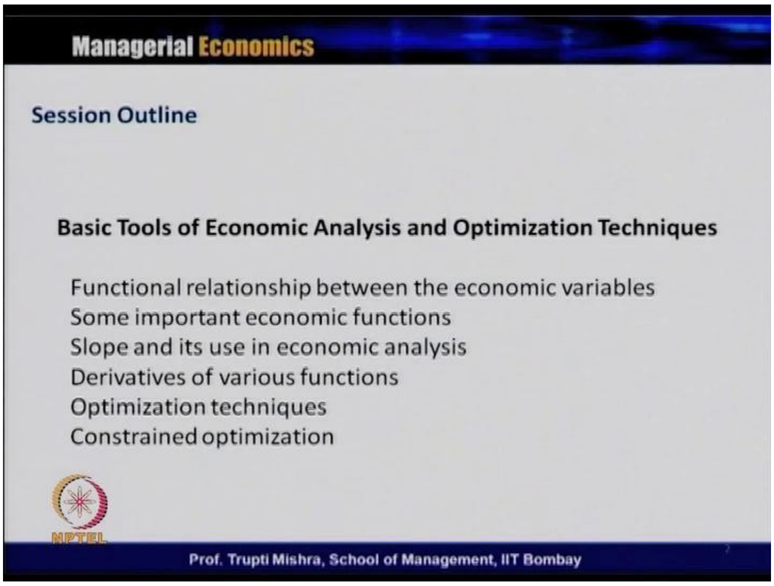
Managerial Economics
Prof. Trupti Mishra
S. J. M. School of Management
Indian Institute of Technology, Bombay

Lecture - 7

Welcome to the fourth session of managerial economics. Basically, we are on the first module of managerial economics, which talks about introduction and fundamentals to managerial economics.

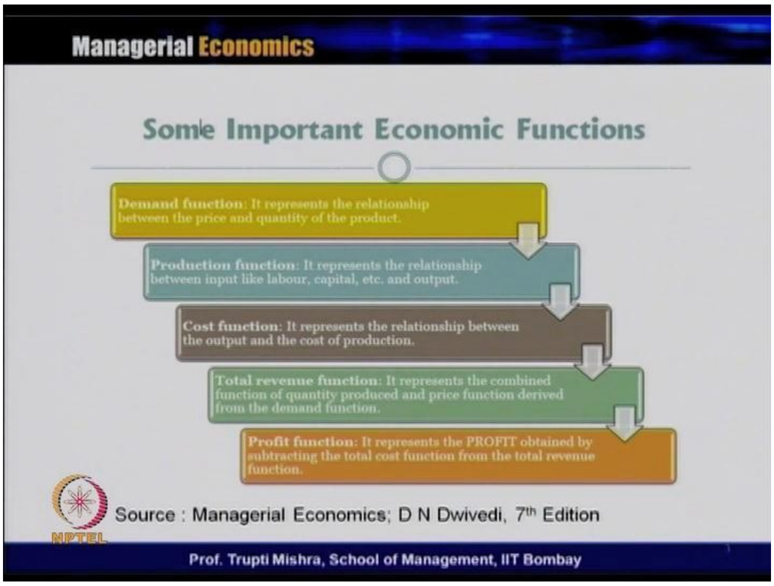
(Refer Slide Time: 00:33)



So, if you remember in the last class, we just discussed about the functional relationship between the economic variables, how they are related, and what are the different forms to represent them. Then we discussed some of the important economic functions like demand function, bi-variate demand function and multivariate demand function.

So, today's session we will focus on the different types of function that gets used typically in a demand function, how to measure a slope and what is its use in the economic analysis, different methods to analyze the slope and find out the slope or measurement of the slope. Then derivative of various functions and in the next session, we will basically take the optimization technique and constant optimization.

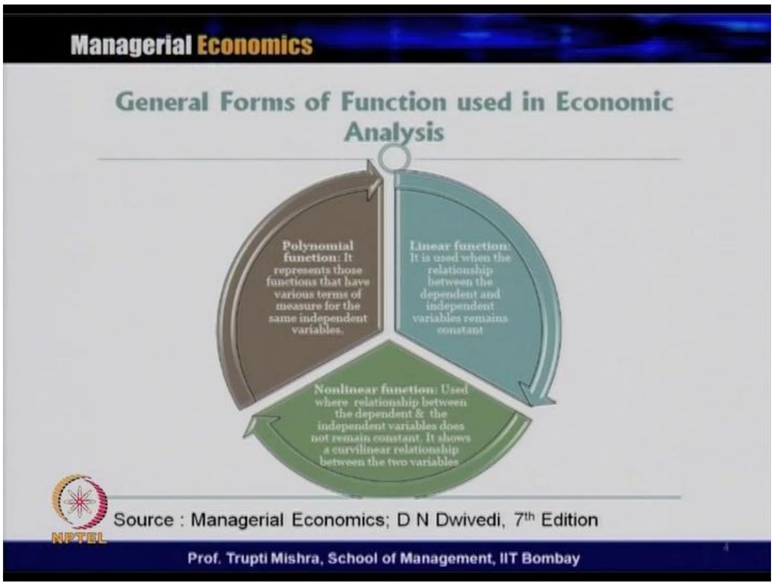
(Refer Slide Time: 01:25)



So, till now, all our discussions, if you look at it just focuses on the demand function. But apart from the demand function, there are certain other topics also where we generally use the relationship between two variables in a functional form, like production function which represents the relationship between the inputs like labour and capital with the output.

We talked about the cost function, where it is basically the relationship between the output and the cost of the production associated with that. When you talk about the total revenue function, it represents the combined function of quantity produced and price function is based on the demand function. Sometimes also we talk about a profit function. This is the profit basically, as you know it is the difference between the total revenue and total cost function. So, whenever there is a change in the total revenue and wherever there is a change in the total cost, it generally affects the profit. So, profit function is basically the relationship between the profit revenue and cost.

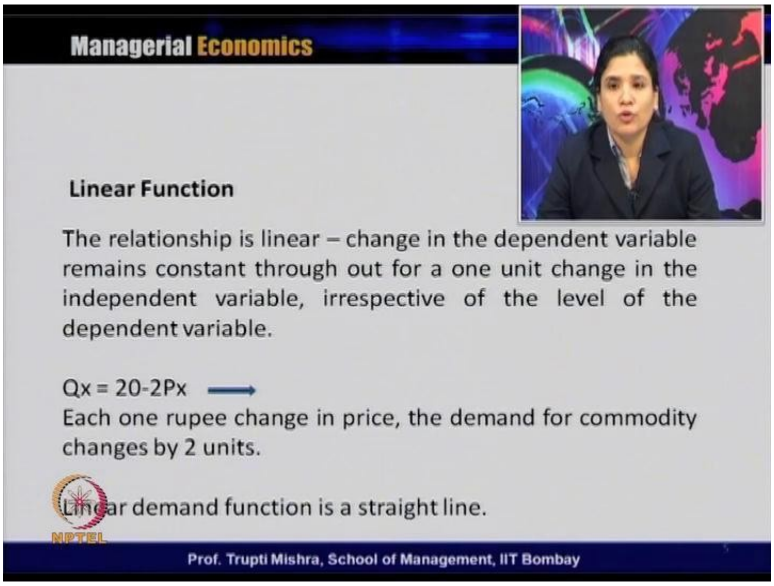
(Refer Slide Time: 02:24)



Then, we will discuss what are the general forms of function used in the economic analysis. So, one way we are clear that we use a functional form to understand the relationship between two types of variable. The variable, typically in this case, all the variables are economic variables. So, there are three types of function we use in analyzing the relationship between the variables. One is linear function, second one is the non-linear function and third is the polynomial function.

Linear function is used when the relationship between dependent and independent variable remains constant. Non-linear function is used where the relationship between the independent variable and dependent variable is not constant, but changes with the changes in the economic variable. Polynomial function represents those functions that have various terms of measure for the same independent variable.

(Refer Slide Time: 03:24)

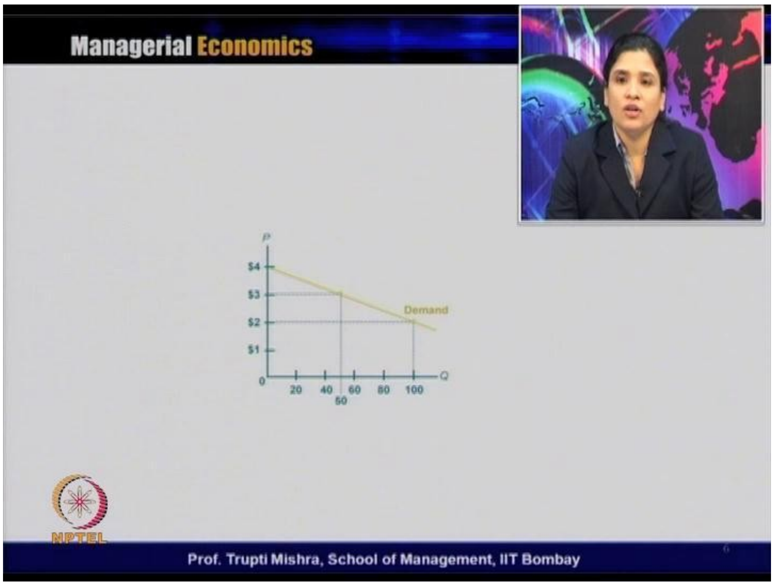


So, we will check all this functions in more detail by taking them individually. So, in a linear function, the relationship is linear. The change in the dependent variable remains constant throughout for one unit change in the independent variable, irrespective of the level of the dependent variable. Whatever the change in the independent variable, the change in the dependent variable remains constant in case of a linear function.

Suppose you are taking a demand function, which says that Q x is equal to 20 minus 2 P x. What does it signify? For each 1 rupee change in price, the demand for commodity changes by 2 units. Because, if you look at the second term of this functional form, there it is minus 2 P x. So, for 1 rupee change in the price, the demand for the commodity changes by 2 units.

When you represent graphically, the linear demand function is always a straight line because the change in the dependent variable remains constant for one unit change in the independent variable.

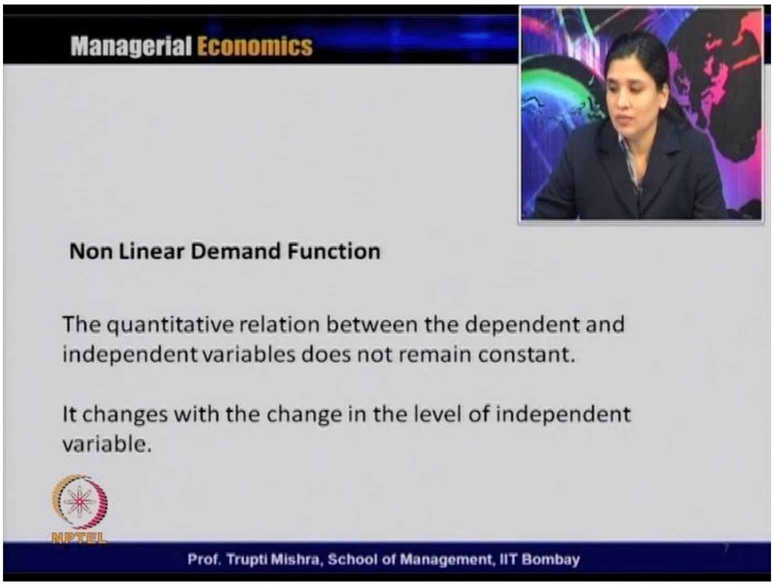
(Refer Slide Time: 04:30)



So, this is just a hypothetical way to understand the linear demand. In the vertical axis, we are taking the price and in the horizontal axis, we are taking quantity. So, if you look at it, when the price is changing, the quantity demanded is also changing. So, initially when the price is 2 dollar, the quantity demanded is 100 units. When the price increases from 2 dollar to 3 dollar, the quantity decreases by 100 units to 50 units.

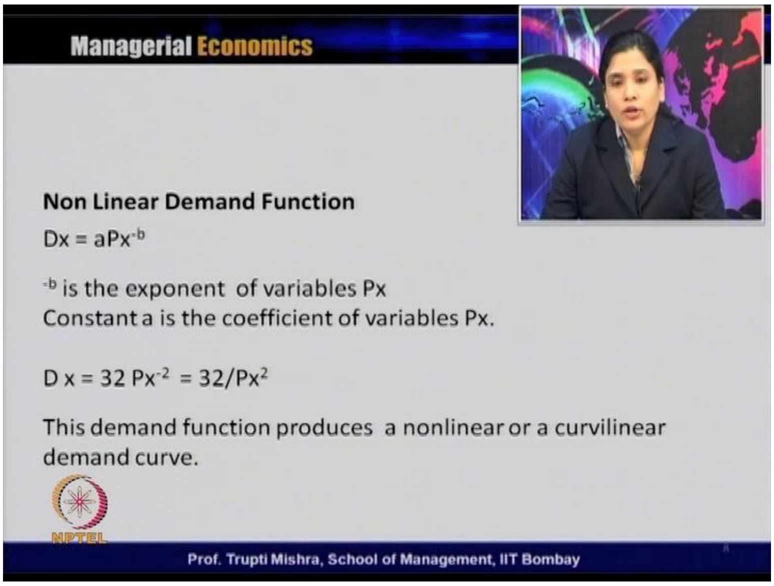
So, if you look at the demand curve, at each point it gives a price and quantity combination. Here, the quantity demanded is the dependent variable. Whenever there is a change in the price, that leads to change in the quantity demanded also. If you look in the percentage wise also, when the price changes from 2 dollar to 3 dollar, there is 50 percent change in the price. When the quantity demanded decreases, it decreases from 100 to 50. Again, this is a 50 percent decrease in the quantity demanded. So, 50 percent increase in the price is leading to 50 percent decrease in the quantity demanded. At this point, the relationship between these two variables is linear. As it is constant, the price point changes from one to another.

(Refer Slide Time: 05:55)



Then, we will discuss the non-linear demand function, where the relationship between the dependent and independent variable is not constant. It changes with the change in the level of independent variable. So, in the previous case, we are discussing that 50 percent change in the price will bring 50 percent change in the quantity demanded. However, in case of non-linear demand function, the unit of change may not constant with each change in price. When the price changes from 1 dollar to 2 dollar or 2 dollar to 3 dollar, that may not necessarily be the same kind of change in the level of the quantity demanded.

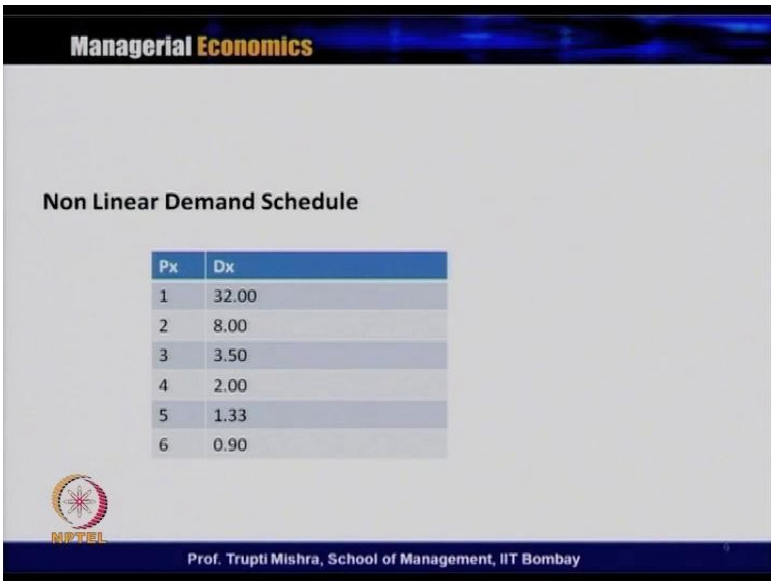
(Refer Slide Time: 06:30)



So, if you are taking a non-linear demand function, that is d x which is a function of price x. So, here x is the product, P x is the price of x and d x is the quantity demanded of x. Taking the functional form in a non-linear, d x is a P x to the power minus P. Here, a and b they are the constants. Minus P is the exponent of variable P x and constant a is the coefficient of variable P x.

If you simplify it further, may be you are taking a number term over here. Suppose, d x is 32 P x to the power minus 2. Maybe, we can take just a reciprocal of this 32 minus P x square. So, in this case, the demand function produces a non-linear or curvilinear demand curve. It means it is not a straight line. The change in the independent variable is not constant throughout whenever there is change in the price.

(Refer Slide Time: 07:33)



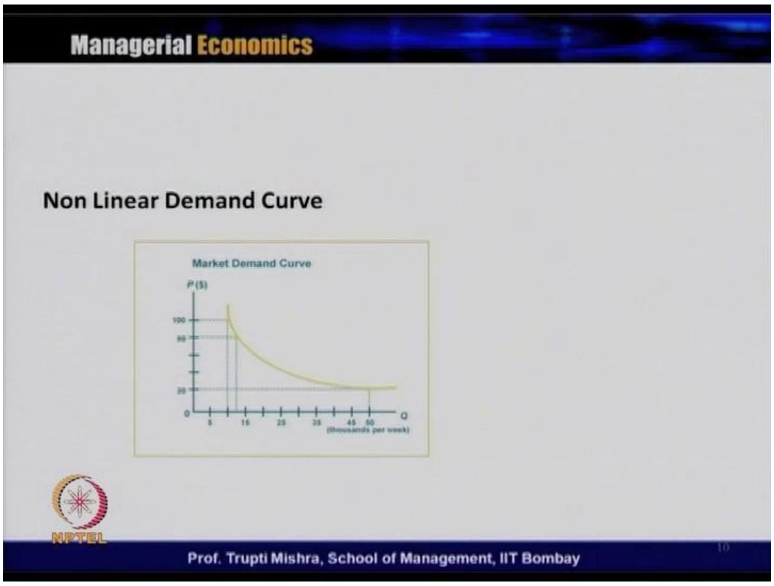
Px	Dx
1	32.00
2	8.00
3	3.50
4	2.00
5	1.33
6	0.90

So, this is an example of a non-linear demand schedule that shows how it changes when there is a change in the price. So, when price is 1, quantity demanded is 32. When price is 2, quantity demanded is 8 and when it is 3, quantity demanded is 3.5. Similarly, for 4, 5 and 6, if you look at the trend, the quantity demanded is going on decreasing when the price is increasing.

But, here the point is not to establish a negative relationship or inverse relationship between the price of x and d x. The point what we are discussing here is that, with each change in the price point, the change in the quantity demanded does not remain constant. The change in the quantity demanded with respect to each price point becomes different.

This is a typical feature of a non-linear demand curve. When you plot this in a graph, we generally get a curvilinear relationship, which is in the form of a curve. We do not get a line. We do not get a straight line, which is generally the representation of a linear demand curve.

(Refer Slide Time: 08:39)

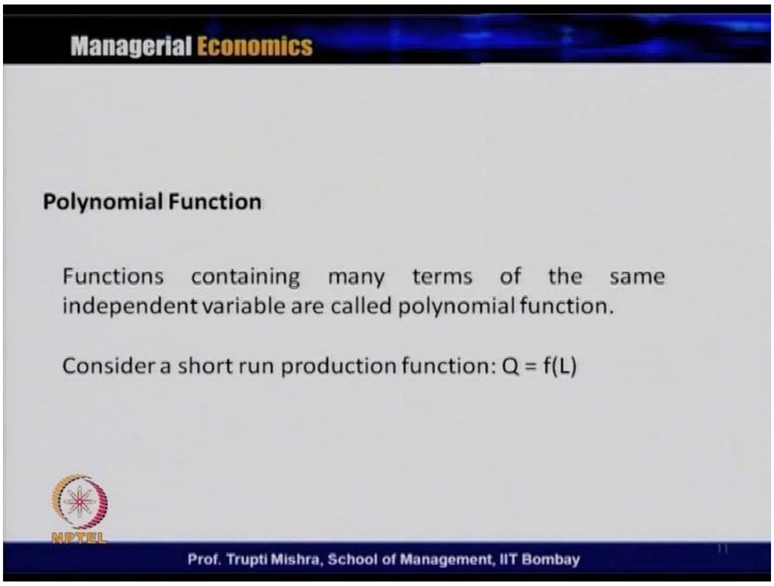


So, this is the graphical representation of a demand curve. If you look at the different points in the demand curve, the change in the quantity demanded does not remain the same. So, if you are taking the p here which is the price, it is represented in the vertical axis and q is the quantity, which is represented in the horizontal axis. When the price is changing from 100 to 80, the quantity demanded is increasing. Again, when it is decreasing from 80 to 20, the quantity demanded is again increasing.

But, if you look at the change in the price point from 100 to 80 and the corresponding change in the quantity demanded from may be 10 to 12, that does not remain constant. With the next change in the price point from 80 to 20, there is a significant amount of change in the quantity demanded, that is from 12 units to 50 units.

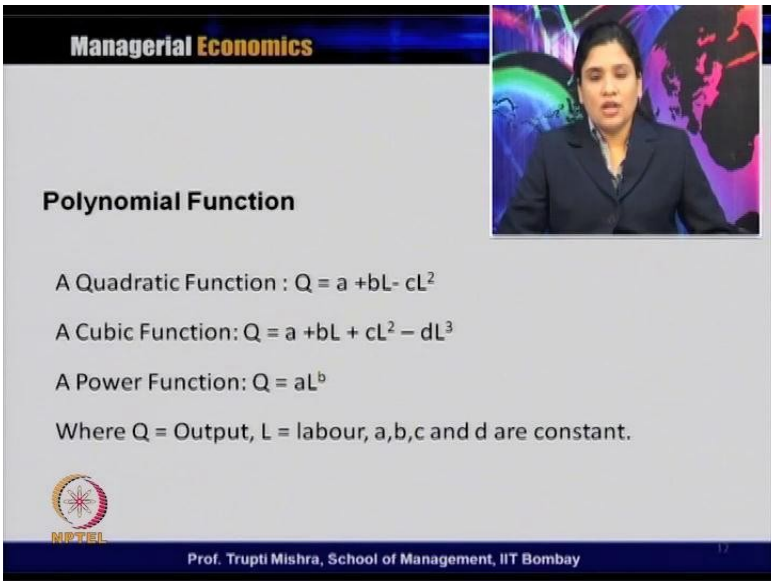
So, in case of a non-linear demand curve, even if the demand changes along with the change in the price point, there is always a difference in the amount of change at different price point.

(Refer Slide Time: 09:49)



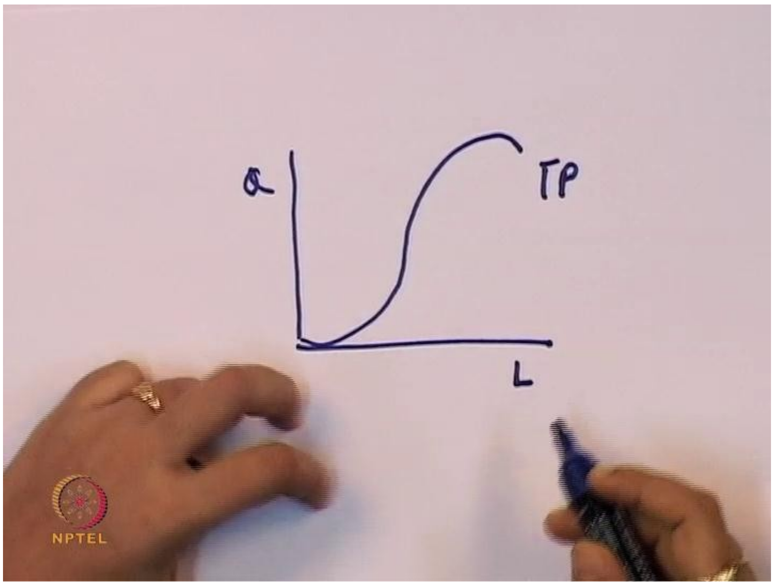
The third kind of function generally used in the economic analysis is polynomial function. What is polynomial function? The function that contains many terms of the same independent variable are called polynomial function. So, we consider a short term production function here, where output is a function of the labour and output is represented as Q and labour is represented as L. So, putting it in a functional form, Q is the function of L over here. The polynomial function takes different types of functional forms such as quadratic functions, cubic functions, and power functions.

(Refer Slide Time: 10:18)



So, taking the example of the same short run production function, where Q is the output, L is the labour and a, b, c and d are constants associated with the different coefficients. It takes a quadratic function or it takes the form of a cubic function or it takes the function of the power function. So, when it is becomes a quadratic function, Q is equal to a plus b L minus c L square, where a, b, and c are constants. When you take a cubic function, then it is a plus b L plus c L square minus d L q, where again a, b, and c are the constants associated with the coefficient. When it takes as power function here, it is a L to the power b, where a and b are the constants and b is the coefficient associated with variable L.

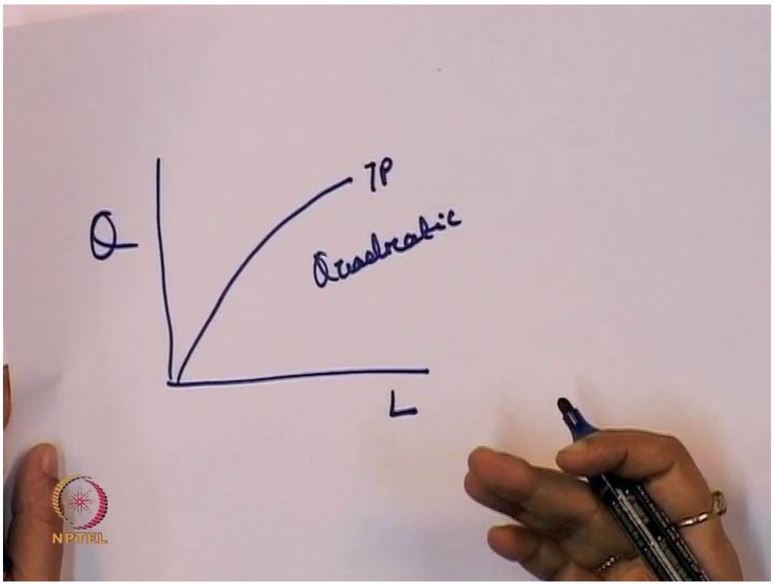
(Refer Slide Time: 11:38)



So, polynomial function may take a quadratic function or a cubic function or a power function. You can represent this polynomial function graphically, with all these three types of function, whether its quadratic cubic and function. So graphically, if you look at a cubic function, when the polynomial function takes a cubic function, suppose we take L over here, L is the labour and Q is the output. Now, the cubic function takes this type of shape. Now, what is this curve? This curve is the total product curve and total product is dependent on the output and the labour.

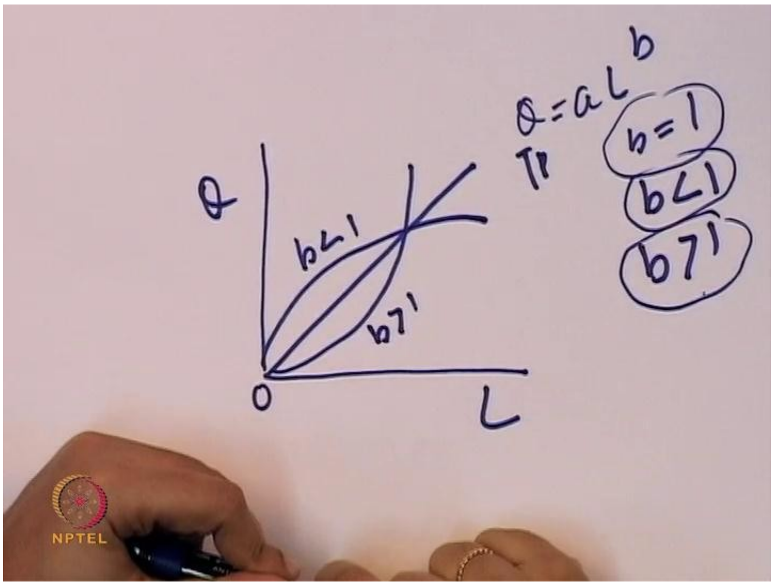
So, if you are taking Q over here and L over here, cubic function takes a form which may be not a straight line and not exactly a curve. It follows a different kind of change at each change in the L. So, how this Q and L are related here? L is the independent variable and Q is the dependent variable. So, whenever there is a change in L, that will bring in change in Q. So, in this case of a cubic function, L changes when Q changes. But the change in the Q is not constant with each change in the labour.

(Refer Slide Time: 12:45)



Now, take a case of a quadratic. So, with the same short run production function, we take L in the x axis and Q in the y axis. Now, it is a quadratic. So, you just follow it. There is no cyclical function over here and there is no much fluctuation here. Total product curve is this and this is a typical example of a quadratic function.

(Refer Slide Time: 13:26)

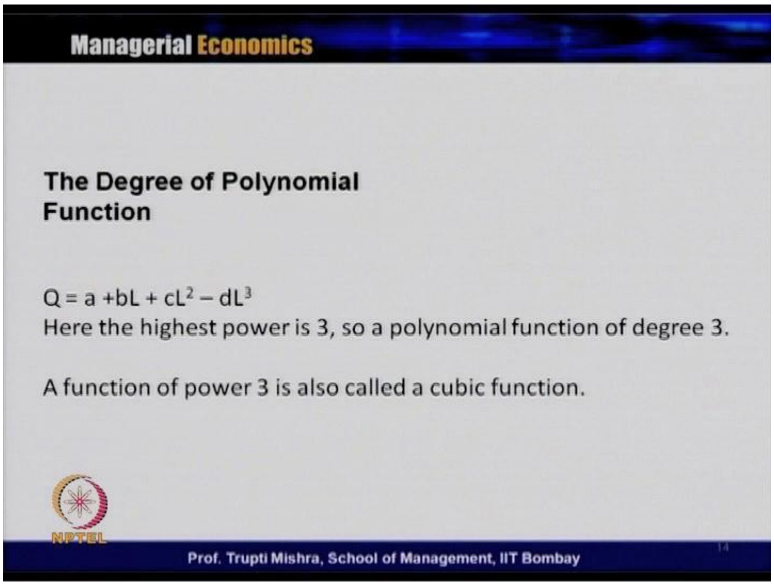


Now, graphically we represent the power function of the polynomial function. So, the power can take any value. The coefficient associated with b or the coefficient b associated with L can take any form. So, if you remember the power function is Q, it is equal to a L to the power b. So, b can take a value which is equal to 1, less than 1 or greater than 1. So, in this case, if you represent graphically again by taking the same formulation, here it is labour and here it is output. When we get the value of b equal to 1, it is a straight line. The total product curve is a straight line. When b is less than 1, we get this kind of shape and when b is greater than 1, we get this type of shape.

So, if you take a power function in case of a polynomial function, the power associated with the coefficient b can take any value. May be, sometimes it is 1 or sometimes it is less than 1 or sometimes greater than 1. So, whether it is 1 or less than 1 or greater than 1, when you represent that graphically or when you represent that geometrically, this is the shape that we get for different kinds of function.

So, polynomial function takes the quadratic function. Polynomial function can also be represented through cubic function and polynomial function can also be represented through a power function. Each time the value of b changes, the graphical representation also changes dependent on the value of the coefficient.

(Refer Slide Time: 15:21)

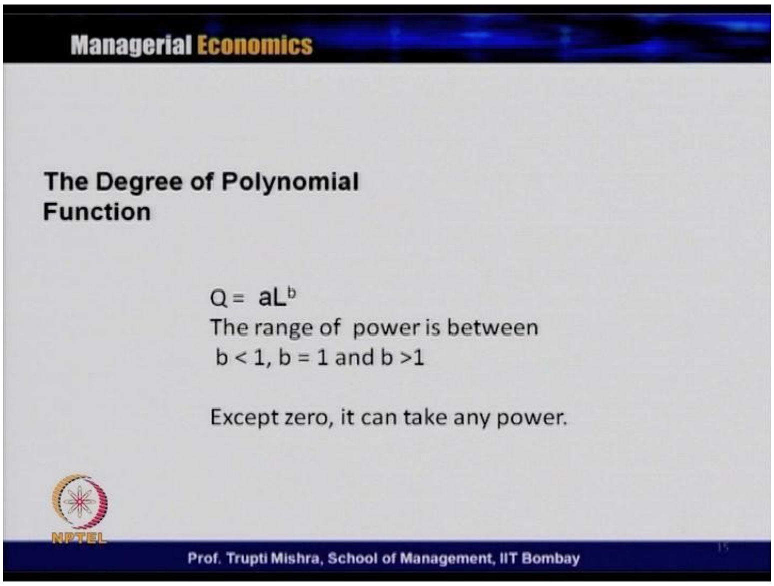


Now, how to find what is the degree of a polynomial function. So, degree of a polynomial function is, if we are taking a functional firm, which is q, it is equal to a plus b L minus c L square. Here, the highest power is 2. So, this is a polynomial function of degree two. A polynomial function of power two is also called a quadratic function.

So, in order to identify what is the polynomial function, it is always the highest power associated in this functional form. So, in this case, the highest power is 2. So, the polynomial function of degree 2 we can say is having this functional form. So, polynomial function of power root is also called a quadratic function.

Let us take one more example in terms of a cubic function. So here, what is a functional form? The functional form is q is equal to a plus b L plus c L square minus d L cube. Here, the highest power associated with the coefficient is 3 and this is a polynomial function of degree 3. A function of power 3 is also called cubic function. So, in the previous example, in the functional form, the highest power is 2. So, that is why it was a quadratic function of degree 2 and in this typical functional form, the highest power is 3. That is the reason, it is called as a cubic function because the function of power is 3.

(Refer Slide Time: 15:51)

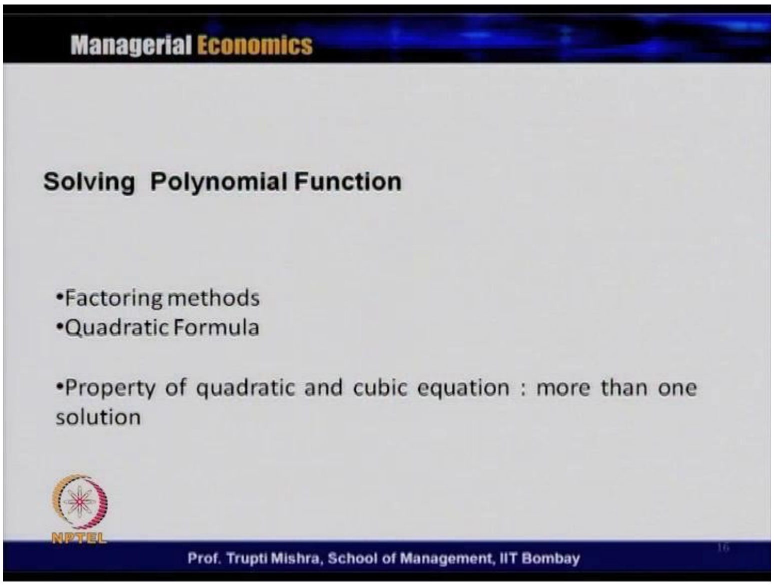


Then, we will see what is the degree of polynomial function, when the polynomial function is in terms of a power function. So, here in this case, the functional form q is equal to a L to the power b. The range of power is between b greater than 1, b is equal to 1 and b less than 1.

So, in this case, except 0 it can take any power. So, it may be less than 1 in the negative form, equal to 1 or it may be greater than 1. So, in this case, b taking the value of 0 is not possible. It takes any of the value and this is the example of a power function under the polynomial function.

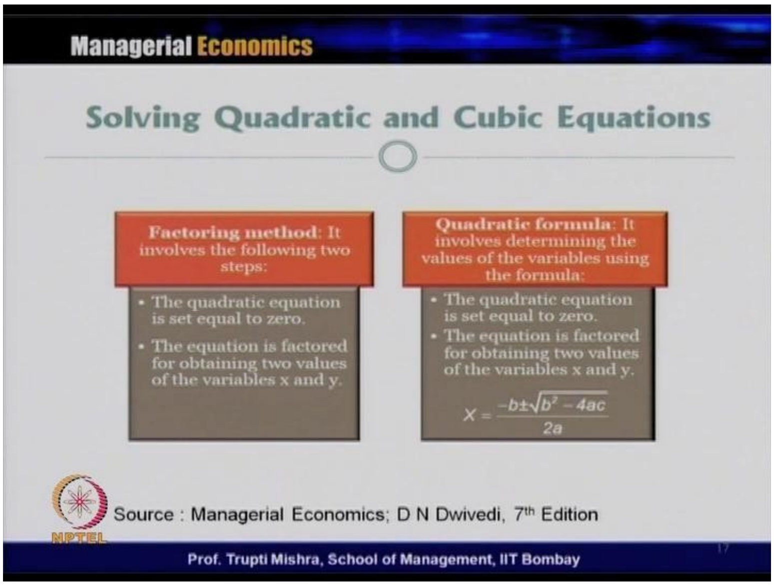
So, there are three types of functions. One is linear, second one is non-linear and third one is polynomial. In case of polynomial, in polynomial function again we represent in terms of quadratic function, in terms of a cubic function or in terms of a power function. Every time, the degree changes on the basis of the power associated with the functional form. The highest degree in case of a quadratic function is 2 and the highest degree in terms of a cubic function is 3. The highest degree in terms of power function is greater than 0 or it may take a negative value or a positive value.

(Refer Slide Time: 18:16)



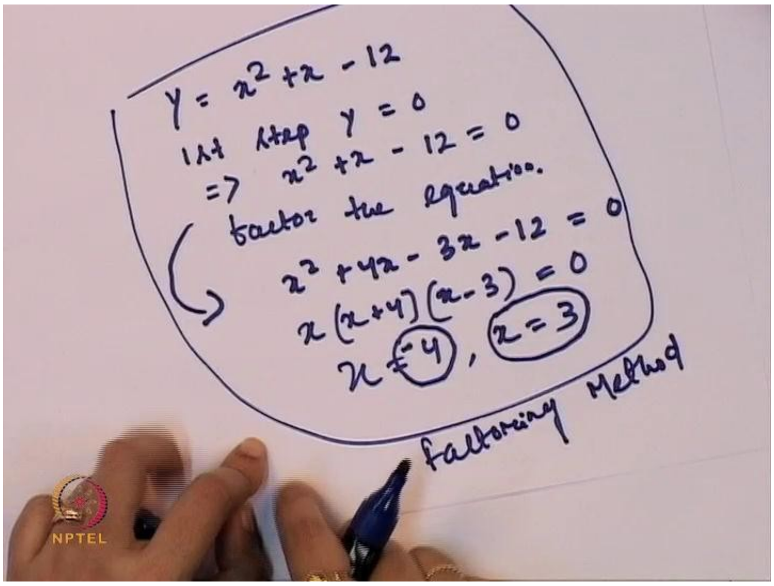
Now, how to solve a polynomial function? It can be solved either through the factoring methods or through the quadratic formula. What is the property of a quadratic or a cubic equation, when there is more than one solution? So, polynomial function can be solved by factoring method or by quadratic formula. It can be solved and the property of quadratic and cubic equation is that it has more than one solution.

(Refer Slide Time: 18:46)



Now, solving this quadratic and cubic equation, we have two methods. One is factoring method and second one is the quadratic formula. Now, we will see what is factoring method and what is cubic equation or what is the quadratic formula to solve this cubic equation.

(Refer Slide Time: 19:05)



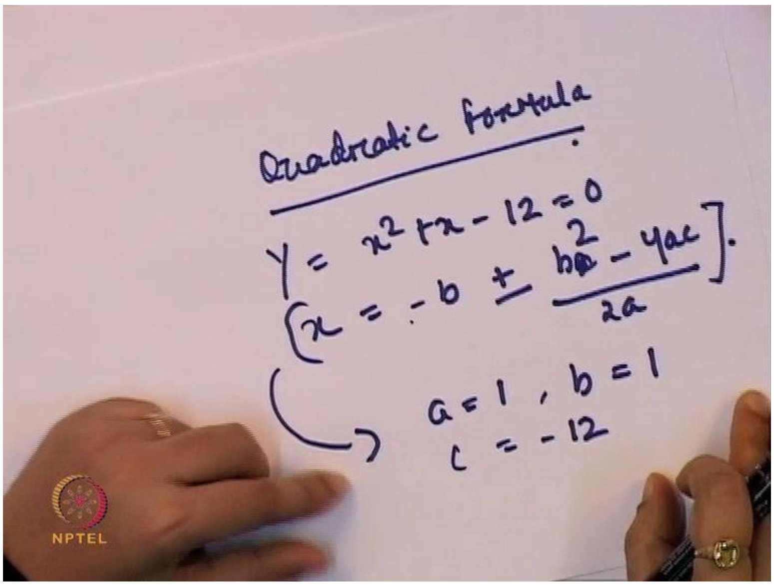
So, we will take a function. That is, y is equal to x square plus x minus 12. So, in the factoring method, what is the first step? The first step is that we have to set y is equal to 0. So, taking this x square plus x minus 12 is equal to 0, now what is the second step? We have to factor the equation.

So, this x square plus x minus 12 can be also represented with x square plus 4 x minus 3 x minus 12, which is equal to 0. Now, simplifying again, this takes x plus 4 and x minus 3, which is equal to 0. If you simplify, we get two values of x over here. One is x is equal to minus 4 and the second is x equal to 3. If you look at minus 4 has no meaning in economic analysis.

So, basically we will go with the positive value, that is x is equal to 3 and we solve this functional form with a value of x which is equal to 3. So, even if we are getting two values, one is minus and second one is plus. Typically, since we are applying this in economic analysis, there is no significance when we get a negative value of any variable. So, that is the reason we are ignoring the first value of x, which is minus 4 and we are going with the second value of x, which is equal to 3. So, if you look at it, this solution is through factoring method. So, this is the solution of a polynomial function by using the factoring method. Now, we will check the second one through the quadratic formula.

Now, what happens in case of a quadratic formula? The quadratic equation is set equal to 0. That is the first step and the equation is again the factor for obtaining the two values of the variables x and y. Following the formula, that is minus b plus minus b square minus 4 a c divided by 2 a. So, let us see, how we can solve a polynomial function through the quadratic formula.

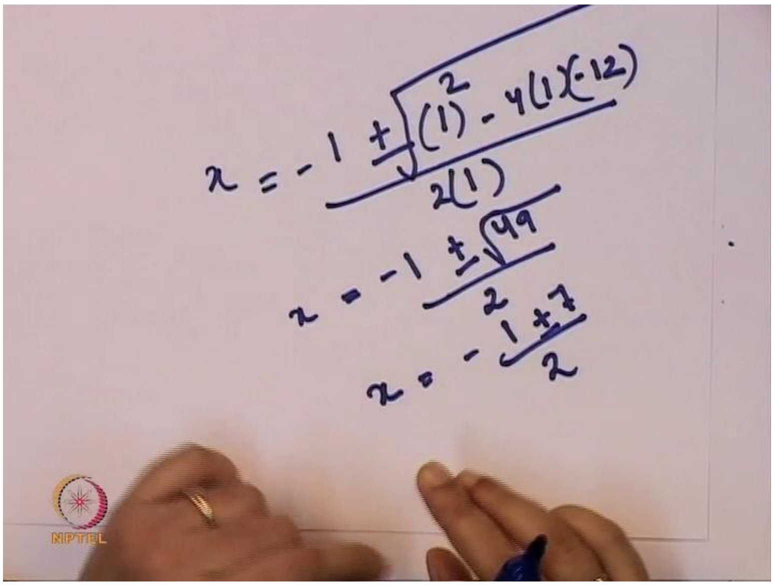
(Refer Slide Time: 22:02)



Now, what is the functional form over here? The functional form is y is equal to x square plus x minus 12, which is equal to 0. Because, what is our first step? The first step is to set the quadratic equation or whatever the functional form, we have to set that equal to 0. Now, what is the implication of this equation? Now, x is equal to, how to factor it again to get the value of x? Because, the first step is always to set it is equal to 0 and second, we factor out this equation in order to get the values of the variable x and y or if there is only one value, the value of the x.

So, if you are following, then this is minus b plus minus b square minus 4 a c by 2 a. Now, what is the implication of this equation? Now, a is equal to 1. This formula is taking this equation, that is a is equal to 1, b is equal to 1 and c is equal to minus 12. So, this is the formula to factor out the equation. This is the second step. The first step is to set this equal to 0, that is x square plus x minus 12 is equal to 0. From this equation, we get the value, and that is a is equal to 1, b is equal to 1 and c is equal to minus 1.

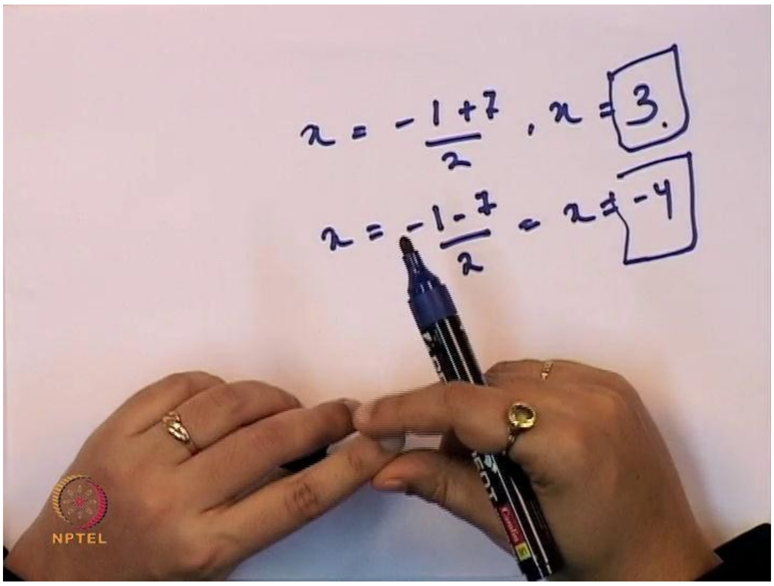
(Refer Slide Time: 24:09)



Now, we will substitute this value of a, b and c, in case of the quadratic formula. So, it is x is equal to minus 1 plus minus 1 square 4 a c and 2 a. So, this is 2 and 1. So, x is equal to minus 1 plus minus 49 root divided by 2, which you simplify again, this is minus 1 plus 7 by 2.

So, in the previous case, once we have identified the value of a is equal to 1, b is equal to 1 and c is equal to minus 12, we will put this value into the quadratic formula and we are getting the value of x. Now, this is 1 plus minus, it means x is having two values here, which satisfies the quadratic equation. Because, this is minus 1 plus minus 7, which is divided by 2. So, we will get two values of x and it satisfies the quadratic equation.

(Refer Slide Time: 25:29)



Now, if we take the first value, that is x is equal to minus 1 plus 7 by 2, we get x is equal to 3. If we take the second value, that is minus 1 minus 7 divided by 2, then we get x is equal to minus 4. So, we are getting one negative value and one positive value. Anyway, this is negative and still it is satisfying the quadratic equation. So, we have two values. One is positive and one is negative.

So, basically we ignore the value, which is with negative sign. We always go for the positive sign value because it makes some sense in the economic analysis when we go for the positive value. So, if you remember, in the previous solution that we did through the factoring method, we also got two values of x, that is 3 and is minus 4.

So, whether you solve the quadratic or cubic equation or whether you solve the polynomial function, either by taking factoring method or by taking the quadratic formula, you always get two values of x. On the basis of the value we get for x, we decide which value to take for further analysis and which value to ignore. So, polynomial function can be solved by using two methods. That is, one is the factoring method and second one is the quadratic formula. We get the same value for x taking any specific formula, either through the factoring method or through the quadratic formula.