Managerial Economics
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Lecture - 8

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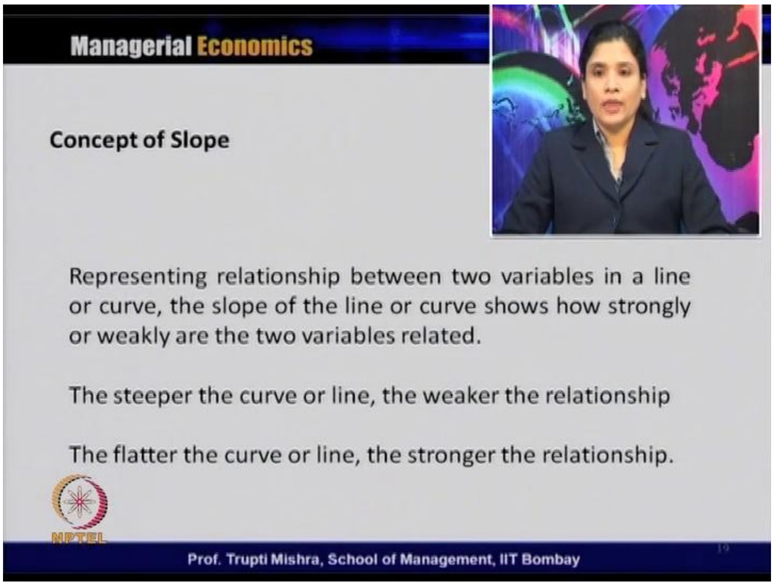

Then, we will discuss the concept of the slope over here. Now, what is slope? If you remember your marginal analysis what we discussed before, may be few sessions back or may be the last session, the marginal change is always whatever change in the dependent variable due to one unit change in the independent variable.

So, slope is to measure the relationship between the marginal changes in two related variables. It can be also defined as the rate of change in the dependent variable as a result of change in the independent variable. So, through marginal analysis, we know how the dependent or the independent variable changes, when there is a change in one variable. Through the marginal analysis, we know that when there is a change in the independent variable, it leads to change in the dependent variable.

But, through slope, we can measure the exact nature of the relationship, that is whether they are positively related or whether they are negatively related. We can also quantify the change, that is what the percentage change is or what is the amount of change that has taken place in the dependent variable due to change in the independent variable.

So, geometrically if you look at what is a slope, it represents the relationship between two variables in a line, in case of a linear relationship and or as a curve in case of a non-linear relationship. So, the slope of the line or the curve shows how strongly or weakly two variables are related. So, in order to find out the slope, we represent graphically the relationship between the two variables. So, if the two variables are linear variables, or linearly related, we get a line. If the two variables are nonlinearly related, we get a curve. Slope generally says, how strongly or how weakly these two variables are related to each other.

The steeper is the curve or the steeper is the line, the weaker is the relationship. Implication for this is that they are not strongly related or they are not related, if there is a steeper line or steeper curve. It means that there is no change or no more change or no significant change in the dependent variable, even if there is a change in the independent variable.

However, if the curve or the line is more flat or becomes more flat, it signifies that there is a stronger relationship between these two variables. Or we can say if there is a small change in the independent variable, it leads to a greater change in the dependent variable. So, when two variables are represented through a line or a curve, the slope measures the change between these two variables. That is, the amount of change, the nature of change or in the other words, we can say that they can quantify the relationship between these two variables. So, if it they are steeper, they are not much related and if they are flatter, then they are related to each other.

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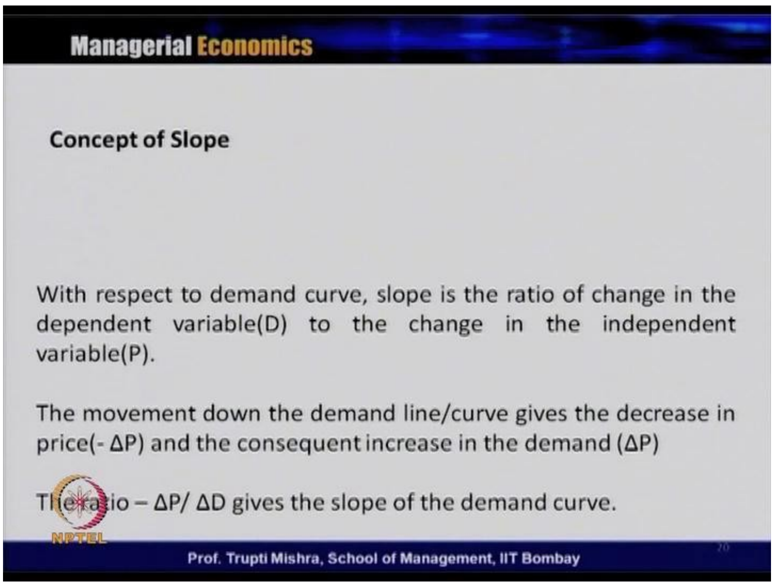

Now, taking a typical example of a demand function over here or demand curve over here, the slope is the ratio of change in the dependent variable and change in the independent variable. So, if you look at in case of a linear demand curve, we always get a straight line demand curve and in case of a non-linear demand curve, we get a curve.

So, with respect to demand curve, what is the slope? Slope is the ratio of change in the dependent variable D to the change in the independent variable. So, movement down the demand curve gives the decrease in the price and if it is upwards, there is an increase in the demand. The ratio of the change in the price and the change in the demand gives the slope of the demand curve. So, demand function is a function of price. So, price is independent over here and demand is dependent over here.

So, how to measure the slope over here? The change in the demand due to change in the price becomes the slope of the demand curve. Because, this slope measures the change in the demand curve due to change in the price. So, in this specific case, the slope is the change in the demand due to change in the price. So, we will see how generally we get a slope in case of a linear demand curve and in case of a non-linear demand curve.

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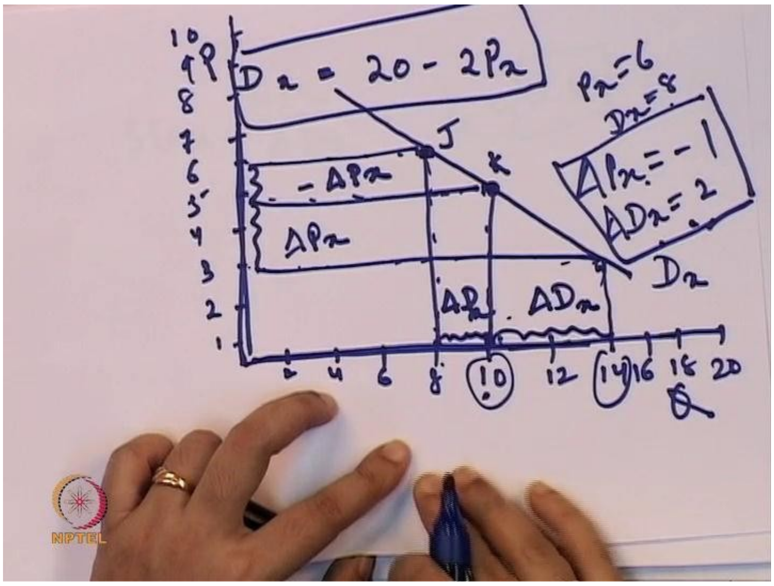

Suppose, we take the example of a linear demand function and that is, d x is equal to 20 minus 2 p x. This is a demand function. Now, how to find out the demand curve from this demand function. So, let us say this is 2, this is 4, this is 6, this is 8, and this is 10, 12, 14, 16, 18 and 20. Similarly here, we can say 1 2 3 4 5 6 7 8 9 and 10.

So, when the price is 6, suppose we say the quantity demanded is 8 and when the price is 5, the quantity demanded is 10. When the price is 3, the quantity demanded is 14. So, price is 6 and quantity demanded is 8. We get one point of the demand curve. Then price is 5 and quantity demanded is 10. We get the second point of the demand curve. Price is 3 and quantity demanded is 14. We get the third point of the demand curve. If you join these three points, we get the demand curve. So, maybe this is point j, and this is point k.

Now, what is the demand curve showing over here? If it is demand for x suppose, what is the demand curve showing over here? This is the change in the price of x and the consequent change in the quantity demanded of x. Price, we are considering here and quantity we are considering here. So, if you look at y axis is p and q is represented in x axis. So, this demand curve is essentially showing the relationship between the change in the quantity demanded due to change in the price. So, suppose the initial price, as we mentioned the initial price p x is equal to 6. So, the quantity demanded is 8 with respect to that.

Now, suppose p x decreases from 6 to 5 and quantity demanded increases from 8 to 10. So, this is what this is the change in the p x and this is the increase in the d x. So, this is minus because there is a decrease in the price and this is positive or this is plus because there is an increase in the quantity demanded.

So, when price of x decreases from 6 to 5, quantity demanded increases from 8 to 10. So, what is the value of del p x? del p x is equal to minus 1 because it changes from 6 to 5 or we can say, maybe it is from 5 to 6 and then it becomes 1. Then del d x is from 10 to 8 because this is the change in the x. So, del d x is 2. So, p x is the independent variable and d x is the dependent variable. Due to change in the p x, there is a change in the d x. So, given the value of del p x is equal to 1 and del d x is equal to 2, what is the slope of a straight line demand function?

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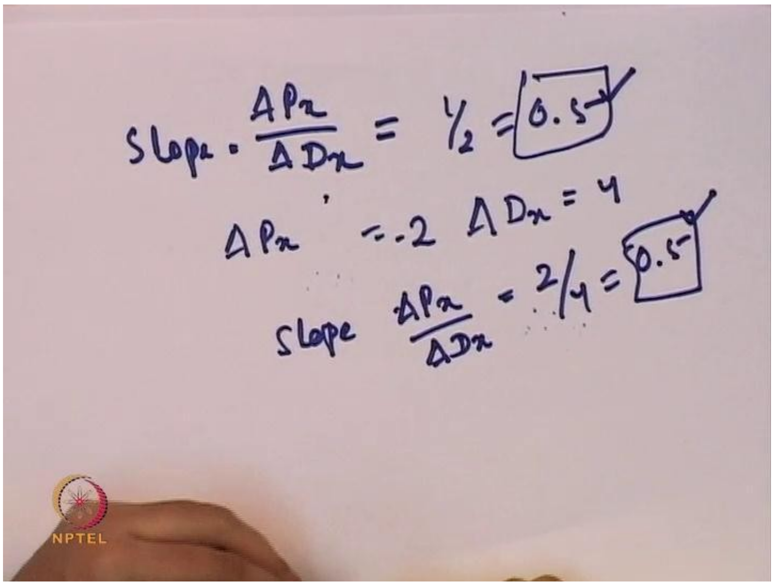

The slope of a straight line demand function is the ratio of the del p x by del d x. So, this is the slope of the demand function. So, del p x and del d x becomes the ratio and through this ratio, we can find out the slope of a straight line demand curve between the point j and k. So, if you see the previous curve, this is the point j and k. So, through this ratio, we can find out the slope between the two points j and k and this becomes 1 by 2, which is equal to 0.5.

So, given the demand function d x is equal to 20 by 2 p x and p x is equal to 6 and d x is equal to 8 initially, there is a change in the p x from 6 to 5 and change in the quantity demanded from 8 to 10. That leads to the change in the price of x, that is del p x and change in the quantity demanded d x. So, which is 1 by 2 and the slope is 0.5. This slope, if you look at it, this is the case of a linear demand curve. The slope is constant throughout the demand curve.

Now, suppose we consider that if price of x decreases from 5 to 3. This is again the change in the price of x and the quantity demanded changes from 10 to 14. So, this is 10 and this is 14. This is the amount of change in the quantity demanded of x. So, if price decreases from 5 to 3 and quantity demanded increases from 10 to 14, we will find out what is del p x over here and what is del d x over here. So, del p x is 3 minus 5 and that is minus 2 and del d x is 14 minus 10 and that is 4. So, in this case, when you identify

what is the slope between these two points, then this is again the ratio of del p x by del d x and which is again 2 by 4 and we are getting a value, which is 0.5.

So, in case of a linear demand curve, you get a constant slope throughout all the points of the demand curve because the change in the dependent variable remains constant with respect to change in the independent variable.

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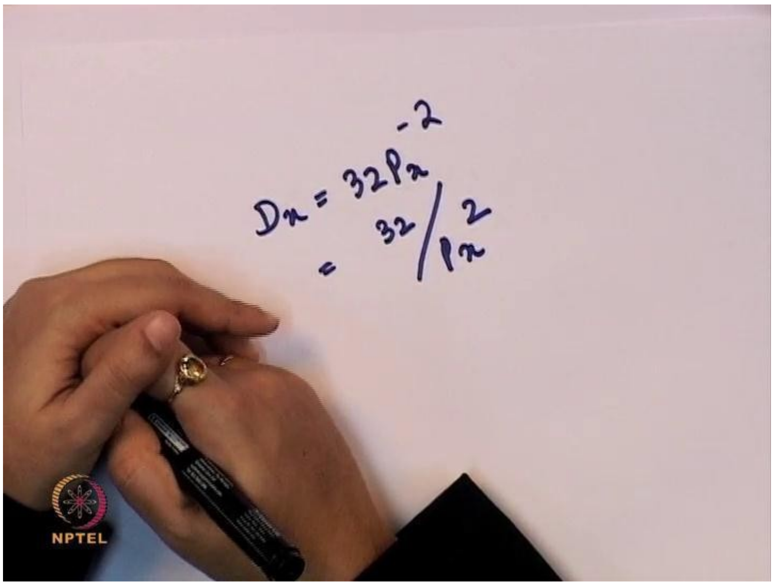

Next, we will see how we measure the slope of a non-linear demand function. Let us take a functional form, that is d x is equal to 32 p x minus 2 or we can say, this is 32 by p x square. Now, in case of a non-linear demand curve, the slope of the curve can be measured between any two points and then we can compare what is the slope between these two points.

What is the essential difference between a linear demand curve and a non-linear demand form? In case of linear demand curve, the change in the dependent variable remains constant throughout the entire demand analysis or entire analysis period. But in case of a curvilinear or in case of a non-linear demand curve, the dependent variable changes in a cyclic manner or in a different proportion at each point of the demand curve. That is the reason to necessarily measure the slope between the two different points and again compare whether the slope remains the same or slope is decreasing or slope is increasing or to identify what is the trend of the slope between different points of the demand curve.

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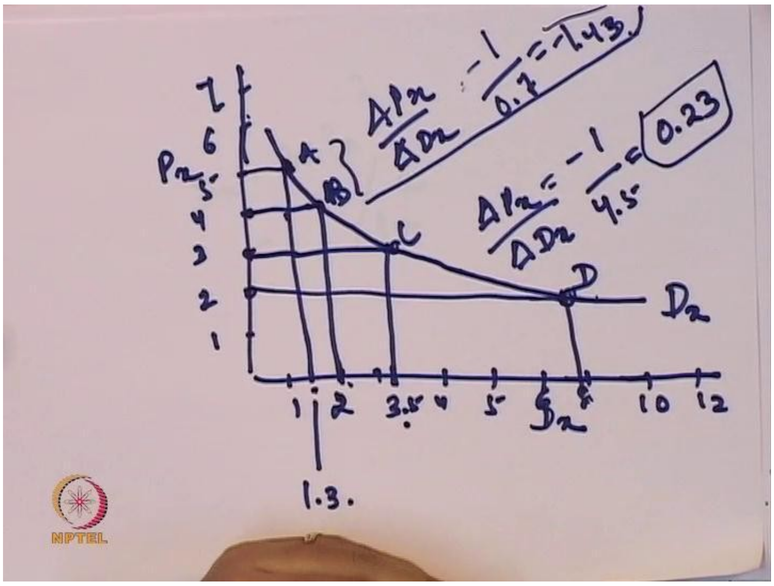

We take p x on the vertical axis and d x on the horizontal axis. So, here we get 1 3 or maybe, we can get 1 2 3 4 5 6 8 10 12 and so on. In case of p x, we can say this is 1 2 3 4 5 6 and 7. When price is 5, the quantity demanded is somewhere between 1 and 2. So, let us say this is 1.3. When price is 4, the quantity demanded is 2 and when price is 3, the quantity demanded is 3 or maybe, say it is somewhere 3.5 and when price is 2, then the quantity demanded is 8.

Basically, if you join these points, suppose this is point A, this is point B, this is point C and this is point D. If you join all these three points, all this four points rather, we get a non-linear demand curve. Now, how we will identify or how we will measure the slope between these two points? Now, what is the slope between point A and point B? Now, what is del p x over here? del p x is the difference between 4 and 5, that is the change in the price and that is 4 and 5. So, from point A to point B, what is the slope? That is, del p x by del d x. What is del p x? It is the difference between 4 and 5. So, that comes to minus 1. What is the difference in the quantity demanded? That is, the difference between 2 and 1.3. So, that comes to 0.7. Now, what is the slope over here? The slope over here is minus 1.43. That is, between point A and point B, the slope is 1.43. Now, what is the slope between point C and point D? So, what is the change in del p x? The change in del p x is between price 2 and price 3. So, this is minus 1. What is the change in the demand? The change in the demand is between 8 and 3.5. So, that leads to 4.5. So, this comes to 0.23.

So, if you look at it in a non-linear demand curve, the value of the slope changes or the value of slope is not constant at all points of the demand curve. So, when we measure or when we calculate the slope between point A and point B, we got a figure which is 1.43. When we calculated the slope between point C and point D, the value of the slope is 0.3. So, we can say that the slope of the non-linear demand curve is different between the different points.

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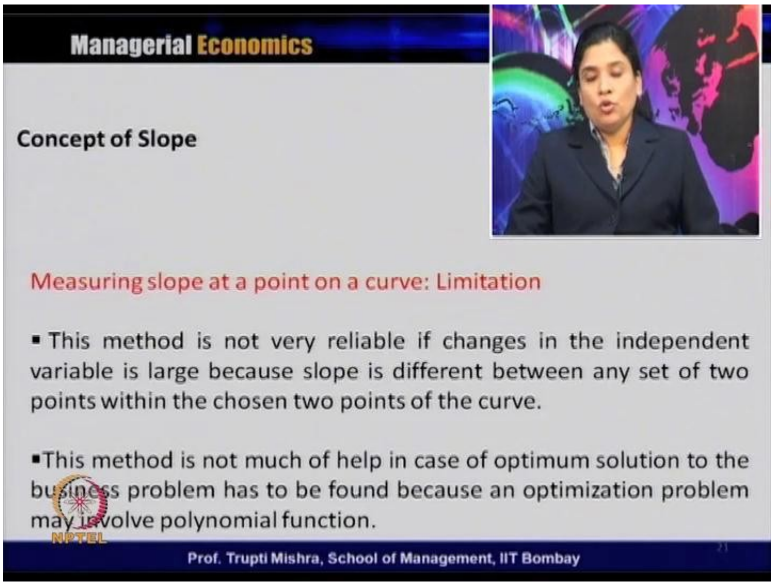

Now, when you measure the slope at a point of the curve, what may be the limitation or what maybe the constant over here. In case of a non-linear demand cure, what we do? We calculate the slope at two different points taking the change in the price and change in the quantity demanded and then we measure the value of the slope.

So, what are the limitations when you measure the slope at a point on a curve? This method may not be reliable because particularly in this case, when the change in the independent variable is large because slope is different from any set of two points within the chosen two points of the curve. This method is not much of help in case of a optimum solution to the business problem, that is to a firm, because an optimization problem may involve a polynomial function.

So, measuring slope, particularly for a linear and non-linear, it is possible when it comes to polynomial. It is basically difficult to use the same method to measure the slope and that is the reason, the difference between two variables may be sometimes too large that is difficult to do analysis by measuring the slope in this way.

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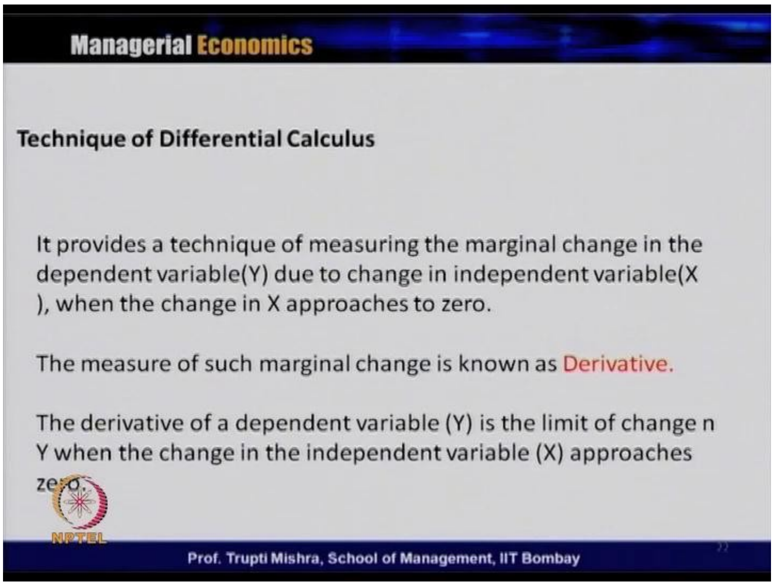

That is the reason that there is a technique of the differential calculation has come into existence, in order to understand the margin or in order to measure the marginal change in the dependent variable, due to change in the independent variable. Particularly, when the change of change in the independent variable approaches 0 and the measure of such marginal change is generally known as the derivative.

The derivative of a dependent variable y is the limit of change on y when the change in the independent variable x approaches 0. So, because of the limitation to measure a slope at a point in a curve, the technique of differential calculation generally comes into picture.

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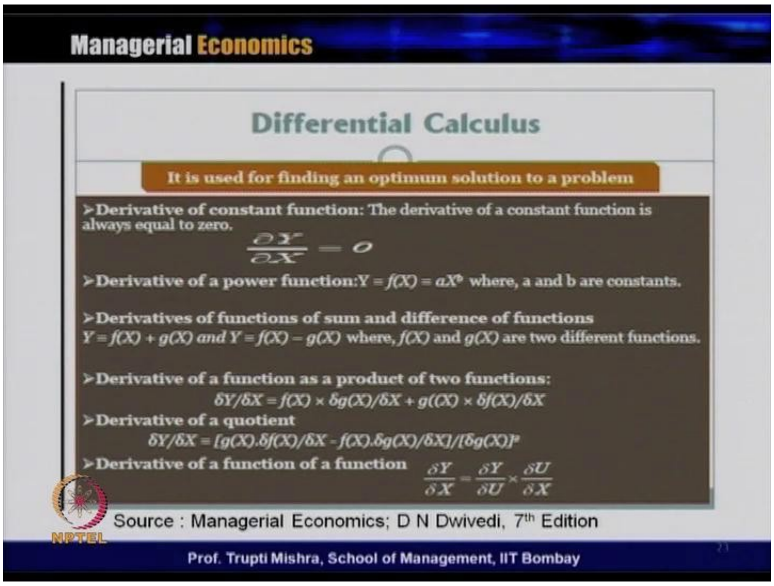

So, differential calculation is generally used to find an optimum solution to the problem. This is used in the derivative of a constant function, derivative of a power function, derivative of a function of the sum and difference of the function. Function is a product of two functions, and that is derivative of a quotient, and derivative of a function of a function.

So, we will check each function individually and how we use differential calculus over there. But before that, we will see how we can find out the differential calculus or how we can represent the differential calculus graphically. So, we are considering y is the dependent over here and x is the independent over here.

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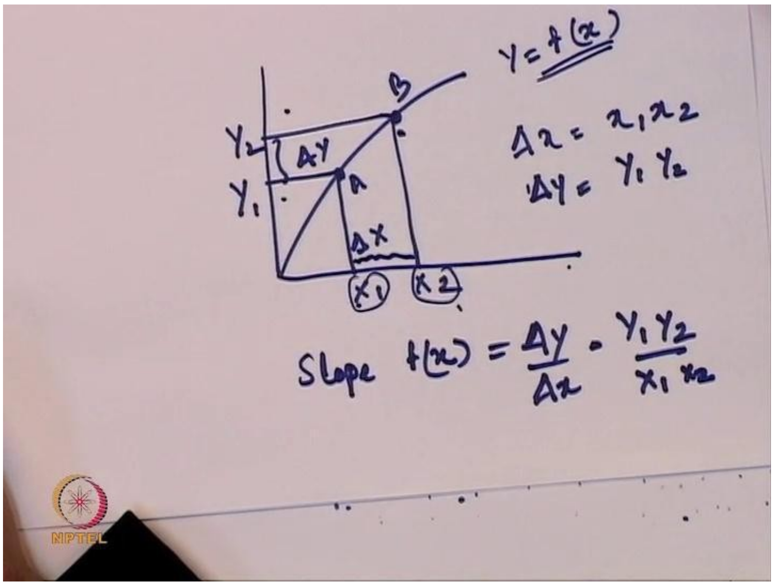

So, when we represent this in a graph, we take a function and that is y is equal to function of x, x 1, and x 2. This is y 1 and this is y 2. Now, what is the change in the x? That is, what is change in x from x 1 to x 2? This is the change in the y from y 1 to y 2. So, this is point A and this is point B.

Now, when x increases from x 1 to x 2, y increases from y 1 to y 2. So, demand function shifts from point A to point B. So here, what is del x now? del x is x 1, x 2 and del y is y 1, y 2. How we will identify what is the slope of this function? So, slope of this function is del y by del x, which is y 1, y 2 by x 1, x 2

So, when the change in the dependent and independent variable is very small, the slope can be calculated from the method of the differentiation or by the method of the differential calculus. Now, we will take the same example here. We have just taken a general function. Now, we will take a function specifically to the demand function to understand how differential calculus is being used in order to calculate the marginal change or in order to measure the changes between the two variables, that is dependent variable and the independent variable.

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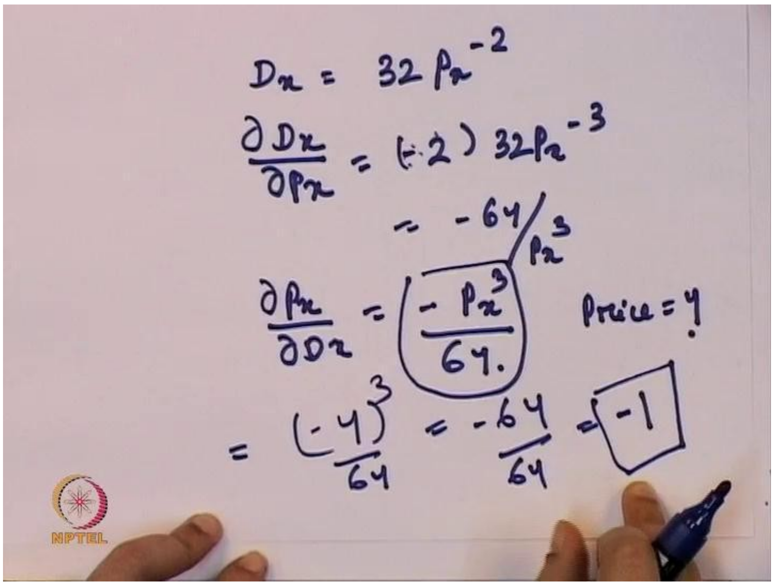

So, let us take a demand function, that is D x is equal to 32 P x to the power minus 2. So, this is a demand function and we will use the differential calculus to find out the slope. When this differential calculus is required or when this differential calculus is helpful? When the change in the independent and dependent variable is very small and it is difficult to find out the value of slope by the formula what we had discussed earlier.

So, if you are taking the first order derivative equal to 0, then it becomes del D x with respect to del P x. So, this comes to minus 2 32 P x minus 3 . So, this comes to minus 64 by P x cube.

So, if we take the reciprocal of the above equation, then this is del P x del D x and this is minus P x cube by 64. So, if you take this point in point B, considering this function is point B, and the graphical representation that we did earlier and substituting price is equal to 4. So, taking this and substituting price is equal to 4, now what will be the value of this equation? That is, minus 4 cube by 64. So, this is minus 64 by 64, which is cube equal to minus 1. So, the slope of the demand curve using the differential calculus, both at the tangent method and by the differential calculation, this is 1.

So, in this case, what we did is, we took the same demand function what we did earlier by taking the general tangent method to understand the slope and we found the value of the slope is equal to minus 1. Now, we have taken a different formula or may be a different method and that is the rule of differentiation or the difference calculation or derivative to understand or find out the slope. Following the differential calculus, taking the first order derivative and putting the value of p, we got a value of slope which is equal to minus 1.

So, if you remember in case of a tangent method also, the value of slope is minus 1. So, whether we follow the tangent method or whether we follow the differential calculus method by taking the same demand equation, the slope becomes equal. The only difference here is that we cannot use the tangent method with all changes in the dependent or independent variable. In the dependent and independent variable, the change related to is small, in that case only the differentiation or the differential calculation method can be useful.

So, in the next session, we will look at what are the rules of differentiation. As we discuss that, how we deal with the derivative or a constant function, power function, function of sum and differences of equation, quotient, and then power function, everything we will discuss in the next class related to the rule of derivatives or the rule of differentiation.