Quality Control and Improvement with MINITAB
Prof. Indrajit Mukherjee
Shailesh J. Mehta School of Management
Indian Institute of Technology, Bombay

Lecture - 11
Individual Moving Range Chart

Hello and welcome to session 11 of our course on Quality Control and Improvement using MINITAB. I am Professor Indrajit Mukherjee from Shailesh J. Mehta School of Management, IIT Bombay. So, previous session what we have seen is basically control charts, we are discussing about control charts.

So, we have talked about X bar R and X bar S type of control chart to monitor CTQs and figure out where there is some assignable cause because of which the mean is shifting or variation is shifting to abnormal scenarios. In this section, we will talk about one more control chart and then we will move towards attribute type of charts which is also very relevant and frequently used in controlling quality.

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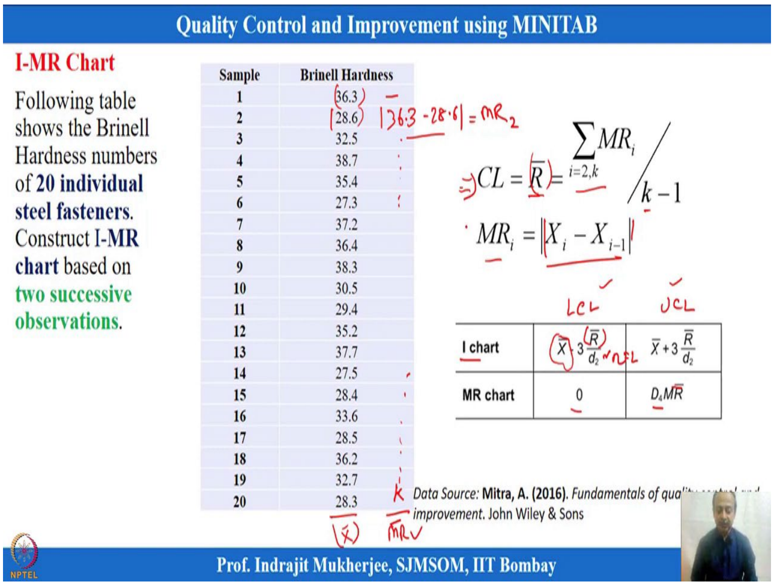

Sometimes I cannot collect multiple observations of subgroups at a given time point. And sometimes it is irrelevant also to take samples when the readings will not differ much, for e.g. in viscosity in a chemical process. So, if I go at a certain instance and if I take 5 samples, all samples will be more or less very close. Readings will be very close, there will not much difference over here. For e.g. in a manufacturing process hardness by treatment is done, we expect Brinell hardness to be more or less same.

So, one sample is sufficient because we want to save the cost in that case. So, we do not need multiple samples. So, single sample observation is there and in that case what is to be done, what type of control chart I should implement and try to figure out whether the process is going out of control or is in control.

But in this kind of scenarios also you have to remember that the data points may be first observation with the second time t_2 observation, there can be some correlation which can exist between the data or auto correlation we talk about. So, there can be scenarios where these 2 data points may be interrelated.

So, readings of initial observation t_1 and second observation may be somewhat related with each other. When we have such kind of scenarios, in that case we may use different types of charts. So, one of the options that is available is individual moving range charts. So, here you can see 20 observations are taken in a given process at different time points, t_1 to t_{20} let us say.

And only one observation was selected over here, and we want to monitor the mean and variance of this. So, we can calculate the overall mean $(\overline{\mathrm{X}})$. Upper control limit line lower control limit lines are calculated based on some measures which is

$$\overline{\mathrm{R}} = \sum_{i=2,k} MR_i \Big/ k-1$$

And this $\overline{\mathrm{R}}$ is basically calculated based on moving range.

$$MR_i = |X_i - X_{i-1}|$$

So, how moving range is calculated over here? 36.3 over here minus 28.6 that is the moving range for 2nd observation.

So, from 2nd observation we will start calculating moving range and let us say there are k observations, $k-1$ moving average we are having. Average of moving range will be treated as $\overline{\mathrm{R}}$ over here.

o, average of moving range is calculated over here and then with this average which is treated as $\overline{\mathrm{R}}$, I can calculate the limit lines for individual chart and which is $\overline{\mathrm{X}} \pm 3\frac{\overline{R}}{d_2}$, where d_2 will be as defined in X bar R chart. The d_2 value will be influenced by the value of n. So, corresponding to n = 2, I will figure out d_2 value, because every 2 observation we are calculating the moving range. So, this subgroup size will be treated as 2 over here. So, in this case what will be happening is that $\overline{\mathrm{X}} + 3\frac{\overline{R}}{d_2}$ and $\overline{\mathrm{X}} - 3\frac{\overline{R}}{d_2}$ will be the upper and lower control limit lines, respectively.

Similarly, here also you will find $\mathrm{D}_3\overline{R}$ and $\mathrm{D}_4\overline{R}$ similar to in X bar R charts. Because D_3 is 0 here for n = 2. So, in this case, this becomes 0 and this is $\mathrm{D}_4\overline{MR}$ which is the average of moving range basically which we are calculating over here, ok. So, overall average $\overline{\mathrm{X}}$ of the individual observation we can get and then we can define the upper control limit line, lower control limit line. And similarly moving range average will be done we can calculate over here.

So, moving range can be calculated over here and this is the expression $MR_i = |X_i - X_{i-1}|$. So, previous observation absolute value of that. So, in this case, this will give me moving range. An average of this moving range we are just monitoring over here. So, their upper control limit line will be there, lower controlling limit line will be there and the central line will be MR average basically.

So, average of this or R will be the average of this is a central line what you see over here, ok. So, MINITAB does it automatically for you, and in that case we will try to demonstrate how the MINITAB gives the output when we are just defining that one. So, I am opening a file where this information is already there like that.

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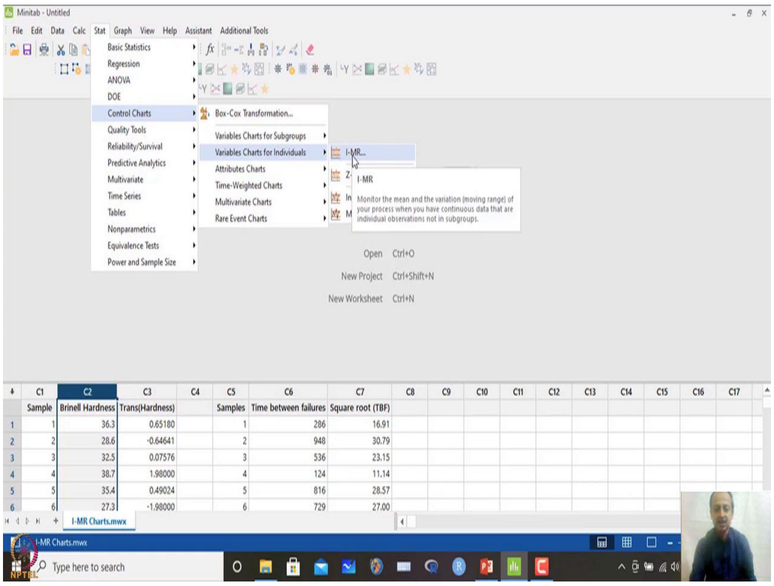

So, I am just opening that one. So, in this case, data is in C2 column, and we can go to stat and then control chart and then variable chart for individuals over here I-MR chart option is going to come over here.

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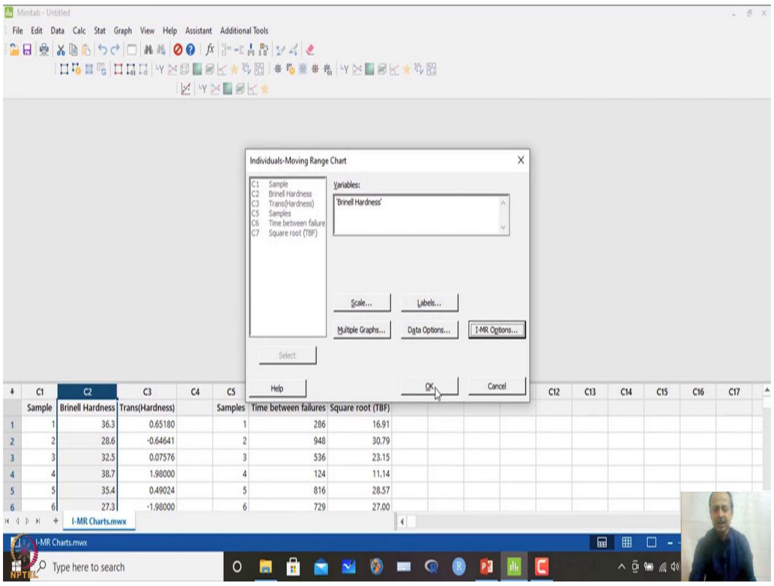

So, I-MR chart when you click and then identify the variable that Brinell hardness we want to monitor over here.

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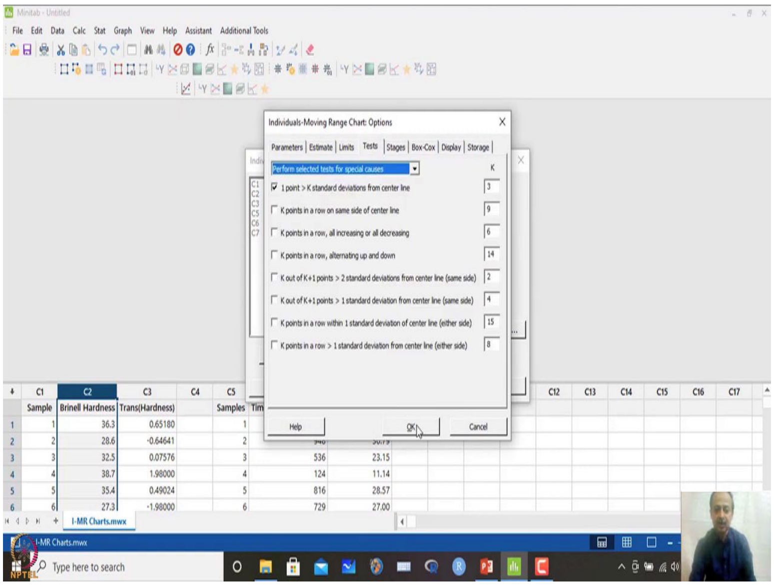

And I-MR option will we also test that 1 point goes outside or not, everything else remains same over here. And then I click ok.

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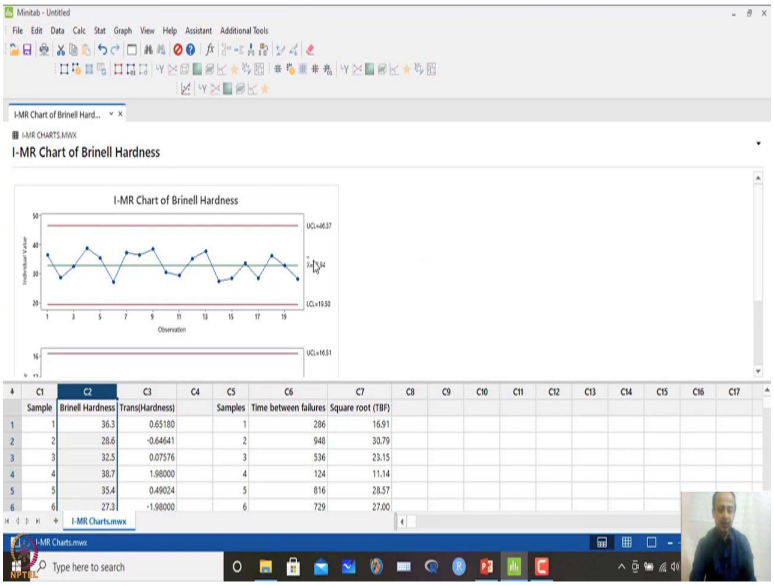

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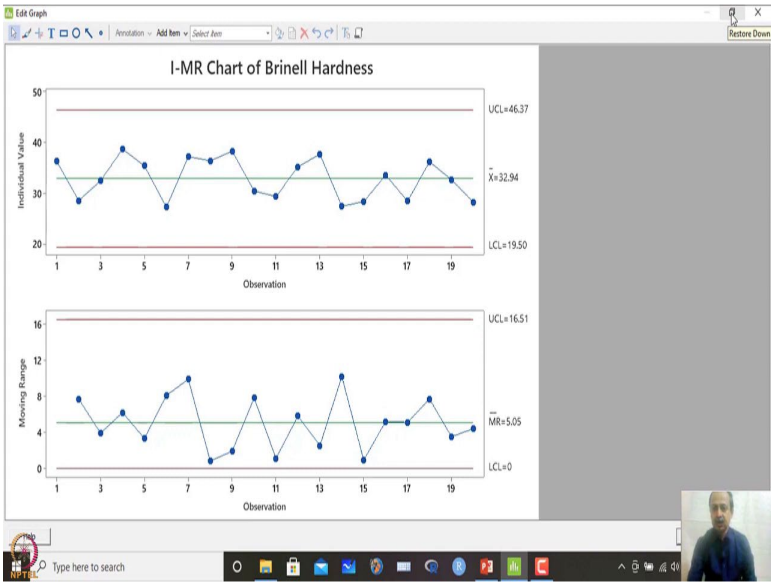

And immediately what will happen is that you will get an output in MINITAB and in that case, you will try to figure out that these observations are shown in this. On the top it is individual values and the moving range is shown over here. So, moving range is having a limit over here. And also individual chart is having a upper control limit line. So, all points are within the control limit line. So, there is no problem and the process is under statistical control and if this is a stable process we can just define like that.

And although this charting techniques is quite robust, but we have to remember that charts also are build based on certain assumption like $\pm$ 3 standardization what we are considering over here and that can be violated also. So, in this case, we need to cross check that whether the data of hardness over here follows normal distribution or not. So, how do we check that? There are ways different ways of checking that one probability plot papers are there.

Otherwise, statistical tests are also there, which confirms that these data sets are non-normal or normal. For that knowledge of hypothesis testing is required which we will discuss because design of experimentation requires knowledge like that.

And so, we will give brief about that afterwards. But if the data is non-normal although I-MR chart can handle moderate deviation from normality over here, but if scenario comes that it is skewed in that case there are options that we can change these dataset into a different variable.

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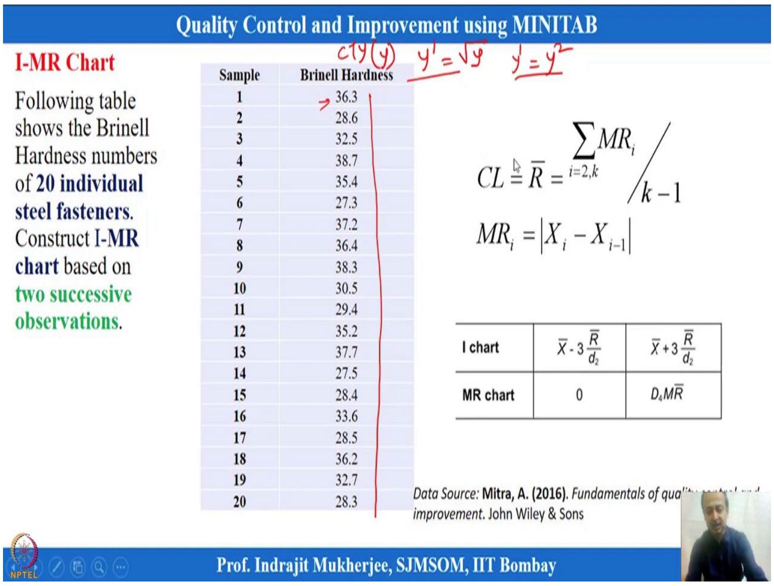

So, if we are saying this is the CTQ which we can define as y, we can change these dataset or convert these dataset by some transformation over here, so which can be y' over here. So, we will use some transformation:

$$y' = \sqrt{y}$$

And in that case, what happens is that the converted data will become normally distributed. Then, what we do is that this can be implemented in X bar R also because if the data is non-normal in the case we have to we need to do something on the data.

So, there are many ways of doing this and we will discuss this afterwards. But at present I can assure you that there are checks which ensures that whether the data coming from normal distribution or not.

So, in this case, what happens is that I have checked the data initially, and I saw that it is deviating from normality over here. So, some transformation was used here. So, I am writing the as trans hardness over here, some transformation was done over here and the data seems to follow normal after I have done some transformation. So, 36.3 let us say some transformation was done over here with some expression and that is the standard way we do transformation of the data set.

Then, what we do is that on the transform data when it is normal then we plot the transform data and try to figure out whether that is following and that is under control or not. So, what we do is that first we check the data normality, if it is normal we plot it in I-MR chart, but if it is non-normal in certain scenarios what we will do is, we will try out some tricks and we do the transformation of the data, it can be linear or non-linear transformation of the data.

And when we transform the data into some other variables and then we try to monitor and that follows normal distribution, then we will apply our control chart techniques on transformed data. So, I-MR chart will be used over here. Now, I will use transformed data hardness.

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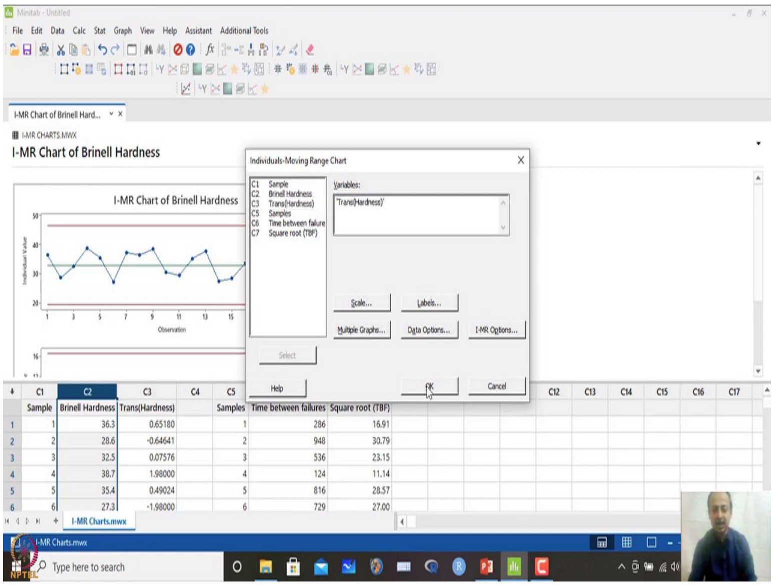

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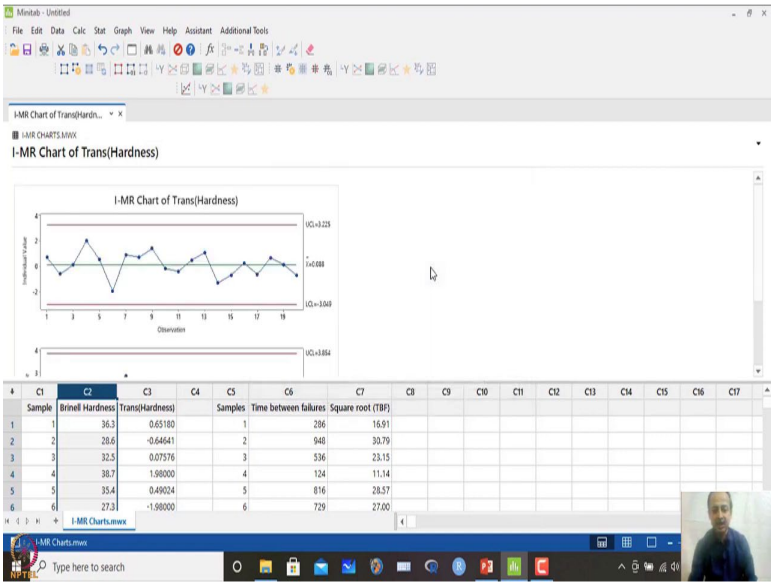

So, then I will again click ok, and try to see whether that is following within the control limit line or not. So, this is only done when there is a higher amount of skewness, but I-MR chart can handle moderate deviations. So, that you have to remember.

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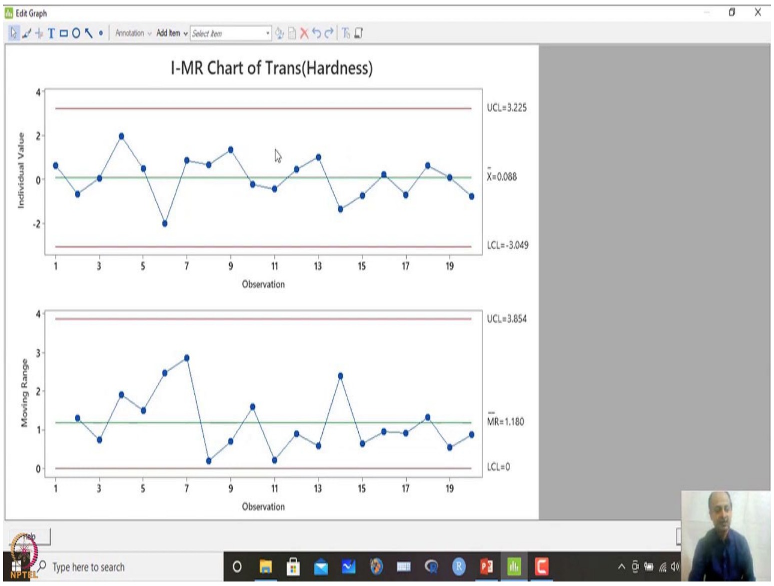

And in this case also, after transformation also what we see is that data is within the control limit line in individual and also in range over here. So, we can assure that the process is in statistical control. So, this is a stable process. So, whenever you find that there is some deviation you can always ensure that whether I can do transformation on the data set which will assure the normality assumptions.

And then I can apply the charting techniques what we are using over here I-MR like that. So, everywhere it is true. So, certain scenarios we need to transform. It is non-normal scenarios because everything is based on assumption of ± 3 standard deviations that assumptions of building the upper limit and lower limit which differentiates between control and out of control.

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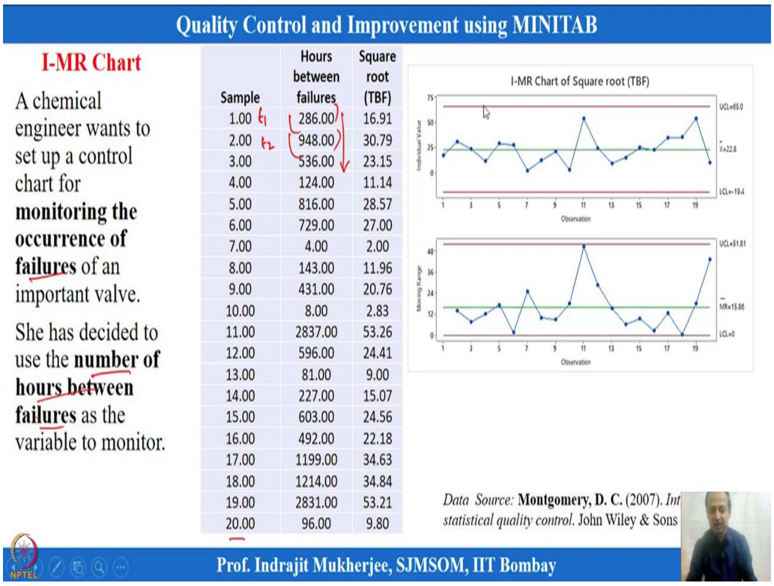

So, I have taken another example over here. So, this was taken from Amitava Mitra's book. And next example also we have which talks about I-MR implementation. Say here hours between failure was monitored over here; so, monitoring the occurrence of failure over here. So, how many hours before it fails like that, so this observation was collected over here.

And there are 20 observations sample, 20 samples observation. And this is also a single observation that we are having at it any given time point like that. So, this is t_1, this is t_2 like this. And here also with the original dataset we can try a I-MR chart and because the data was seen to be not normal in that case we have also transformed the data. Here one transformation was used which is known as which is square root of the original values like that. So, if it is 286 , square root of that will be 16.91 like that.

It was also not following normal distribution, so we will do that afterwards and try to figure out that how to check normalities because we need to understand hypothesis and then only we can understand the statistical test which confirms whether it is normal or non-normal like that scenario.

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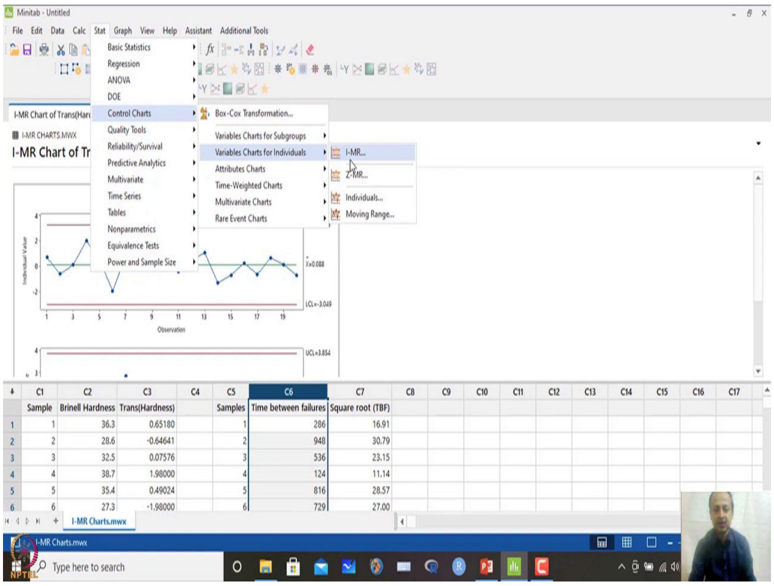

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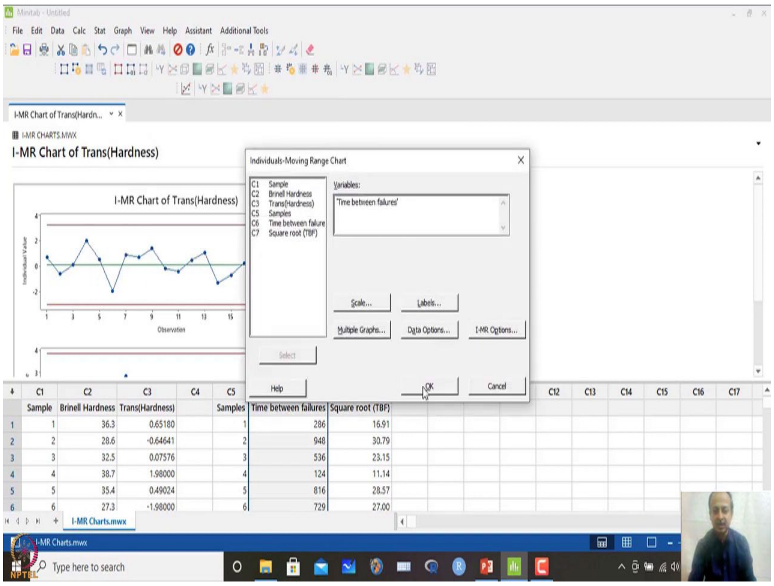

So, here what we are assuming that this moderately it is deviating from normalities over here and then we have applied control charts. So, I-MR chart we will apply first, and let us try to see that whether everything is in control. So, when we do that we see some amount of, some amount of observation which is going outside the control limit line for time between failures over here. And after transformation, let us try to see then what happens.

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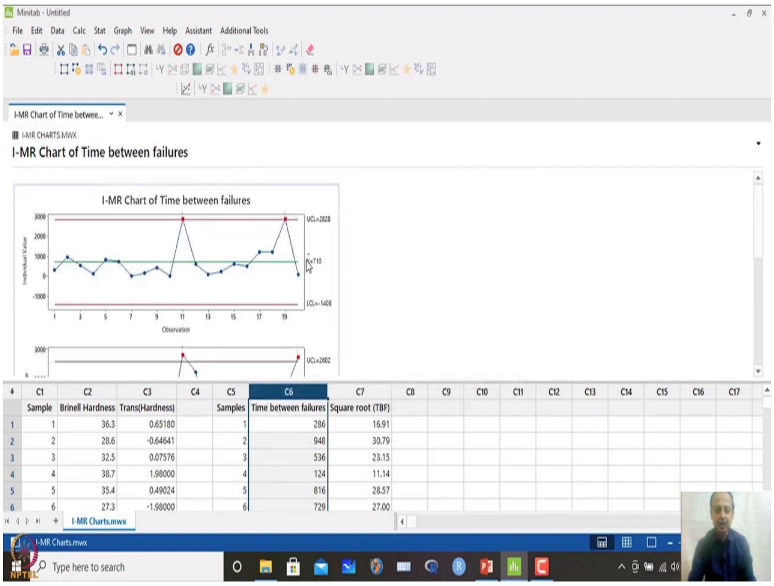

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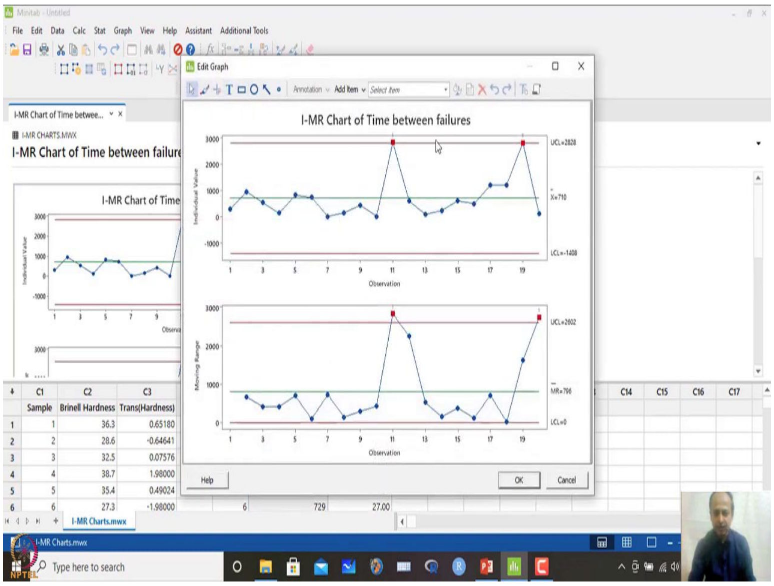

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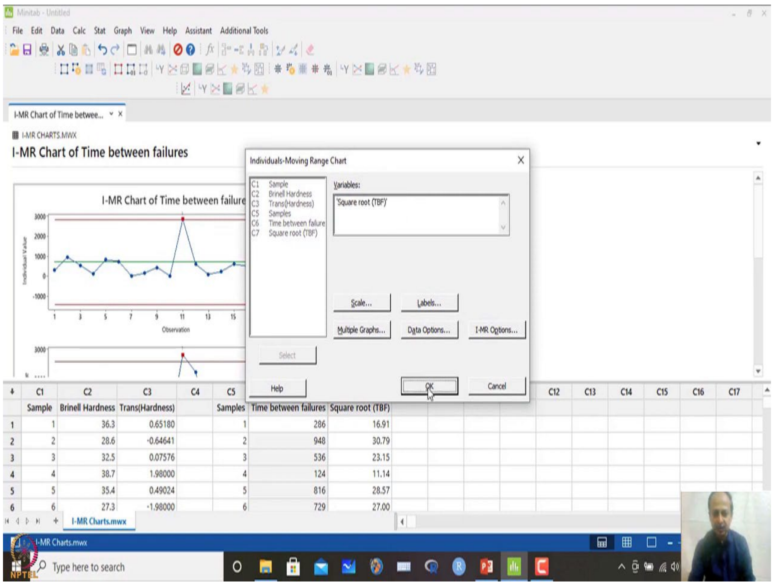

So, here it is reflecting that without transformation some values are going outside and but after transformation something extraordinary happened over here.

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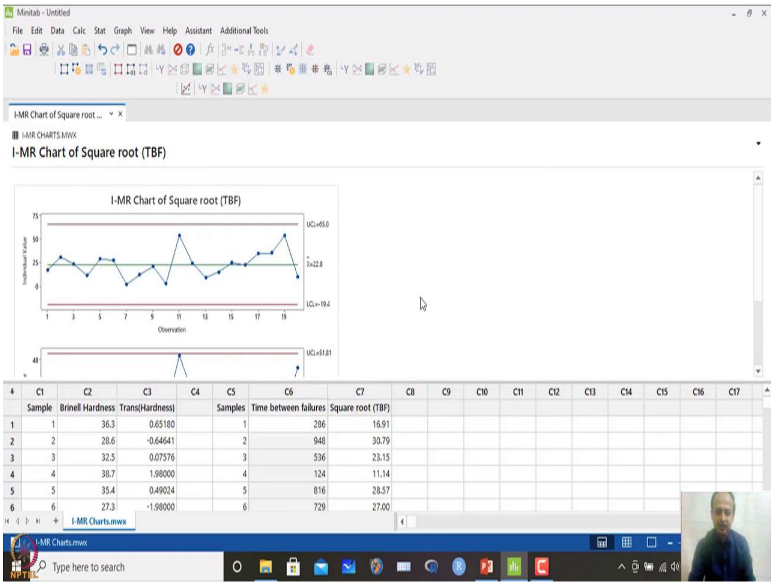

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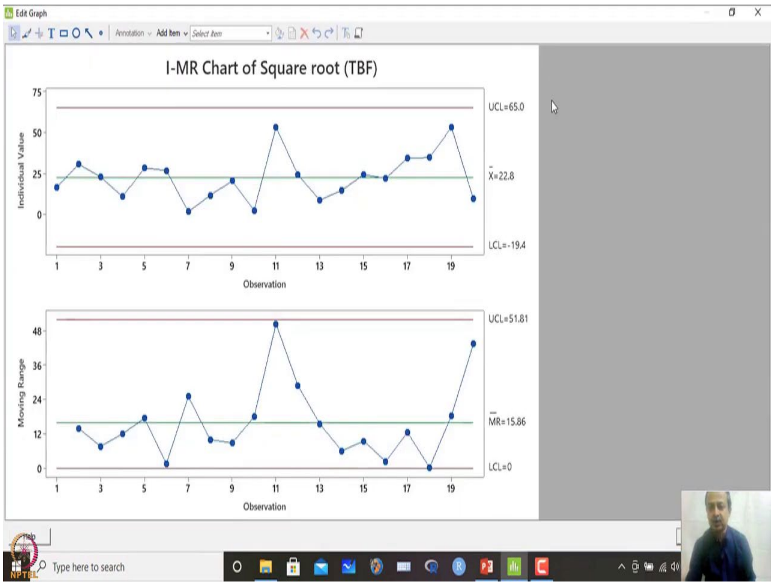

So, in this case what is happening is that all the data points are falling within the control limit line. Initially, without transformation we are seeing that abnormal scenarios and there is a there are observations which is going outside, but when I have converted the data and it follows normal distribution what happened is that I saw and I plotted the I-MR chart then everything is in control. So, if we have taken the action based on the assuming that is normal, we may have over adjusted the process because process has not gone outside. But because the data is non-normal it is showing outside of the control limit line. But when I convert the data and converted data is plotted I-MR chart what happens is that everything is fine. So, I do not need to adjust the process.

So, there is one quick observation, what we can make out of this is that first we have to see the assumptions of normality is everything is fine, no problem. And if it is not there let us convert the data, so that it is normal. Some transformation we will apply over here, linear or non-linear transformation or log transformation whatever transformation is required. And that we will understand what are the different types of transformation that can help in this regard.

And whenever I have done that and then plot the I-MR or any types of control chart and then we will see that whether actually we need to take any action or we do not need to take any actions. So, we have applied such kind of method. So, this is an observation and you will find in any books that they will refer that. You can convert the data into normal, so that it follows normality, and then you apply the control limits or apply the control chart techniques like that on the converted data like that, ok.

So, these are the main charts which are generally used in manufacturing, any other processes. So, in that case, and the magnitude of shift is quite large and in that case we prefer to use this one. But if the magnitude of shift is less we have other types of control chart to monitor that one. So, those things, those I will not discuss over here like some chart exponentially waited moving average chart. So, some charts are available even their robust against failure of normality assumptions like that.

So, there researchers has proved that they are quit efficient like that but this that will not be our topic of discussion over here. So, we will stop over for the variable type of control chart we have only discussed X bar R, X bar S, what scenario we will apply, then individual moving range when I have single observations, and then how to deal with that.

And then we will discuss about another types of control chart which is attribute control chart and that will be our discussion topic now onwards, ok.

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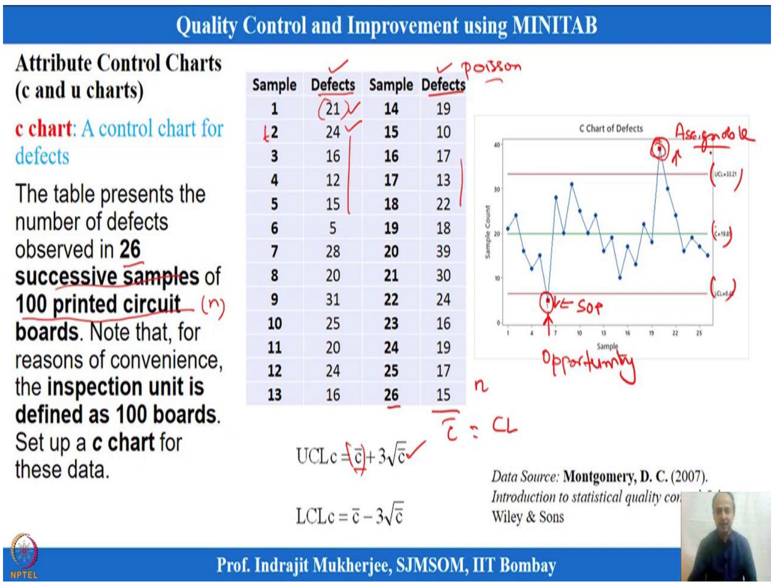

So, I-MR chart; and now we will discuss about an attribute control chart. So, what is attribute control chart? Here generally what we do is that this is the initial phase of analysis when we implement preventive measures. What we do is that in a process we try to figure out or in assembly lines how many defects are coming in the process like that, ok. So, out of 100 assemblies that is inspected and within that how many defects or defective items is coming in the process.

So, defects arrival of the defects, so if you have done statistical course you know that arrival of defect follows Poisson distribution. And this chart is approximation of Poisson distribution and based on that ± 3 standardization was implemented and the formula is given over here to calculate the upper limit line and lower limit line.

So, this dataset what you are seeing is that at a given time point we are taking 100 printer circuit board and within the circuit board how many defects was observed like that. So, this term you have to know this is for defects only, ok. These are the defects and not defectives over here.

So, when it is defects it follows a Poisson distribution and arrival of the defect is basically Poisson that is the underlying assumption. And then what we do is that we try to build the control charts for that and this is the rate of defects arrivals like that.

So, this is $\bar{c}$ or average of defects that you count over here. So, summation of this and summation of this and divided by total number of observation that is 26 that will give me $\bar{c}$ over here. And $+3\sigma$ of this process Poisson process, so that is also mean and you understand that mean and variance of the Poisson distribution is same. So, in that case those are same. So, this is $\bar{c}+3\sigma$ and this $\bar{c}$ can be calculated easily.

So, here there is no need of putting any n over here. So, because n is constant over here. So, we do not need to take care of that. So, over here every time I have taken 26 successive samples over here of 100 printed circuit board. So, this is the sample size n that we are taking at a given time point like that. So, at t_1 time point 100 was inspected out of that 21 was defective. Then again, 100 was inspected at time point t_2 and what has come out is 24 defects over here.

Then, average of the defects will give me $\bar{c}$ observation over here. So, summation of this plus this will give me $\bar{c}$ observation. So, we will get $\bar{c}$ observation over here. And then I can calculate the upper control limit line and this will be the central line over here, $\bar{c}$ will be the central line.

So, $\bar{c}$ is the central line, upper control limit line and lower control limit line that will give me some idea that whether the defects had shoot out in certain scenarios like that. Here you can see that when we plotted the data in control charts, one has gone outside the upper control limit line and one has gone below the lower control limit line over here.

Now, if it goes below the lower control limit line you see number of defects has gone down drastically over here. So, in this case what happens is that I can take this as an opportunity. So, this is an opportunity. So, I need to understand why defects has gone down over here. So, sometimes what happens, operator may not be able to differentiate between defects. So, if that is the scenario then the way we are measuring defects is something is going wrong, so we need to correct that one.

But otherwise if the process everything is fine same inspector, same personnel is doing the inspection over here and it has gone down, so what was the scenario in the process at the time point? So, what is the process condition at the time point? That can be taken as an opportunity and that may be the optimal scenario what we will try to implement in every time we start the process.

So, this is not an abnormal situation for me until and unless there is evidence that the reading that was taken was quite incorrect. So, then we need to correct that one, otherwise this gives me an opportunity to develop the standard outputting practice like that which may be the optimal one and to arrive at the optimal scenarios like that.

But if this is shooting out over here and defects are very high over here and in that case this is a assignable cause this is a assignable cause, and we need to correct this one what has happened over here we need to cross check and try to see that one. So, this data I have in MINITAB and we will try to plot and see c charting of that.

So, here you try to understand that here n is not used to calculate the upper limit line lower limit line, n is not required because this is constant and because we are using Poisson assumption in that case n is not required. It will be required at certain other instances like that, ok.

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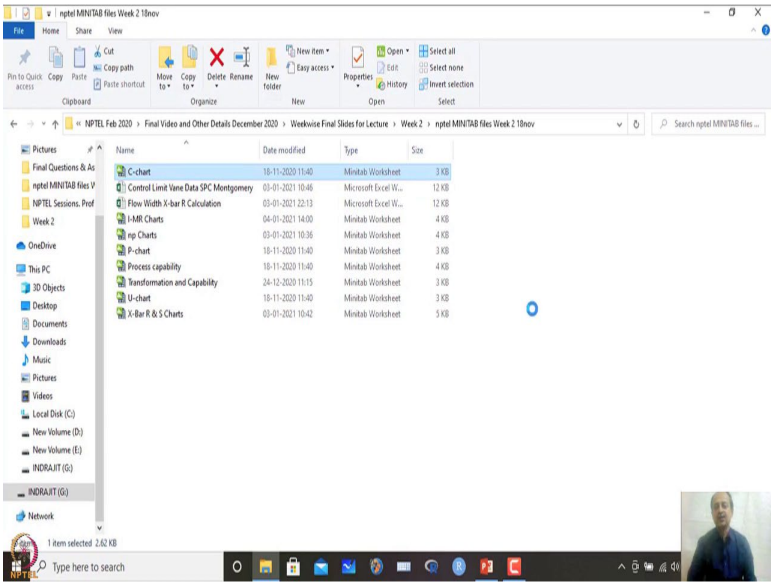

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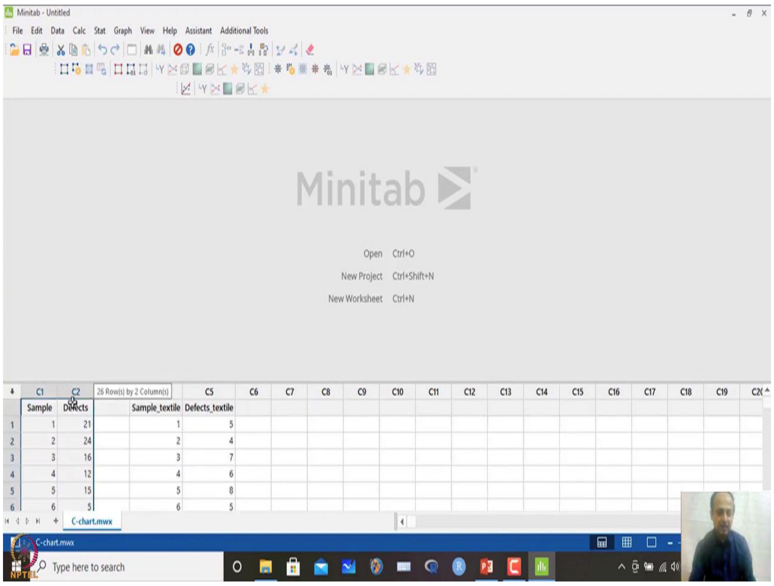

So, I will go to the MINITAB option. And then we go to another file where we have C charts observations that is already given there, and we will try to illustrate C over here. So, in this case 21 defects. So, let me go back to the, is it the same example yeah, 26 observations are there and we have over here 26 observation.

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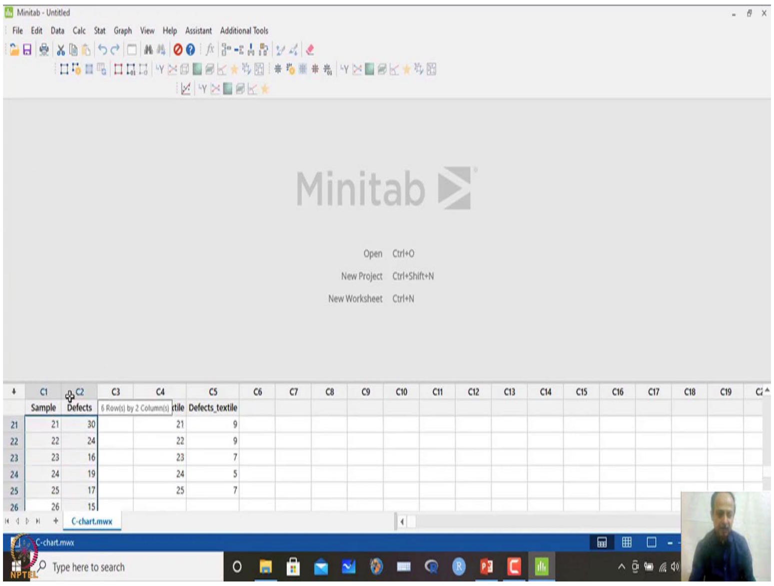

So, this is the dataset that we have taken over here and this is taken from Montgomery's book and then we try to see whether it is under control or there is some abnormalities in the process. So, I go to stat and we have seen the on. So, then what we do is that control charge, I go to control chart attribute chart and then there are options of C at the bottom over here you will find. So, when I click C over here it will ask which is the variable, I will say defect is the what I want to monitor.

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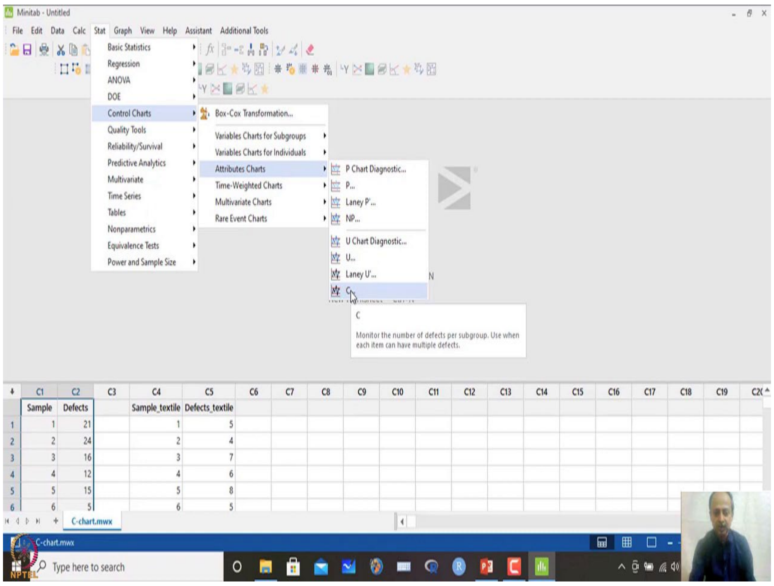

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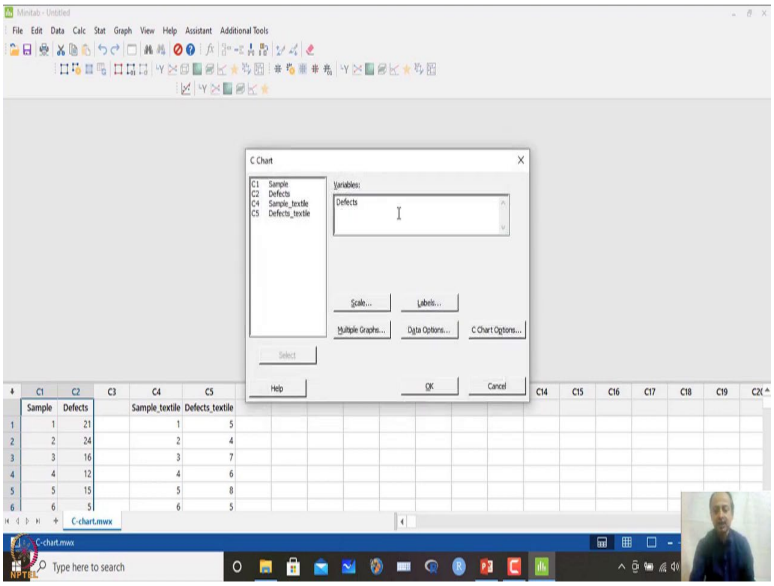

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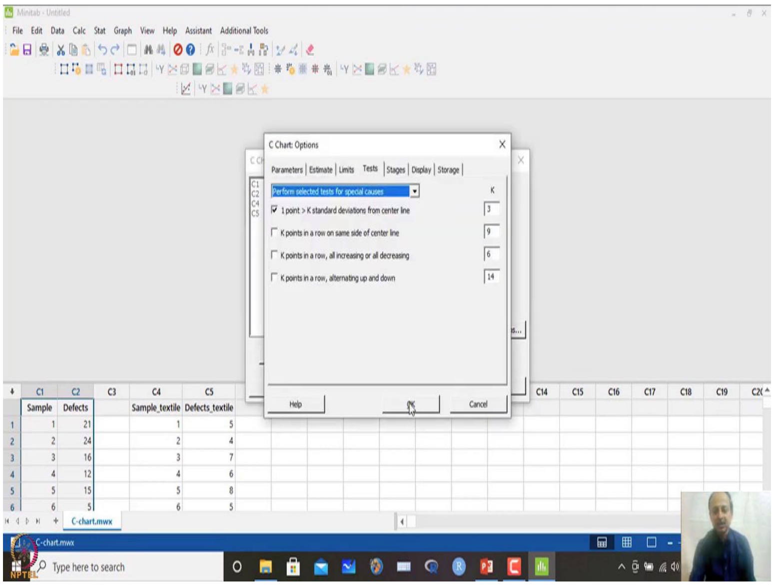

And C chart option again test will be one point will going outside. So, other things you can just explore. So, I am just showing that. Only I am interested in defects. And in this case defects follows Poisson and that is the based on that control limits will be developed, I click ok, and what will happen is that I will get a chart like this.

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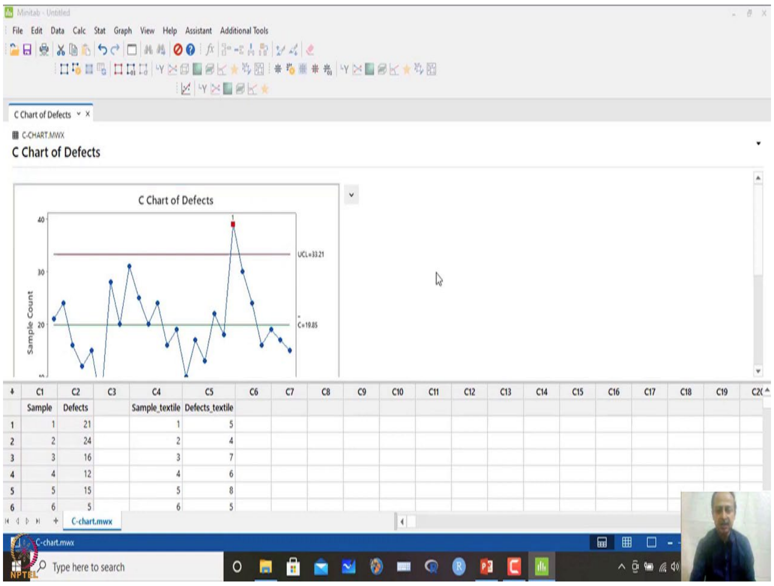

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So, same chart what we have generated and shown in the power point this is same. 2 points have gone outside over here. So, if you have to implement that you want to build the natural limit lines like that and give to the process for next, so what we will do is that we will eliminate this point, we will eliminate this point and reevaluate whether all the points are within the control limit line like that.

So, if that is so then those other limit lines we will use. So, because of this point this limit line is changing over here. So, if I eliminate this one limit line will automatically change again. So, then again I will see whether all the points are I will review this point on this point over here, I want to capture the natural variation to build up the control limit line and that will be used in actual manufacturing and that I will hand over to the process personal ways monitoring or owner of the process basically.

So, in that case what we will use that we will eliminate and recalculate. So, that was the trail control limit line concept that we have used. Over here two points are going outside that we have noted down over here, ok. So, n is not changing over here. So, in this case similarly defects and sample in textile this was another example.

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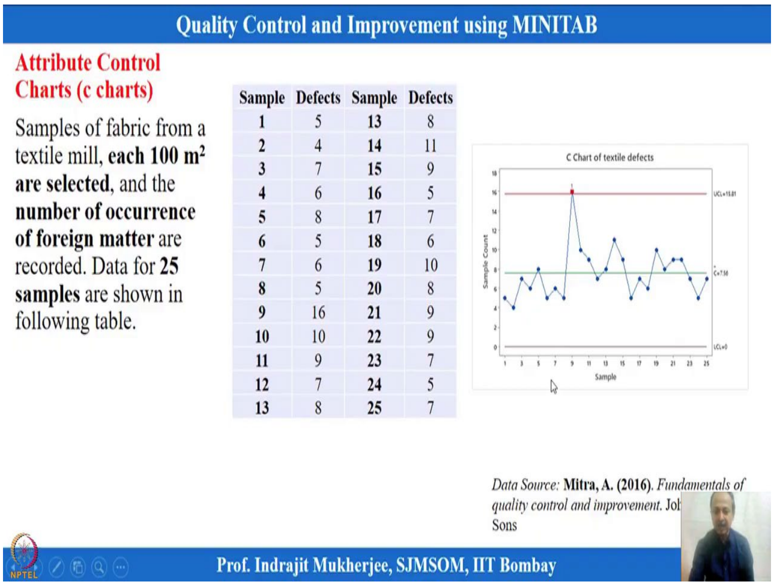

So, maybe. So, this is the second observation over here and also here also you see 25 samples and there are every time I have number of occurrence of foreign particles are recorded, 25 samples, each is 100 meter square as selected. So, this is the units that we you are selecting over here, 100 meters square like that. So, in this case also we are counting the defects only and units remain same, n remain same, so in this case textile we can again draw this.

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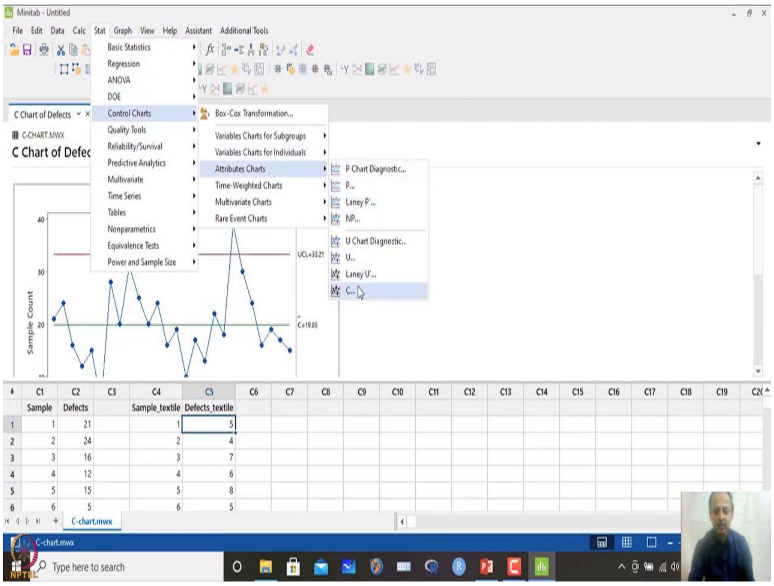

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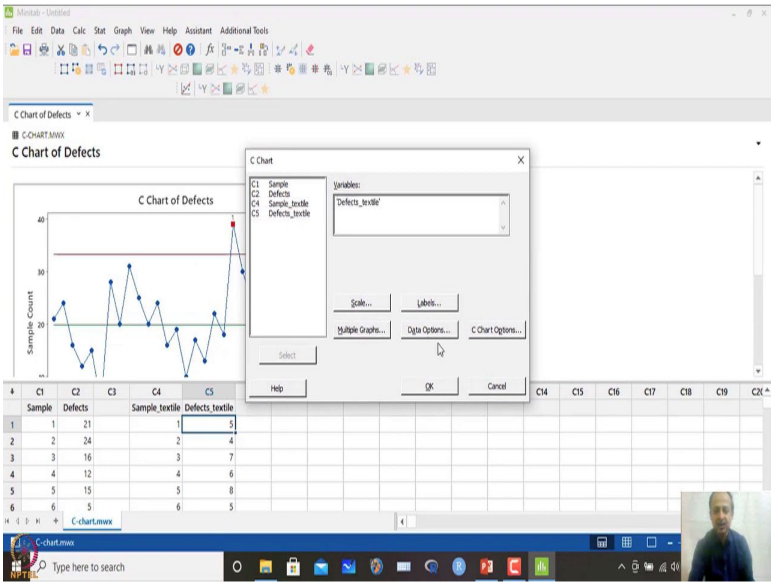

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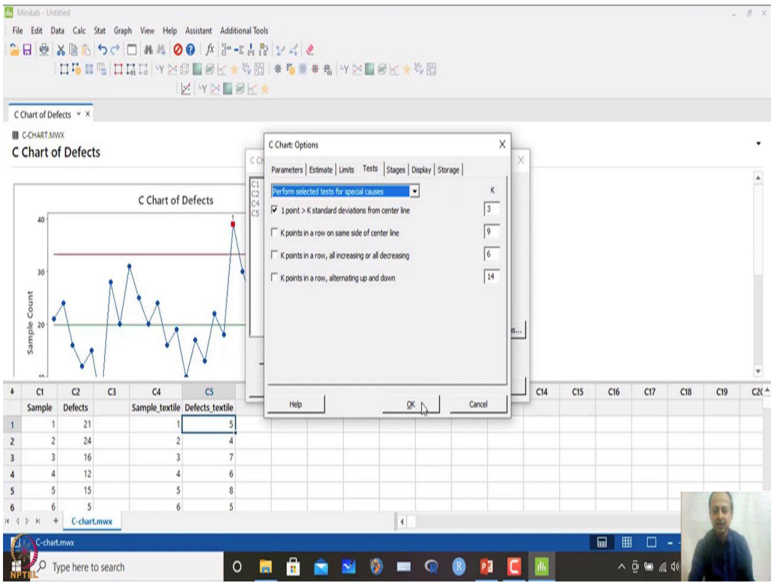

So, what I have done is a control chart, attribute control chart. And I go to C type of control chart, because then defect textile over here and options remain same. So, I am not changing the options over here and I click ok. And what I get is that one point is only falling outside the limit line.

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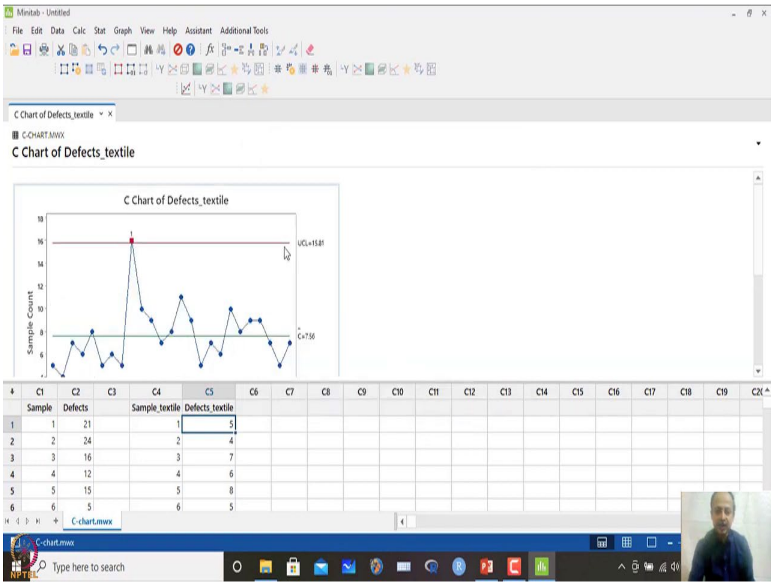

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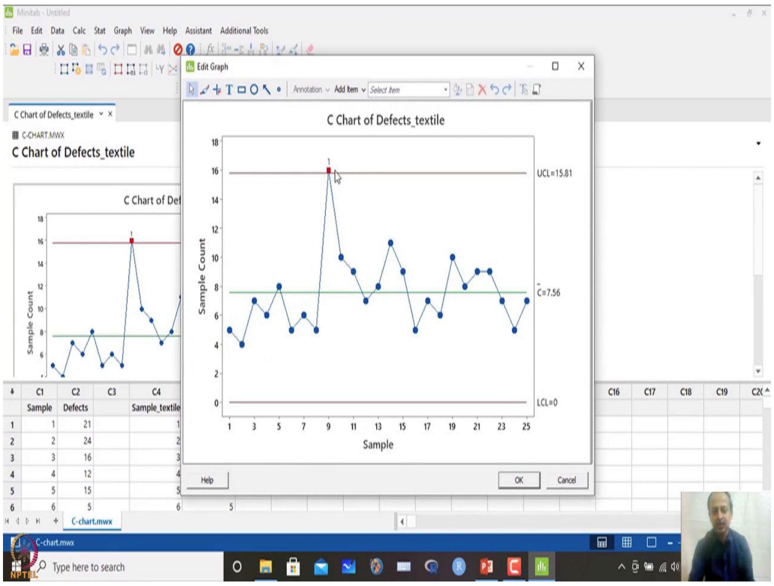

So, that is an abnormal scenario. So, we need to take care of that and try to figure out what has gone wrong in this case. And always you see these lines are joint over here that does not mean that this is a continuous over here, just for visualization of this, all are discrete points over here.

Because in control chart what we do is that we do not continuously monitor the process like that. We take some observation and then what we do is that we try to go to the process again after 30 minutes, 25 minutes or 15 minutes based on the frequency what is decided and that depends on the number of samples to be taken and the total production lot that is happening. And based on that these are all discreet time points we are making an observation I am just plotting that one.

But for visual impacts what we are doing we are joining the points to see any trend is appearing or not like that. Their different trends like cyclic behavior is there or not, or continuous increasing trend of defect is happening or not, and all these things also indicate some abnormities.

There are different trends that can be trend analysis also can done on this control charts and that can be seen also whether there any have any other different types of abnormalities there or not. Even if it is within the control limit line we do those analysis. So, this is C type of control chart when we are talking about defects, when we are talking about defects C type of control chart and the n is fixed.

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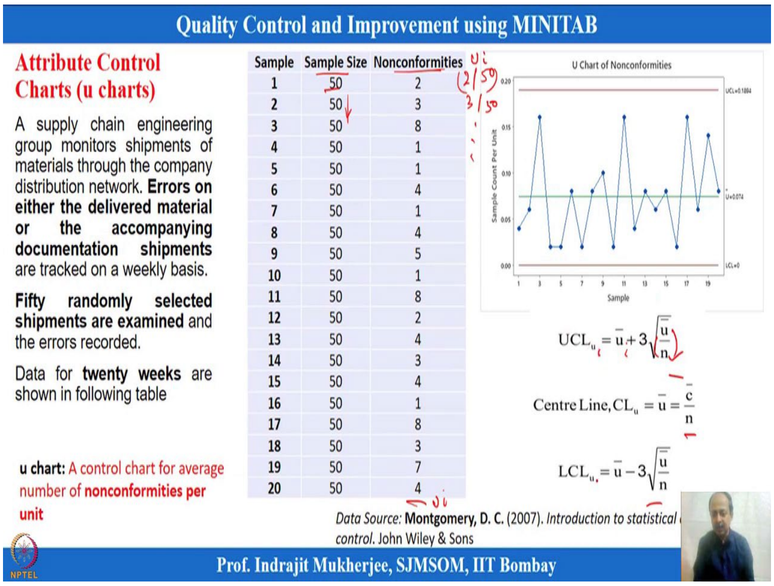

Scenario can be the defects, number of defects may not be fixed what you are seeing over here. Every time I go to the process, and in that case what will happen is that sample size can also change like that. What you see over here is fixed that is sample size, but this can always change, this sample size can always change like that.

Here it is 50 that you can see it is non-conformities over here is given. So, sometimes somebody may be interested that I am going to plot that per unit with respect to this is the sample number over here I am checking. So, how many per unit. So, in this case $^{2}/_{50}$ like these, this is $^{3}/_{50}$ like this. So, I will get some u_i measures over here.

So, then what I can do is that I will monitor this because that makes more sense per unit sometimes what happens that gives more impression. So, sometimes somebody also prefers to use per unit. So, in that case it is not much difficult. Formula remains same, so in this case only n comes in picture.

So, in this case number of observations as a unit what I am taking over here that will that will be counted over here. So, average per unit will be counted over here. So, this is the formulation that you see for U chart which is known as U chart. But mostly used when the sample size changes.

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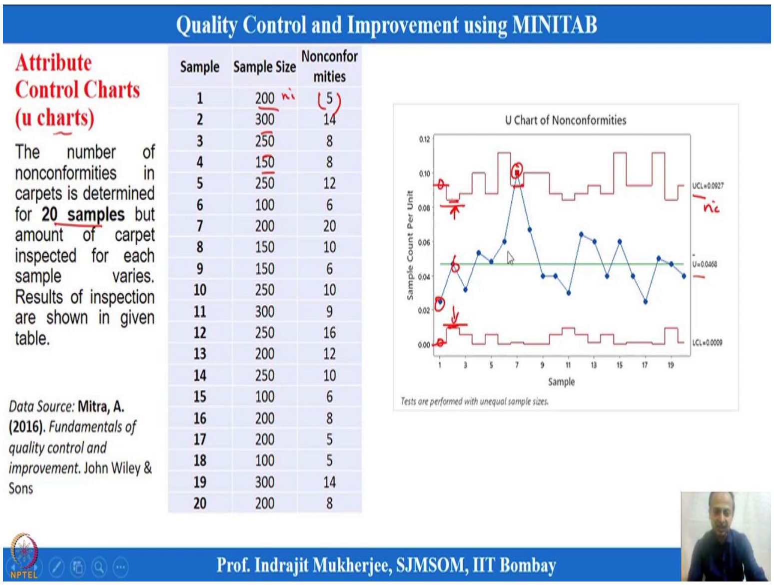

Whenever sample size changes like this example, where 20 samples are taken and every time sample size is changing over here, in this type of scenarios the formulation that you have seen, so here it will be u_i .

So, every time the upper control limit line and the lower control limit line will keep on changing over here, ok. So, what will happen is that we will have $\bar{u}$ as the central line over here. So, every time point, when you go this is the first point and the observation is this is the upper limit line and this is the lower limit line like that, ok. So, then this is the point over here, this is the upper control limit line and this is the lower control limit line, ok. So, limit position is changing basically.

Earlier it was this like this for the first point, second point it is from here to this point, right that, ok. So, this is the, this is the area where we need to concentrate and see whether the point is going outside or not. Here you see the limit line is somewhere below over here, point has gone outside. So, this is a variable control limit line that can be generated.

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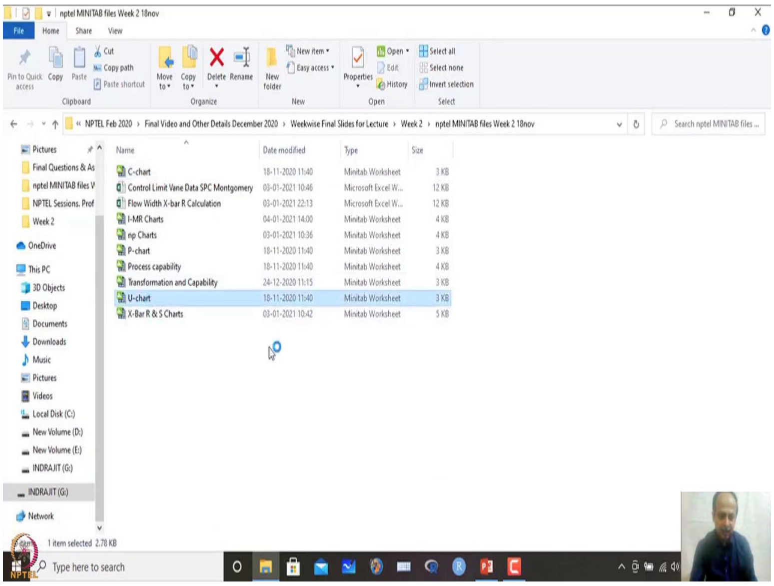

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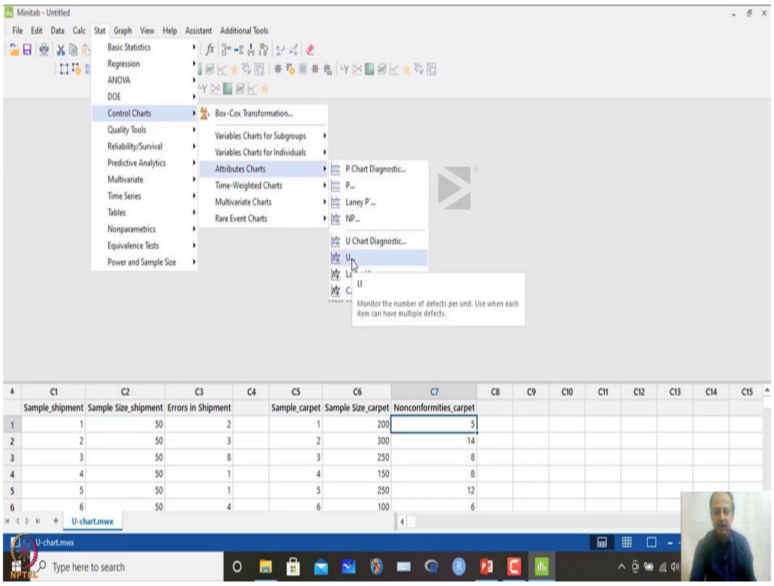

And we can see this generation when we are using MINITAB, same data points what we want to use over here, and maybe U chart that dataset may be over here with variable sample size like that. And this is the sample that is C5 to C7 and sample size is changing and I am monitoring non-conformities and that is defects let us say. So, in this case stat if I go and quality tools and I go to this control chart and I go to attribute chart and you go to U charts like that one option is there.

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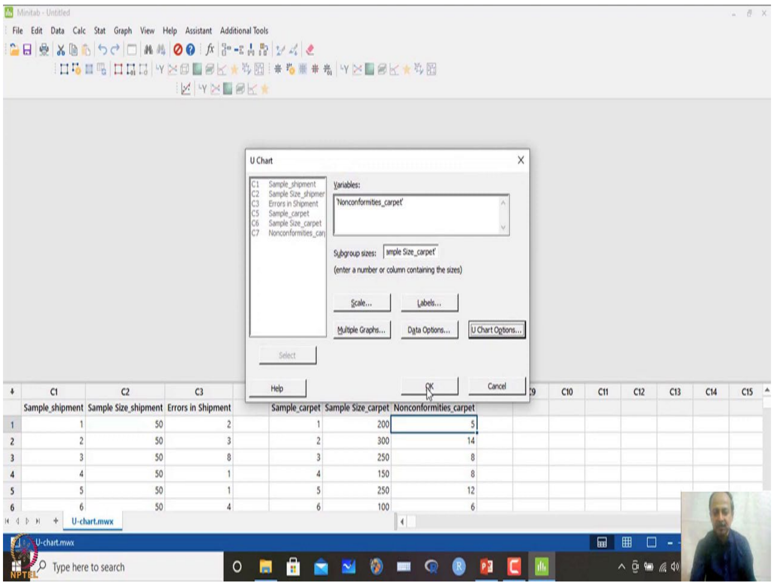

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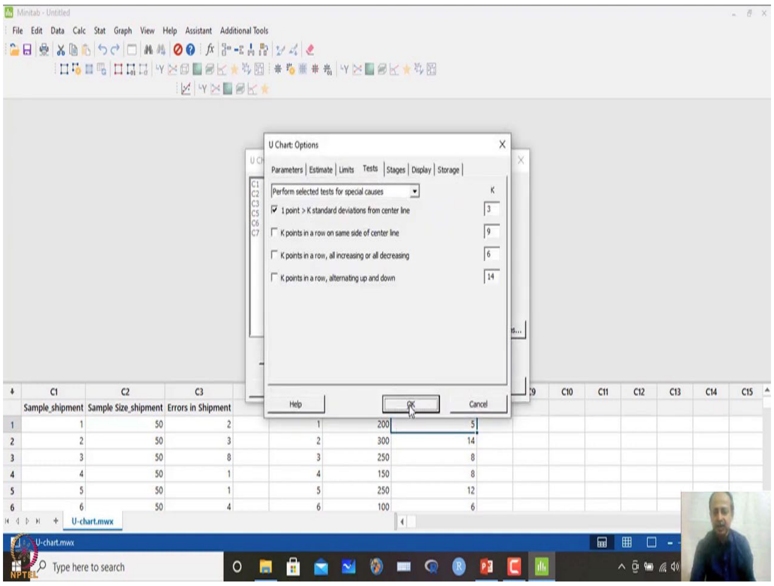

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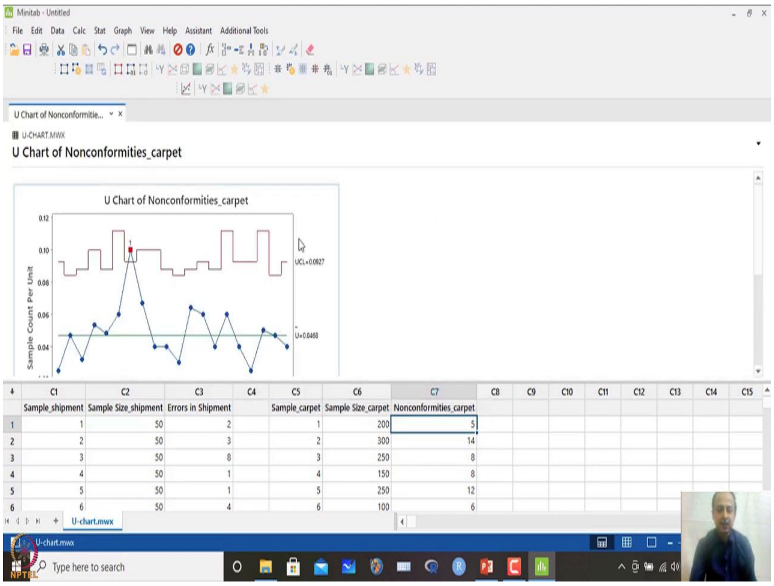

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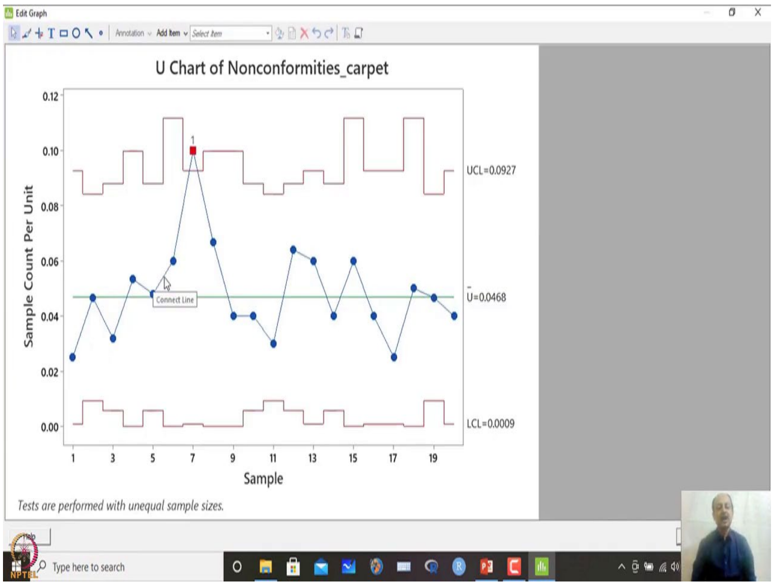

And then, you mention that variable is non-conformities and subgroup size is given in sample size carpets like that. So, if you give this one; and options over here I am doing the testing of one point going outside, so that is the only condition I am taking. And when I click ok over here what you see that this is the same graph what I have just plot it like that. So, every time the control limit lines are changing and because of sample size is changing over here and we have different control limit line.

But operator does not prefer to do like that way because every time they have to change the control limit and check that one. So, sometimes what happens is that we take average of this control limits that you are seeing. So, some average line will be plotted over here. So, one straight line may be plotted.

So, if I go to this one sometimes what operator does for their benefit or something like that, even the quality personnel can do is that they can take an average line over here. Similarly, they can take an average line over here based on the data and information, so that it is easy for the operators to see also.

So, there will be mistakes for this, but sometimes maybe the maximum of these points maybe that will be the upper limit over here and this is the lower limit I will follow like that. So, we can implement those types of; otherwise the best one is to have variable control make line like that variable control limit lines, ok.

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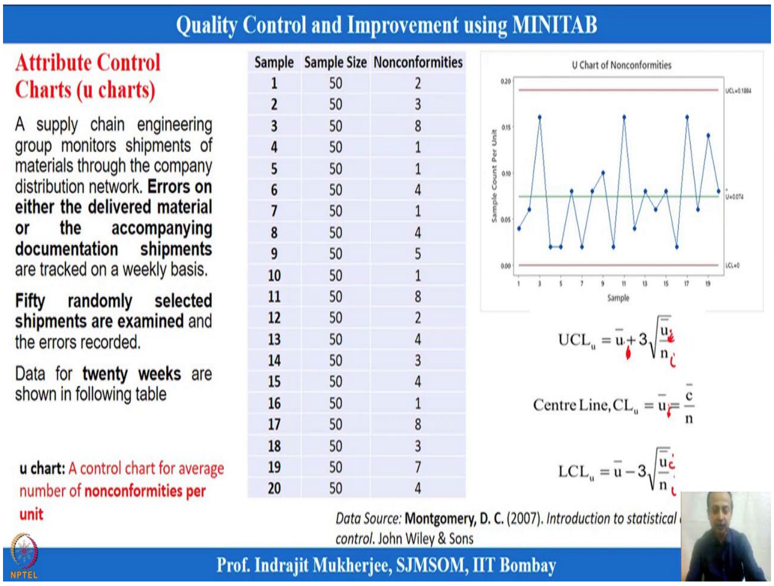

Sample	Sample Size	Nonconformities
1	50	2
2	50	3
3	50	8
4	50	1
5	50	1
6	50	4
7	50	1
8	50	4
9	50	5
10	50	1
11	50	8
12	50	2
13	50	4
14	50	3
15	50	4
16	50	1
17	50	8
18	50	3
19	50	7
20	50	4

So, limit lines and calculation of this is given by this one. So, this will be u_i over here, this will be central line. So, this this is taken as overall average over here.

So, we will have different upper limit line and lower limit line for this. So, this type of U charts is generally used like in software industries, lines of codes and number of defects that comes like that, at different phases like that. So, I have different phases and each phases we have this many lines of codes and out of that how many defects like that. So, it will have variable number of lines of codes and accordingly how many errors that is appearing like that.

So, we want to represent that one in control chart. So, they use generally prefer to use U type of control chart in their quality control division like that. So, they prefer to use U because sample size changes or the unit changes basically, unit of inspection changes. So, that is why they uses this type of control charts like this.

So, we will continue from here attribute type of chart. So, we have done with defect kinds of chart when there is defects is follows Poisson, and if number of sample is constant or in an unit, unit is fixed like that in that case we use C type of control chart.

And in case the sample size is varying mostly we prefer to use U type of control charts. So, we will continue from here. And we will start with other types of attribute control chart like P chart and NP charts which are also very prominent and used in many scenarios like that. So, we will discuss that in our next session.

Thank you for listening.