Quality Control and Improvement with MINITAB
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Lecture - 33
Measurement System Analysis (Contd.), Introduction to Factorial Experiments

Hello and welcome to session 33 of course on Quality Control and Improvement with MINITAB. I am Professor Indrajit Mukherjee from Shailesh J Mehta School of Management IIT Bombay.

So, we are discussing about measurement system analysis and within that we are discussing about Gage R&R study, repeatability and reproducibility.

So, we took some examples to understand and then we have seen that percentage contribution is one of the measures that needs to be considered over here and study variability can also be considered.

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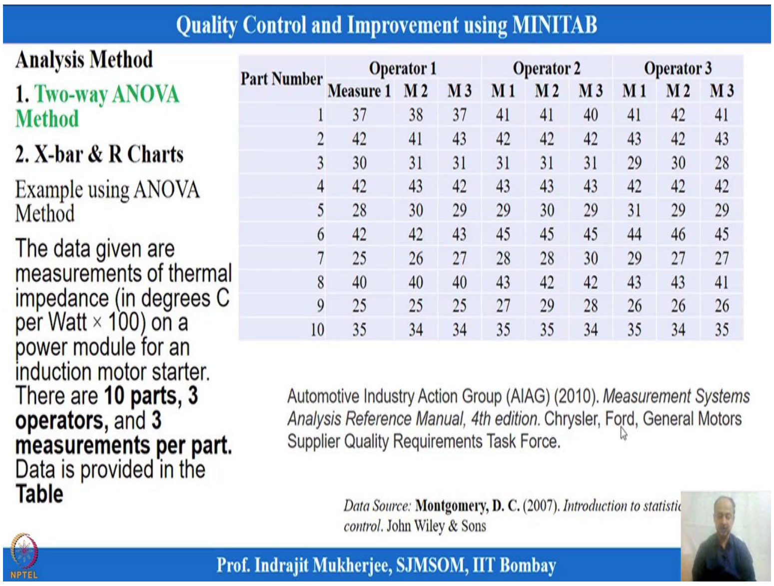

Part Number	Operator 1			Operator 2			Operator 3		
	Measure 1	M 2	M 3	M 1	M 2	M 3	M 1	M 2	M 3
1	37	38	37	41	41	40	41	42	41
2	42	41	43	42	42	42	43	42	43
3	30	31	31	31	31	31	29	30	28
4	42	43	42	43	43	43	42	42	42
5	28	30	29	29	30	29	31	29	29
6	42	42	43	45	45	45	44	46	45
7	25	26	27	28	28	30	29	27	27
8	40	40	40	43	42	42	43	43	41
9	25	25	25	27	29	28	26	26	26
10	35	34	34	35	35	34	35	34	35

We took a specific example where 10 parts are measured and there are 3 operators and each operator is measuring the same part 3 times but randomly.

So, the operator does not know which part it is measuring and a master information of the values is not known to the operator. 1 to 10 parts are covering the process variations basically or tolerance we can think of like that. So, this is as per the AIAG guidelines I am discussing over here. So, this was the basic guidelines and MINITAB adds on to that some of the measures that can be used for measurement system analysis.

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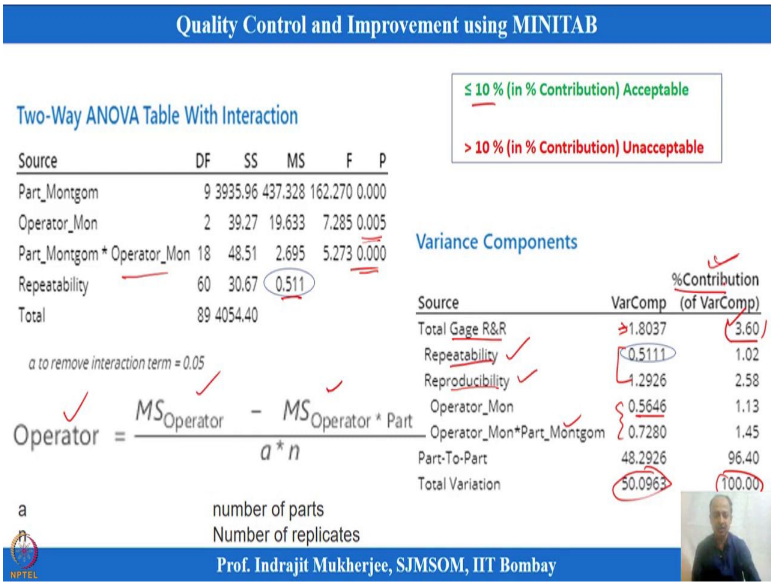

So Gage R&R studies, so over here what you have done is that we have seen when we estimated MINITAB in as a 2 factor experimentations. So, in that case part variation is expected what we see over here. But in this case operator to operator variation is quite significant that is shown and interaction term is also significant that should not happen basically if it is a very good instruments.

However, we have calculated the percentage contribution over here this is around 3.6 and this is much less as compared to the 10 percent criteria that we are using over here. And this repeatability what we have seen is this is the error variation mean square error variations the other measures are also calculated like for operator as follows:

$$\frac{MS_{operator} - MS_{operator*part}}{a*n}$$

What we have seen is that this is 0.56 and out of 100 percent if this is a total variations over here and we can get what is the contribution of total instrument over here that consist of repeatability and reproducibility.

And interaction effects and also operator variability this reproducibility within that we are getting these 2 components over here and these 2 summation of this repeatability and reproducibility gives you the measures of total gage repeatability.

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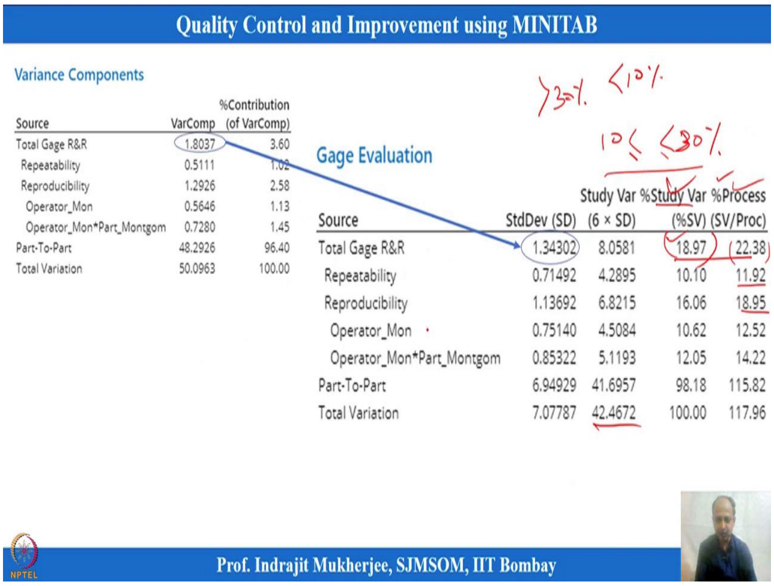

And then what we have seen is that we can also get the study variability and we have historic information of the variable variance if it is given. And then process standard deviation can also be used and in that case we are getting some percentage over here study variability as compared to 42, what is the study variation and also percentage process also we have discussed in our last session that.

And the criteria what I told is that if it is less than equals 30 percent we can go ahead with the instrument, but only thing is that we need to see which is creating problem over here repeatability, reproducibility which percentage is contributing.

So, repeatability is around 80 percent reproducibility about 18 percent over here, if it is less than 10 percent that is the best we can get ok. So, over all it is 30 percent so if it less than so best instrument should be less than 10 percent over here, but 10 to 30 percent we can keep within this if it is within this in that case we can use the instrument.

But if it is more than 30 percent what we can do is that we have to reject the instrument for use in production flow, it has to go to calibration and figure out what is going wrong basically in the instrument what is going wrong in the instrument.

And they will make some corrections over there, either they will send back the instrument to us or they will say that this cannot be used. So, you have to use a new instrument which they will provide to the operator or the production flow so ok.

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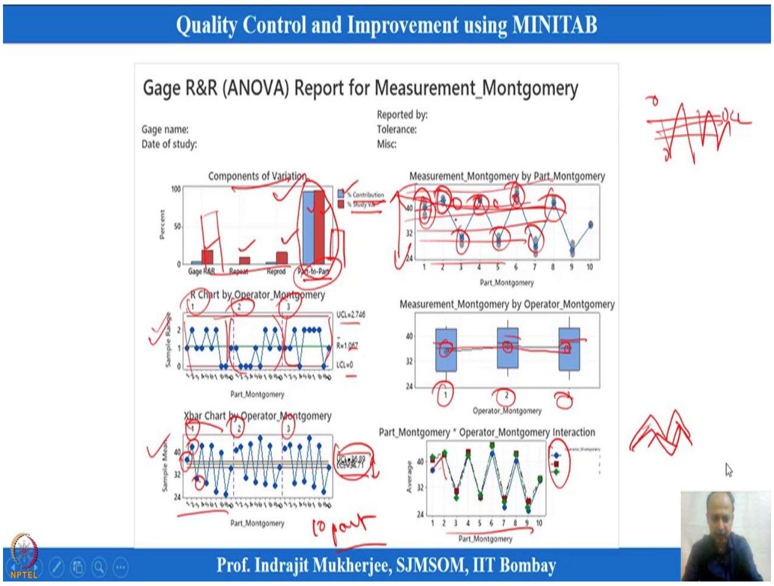

So this study will reflect that one and when you do these Gage R&R study also what we will see is that this whatever we have calculated percentage study variation, process variations like this, this will be represented over here as component of variability over here, percentage contribution and percentage study variations over here these are the measures that is given.

So, part to part it is expected that this will be very high over here and then we will get Gage R&R contribution for the study variations over here and percentage contribution also will be reflected over here.

Similarly, repeatability and reproducibility this will be graphically represented. So, what it says is that most of the contribution is due to percentage. This is the favorable situation. Now, if this graph this increases and this goes down that is not expected over here. So, part to part variation should be the maximum contributed and others will have negligible contributions like that.

So, this graph will tell you how much contribution is there in percentage and how much is the study variation that bifurcation of Gage R&R repeatability reproducibility like that.

Then there is a control chart aspect also you will see over here one is monitoring the mean of the observations and one is monitoring the variability observations within operator to operator. So, operator 1 operator 2 and operator 3 how they are measuring the part that is given the variability within is reflected over here.

Like R chart interpretation we have to use this one and if all the points are within the control limit line that is given over here, then it seems variability is within control. So, there is no problem in this graph over here.

So, operator 1 operator 2 internal variability or within variability is not so significant. And the second one is X bar R chart that you will find that this is operator 1 is measuring operator 2 and operator 3 and the control limit lines will be narrow over here. It is expected like that because parts are from different ranges over here. So, what is expected is that the values of this mean values over here what you see will fall outside the specification outside the control limit line basically.

So, this is expected so whenever you are measuring the parts and you have calculated the control limit line using this R bar information over here and used the control chart. So, part to part variation will be there. So, 10 parts will be outside the specification most of the time you will.

This is not unnatural in case of measurement system analysis. If it is not then there is a problem. So, I have not selected the part which is having a different range like that within the specification.

So, I have to create a range, I cannot select all parts within a single range like that. So, a part should be distinct and should have measurements which are somewhat different from the other one.

So, if that is so if the parts are different in that case I have 10 observations. So, all parts are differently measured. So, in that case it is expected point will fall out the specification limits over here and these are the 10 parts that is measured like that. So, this is measurement 1 measure for the part 2 part 3 these are the points measured over here.

So, measurement of parts this variability is shown over here. So, what is the range of that? So, this is measured at this range this is the value that this part is having. So, this is the second part like that and so measurements are quite scattered within the operating within the may be process variability or the total specifications.

So, parts are selected randomly throughout the space. It cannot happen that all the parts are in this range. I have selected a range on the higher side on the middle side and also on the lower side. This is the way we should do this Gage R&R studies.

Then what is required is that we want to see operator to operator variation, how is the median value moving over here. So, more or less you see that operator to operator not much variation in box plot. What we are seeing is that is also not so prominent over here.

Part and operator interaction is significant what we have observed. Operator to operator there is a significance. But among statistical significance and actual significance means practical significance, we have to consider over here and we have to see that whether to take action or not to take action over here.

We expect that there is little interaction although the significant interaction is shown over, but practically also we have to consider like that.

So, either you go by percentage contribution over here and if this is very high then we will look into the operator and part and operator interaction, otherwise we may ignore this one and we will only go by percentage contribution and based on that we may make a decision out of that.

So, over here we are seeing interaction plot over here, although statistically it is significant, but more or less we can see that there is a there is more or less they are going parallel more or less they are going parallel.

Now, the dataset says that it is somewhat significant statistically, but we have to see practical aspects of that and percentage contribution if it is less. But we can always look back to that and send it to meteorology and try to figure out what is going wrong why interaction is happening like that. So, that has to be considered over here ok. So, that is one aspects over here and there are another aspects also you can find. So, this how this graph is generated in MINITAB.

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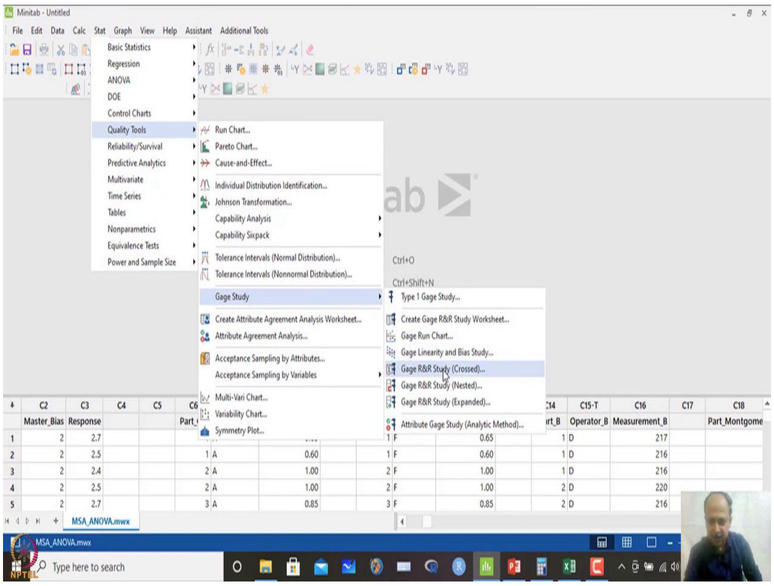

So, when we do the analysis over here and let us say we have taken the same experimental trials over here and gage studies and we have done this Gage R&R cross studies over here.

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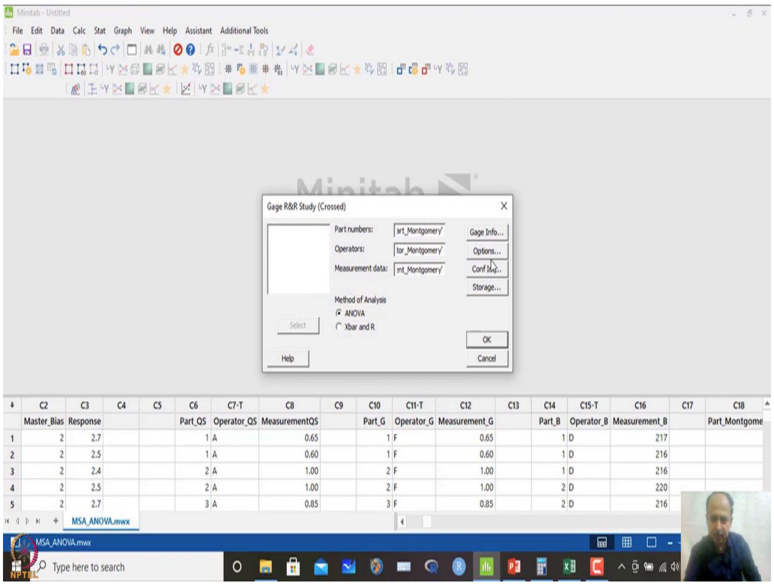

So, part a let us consider this first part this one and then we have operator and then we have measurement over here.

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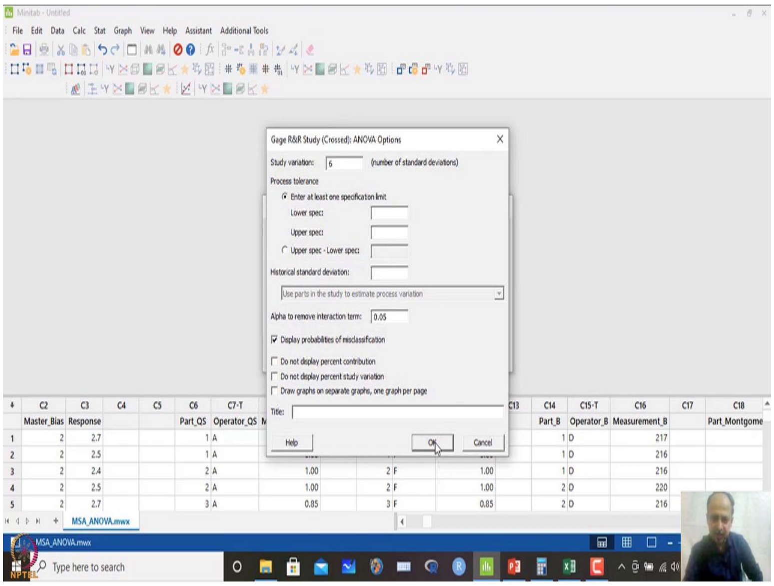

And options what we have given is that this is this I am removing at this current position. So, we are not concerned about this.

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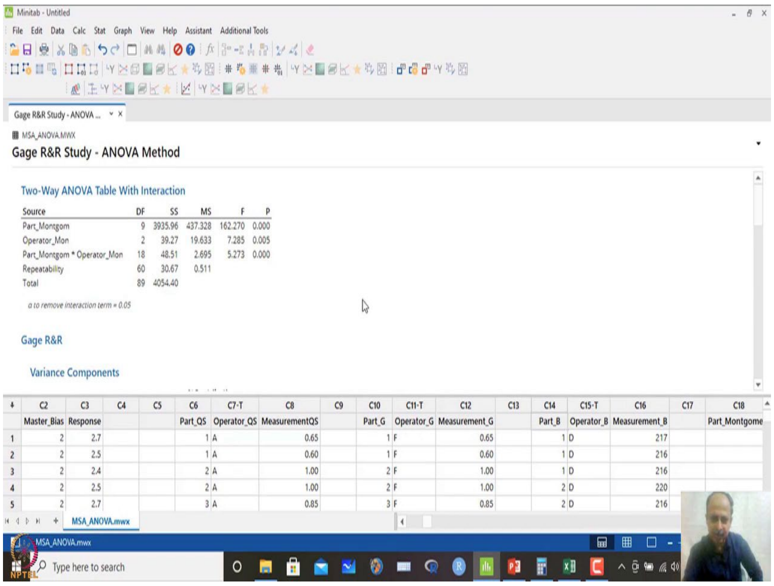

We want to see the graphical interpretation of this.

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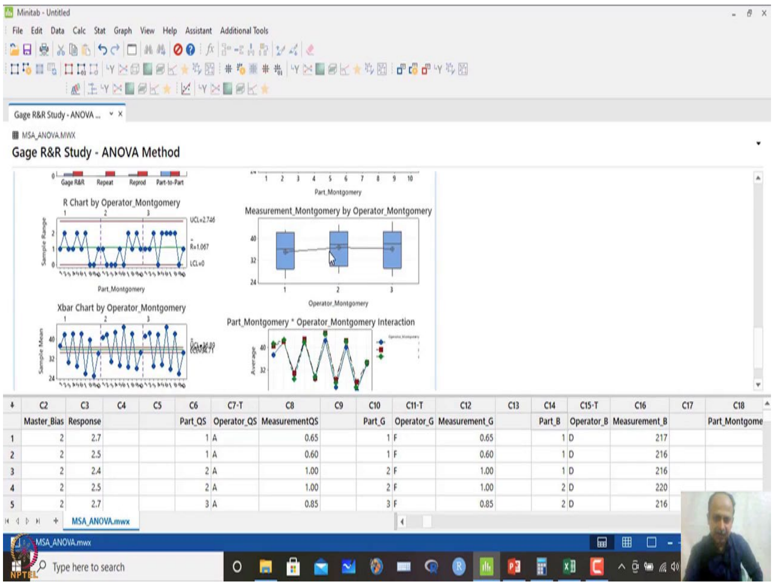

And we will have this graph information over here what you see.

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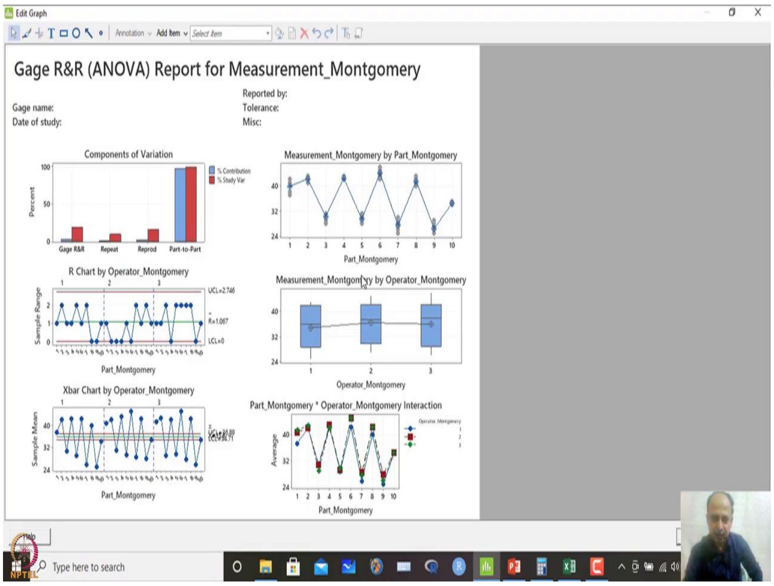

So, this when you plot this one this is the graph that I have just copy pasted in that our PPT what I am showing over here. So, this is the interpretation that we have discussed ok.

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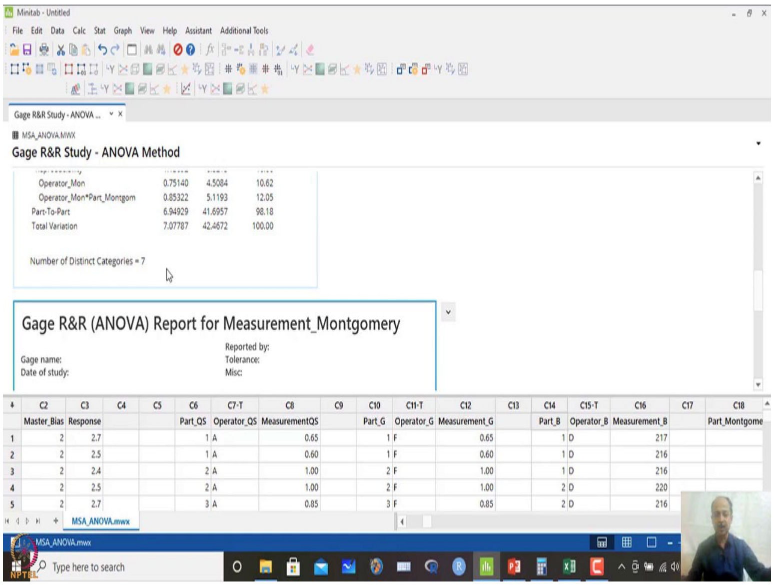

And there will be another important aspect over here number of distinct category over here. I have 10 parts so the instrument should be able to differentiate between the 10 parts and if the measurements are quite in the same range they it may not be possible. But what is important is that number of distinct category should be more than 5.

So, we expect that at least instrument should be able to distinguish in 5 different categories, because all are different categories parts are in different categories ok, some parts may be very closely taken like that.

But this is calculated based on the part variation or standard deviation of the parts and when we divide it by standard deviation of the gage or instruments like that variation of the instruments like that. So, that gives you say formula is given in MINITAB or anywhere you can see like that in manuals also it is given.

So, we will get a number of distinct category. So, it is recommended that greater than equals to 5 will be considered as a good instrument like that. So, this is another important aspects. So, let us take one more example where the instrument is not so perfect. So, this is 14, 15, 16 I am considering over here. So, let us do this second one. So, before that one let me see what is the process variability C16?

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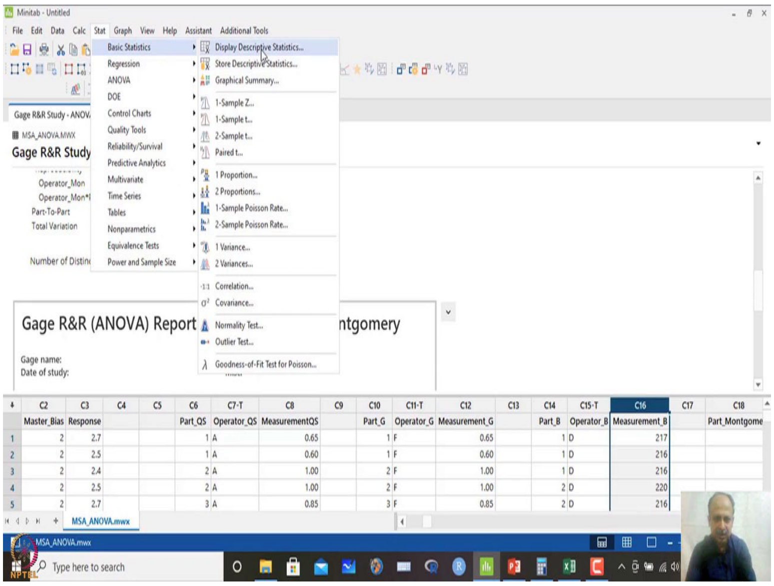

So, we can just see variability of this we can just display the statistics.

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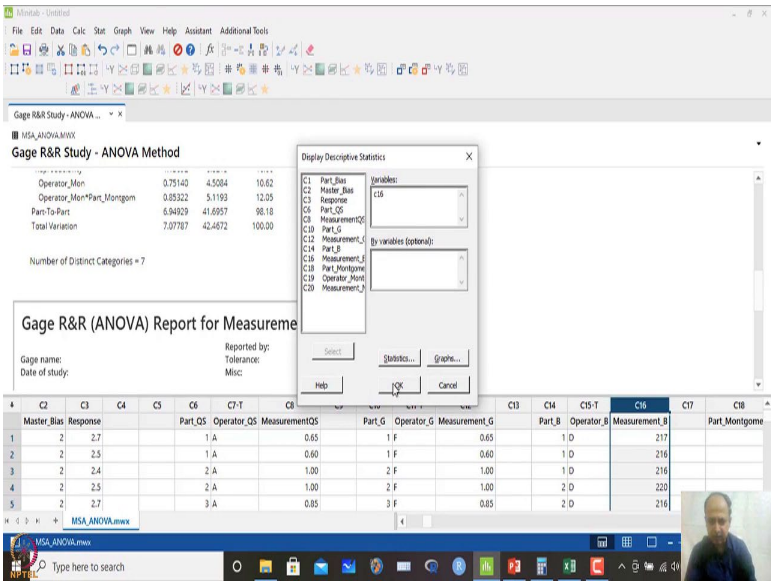

So, we can say C16 we want to see what is the variations of this.

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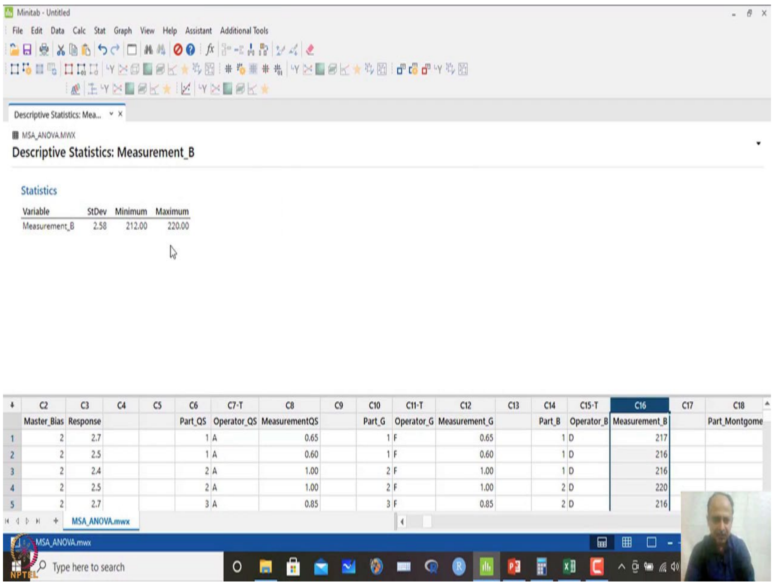

And it is around 2.5, 2.6 we can say the standard deviation is 2.6.

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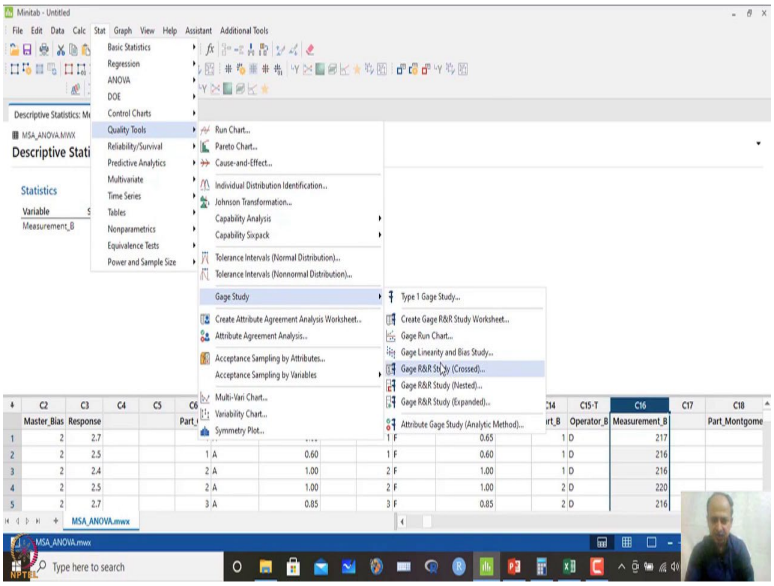

So, what we will do is that we will take this and consider and 2.6. So, gage studies, crossed gage and this is part B.

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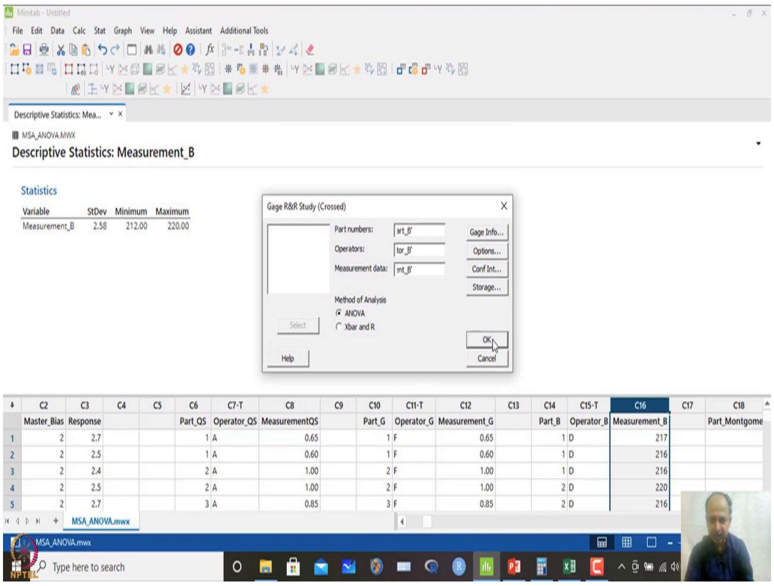

So, this will be operator B and this will be measurement B over here and what we have got is 2.58.

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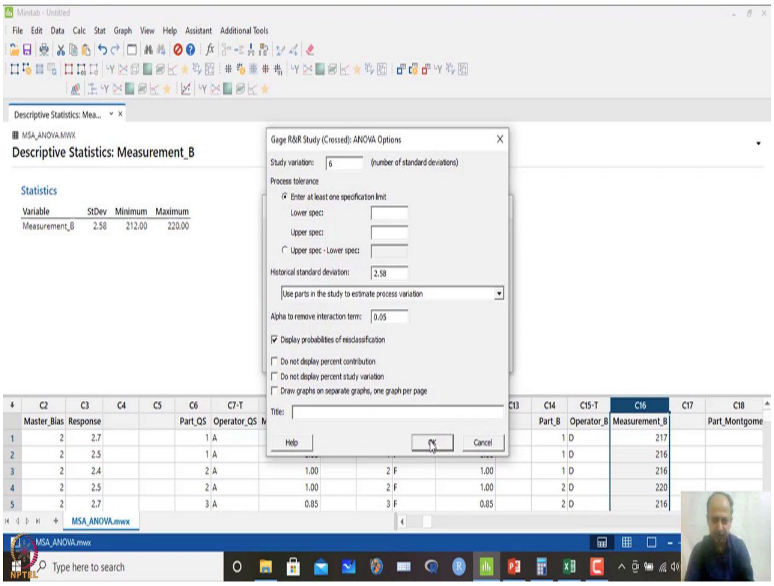

So, options what we can do is that we can write 2.58 over here or use the parts in the study to estimate the process variability. So, this is ok with us and ANOVA analysis we have mentioned that one.

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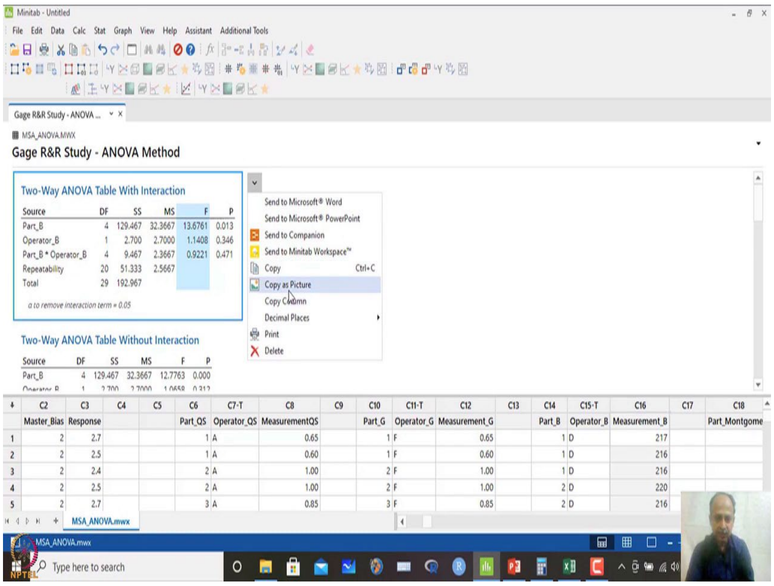

And we click ok what happens is that in this case what we see is that part to part variation is there that is expected operator to operator variation is not there.

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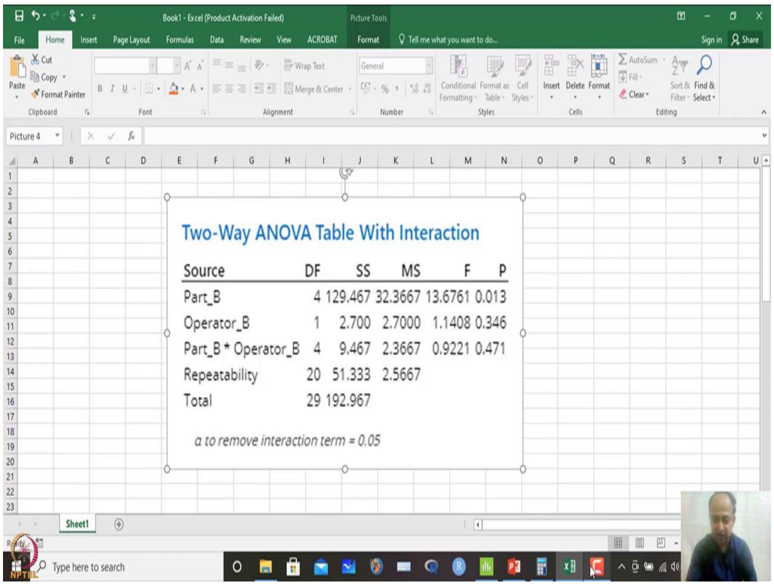

What we can do is that we can place this one and we can show it like this. So, part to part variation is prominent at 0.013. Operator to operator is not prominent and interaction is also not. This is a favorable situation on this aspects at least. So, this is the most suitable scenario over here. So, but we have to see the percentage contributions.

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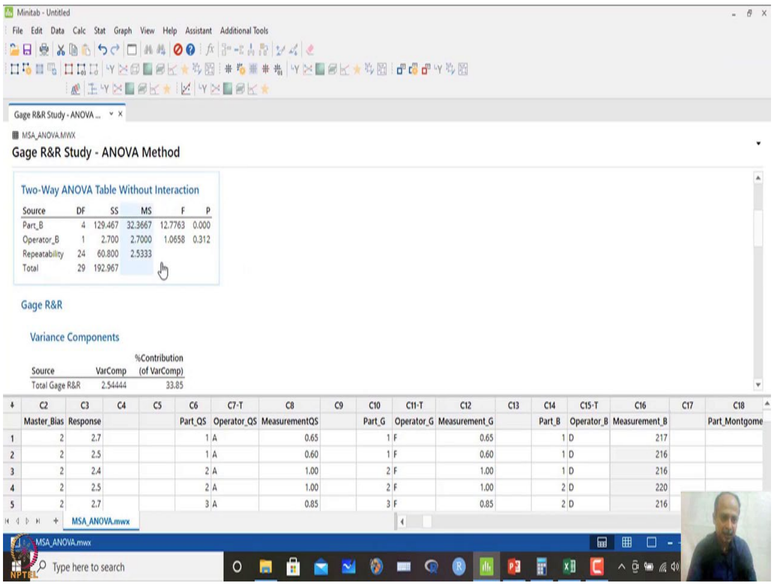

So, we will go back to the results over here and let us do the ANOVA analysis. So, ANOVA when we do the ANOVA and interactions is not prominent over here and is more than 0.25, what happens is that MINITAB automatically combines and gives you a table where only part that p value will be reported and operator will be reported over here.

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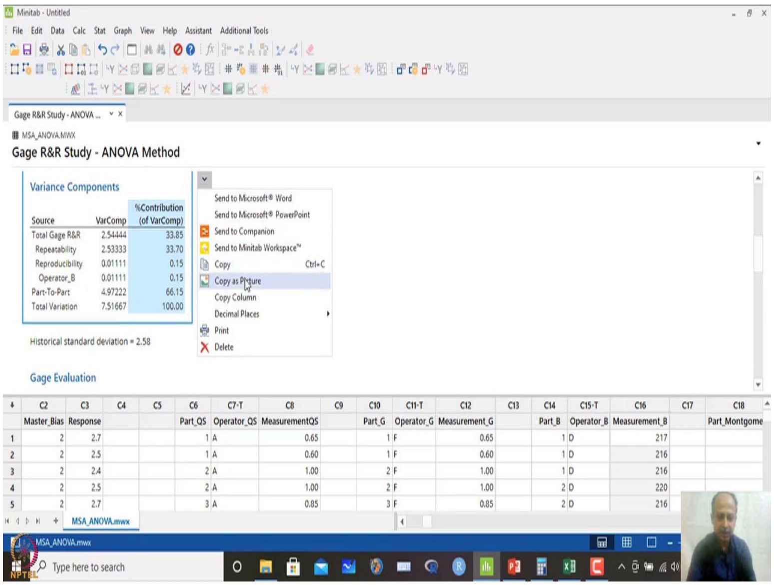

So, that interaction will be taken out. So, without interaction MINITAB will report, and percentage contribution what you can see over here if I copy this one it is around 33 percent over here.

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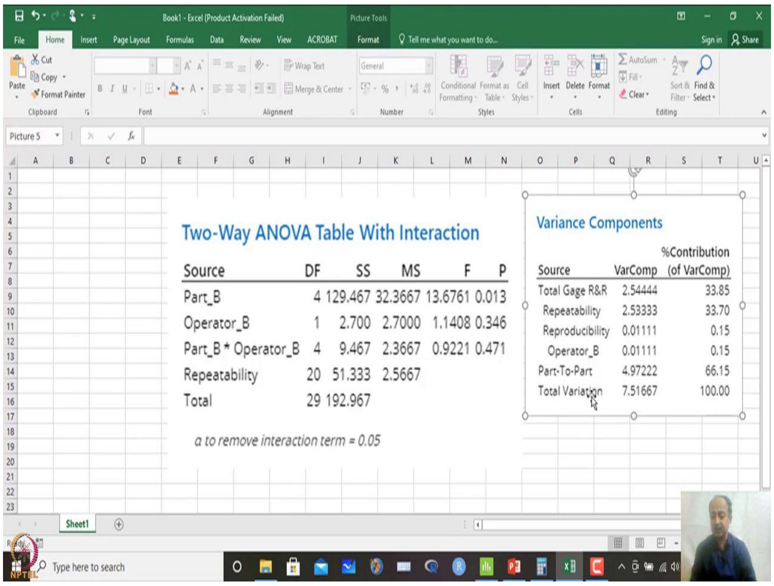

This one is quite high, about 33 percent. I told if it is less than 10 percent we should accept. So in that case instrument is having a big problem when I am considering total Gage R&R variability as compared to the total variation.

Part to part variation should be maximum 90 percent or above and gage should be less than 10 percent, but here gage variability is 33 percent as compared to part variability which is around 66 percent. So that means, instrument has a problem over here and most of the problem is due to repeatability of the instrument.

So, we have to send it to meteorology to look into this what is going wrong basically. If it is due to reproducibility what happens is that operators are measuring it differently. That is not the case repeatability is a problem over here.

And that represents that instrument is having a problem over here not the operator that are used for measuring these. But instrument is measuring differently over here. So, that is an important aspects it has to go to meteorology and we have to check that one, percentage contribution 10 percent less we that is the favorable scenario.

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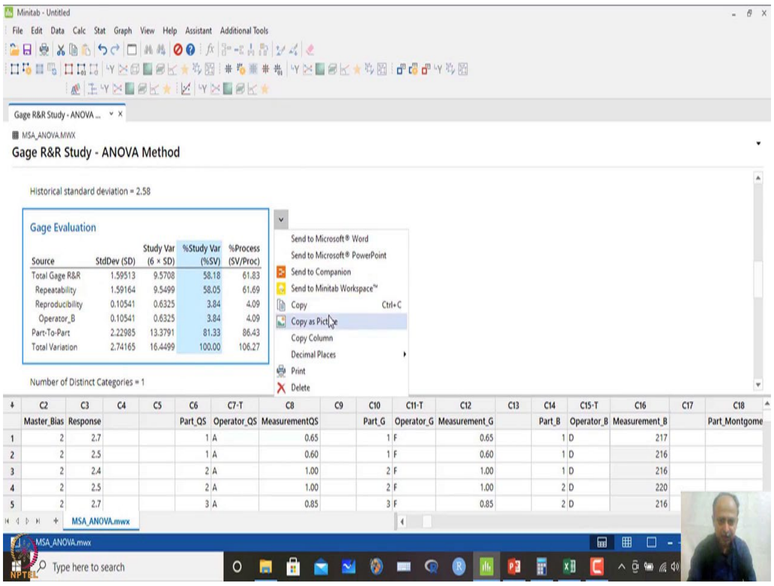

And then another information what we have is that Gage evaluation. If we place that in excel what will happen is that we can enlarge that term that one and see.

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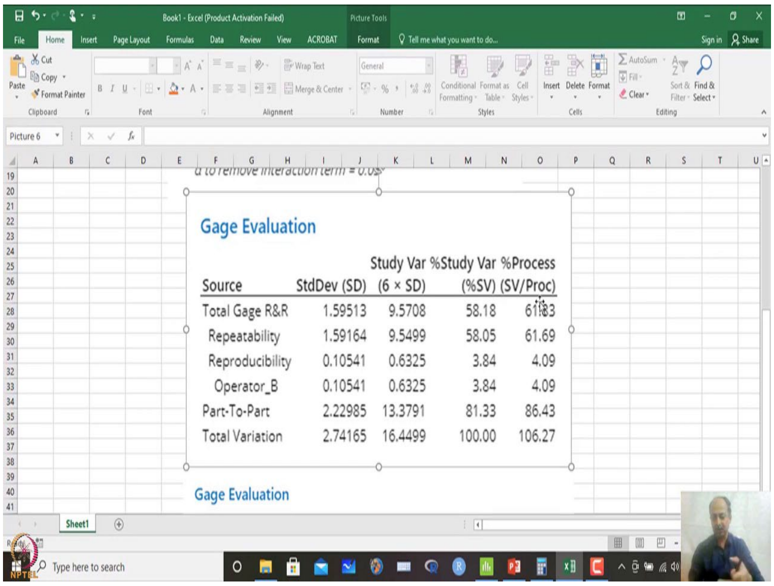

So, here what you are seeing is that study variability will be close to process variability, because we have taken the information from the sample observations that we are having that is the parts information that we are having.

So, here also 58%, 61% that is quite high even 30% more than that one over here, so it is a big problem like that instrument is having a big problem over here. So, we have to send it to meteorology.

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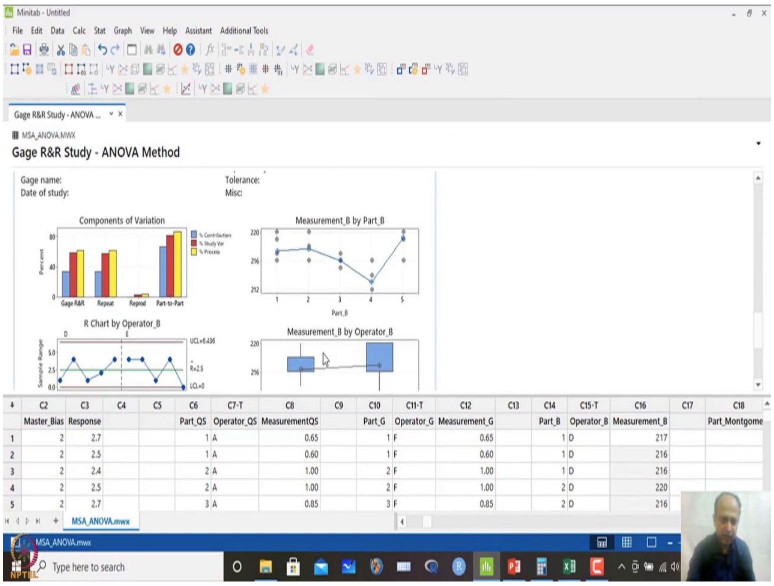

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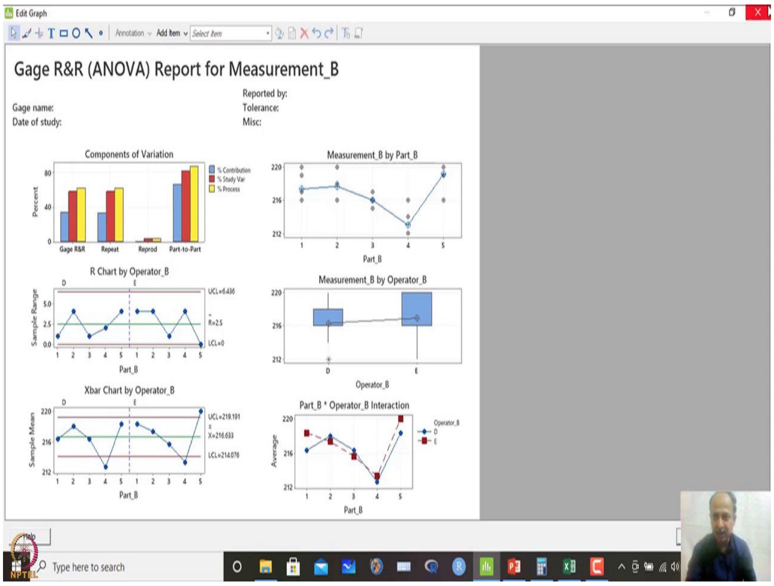

And also what we can see is that when a when I am doing this diagrammatically what you observe over the here is that this is a bad instrument. So, in this case percentage contribution this is high here also you see Gage R&R percentage this is also quite high. So, component of variation what do you see is this should be minimum over here, but this is also quite significant over here and operator to operator that there are 2 operators D and E over here.

So, within operator variation is not so high. But when you see part selection over here, the measurements is within this and it is not expected all the part should inside. Most of the part should be outside the control limit line. So, that is not happening over here and here also measurements there is some difference, but that is not significant, what we have seen that operator to operator variation is not significant D and E are measuring more or less in same pattern.

Interaction is also not prominent that is also ok over here and the parts are measurement are shown over here. So, in this case problem is the contribution component of variation over here.

What is the number of distinct category let us try to see whether what is the calculation? So, number of distinct category is 1 which is less than 5 and that is a concern 4. And this is not acceptable, instrument is not able to distinguish between the parts like that, so that is not acceptable like that ok.

We can take an example from QS manual that is standards for automobile industries. So, in this case this was the example that is given for measurement system analysis so C7, C 8 and C9.

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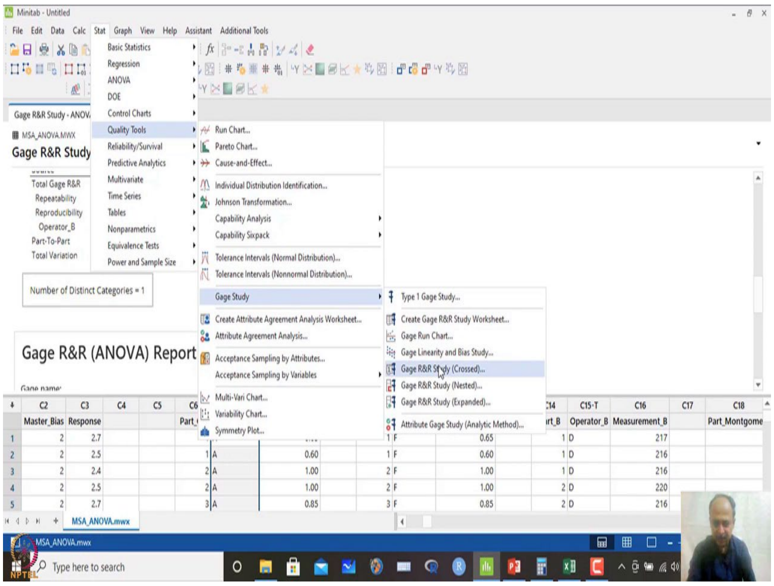

So, in this case also we can see what how interpretations.

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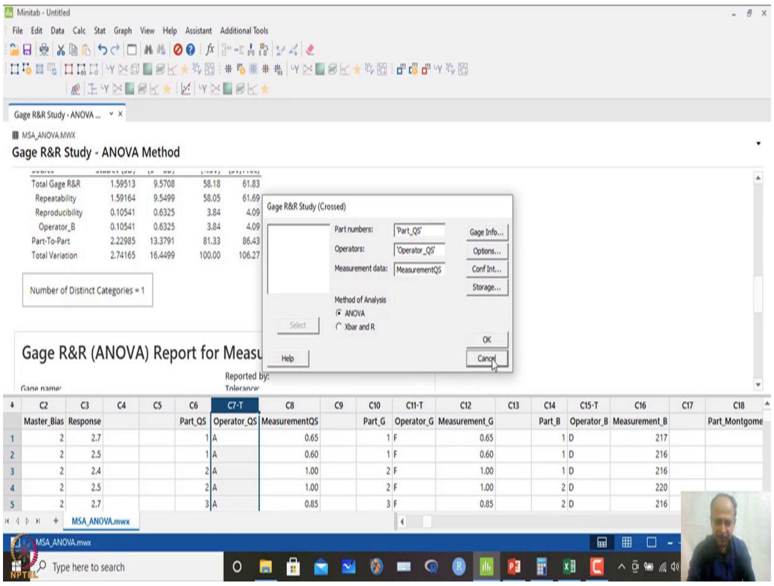

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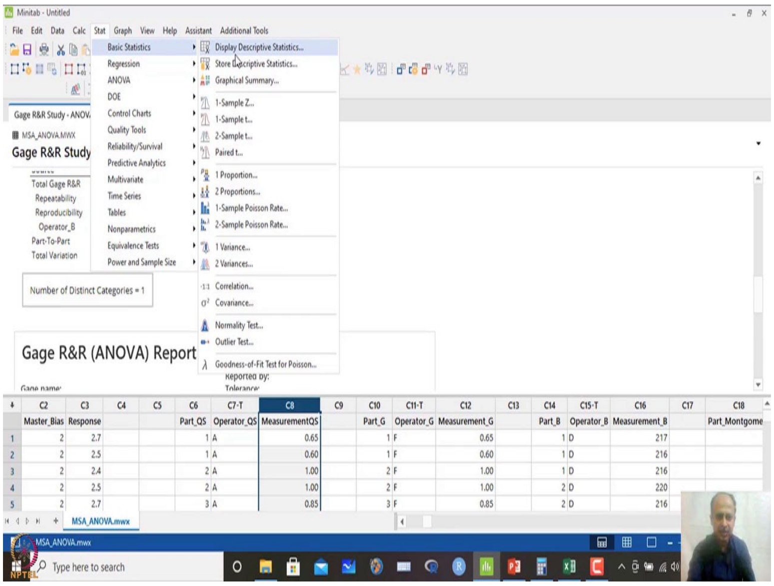

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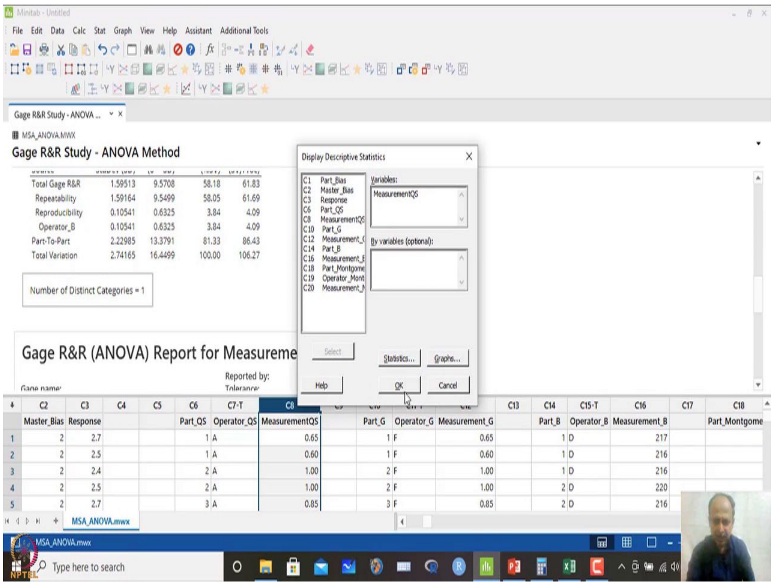

So, let me try to calculate this standard deviation of this.

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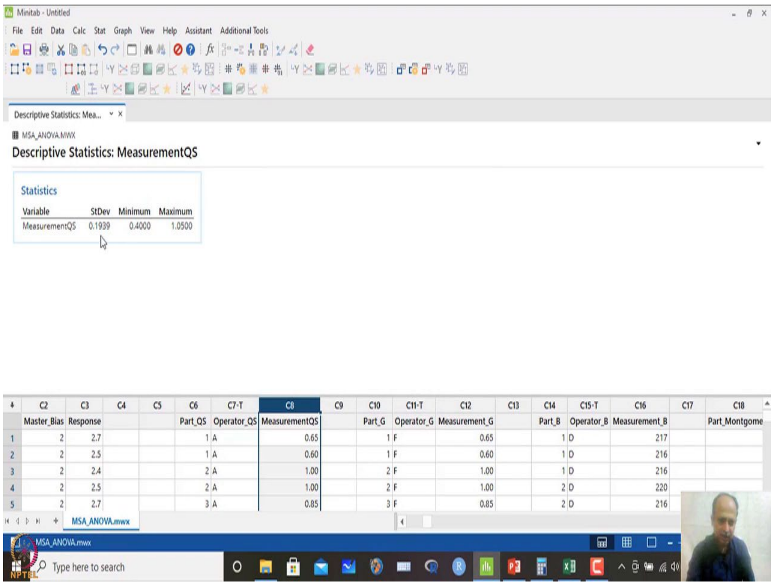

And let me try to see what is the value? So, 0.2 approximately 0.2 we can take.

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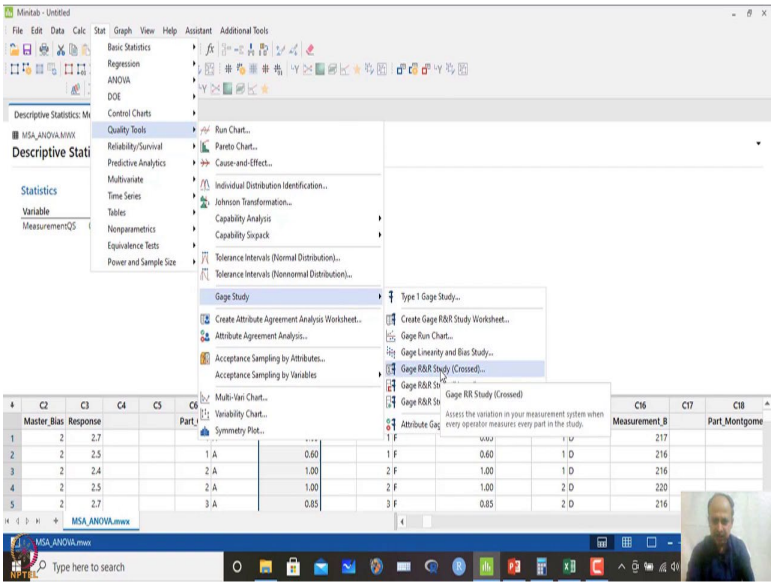

So, 0.2 we can take over here so quality then gage studies.

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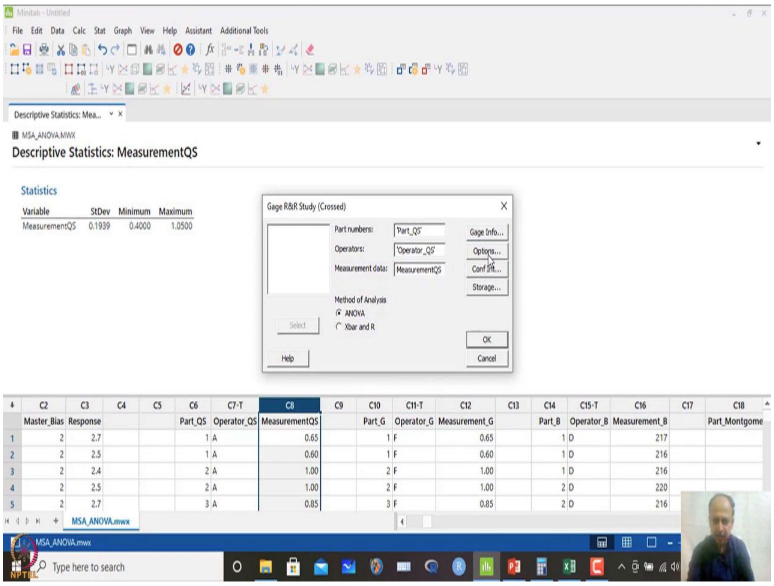

And then Gage R&R and we have taken part as QS operator measurement.

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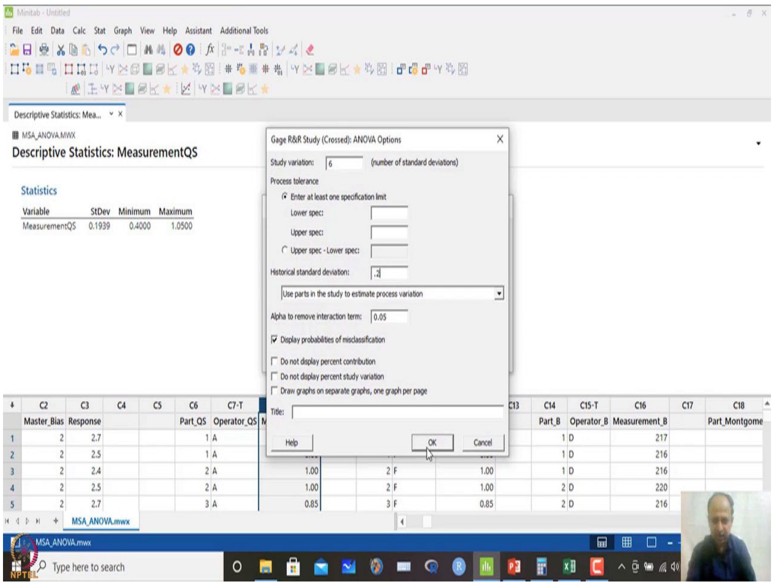

Options is we are getting 0.2 over here that is the estimation, but it can estimate purposes variations over here and I will click ok and I will click ok.

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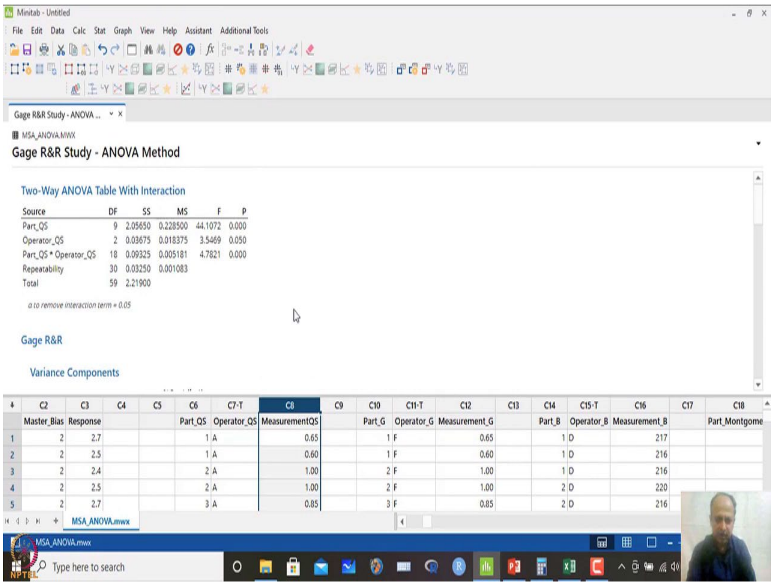

So, in this case what we are seeing is that part to part variation is there that is required, operator to operator variation is also happening this is less than equals to 0.05 and part and operator interaction is also prominent.

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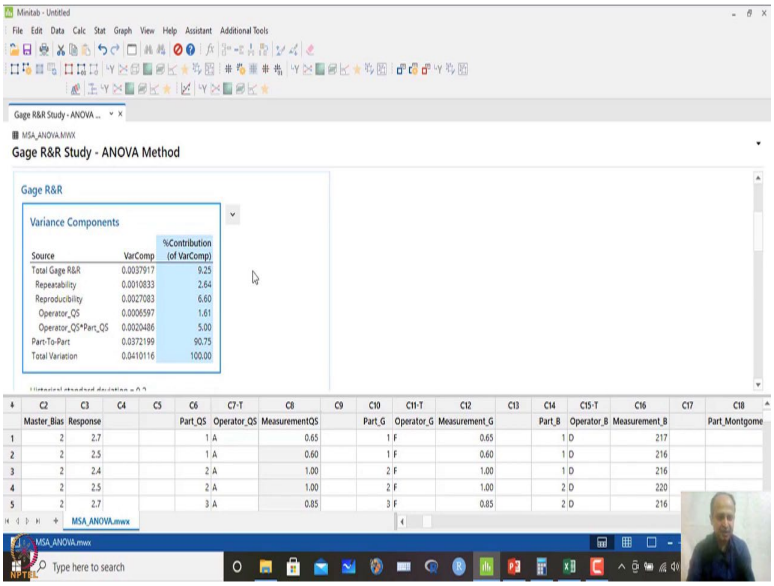

Percentage contribution if you see this one less than 10 percent, so somewhat satisfactory over here. So, this is not a concern for us so this this is favorable situation for us over here.

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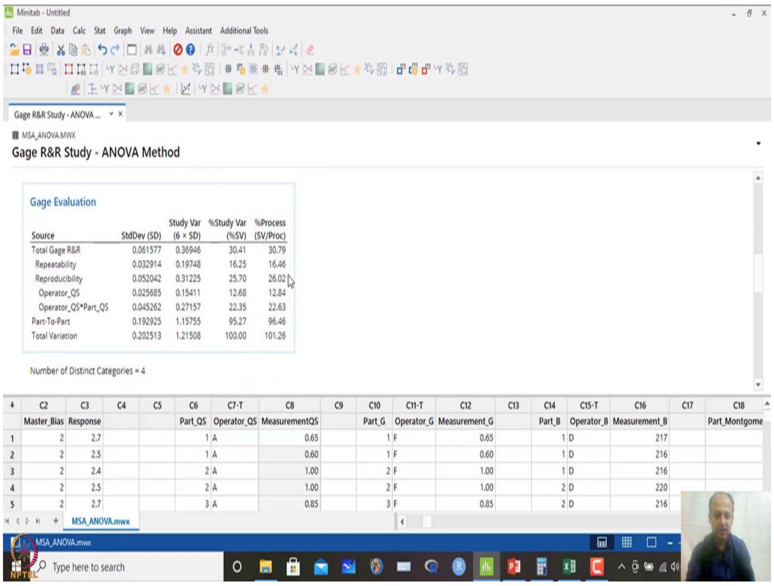

And if you see process variation it has just touched 30 percent and here it is just about 10 percent like that 9.25 and somebody can take 9 as a criteria also. So, in this case somewhat we are at the border line over here. So, we have to see how to reduce this one total variability over here and the main contributor is reproducibility that means that is happening from operator to operator.

Operators are measuring it differently why is it happening that we have to see, we want it to be insignificant because operators are highly skilled. So, there must be some skill difference that is happening over here, that is why reproducibility is giving you a higher value over here.

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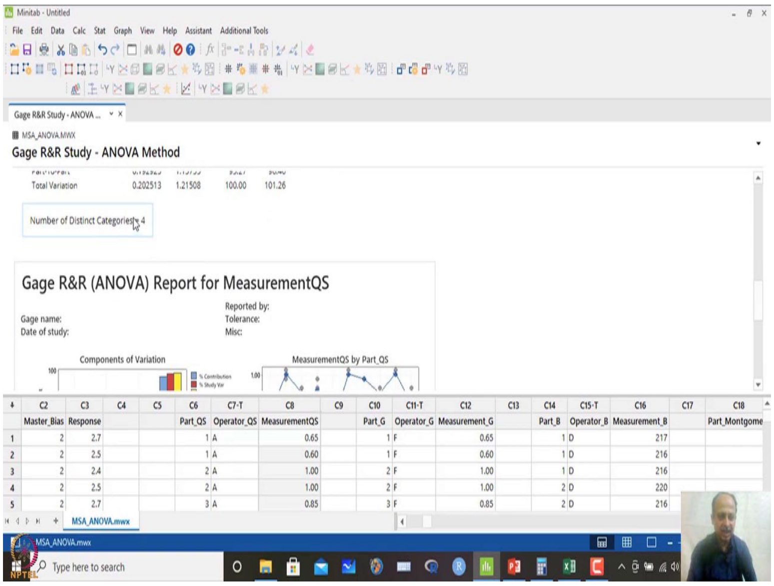

So, you have to take measures over here. So, that it does not happen like that operator to operator variation may be this 10 percent that we are getting around 9 more than 9 percent will reduce over here.

So, so we have to take actions where it is necessary like that. So, reproducibility is a problem over here we can reduce repeatability. So, that will reduce the overall 30 percent criteria that we are having.

So, what action to take? Whether to send to meteorology or it is to operator to operator variation we have to we have to take action, because operator to operator variation is also contributing to the overall variations of the measurements that you are getting.

So, if operator every operator is measuring differently that is not acceptable basically in production or operations, because somebody will reject the variations somebody will accept that this is ok like that. So, capability analysis will be different. So, everything goes wrong if one thing goes wrong everything goes wrong like that. So, we do not want that scenarios to happen.

So, this is what I wanted to emphasize and number of distinct category also is 4. I have to see this because it should be greater than equals to 5, so at least equals to 5.

We have to concentrate why this is happening. May be operator to operator is contributing over here mostly. So, then it is and the part selection is also there is we have to be very precautious about selecting the parts over here.

So, it should cover the operating range the CTQs basically or the specification range of CTQs. This is all what we have to discuss in measurement system analysis and now let us go to another important topic over here which we will discuss now for improvements. So, what we have said is that instrument should be correct, so that we get the exact information of the process variation.

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And now we will enter into an important topic which is known as Factorial Experimentation and we have already entered into that asymmetric factorial experimentation, i.e. 2 way analysis of variance we have done. As we have already covered that one, this part becomes easier to understand and we assure that measurement systems are also ok so there is no problem.

So, we can now interpret and select the factors; factors screening basically what we are doing in factorial experimentation ok. So, what we will discuss is how factorial experiment is done, how it is done in how results are generated and based on that in MINITAB. So, based on that what conclusions to make.

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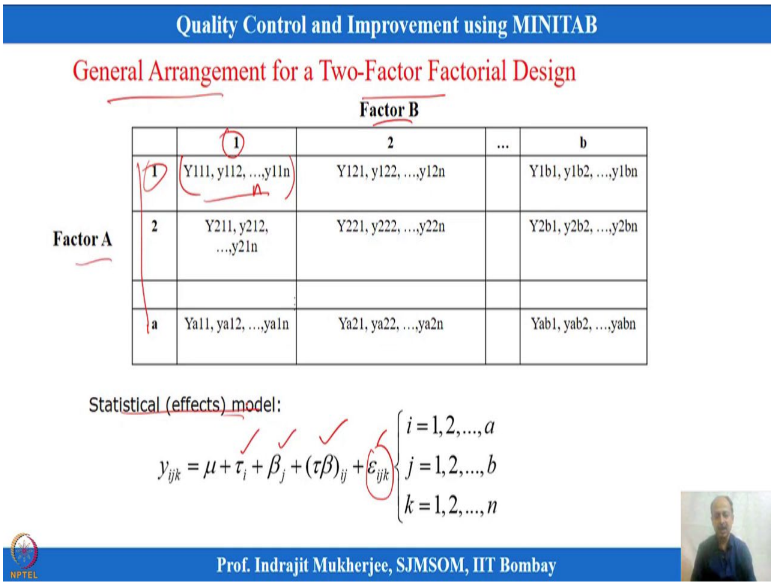

So, we have to explain factorial experimentation first. So, these are factors A and factor B this is the general expression that we are having over here. There will be factor A and factor B over here, there will be observations. We also have replicates at each combination. We will get the total experimentation and the contribution of this factors over here are given over here.

And the interaction contribution is given over here and the overall what is unexplained is the error variability over here and in any factorial experimentation also we have to take care the error assumptions or residual assumption for regression also it is true for factorial experimentation also it is true.

So, this is the mathematical model, statistical effect model that is considered over here and that is what we want to understand in hypothesis testing when we are doing the experimentation. ANOVA analysis will reflect which factor is important and which interaction is important or not.

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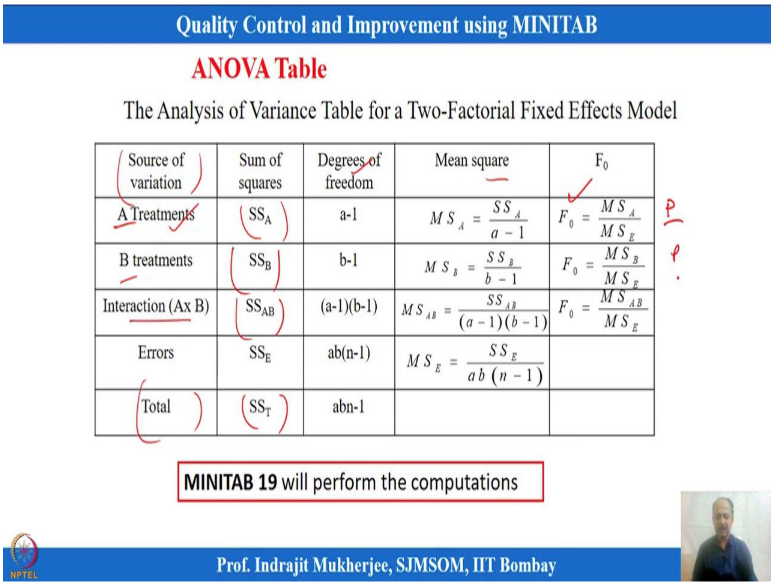

So, there can be A level and B level it can be asymmetric and it can be symmetric also. So, we will discuss about symmetric. So, what is important is that whenever we have done the experimentation we have the sources of variation. So, total variability is known as SS_T that can be we can have if it is 2 factor over here we will have effect of A.

What is that contribution of that effect of B what is the contribution of that interaction effects. What is the contribution of that? So, degree of freedom can be calculated. We have already discussed mean square errors can be calculated and F_0 values can be calculated and based on that p values criteria can be used to see when I change A whether it is impacting the CTQ, when I change B whether it is impacting the CTQ or when I change when interaction effect is prominent or not that also we can check over here ok.

So, those p values will indicate what is happening. This is an approach which is taken in screening experimentation and also may be we want a sub optimal solution, we want to see what is the combination of factor A and B. And but this is not the final optimization what we do generally in screening experimentation factorial design is used and later on we will have we have a technique which is known as response surface methodology which talks about optimization of the system or process like that.

So, in that case sequentially we move towards optimal scenarios like that. So, here it is may be at a snapshot like that what is the optimal combination at this scenario, if this is the operating range these are the 2 factors can you tell me which is optimal over here that may it may be sub optimal solutions like that. Anyway so we are talking about the factor A and B over here and ANOVA analysis will tell me which factor is important.

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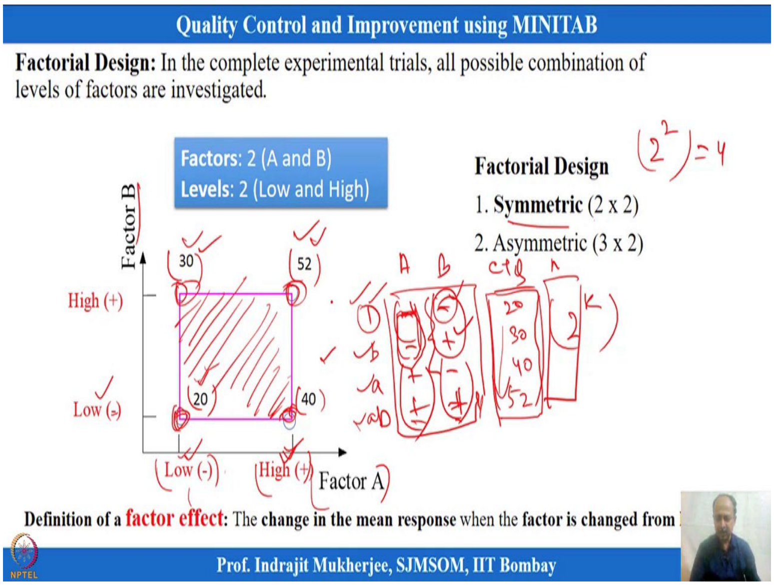

And when we are talking about factorial experimentation it is basically a symmetric experimentation that we are doing. It may be 2 factor at 2 levels. This is known as symmetric experimentation. So, this is let us say a factor A and factor B, so in this case we have 4 corner points that we are experimenting over here. Factor A has same number of levels as factor B over here.

So, levels can be defined over here let us say level over here is defined in some arbitrary definition what we are saying as low level and high level like that. Because there are only 2 levels, so 2 factor at 2 levels so this is 2^2 design basically.

If we have 2 levels and k factors that is a 2^k design, basically that is a general expression when we have 2 levels experimentation. So, 2^2 design, where we have 2 factors at 2 levels and levels are arbitrarily defined as low level and high level over here.

So, it can be like color or something like that categorical variables also, because in experimentation we may have categorical variables we may have continuous variables like that.

If both are continuous variable in that case we can have contour plots and all these options are there response optimization that is possible like that what we have seen earlier also. But sometimes it is categorical whether the factor is important and based on that also we can develop equations and we can optimize the systems like that that is also possible.

So, over here what we are seeing is that we are starting with a simple experimentation and this is 2^2 design basically, where 2 factors are there and both are at 2 levels over here. So, low and low combination is giving you a reading CTQ value of 20 over here and high and low combination factor B is at low level and factor A is at high level over here ok.

So, this is giving me a reading of 40 like this. So, whenever both the factors are kept at high level this is the readings that we are getting over here and when the A factor is kept at low level and this is at high level the reading that we are getting is 30 like that.

All combination is tested over here so, all combination of 2 square means 4 number of trials. So, all the 4 combination reading what you see over here all 4 combination. So, one is minus minus information that is if I write low as minus over here and over here also minus, so that combination we have run so and then may be factor.

So, this is factor A and factor B, so minus minus means low level of A and low level of B like that. So, then we can have a combination of this minus plus then we can have a combination of plus minus over here and the final one may be plus plus like that; that way also we can think of that experimental and this is the matrix that design matrix what we are using over here.

This is the design matrix they say this is the design matrix and we will run the experimental trial and we will measure the CTQ values and we will measure what is the if I run this combination, and if I run this combination this is let us say this is B combination over here and this is A combination over here this AB combination over here.

So, in this case we will measure the values. So, this is minus minus combination so value is 20 over here. So, this is A at minus level over here A at minus and B at plus A at minus and B at plus so in this case the value is 30 over here. So, this is 30 observation and the this is 52 observation and then this is 40 observation over here.

So, these are the observation that we have when we have run the trial like this. So, this is the symbolic notation that I am using over here. 1 means both at low level, B means only B is at high level and A is at low level and then A means A is at high level and B is at low level like that and AB means both A and B at high level that we are experimenting over here.

And these are the results that we have got from the experimentation and this is randomized. So, which trial will run this is not the first trial that we run this will be randomized like that ok.

We can also have a replicates over here that means this is one measurement set that we are getting we can have a second measurement set like that. So, n will be the number of replicates that we do and more and more we replicate we have, accuracy level of the model increases and the interpretation and the final conclusion becomes more accurate as compared to.

Therefore, what is suggested is that you replicate experiments and here what we are doing is that at every level of A we are having all combinations of A. So, if A is at minus level over here and B also at minus and plus and all combinations we are running.

When A is at positive we are also running B at minus level and also at plus level like that, this is known as balanced experimentation that we are doing ok. This is also important aspect that we considered in experimentation.

When we are talking about factorial experimentation it is a balanced experimentation, it is very scientifically designed like that and it gives you information of interactions. So, what is the advantage of that we will see in our next session. Factorial experimentation is symmetric design you can think of like 2 way analysis of variance and factors are at different levels.

We can have 2^k, 3^k designs. This is a general symbolic way we represent factorial experimentation.

So, this can be of numbers this can be of categories like that and these are the observations that we have and we have covered all the points all the extreme corners over here such as surface that is covered over here. Basically this is the experimental zone that we are covering over here for factor A and factor B and a third dimension you can think of z dimension is basically CTQ.

If you are talking about response surface so, in that case A and B are in x y axis and z is the basically response that we are seeing over here and in case it is continuous then we can develop the surface contour plot and everything is possible. We will continue from here.

Thank you for listening.