Managerial Economics
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Lecture - 9

Welcome to the fifth session of managerial economics; we were discussing the first module of managerial economics; that is, introduction to managerial economics. And in this topic, if you remember in the last session, we discussed about the relationship between economic variable, we termed them in term of linear, non-linear. Then we discussed about that, how these variables are related and what is the method to capture the relationship? So, one was, what we discussed in the last class is slope; slope is basically, it captures the change in between the dependent variable and the independent variable. But when the change in the independent and the dependent variable is very small; in that case, the general method to calculate slope or just finding the relationship through the slope sometimes does not serve the purpose. And that is the reason we introduce the concept of differentiation. And differentiation is a method, differentiation is a approach through which we can calculate the or capture the change in the dependent variable, due to change in the independent variable.

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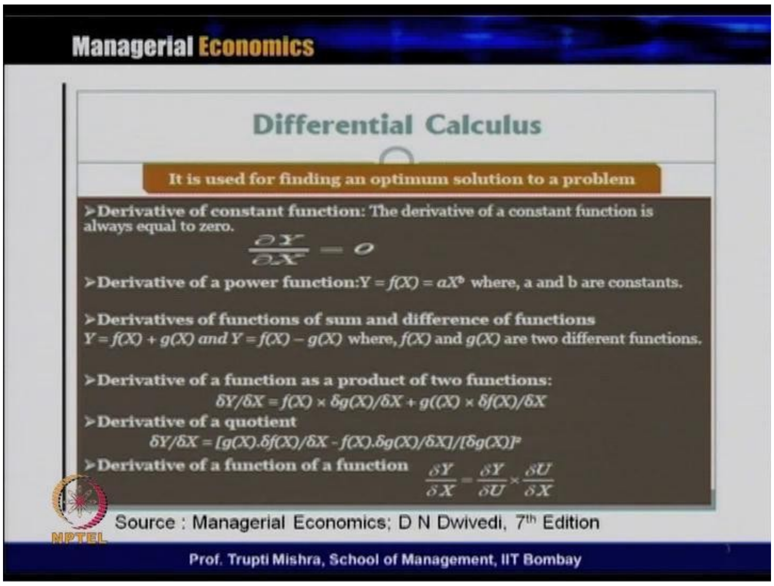

So, in the last class, we have taken just a general functional form, and identified the differential calculus method. In this class, what we will do? We will take some different kind of function like constant function, power function, derivative of function of sums and differences of function. Derivative of function is a product of two function, derivative of a quotient, and derivative of a function of a function, and also we will take a function, where there are multi variables are there. So, we will just check one by one how the differentiation is generally used, or how that through differentiation, how we calculate the relationship between the two different variable; independent and dependent variable, in the different kind of functional form.

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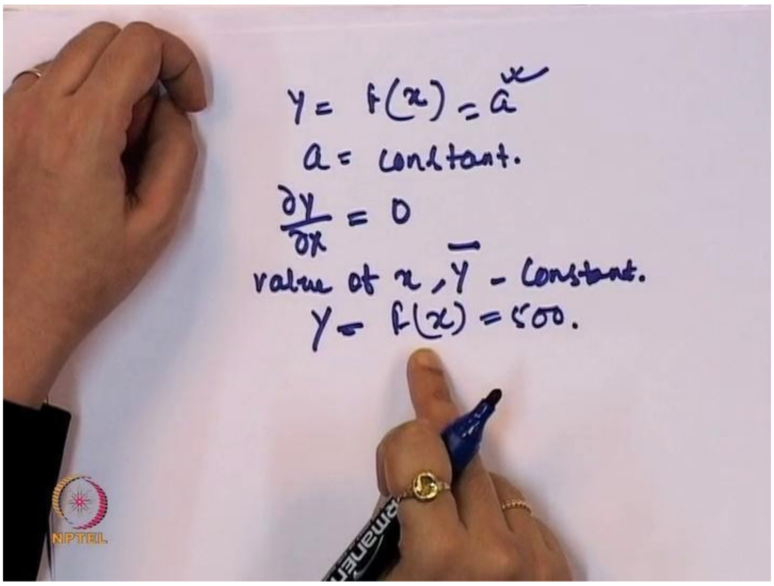

So, we will start with a constant function, and where if we look at functional form is, y is equal to it is a function of x. So, y is a function of x; and which is equal to a. Now what is a over here? a is the constant. Now, if we will take a first order derivative of this functional form. In this case, what is the first order derivative? We have to take the derivative with respect to x, and which will come as 0. Why it will come as 0, because it is a function of x and that is in the form of a constant. And when you are taking the first order derivative with respect to a constant, the derivative the value of the derivative will be equal to 0. So, whatever the value of x, so whatever the value of x, y remain constant here. There is no change in the y, because y is represented in term of a function, and function is a constant over here. So, may be if you modify this weight and if you put a value, suppose you take a functional form; y is a function of x and which is equal to 500.

This is the value of the intercept; this is not the value of the slope here. So, in this case, whatever may be the value of x, there is no change in the y, y remains constant, because this is a derivative of a constant function.

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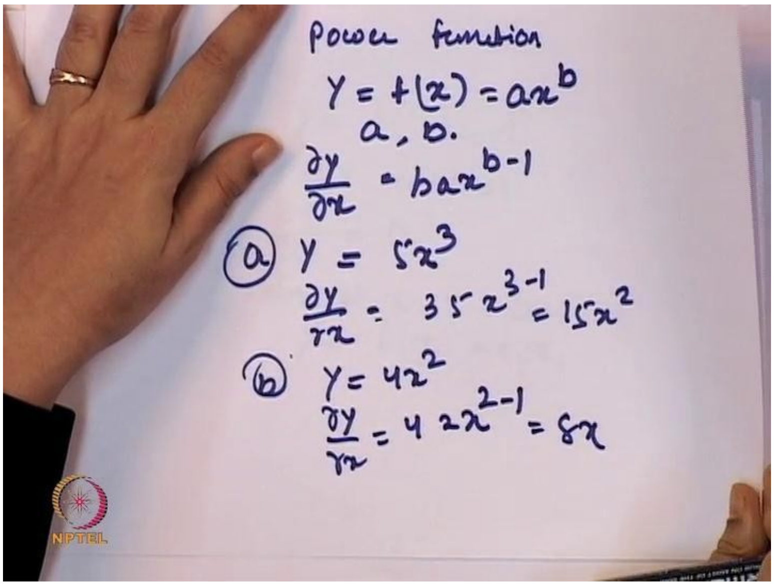

Now, we will discuss the second kind the derivative of a power function. So, here the functional form has a power on it. Suppose here again y is the dependent variable, x is the independent variable. It is a y is a function of x, and which may be a x to the power b. Now here there are two constant; one is a and another is b. So, y is a function of x, which is a x to the power b, and in this functional form we are getting two points; one is a and second one is b. Now if you take a derivative of this, how this will become, what is the outcome over here, del y with respect to del x; that will be b a x b minus 1. Now, let us give a number to this functional form. Suppose take a functional form where y is equal to 5 x cube. So, x to the power cube. So, in this case, y is a function of x and the x has also a power on it.

So, now what is the derivative or how we can check the relationship, del y to the power del y with respect to del x, so that will come as the 3 5 x 3 minus 1. So, this is 15 x square. Similarly, you take one more functional form, where y is equal to 4 x square. Now how we will find the differentiation over here. So, del y by del x is equal to 4, then 2 x, 2 minus 1, so this is 8 x. Similarly, suppose we take one more functional form, which has also a power on it.

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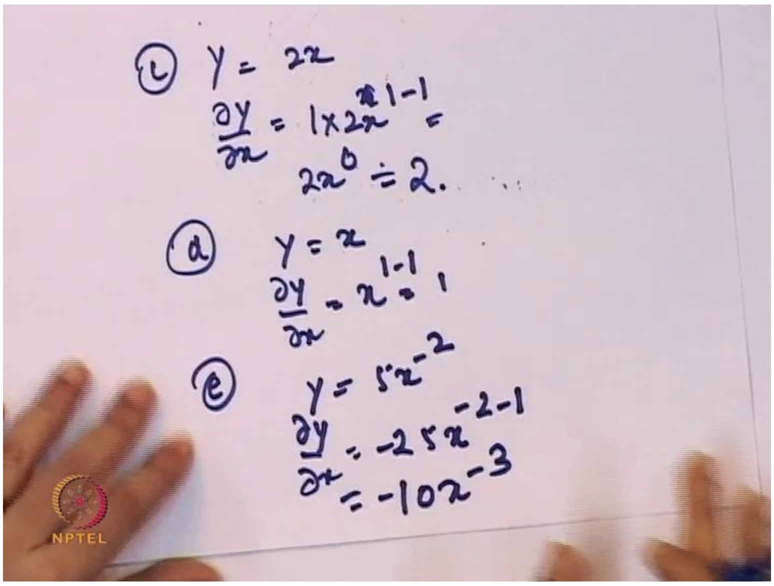

So, if you look at, suppose we take y is equal to 2 x, x to the power 1 over here. Now if you take del y by del x, then this is, this comes to 1 multiplied by 2 x then 1 minus 1, so 2 x to the power 0 which is equal to 2. Suppose, take one more functional form in this category of derivatives; like this is a derivative of a power function. So, let us take y is equal to x. Now what is del y by del x, del y by del x is equal to x 1 minus 1 which is equal to 1. Similarly, if you take one more functional form, which has a negative power y is equal to 5 x minus 2. So, this is del y by del x minus 2 5 x minus 2 minus 1, and this comes to minus 10 x minus 3. So, this is how we solve when we get a power function, and what is the significance of this power function? The functional form has a power, and it can take any value that is from less than 0 or that may be the greater than 0. Now, we will discuss about the third category of the derivative; that is derivative of a function of sum differences of functions. Sometimes if you will find it is not only a single variable, there is also a summation added to it.

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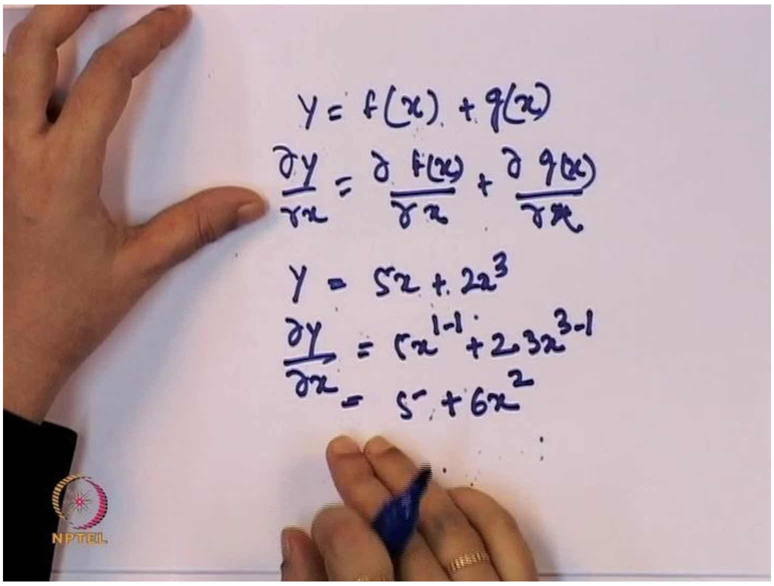

So, if you take a example where y is a function of x and also it is a g is a function of x. So, y is dependent on this x function of x and also function of this x. So, in this case how we take the derivative, in this case the derivative is del y by del x is equal to del f x with respect to del x, plus del g x with respect to del x. Similarly, now if you give a numerical term to this, suppose we take a functional form y is equal to 5 x plus 2 x cube. So, now taking the first order derivative del y by del x is equal to 5 x 1 minus 1, plus 2 multiplied by 3 x 3 minus 1. So, this comes to 5 plus 6 x square. Now what happens if it is not a case of addition, if there is a difference, it is not a sum rather it is a differences of function. Now, we will take a another functional form, in order to show that when there is a subtraction, or when the y is dependent on x and the functional form has a subtractions of, the functional form has a differences between two variables, how to get the derivatives of this.

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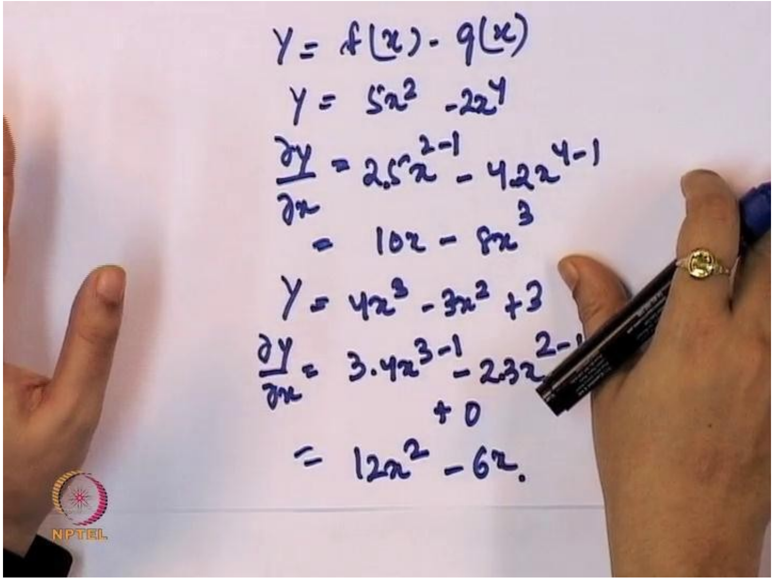

So, let us take here that y is a function of f x minus g x. Let us give a numerical value to this, so y is equal to 5 x square minus 2 x 4. Taking the first order derivative del y by del x is equal to 2, 5 x 2 minus 1, minus 4 2 x 4 minus 1. So, this comes to 10 x minus 8 x cube. So, it is like, if the y is dependent on a function which has the addition or which has the subtraction, in that case we have to take the partial derivative with respect to both the variable. So, in this case like it is f x and in the g x. Now, suppose apart from this two variables, let us add a constant also in the functional form. Let us take a functional form, where y is equal to 4 x cube minus 3 x square plus 3. So, now how we will get the derivatives over here, so del y by del x is equal to 3 4 x 3 minus 1 minus 2 3 x 2 minus 1 plus 0, because the first order derivative of a constant would be always equal to 0.

So, this comes to 12 x square minus 6 x. So, in case of a derivative, where the function is of sums or the differences, we get the functional form in term of the negative value; like there is a differences between two variable or there is a positive value, there is a summation between this two variables. Now, let us check, what is the next kind of a function? So, this is a derivative of a function as a product of two functions. So, in the last case we took care of summation, we took care of the subtraction. Now we will say that, when the derivative of a function, where the function is as a product of two function.

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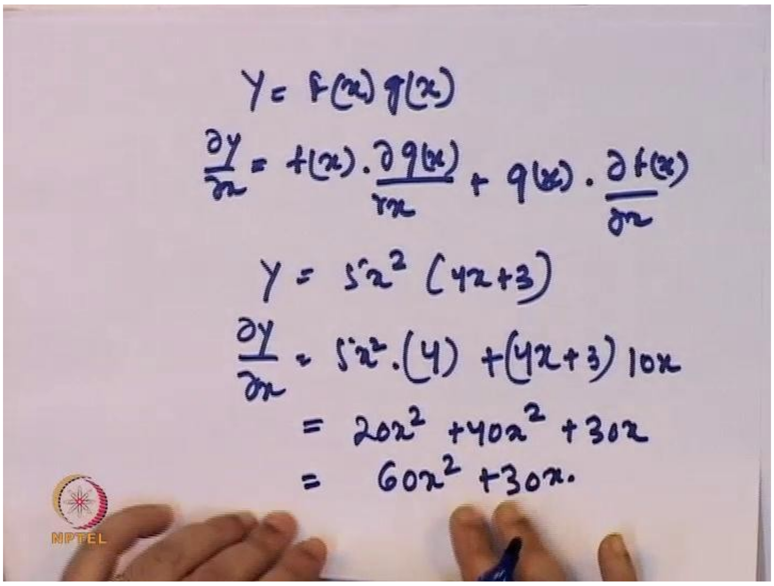

So, in this case how the functional form will be, the functional form will be Y is function of x and g x. So, the functional form is, product of two function; that is f x and g x. Now, how to take the derivative over here? Here, we take the derivative keeping others as constant, and in the second part we take the derivative of the second part keeping the first part as the constant. So, in this case, this is f x d g x, d x plus g x d f x, d x. So, now let us take a numerical function to get more clarity. Suppose we say that y is equal to 5 x square 4 x plus 3. So, here y is dependent on two function; one is 5 x square second one is 4 x plus 3. So, this is a case where the function is a function of 2 other function. So, in this case how to solve, how to find out the derivatives. So, in this case 5 x square plus multiplied by 4; that is the derivative of del 4 x plus 3, plus 4 x plus 3 as constant and taking the derivative of 5 x square.

So, if you're taking the derivative of 5 x square, how much it will come, it will come as 10 x. In the first case we have to keep 5 x square as the constant, and we have to take the derivative of 4 x plus 3, so which will come as 4. So, the first case f x is constant, the derivative is with respect to g x. Second the g x is constant, the derivative is with respect to f x. So, in this case, if you again simplify this, then this comes to 20 x square, plus 40 x square, plus 30 x, so which come to 60 x square plus 30 x. Now let us take another functional form and where the function is again as a function of two other functions. So, it is a product of two other function serve as a functional form for the independent variable, dependent variable where the value is defined by the independent variable.

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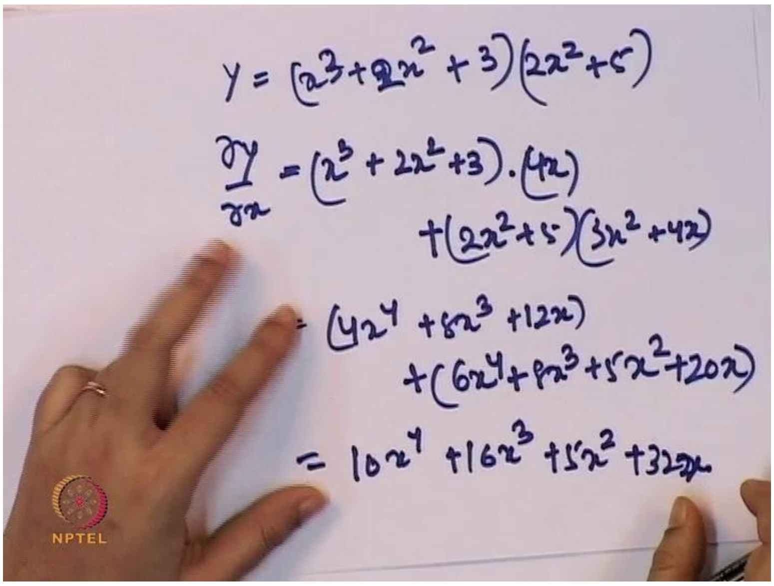

Now, suppose take a numerical example, where y is a function of x square plus 2 x square, sorry, x cube plus 2 x square plus 3. So, this is one function, and the other function is 2 x square plus 5. So, there are two functions over here. So, taking the first order derivative del y by del x is equal to. So, in this case the first one will be constant now; x cube plus 2 x square plus 3, multiplied by the derivative of the second function. So, the derivative of the second function is 4 x. Similarly, now the second term 2 x square plus 5 will be constant, and we have to take the derivative of the first function; that is 3 x square plus 4 x. So, simplifying this we get 4 x 4, plus 8 x cube, plus 12 x for the first one.

For second one, it is 6 x 4 plus 8 x cube, plus 5 x square, plus 20 x; that comes to 10 x 4 plus 16 x cube, plus 5 x square plus 32 x. So, when the functional form is a product of two function, in that case basically we take the derivatives by keeping the first factor, first function as constant and taking the derivative of the second function. And in the second case we keep the second function as the constant and we take the derivative of the first function, when the functional form is product of two functions. Next we will check how we generally solve or how generally find the derivative of a quotient.

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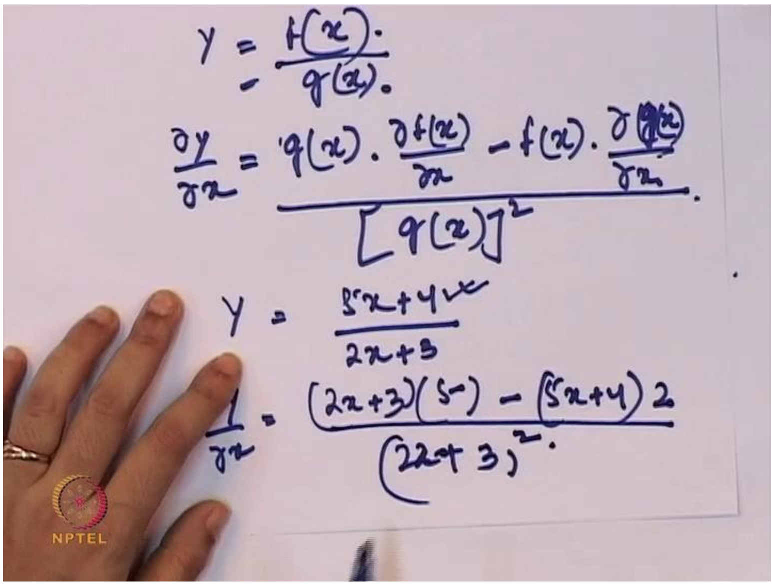

So, here the functional form is y is a function of x, so let us write a functional form. This is a general functional form, but when you take that, this is derivative of a quotient, here the functional form involve a quotient of two function; that is f x and g x. In this case how to get the derivative, what is the formula to calculate the derivative? So, del y by del x is equal to g x multiplied by derivative of f x with respect to del x. In the first case g x is constant the derivative of the f x, and second case f x is constant derivative of derivative of g x, divided by g x to the power whole square. So, when the derivative involves a quotient, or the function of a quotient, two function of a quotient, in this case the derivative is. Derivative generally we calculate derivative by following this formula; that is g x multiplied by d, derivative of f x with respect to x, second term minus second term remain constant, f x remain constant, we take the derivative of g x with respect to d x, as a whole this divided by g x to the power if it is the whole square of this. Now, let us take a functional form to understand this, in a numerical term. So, y is function of 5 x plus 4 and 2 x plus 3. So, in this case, how we will get the derivative del y by del x is equal to 2 x plus 3. Then we take the derivative of this; that is 5 x plus 4 that comes to 5, minus we have to keep f x as constant. So, 5 x plus 4 is the constant, we have to take the derivative of 2 x plus 3, so that comes to 2, as a whole 2 x plus 3 whole square of this.

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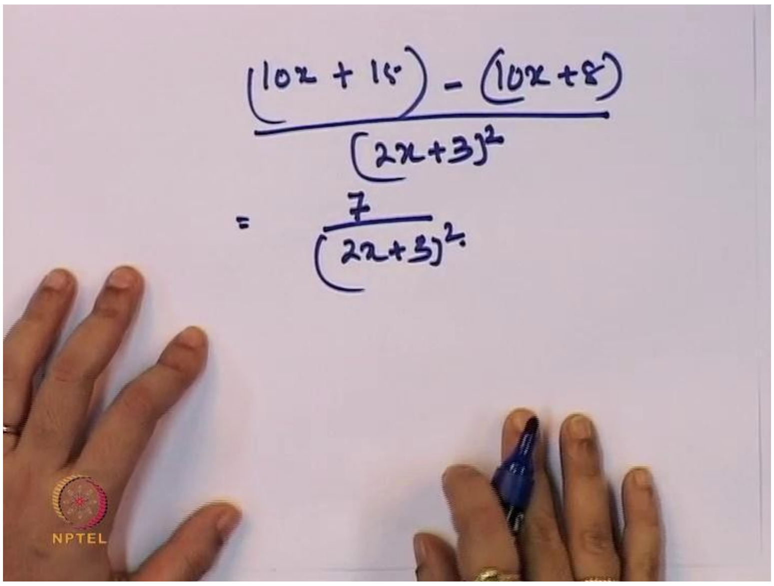

So, if you simplify this, this comes to 10 x plus 15 minus 10 x plus 8 divided by 2 x plus 3 square. So, this is equal to 7 by 2 x plus 3 square. So, in case of a functional form, when there is a it is a quotient of two function, in this case we generally follow a formula, which keep one function constant by taking the other function, derivative of the other function, and finally it is divided by the functional form of the second, the whole square of the second function is divided by the, whatever the derivative and the constant of the other two function. So, now next we will see the last category; that is when the function is a function of function, in that case how to find out the derivative.

So, we discuss about a power function, we discuss about a constant function, we discuss about a function which has sum and differentiation differences, we discuss about a function which has the product, and we discuss about a function which has in the quotient form. So, now we will discuss of a functional form which is function of a function.

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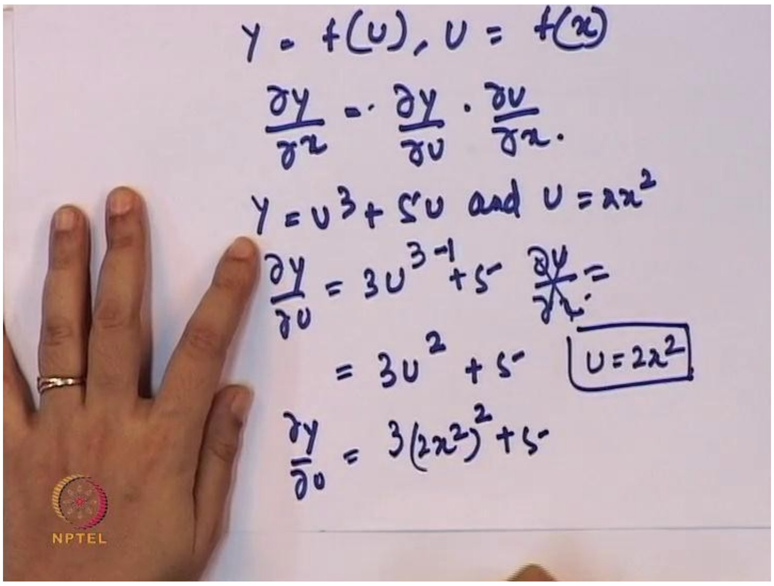

So, now takes a functional form, where y is a function of u and u is a function of x. So, in this case how to find out the derivative; del y by del x is equal to, del y by del u and del u by del x. So, here the derivative has also two term; one is del y by del u, and second one is del u by del x. So, let us take a again a numerical functional form to understand this more. So, y is equal to u cube plus 5 u ,and u is equal to 2 x square. So, del y by del u is equal to 3 u 3 minus 1 plus 5 and del u by del x is equal to. So, before finding this del u by del x, lets simplify it more this del y by del u, so this comes to 3 u square plus 5. Now, what is del y by del u. So, u is equal to 2 x square as we know, this is the functional form u is equal to 2 x square. So, if you put this value over here, so then 3 2 x square, to the power again square plus 5, so which comes to 3 2 x square, again 2 x square plus 5. Simplifying it further, so 3 multiplied by 2 x square again multiplied by 2 x square.

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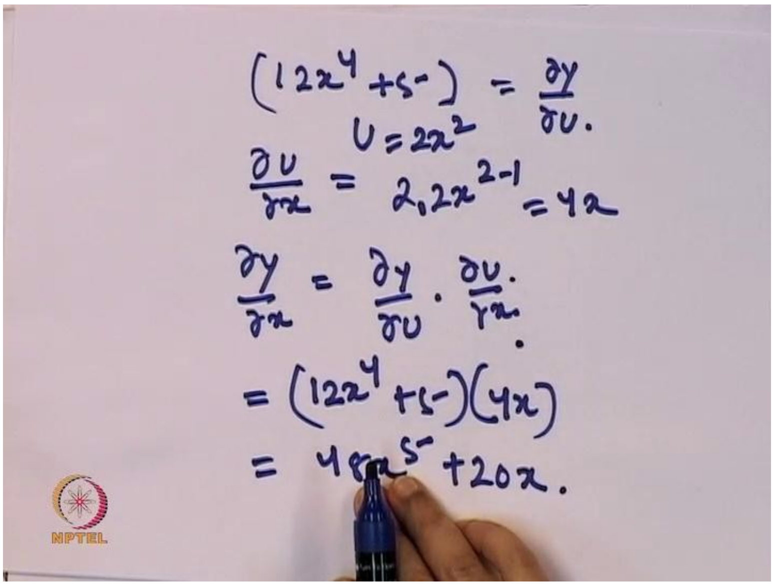

So, if you simplify this again, so this comes to 12 x 4 plus 5. So, this is what, this is our del y by del u, this is the first part of it. Now, coming to the second part, second part is derivative of u with respect to x, why the derivative is u derivative of u with respect to x, because the u is in a functional form, and u here is the dependent variable, the value of the u depend on the value of the x. So, taking the derivative of u with respect to x, we get 2 2 x 2 minus 1 which is equal to 4 x, because u is equal to 2 x square. So, del u by del x is 2 multiplied by 2 x 2 minus 1 which comes to 4 x. So, now taking this del y by del x, which is again a function of del y by del u, and del u by del x. So, that comes to 12 x to the power 4, multiplied by 5 multiplied by 4 x. So, this comes to 48 x 5 plus 20 x. So, what is happening over here. Here the functional form associated with the dependent variable has two functions, it is a function of function. So, the variable is directly not getting, directly not getting generated from the functional form rather, because the functional form is again a function of the other functional form.

So, in that case generally we first find out the value of y, which is with respect to u, then we find out the value of u with respect to x. And then finally, we find the derivative of y with respect to x, which is a product of derivative of y with respect to u, and derivative of u with respect to derivative of x. So, taking the same example it comes to 12 x to the power 4 plus 5 multiplied by 4 x, which come to 48 x to the power of 5 and 20 x. So, in this case, we discuss about a constant function, we discuss about a power function, we discuss about a function with sum and differences, we discuss about a function which has the product, we discuss about a function which has the quotient value, we discuss about a function where the function is again a function of some other variable. Now, let us analyse one more thing, where the functional form has several independent variables. So, here we generally take a functional form where y is a function of x.

Now we will introduce a case, where the value of y is not only getting valued in the form of the value of x rather than, there are several other independent variable those were deciding the value of y. We'll take a value, we take a functional form with the multivariable and then we will see how to find out, how to use the differential calculation, or how to find out a derivative the first order derivative in order to understand the relationship between the independent variable and the dependent variable. So, in this case how the functional form will look like. So, let us take a case of a demand function, because in case of demand function the primary variable it effects demand is price, but if you look at there are several other in variables, those who effect the demand for the product. So, we will take a case of a typical demand function, where we assume that demand is not only influenced by the price, there are certain other variables also influence the demand. And we will see how we find out the relationship by taking the derivative with respect to different variable.

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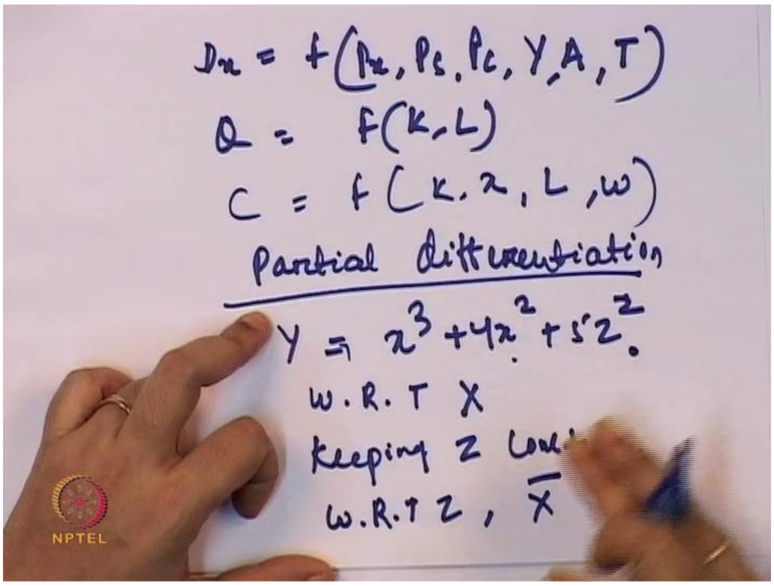

So, here if you're taking lets write a demand function; that is demand for x, it is dependent on price of x, dependent on price of the substitute goods, dependent on the price of the compliment goods, dependent on the income, dependent on the advertisement, and dependent on the taste of the consumer. So, here p x is the price of product, p s is the price of substitute good, p c is the price of compliment good, y is the income a is the advertising expenditure, and T is the taste of the consumer. So, similarly there are some other function what gets used in economics typically, where the dependent variable is dependent variables are dependent on many independent variables not only one independent variable. So, this is one example is demand function, similarly there is also example of production function. So, if you take a production function, it is always a function of capital and labour.

So, at least typically in the long run, the function is not only capital or the function is not only labour rather it's a function of both capital and labour. So, if in this case if you consider Q is the output, which is dependent on the capital and the labour. In this case, the value of output dependent on two independent variables; that is value of capital and value of labour. Similarly, we can take a cost function, where it is a function of capital, where it is a function of the rent what we pay for the capital, it is a function of the whatever the price we pay to the labourer, it is a function on the whatever the wages and the salary we are paying to the. So, in this case particularly, when the functional form has many independent variables. In this case generally we use the partial differentiation partial differentiation, in order to understand the relationship between the two variables; that is x and y.

Now, let us take a functional form. So, here y is a function of x cube plus 4 x square plus 5 z square. So, we have two variables here, those who are influencing y, y is the dependent variable and x and z they are the independent variable. So, now how we will find the derivative or how we will establish the relationship between y and x, and x and z. So, the first thing what we will do, we will find out the derivative with respect to x keeping the z constant. And in the second case we will find out the derivative with respect to z keeping the x as the constant. We need to do a partial differentiation with each variable, in order to understand their relationship with the dependent variable, and we cannot take the derivative of both the variable simultaneously, rather we will keep one as the constant and the derivative of the other in order to understand the relationship.

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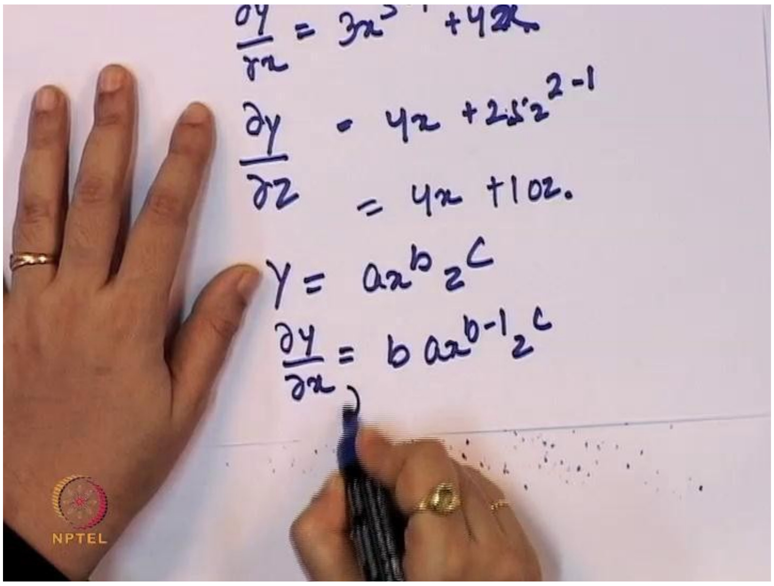

Now, taking the same functional form, where y is a function of, y is a function of x cube plus 4 x square plus 5 z square. We will first find out del y with respect to del x, so this will come as 3 x, 3 minus 1 plus 4 z. Similarly, how we will find the second one; that is del y and with respect to del z, which is 4 x plus 2 5 z 2 minus 1. So, this is 4 x, because z is constant. So, in this case we get 4 x plus 2 x, 2 multiplied by 5 z; that is 2 minus 1, so that comes to 4 x plus 10 z. Similarly, if you take a functional form, suppose y is equal to a x to the power b, and z to the power c. So, in this case, this is a case of your power function, where the y is dependent on x and z, so in this case how we find the derivative. Derivative of y with respect to x keeping the value of z constant; that is b a x b minus 1 z c. And second is del y and del z keeping the value of x remain constant. So, that comes to a x b c z c minus 1. So, when you have a functional form which has many variables, and many variables particularly in the independent variable category, and where the dependent variable is dependent on many independent variable.

In that case in order to find out the derivatives, first we need to keep one variable constant, taking the derivative of y with respect to the other variable, and second case we need to keep the other variable as constant, and taking the derivative of y with respect to the present variable. So, with this, we almost completed that what kind of derivative or how the derivative is used in case of different functional form, whether the functional form has the constant, it has the power function or maybe it is a addition subtraction product quotient, or if it is a function of function.