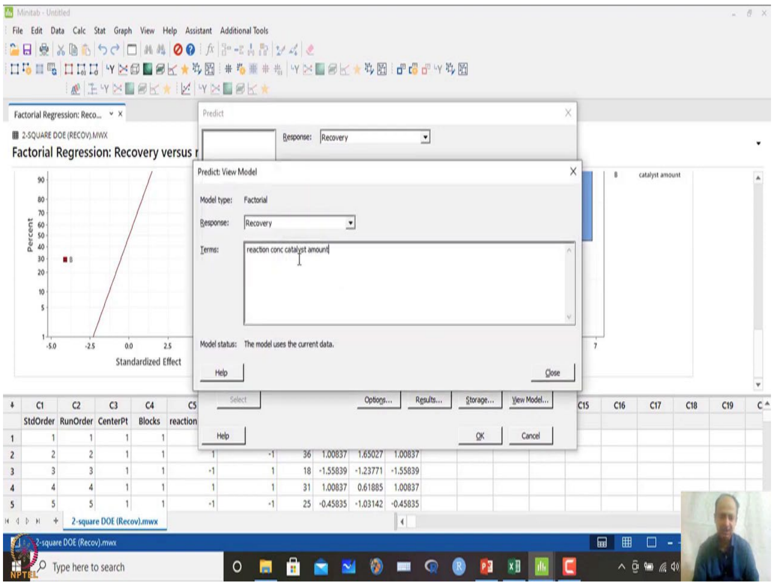
☐ Predicted fits

☐ Standard error of predicted fits

☐ Confidence limits

☐ Prediction limits

(Refer Slide Time: 20:29)



Predict: View Model

Model type: Factorial

Response: Recovery

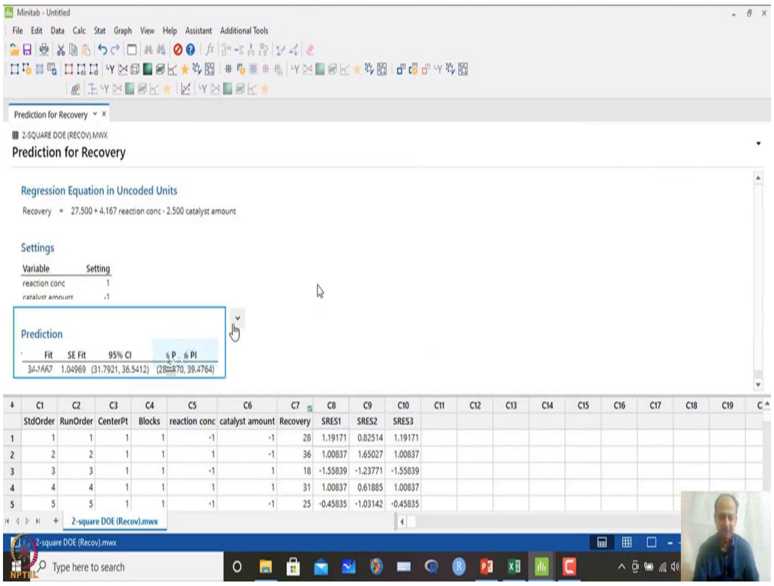
Terms: reaction conc catalyst amount

Model status: The model uses the current data.

What we can do is that we can also make predictions so, that if it is 1 and minus 1 that is what is expected value? So, in this case recovery, I want to maximize, but I am getting that this is the combination. So, this will be reaction time will be 1 and this will be minus 1 let us say.

So, this is the combination give me tell me that what is the expected value of the CTQ, what is the expected recovery value of the CTQ. In options, you do not change anything results we keep it as it is. So, storage we are not saving anything over here and view the models that is with this. So, there is no interaction terms in this model that we have considered and with this what is the prediction that I want to see over here.

(Refer Slide Time: 20:34)



Prediction for Recovery

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount

Settings

Variable	Setting
reaction conc	1
catalyst amount	-1

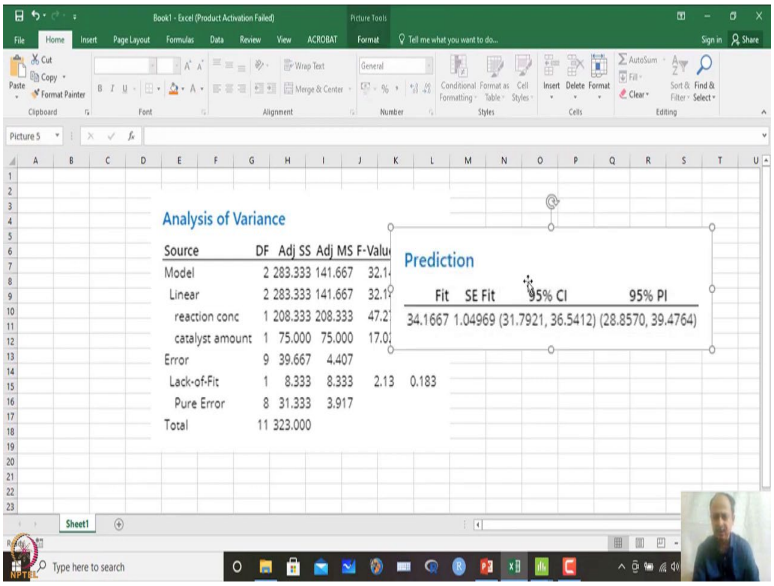
Prediction

Fit	SE Fit	95% CI	95% PI
34.1667	1.04969	(31.7921, 36.5412)	(28.8570, 39.4764)

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst amount	Recovery	SRES1	SRES2	SRES3
1	1	1	1	1	-1	-1	28	1.19171	0.82514	1.19171
2	2	2	1	1	1	-1	36	1.00837	1.65027	1.00837
3	3	3	1	1	-1	1	18	-1.55839	-1.23771	-1.55839
4	4	4	1	1	1	1	31	1.00837	0.61885	1.00837
5	5	5	1	1	-1	-1	25	-0.45835	-1.03142	-0.45835

So, then you will find that there is a; there is a MINITAB, MINITAB will give you what is the prediction value over here.

(Refer Slide Time: 20:47)



Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Valu	
Model	2	283.333	141.667	32.1	
Linear	2	283.333	141.667	32.1	
reaction conc	1	208.333	208.333	47.2	
catalyst amount	1	75.000	75.000	17.0	
Error	9	39.667	4.407		
Lack-of-Fit	1	8.333	8.333	2.13	0.183
Pure Error	8	31.333	3.917		
Total	11	323.000			

Prediction

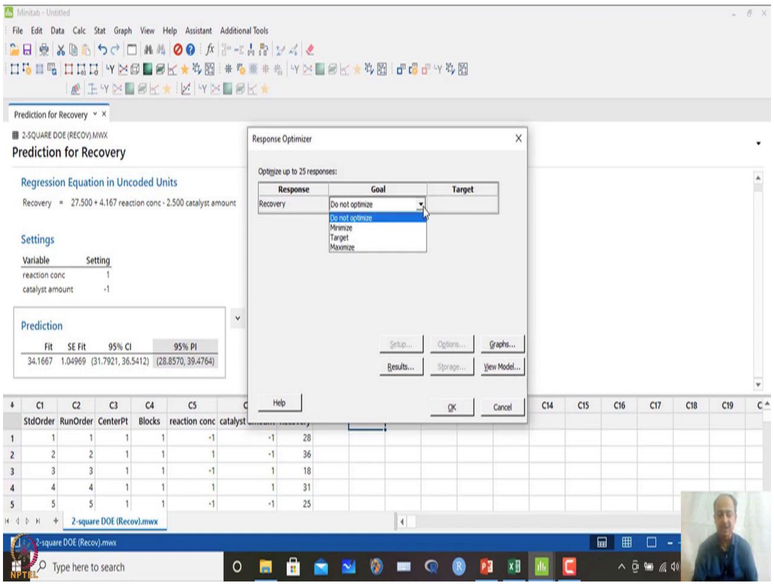
Fit	SE Fit	95% CI	95% PI
34.1667	1.04969	(31.7921, 36.5412)	(28.8570, 39.4764)

So, if you paste this one and enhance this one what will happen is that you will see that the fit that you are seeing is 34.16. This is the prediction that if you run this combination on an average, it is expected that you will get a fit value you will get a CTQ value on an average is about 34.

And there is a prediction interval and confidence interval calculation over here. So, your prediction interval says that the value should lie between 28 and 39 mostly it will vary because of uncertainties; that means, this value can be not exactly 34, it can vary between 28 and 39. So, this value is the 95 percent prediction interval that we have considered over here ok.

So, in this case this is possible over here and these are the residuals. So, we do not need the residuals over here and then what we can do is that, we can we have seen the surface plot also. And then what else we can do is that, we can make a response optimization; we can well we can also see that what is the optimal value.

(Refer Slide Time: 21:43)



2-SQUARE DOE (RECOV).MWX

Prediction for Recovery

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount

Settings

Variable	Setting
reaction conc	1
catalyst amount	-1

Prediction

Fit	SE Fit	95% CI	95% PI
34.1667	1.04969	(31.7921, 36.5412)	(28.8570, 39.4764)

Optimize up to 25 responses:

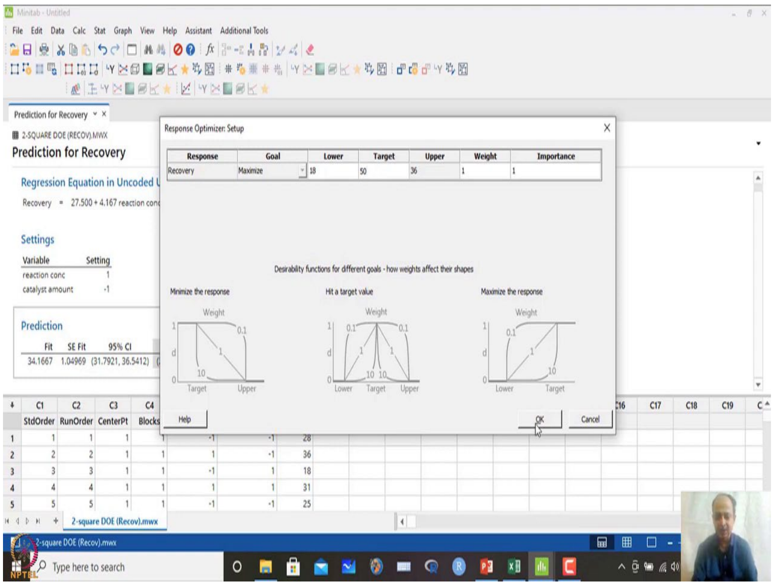
Response	Goal	Target
Recovery	Do not optimize	

Do not optimize
Minimize
Target
Maximize

	C1	C2	C3	C4	C5	C	
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst	
1	1	1	1	1	-1	-1	28
2	2	2	1	1	1	-1	36
3	3	3	1	1	-1	1	18
4	4	4	1	1	1	1	31
5	5	5	1	1	-1	-1	25

If I go by formal way of optimization using one desirability function approach that MINITAB uses. So, in this case desirability we will discuss afterwards. So, in this case also you can get the optimal combination because if I assume both are continuous over here and in that case MINITAB will work automatically and figure out which is the optimal combinations like that using a desirability function over here.

(Refer Slide Time: 22:14)



Prediction for Recovery

2-SQUARE DOE (RECOV).MWX

Regression Equation in Uncoded U

Recovery = 27.500 + 4.167 reaction conc

Settings

Variable	Setting
reaction conc	1
catalyst amount	-1

Prediction

Fit	SE Fit	95% CI
34.1667	1.04969	(31.7921, 36.5412)

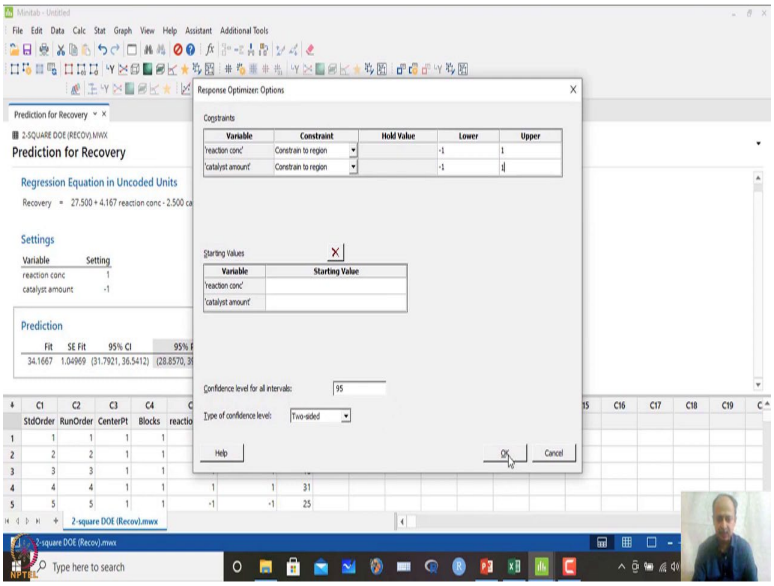
Response Optimizer: Setup

Response	Goal	Lower	Target	Upper	Weight	Importance
Recovery	Maximize	18	50	36	1	1

Desirability functions for different goals - how weights affect their shapes

So, I want to optimize let us say what is the final optimize value like that. So, in this case, I am using recovery to be maximized let us say and then set up what I will do is that, I can change the lower recovery over here is to maximize over here. So, what is the lower value higher value like these targets over here. So, you can change this one, I can make it 50 also. So, whatever it is, how much it will reach that is up to the optimization algorithm how much it can reach.

(Refer Slide Time: 22:33)



Prediction for Recovery

2-SQUARE DOE (RECOV).MWX

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 ca

Settings

Variable	Setting
reaction conc	1
catalyst amount	-1

Prediction

Fit	SE Fit	95% CI	95% P
34.1667	1.04969	(31.7921, 36.5412)	(28.8570, 3

Response Optimizer: Options

Constraints

Variable	Constraint	Hold Value	Lower	Upper
'reaction conc'	Constrain to region		-1	1
'catalyst amount'	Constrain to region		-1	1

Starting Values

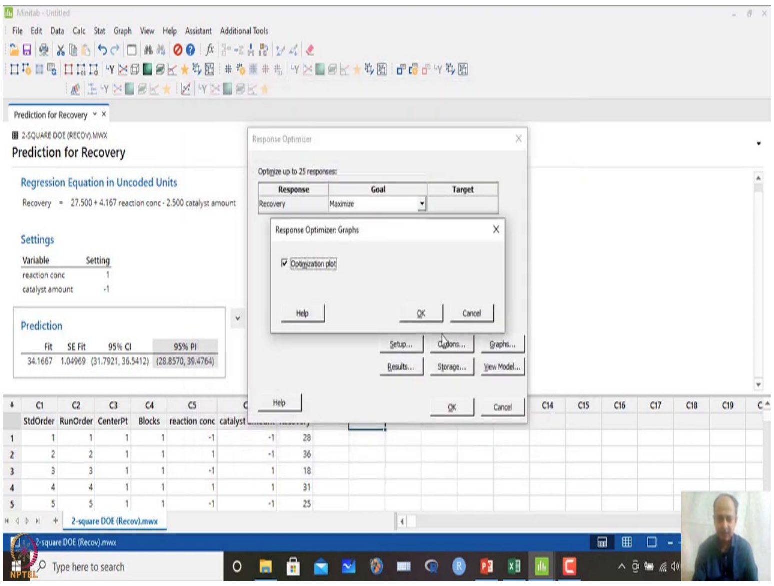
Variable	Starting Value
'reaction conc'	
'catalyst amount'	

Confidence level for all intervals: 95

Type of confidence level: Two-sided

So, in options what we can do is that reaction time, we can mention that constraints within the region. So, this will be from minus 1 to plus 1, keep it within minus 1 or plus 1 and then second also, we can mention that catalyst amount also should vary between these coded variables. So, this will be minus 1 to plus 1 over here and then we will say ok over here.

(Refer Slide Time: 22:55)



Prediction for Recovery

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount

Settings

Variable	Setting
reaction conc	1
catalyst amount	-1

Prediction

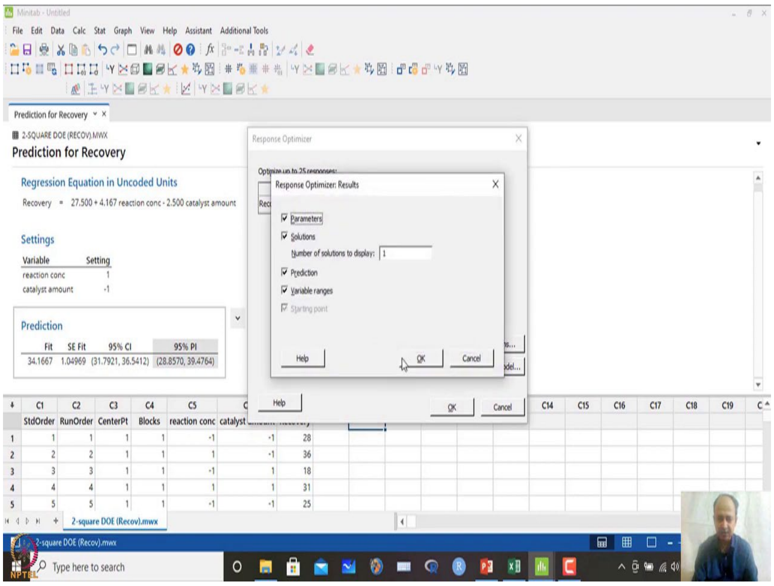
Fit	SE Fit	95% CI	95% PI
34.1667	1.04969	(31.7921, 36.5412)	(28.8570, 39.4764)

Optimize up to 25 responses:

Response	Goal	Target
Recovery	Maximize	

	C1	C2	C3	C4	C5	C
	StdOrder	RunOrder	CenterPt	Blocks	reaction conc	catalyst
1	1	1	1	1	-1	-1	28
2	2	2	1	1	1	-1	36
3	3	3	1	1	-1	1	18
4	4	4	1	1	1	1	31
5	5	5	1	1	-1	-1	25

(Refer Slide Time: 22:57)



Prediction for Recovery

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount

Settings

Variable	Setting
reaction conc	1
catalyst amount	-1

Prediction

Fit	SE Fit	95% CI	95% PI
34.1667	1.04969	(31.7921, 36.5412)	(28.8570, 39.4764)

☑ Parameters
☑ Solutions
Number of solutions to display: 1
☑ Prediction
☑ Variable ranges
☑ Starting point

And graphs, this is the optimal graph over here; results will be mentioned over here, storage we do not want to store this one so, then weightage because there is only 1 response over here. So, we are putting a weightage and importance all importance to this recovery over here. So, in this case I will click OK and so, I will click OK over here.

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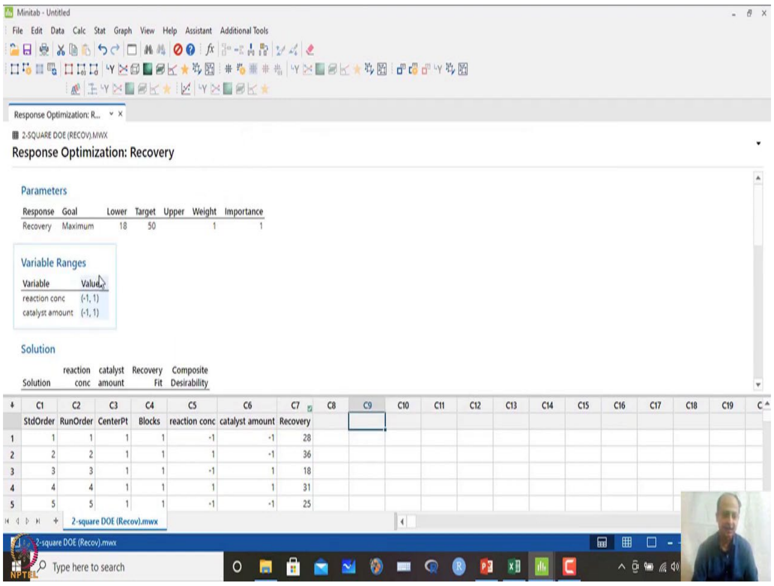

Response Optimization: Recovery

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
Recovery	Maximum	18	50		1	1

Variable Ranges

Variable	Values
reaction conc	(-1, 1)
catalyst amount	(-1, 1)

Solution

Solution	reaction conc	catalyst amount	Recovery Fit	Composite Desirability

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery
1	1	1	1	1	-1	-1	28
2	2	2	1	1	1	-1	36
3	3	3	1	1	-1	1	18
4	4	4	1	1	1	1	31
5	5	5	1	1	-1	-1	25

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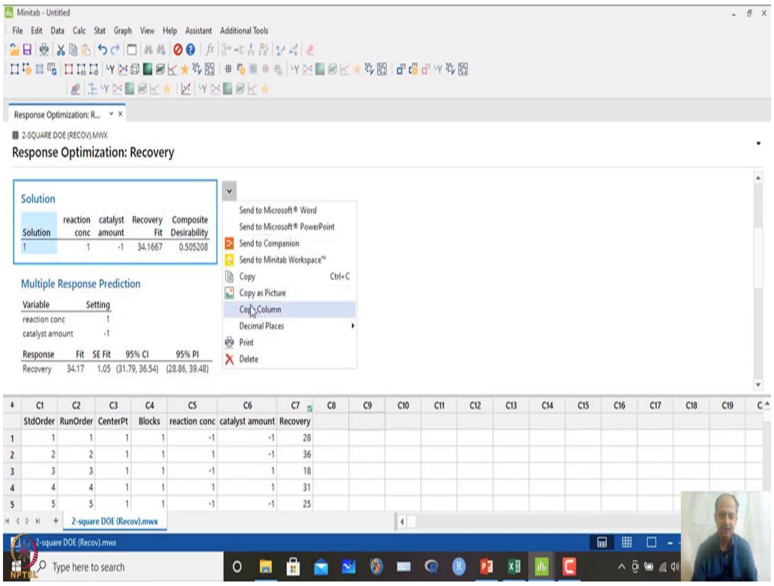

Response Optimization: Recovery

Solution

Solution	reaction conc	catalyst amount	Recovery Fit	Composite Desirability
1	1	-1	34.1667	0.505208

Multiple Response Prediction

Variable	Setting
reaction conc	1
catalyst amount	-1

Response	Fit	SE Fit	95% CI	95% PI
Recovery	34.17	1.05	(31.79, 36.54)	(28.86, 39.48)

	C1 StdOrder	C2 RunOrder	C3 CenterPt	C4 Blocks	C5 reaction conc	C6 catalyst amount	C7 Recovery
1	1	1	1	1	-1	-1	28
2	2	2	1	1	1	-1	36
3	3	3	1	1	-1	1	18
4	4	4	1	1	1	1	31
5	5	5	1	1	-1	-1	25

So, let us see what happens with this combination. So, MINITAB is recommending over here reaction concentration as one same what we have interpreted earlier also, solution 1 is mentioned.

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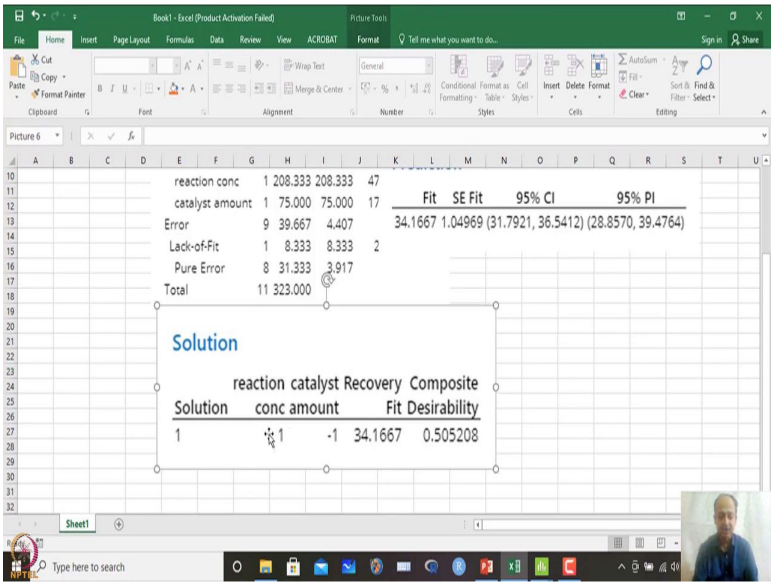

reaction conc	1	208.333	208.333	47
catalyst amount	1	75.000	75.000	17
Error	9	39.667	4.407	
Lack-of-Fit	1	8.333	8.333	2
Pure Error	8	31.333	3.917	
Total	11	323.000		

Fit	SE Fit	95% CI	95% PI
34.1667	1.04969	(31.7921, 36.5412)	(28.8570, 39.4764)

Solution

Solution	reaction conc	catalyst amount	Recovery Fit	Composite Desirability
1	1	-1	34.1667	0.505208

So, if I click this one and copy this one as picture and if I want to paste this one, we can just do that and we can see that this is the final combination optimal combination what MINITAB is mentioning this is equals to 1 and minus 1, this is the only solution. And recovery will be around 34 and some desirability index it is showing.

So, we can ignore this one. So, if it is towards one that is the best scenario so, in this case whatever is given, we have reached up to this using desirability index, but the combination is 1 and minus 1 that we are getting over here. So, that is the combination we have also seen in this case also. So, this is the so, that is also possible over here. So, with this that is plus and minus 1 combination what we have seen is also prominent over here.

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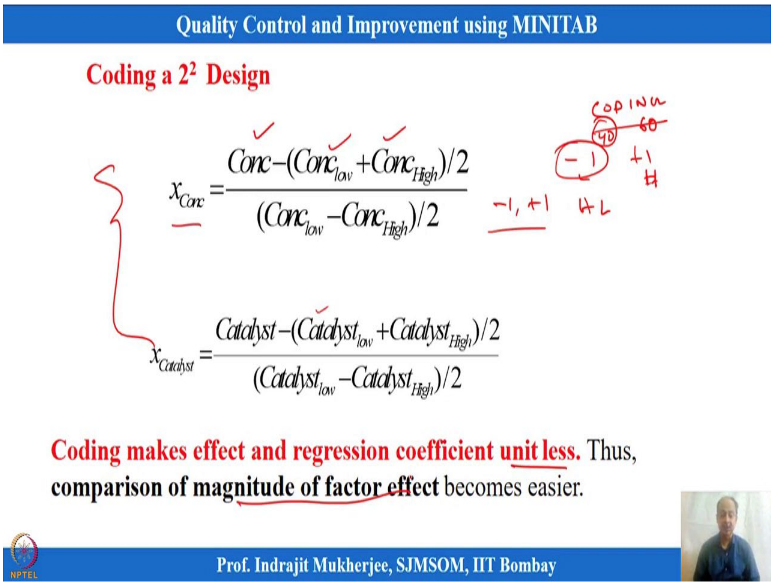

Quality Control and Improvement using MINITAB

Coding a 2^2 Design

$$x_{Conc} = \frac{Conc - (Conc_{low} + Conc_{High})/2}{(Conc_{low} - Conc_{High})/2}$$

$$x_{Catalyst} = \frac{Catalyst - (Catalyst_{low} + Catalyst_{High})/2}{(Catalyst_{low} - Catalyst_{High})/2}$$

Coding makes effect and regression coefficient unit less. Thus, comparison of magnitude of factor effect becomes easier.

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And one important aspects that we have not told is that how they are coding basically to minus 1 and plus 1 conversions like this. So, Montgomery gives you minus 1 to plus 1 so; that means, this is when I am coding so, coding the variables like that. So, when we have a high level and low levels like that each is having high level and low level like that. So, this is the low level and this is high level. This can have actual values, this can have actual value may be 40 and this may be 60 like that.

So, with actual value how do I convert into the minus 1 and plus 1? So, that I can compare the effect estimations like that. So, there is a formulation that is given over here. So, this is x concentration over here, actual concentration over here you have to take, what is maximum minimum like this divided by 2 and then we can get the values that is within minus 1 and plus 1 like that.

So, all the values will be converted into minus 1 and plus 1 similarly so, actual value. So, this is for concentration one of the variables is used is concentration over here and this is the catalyst amount. So, in this case second one is catalyst amount. So, these are the two formulations which can be used to standardize is to standardize within or coding the variables within minus 1 and plus 1.

So, we can convert that one. Why we convert that one? Because if we code this one the, it becomes unit less. So, coefficient becomes unitless. So, we can compare the magnitude of the factor effect basically then we can compare. So, when we do design of experiments, what we will see is that most of the time people are coding that one. So, they are converting the variables in the coded variables, then it is easier to compare the effects; it is easier to compare the effects.

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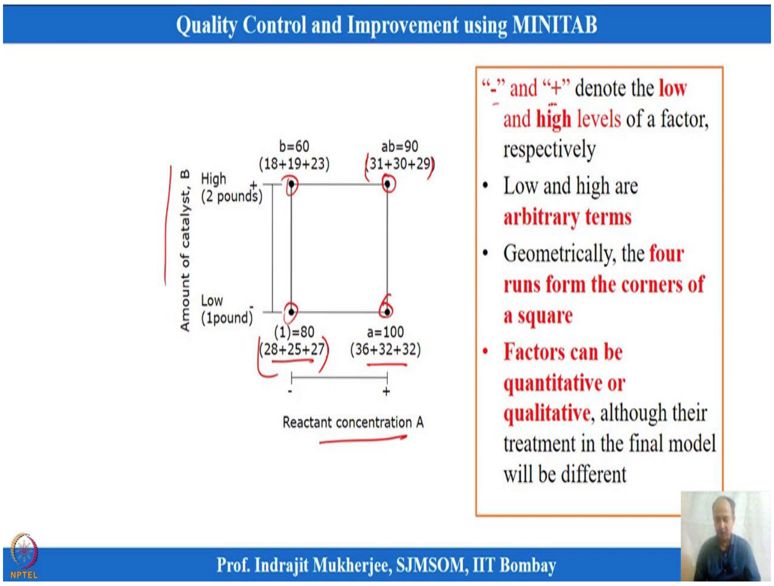

Quality Control and Improvement using MINITAB

"-" and "+" denote the low and high levels of a factor, respectively

- Low and high are **arbitrary terms**
- Geometrically, the **four runs form the corners of a square**
- **Factors can be quantitative or qualitative**, although their treatment in the final model will be different

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So, that is one aspects we have to understand and when we are replicating basically at the corner points, what we what is happening is that we have a total values of this. So, this is replicated 3 times over here, this is replicated 3 times and these are the 4 corner points that is for these 3 replicated trials of recovery.

So, if you 2 dimensional view of this is that this is the reaction time in x axis let us say and this is another y axis is given over here and z axis will be basically CTQ values that you are observing over here. So, these are the values we can think of as Z axis over here.

So, this is symbolic notation minus 1 is arbitrary that I told and if it is continuous variable, it will it can be coded like that way or if your qualitative factors if you are considering also so, there is no as such meaning for quality factors which is plus 1 which is minus 1.

So, one can be taken as plus other one as. So, this is arbitrary completely like that ok. So, qualitative and quantitative factors can also be considered in the experimentation. So, that is possible like that.

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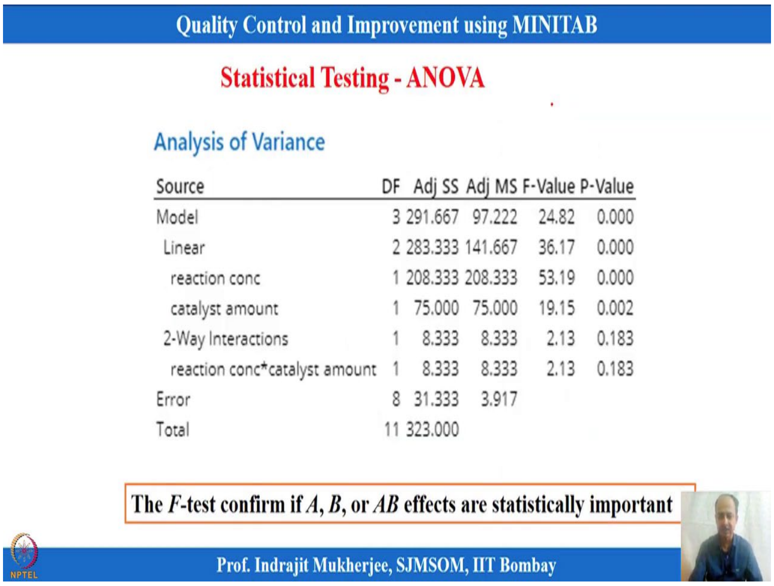

Quality Control and Improvement using MINITAB

Statistical Testing - ANOVA

Analysis of Variance

```
Source                        DF  Adj SS   Adj MS  F-Value  P-Value
Model                          3 291.667   97.222    24.82    0.000
  Linear                       2 283.333  141.667    36.17    0.000
    reaction conc              1 208.333  208.333    53.19    0.000
    catalyst amount            1  75.000   75.000    19.15    0.002
  2-Way Interactions           1   8.333    8.333     2.13    0.183
    reaction conc*catalyst amount  1   8.333    8.333     2.13    0.183
Error                          8  31.333    3.917
Total                         11 323.000
```

The *F*-test confirm if *A*, *B*, or *AB* effects are statistically important

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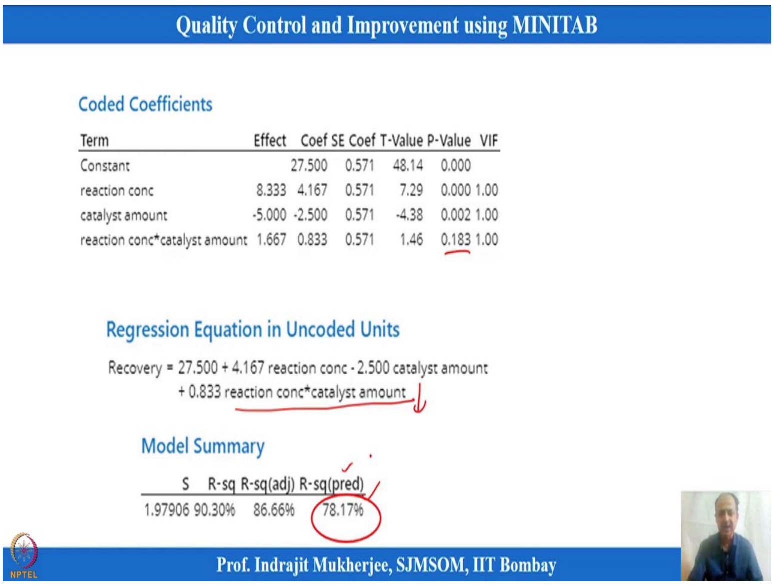

Quality Control and Improvement using MINITAB

Coded Coefficients

```
Term                           Effect    Coef  SE Coef  T-Value  P-Value   VIF
Constant                               27.500    0.571    48.14    0.000
reaction conc                   8.333   4.167    0.571     7.29    0.000  1.00
catalyst amount                -5.000  -2.500    0.571    -4.38    0.002  1.00
reaction conc*catalyst amount   1.667   0.833    0.571     1.46    0.183  1.00
```

Regression Equation in Uncoded Units

```
Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount
           + 0.833 reaction conc*catalyst amount
```

Model Summary

```
      S    R-sq  R-sq(adj)  R-sq(pred)
1.97906  90.30%     86.66%      78.17%
```

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And this already we have mentioned that P values will indicate which is significant which is not and if we have not lost much in the R square predicted so, whether to keep this one or not to keep this one. So, we have drop this one because this is not making significant impacts like that. So, P value is not significant.

So, in this case whether to drop this one or keep this one maybe R square predictor is the criteria which can be used to see that whether it is improving or not, but everything cannot be based on statistics sometimes real life interpretation is also required. So, in this case if I am placing more emphasize on the more emphasize if we are putting on the prediction over here prediction model should be correct like that.

So, in that case to improve the prediction model maybe we can consider that one. But most of the time we will see that they will drop some amount of information we will lose well that is ok from the perspective of prediction and all these things considering all of this ok.

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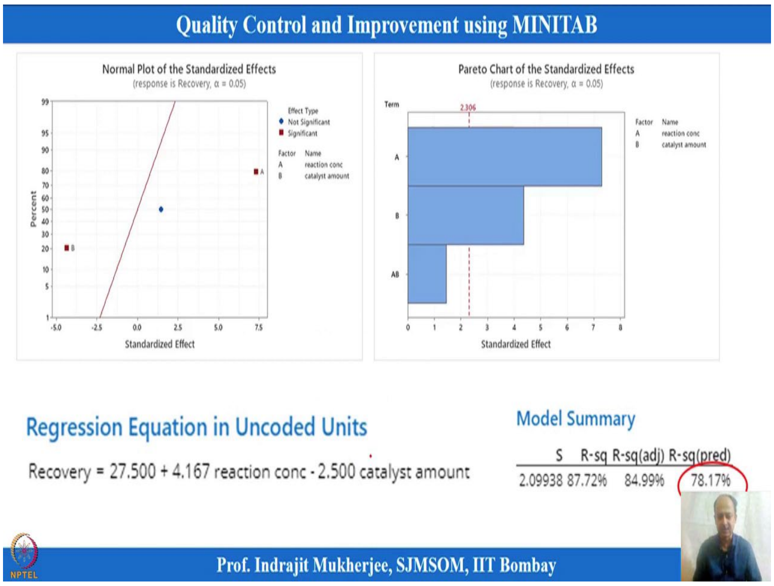

Quality Control and Improvement using MINITAB

Regression Equation in Uncoded Units

Recovery = 27.500 + 4.167 reaction conc - 2.500 catalyst amount

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.09938	87.72%	84.99%	78.17%

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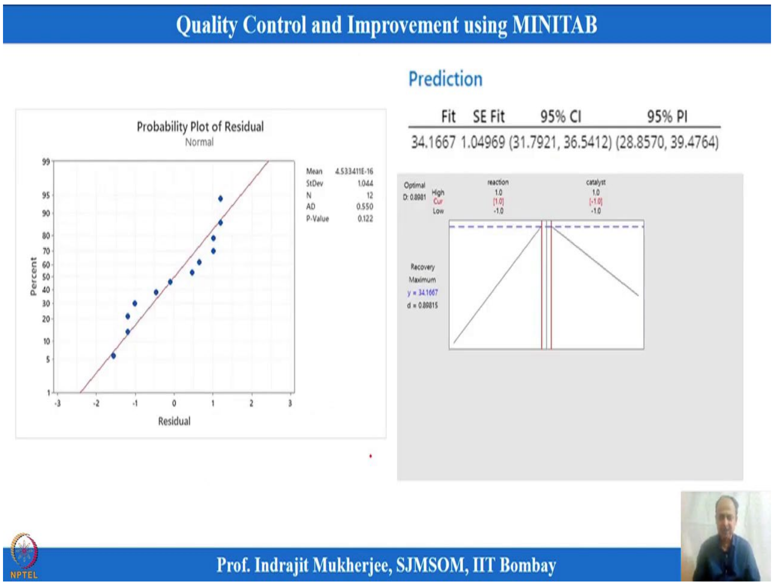

Quality Control and Improvement using MINITAB

Prediction

Fit	SE Fit	95% CI	95% PI
34.1667	1.04969	(31.7921, 36.5412)	(28.8570, 39.4764)

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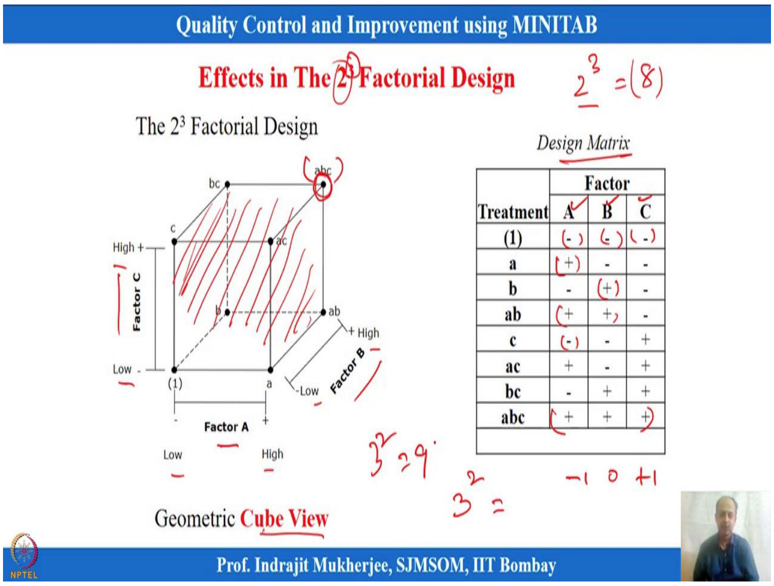

Quality Control and Improvement using MINITAB

Effects in The 2³ Factorial Design

$2^3 = (8)$

The 2³ Factorial Design

$3^2 = 9$

Design Matrix

Treatment	Factor		
	A	B	C
(1)	(-)	(-)	(-)
a	(+)	-	-
b	-	(+)	-
ab	(+	+)	-
c	(-)	-	+
ac	+	-	+
bc	-	+	+
abc	(+	+	+)

$3^2 =$

-1 0 +1

Geometric Cube View

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So, this is the standard effects that we have seen. So, this is also we have seen like that and then what we can do is that we can take another example where I have 3 factors. When I have 3 factors over here so, complexity is increasing you see, I have taken 3 factors over here all at 2 levels. So, this will be 2 levels and 3 factors over here. So, minimum number of trials is 8 over here.

So, I have a factor A, B and C like this and the trials will be like this all at low level like that and then one will be at low level, then second one will be at low level then interaction effects of this will be calculated, then c will be like this. So, symbolically we can develop this design matrix we can develop this design. Only thing is that earlier it was in two dimension, now it is in three dimension.

So, there is a factor A, factor B and factor C which is at low level high level and this is also at low level and high level, this is also at low level and high level like that. So, ABC is the maximum interaction terms over here. There are three factors A, B and C. So, A B and C all at high level so, this is the combination that was done and in the cube, this is the extreme point that we are seen.

So, now it is a cube basically, earlier it was a surface and this is a cube that is generated over here if I have three factors like that. So, the total surface that is generated over here is basically a cube. So, ok and we cannot see 4 dimensions over here; that means, CTQ if

I place it another dimension, it is not visible we are not able to visualize that one, but we can visualize if there is 3 factors what is the surface.

So, earlier it was just flat surface that we are seeing like that in two dimensions A and B factors like that. Now, it is a cube and we will experiment within this cube. So, you can think of a cube as a an experimental ground basically. So, this within this cube, I can move anywhere within this cube I can move anywhere. So, this is the cube views over here that will be used over here. But only thing is that the analysis of this will we have to see.

So, this is 2 cube factors. Similarly we can also see experimentation with when we have 3 levels over here instead of 2 levels. So, 3 levels 2 factors like that also we can see. So, some another examples, we will take over here before we move on to some other topics like that. So, we will take 2^3 design over here and we will also take let us say 3^2 design; that means, level will be 3.

So, this will have 3 levels like that. So, maybe we can say minus 1 0 and plus 1 that are the levels that we can think of and there may be 2 factors like that. So, k will be equals to so, this will be written as 3 square; that means, 9 experimental trial minimum is required, we can replicate that one.

So, what we will do is that we will close this session over here and we will continue discussing on 2^3 and 3^2 designs like that and how to analyze. So, C square we will place more emphasis on 3^2 square design like that and then move forward with other topics like that.

Thank you for listening.