Managerial Economics
Prof. Trupti Mishra
S.J.M School of Management
Indian Institute of Technology, Bombay

Lecture - 45
Theory of Cost (Contd…)

In today's session we will continue our discussion on the Theory of Cost, we are towards the end of theory of cost which talks about, and the economies of scale. And the economies of scale, if you remember in the last class also we discussed, the basis for that why there is a use of long run average cost curve.

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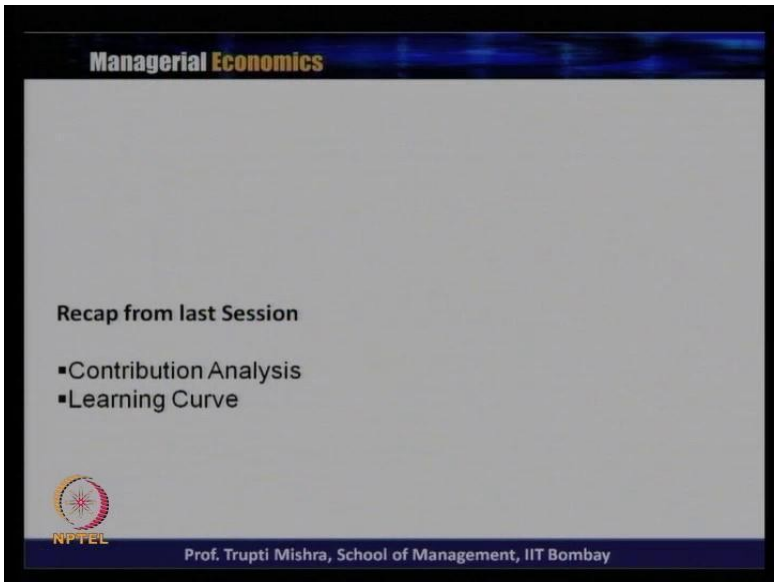

So, in the previous class we discuss about different cost analysis like short run and long run cost analysis, then we discussed that why long run average cost curve is u shaped, what is the evidence of economies of scale, and diseconomies scale. And in the different kind of average cost, then we talked about the contribution analysis specifically the p v ratio, and then we talk about the learning curve, if you look at that is the other way to analyze the long run average cost curve.

So, in one case we say that, when there is a economy of scale long run average cost curve is u shaped, because there is a economies of scale. And this economies of scale whereas, in this case of learning curve, the long run average cost goes on decreasing and there is no point where it increases again. So, in today's class we will focus more on the or mostly on the

economies of scale, different types of economies of scale, how the large firms or how the large large production amount bring some brings some cost advantages, and that is how the the firm produces at a lower unit to cost production that is how we are going to discuss today.

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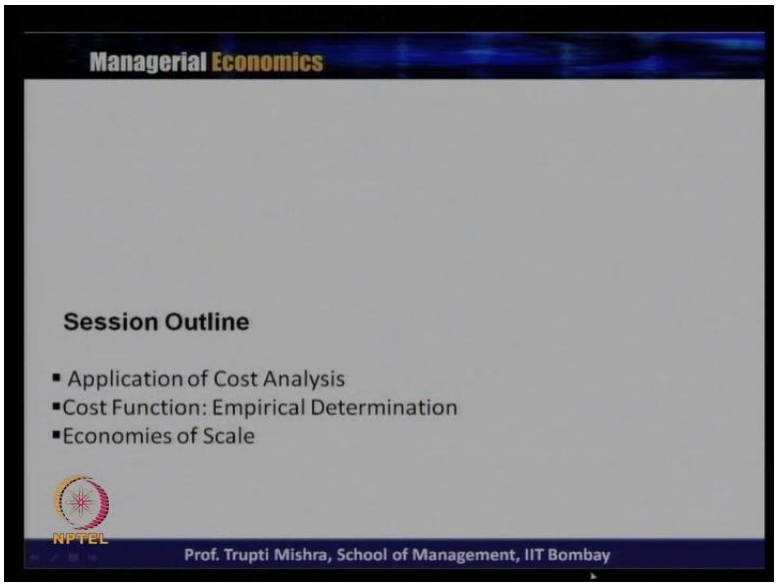

So, the focus or the session coverage is application of the cost analysis, we will take some the cost function the empirical determination of it, and then we will talk about the economies of scale.

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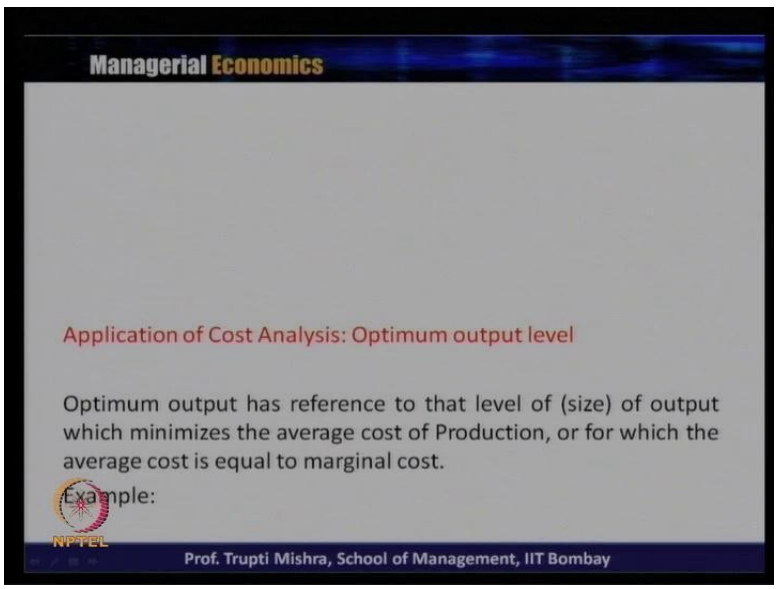

So, application of cost analysis if you look at this is the first case in case of optimum output level, we generally use the cost analysis to identify what is the optimum output level. So, optimum output has the reference to that level of that level of size of output, which minimizes the average cost of production, and for which the average cost is equal to the marginal cost. We have discussed several time this optimal production or optimal output, optimal output is one which can be produced at a lowest minimum cost that is the basic understanding of the optimal output.

And here if you look at optimal output is that level of output, which minimize the average cost of production, because here we are trying to fit the optimal output taking the constant as the average cost of output. So, optimal level of output is 1 which produce a produce that level of output which minimizes the average cost of production or maybe the another condition for is that, where the average cost is equal to the marginal cost. So, we will just take a example that, how to find out empirically taking a typical cost function, how to find out the optimal optimal output level which minimize the average cost at that point of time.

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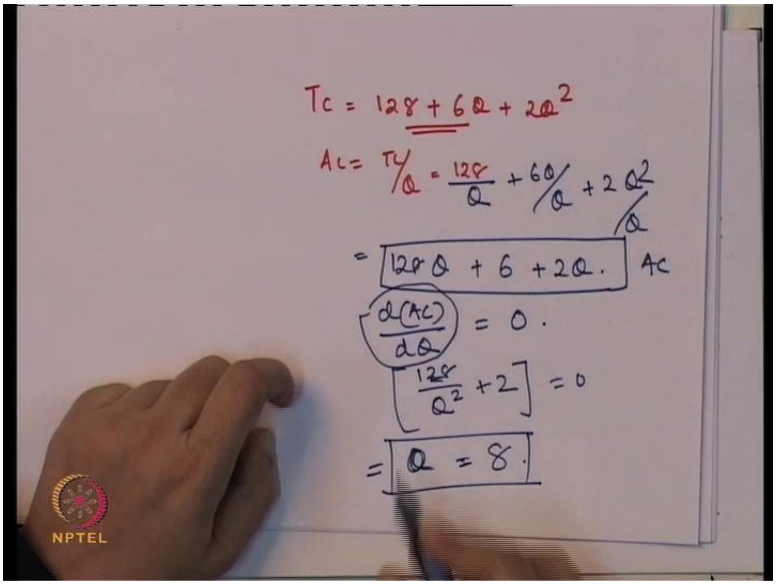

So, if you will take a cost function like that is total cost total cost is equal to $TC = 128 + 6Q + 2Q^2$ if this is the cost function, then in order to find average cost as we know this is total cost divided by Q, so this is 128. So, generally when we find this average cost (Ac), we generally divide it by that the unit of output, so this is $AC = \frac{128}{Q} + \frac{6Q}{Q} + \frac{2Q^2}{Q}1$, so

that comes to $AC=\frac{128}{Q}+6+2Q$, so this is the average cost, this is the average cost for the cost function.

Now, to find out at which point average cost is minimum, we need to take the first order derivation that is first order derivative of average cost with respect to Q. And if you equalize that equal to 0, $\frac{d(AC)}{dQ}=0$then this comes as $\frac{128}{Q^2}+2=0$, and if you summarize or simplify it again we get Q is equal to 8. Now, how do you interpret this Q is equal to 8, so when we know that average cost is minimum average cost is minimum mathematically, we can find out that by taking the first order derivative of average cost with respect to Q.

If you take this with respect to $\frac{128}{Q}+6+2Q$, in this case we get the Q value of Q which is equal to 8, and value of Q if it is equal to 8, then this is the level of output where the average cost is minimum. Now, let us understand the same cost function, taking in term of the marginal cost or when we find out the marginal cost and equalizing the marginal cost and average cost again, whether we get the same level of output, where the cost of production or the where the average cost is minimum.

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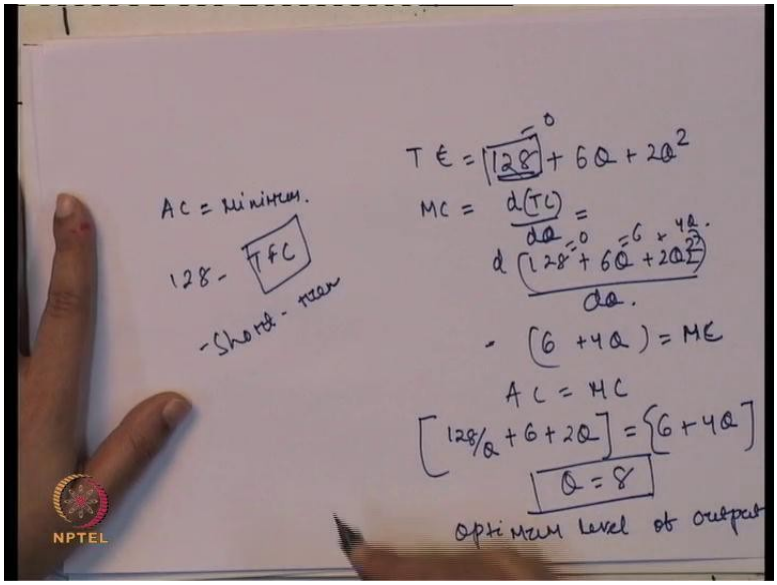

So, taking the same cost function again, suppose this is total total cost is equal to $Tc=128+6Q+2Q^2$, here will find out the marginal cost and how to find out the marginal

cost? Generally we will take the first order derivation of total cost with respect to d Q $MC=\frac{d(TC)}{dQ}$. So, in this case how do you find out this, so this term becomes 0 $\frac{d(128+6Q+2Q^2)}{dQ}$ as compared to with respect to dQ, so if you do this, then this term is equal to 0 calculus. So, this is equal to 6 and this is equal to 4 Q, so $MC=6+4Q$is our marginal cost, we have average cost, we have we have marginal cost.

Now, if you equalize this average cost is equal to the marginal cost, then in this case what is the value we are going to get, as we know the average cost is $AC=\frac{128}{Q}+6+2Q$, because this is nothing but $AC=\frac{TC}{Q}$which is equal to$MC=6+4Q$. So, if you if you make a equalization of average cost and marginal cost and simplify this,$AC=MC$ then again we get a value of Q which is equal to 8, so Q is, Q = 8 is what Q = 8 is the optimum level of output Q eight is the optimum level of output, where average cost is minimum. And we can say that this is a short run cost, because we have some amount of 128 over here.

So, what is 128, 128 is the total fixed cost, and if you remember we get the amount of fixed cost only in case of short run, so since it is a case of total fixed cost, we can say this is a short run analysis. In case of short run cost analysis, Q = 8 is the level of output or 8 is the level of output where the average cost is minimum. So, the point over here we want to focus is that, what is the role of this average cost, marginal cost or total cost or how it helps in taking the economic decision and business decision, or how it helps to generally using it for the different kind of economic analysis.

So, for to summarize this we can say that optimum output level is 1, where the average cost, where the cost to be minimum and that we can check from a cost function by taking the taking out the average cost, the marginal cost, and solving the value for the Q.

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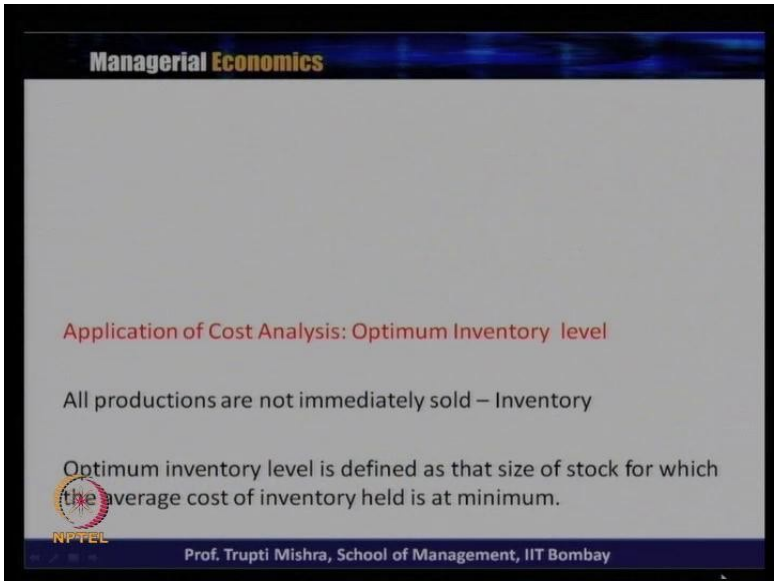

So, next will see this optimum cost analysis in case of the optimum inventory level, so when this concept of inventory comes generally, inventory comes if all the production are not immediately sold. And if you look at whatever is produced, and whatever is sold there is a gap in between and the gap whatever the amount, that generally goes to the warehouse, that is generally goes to the stock. And in economics term we called it as the inventory, because that is the stock kept in the warehouse which is not coming to the open market for getting sold.

Now, how to define the optimum inventory level? Optimum inventory level is defined as the size of stock for which the average cost of inventory is minimum; so if you look at it is just generalized into the same concept of optimum output level is generalized in to the optimum inventory level. And what is in optimum inventory level? The size of the stock or the size of the inventory where the average inventory cost is generally minimum. So, the way we analyze the optimum output level that is that is the level of output where average cost is minimum; in similarly here it is the level of stock where the where the average inventory average cost of inventory is generally minimum.

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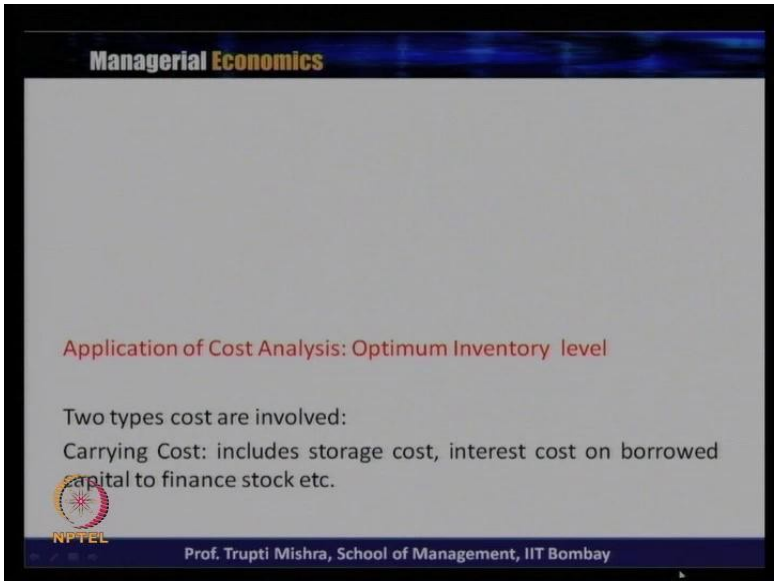

So, let us look at how to find out the optimum inventory level, so there are two types of cost involved over here, one that is carrying cost, generally it includes the storage cost, interest cost on borrowed capital to finance stock etc. And second type of cost is reorder cost which includes the book keeping cost, telephone charges, and some variable cost. So, there are two kinds of costs are involved in case of a when they are finding out the optimum inventory level, one is the carrying cost and second one is the reorder cost; both has to be the part of the cost function, when we are finding out the optimum inventory level.

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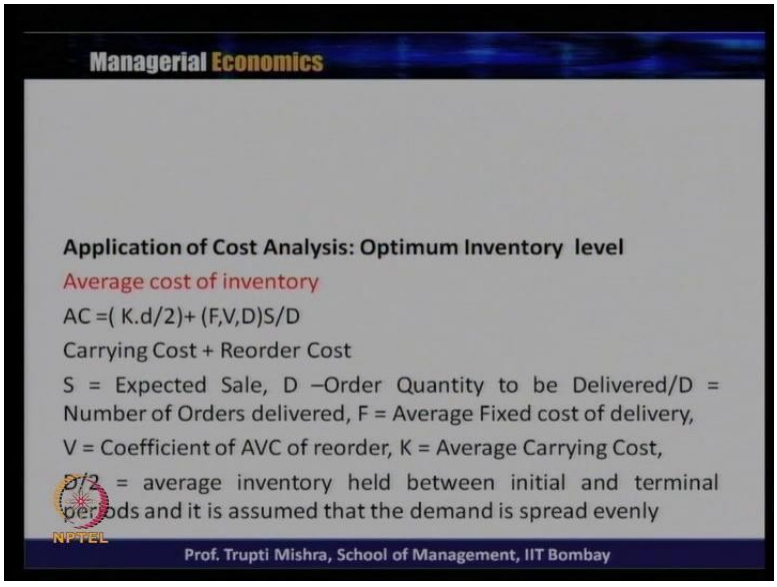

So, average cost, how to find out the average cost of inventory, the average cost of inventory $AC=\left(K.\frac{d}{2}\right)+\frac{(F,V,D)S}{D}$ so the first part of this equation is the carrying cost, and the second part of this equation is the reorder cost. Now, we will see what is the terminologies stands for, S is the expected sale, D and why we are taking the expected sale, because that will tell us that which are which is the what are the amount that is going to be sold, and what are the amount that is going to be stock or that is going to be in the inventory.

So, S is the expected sale, D is the order quantity to be deliver, and D is the number of orders deliver, F is the average fixed cost of delivery, V is the coefficient of AVC of reorder and K is the average carrying cost . So, if you look at this average cost consist of both carrying cost and reorder cost, and different types of cost either coming under carrying cost or reorder cost that is added in the equation.

And what is d by 2 that is the average inventory held between initial and terminal period, and it is assume to that the demand is spread generally evenly. Because this is since we need to find out what is the average inventory cost, we also need to find out the average inventory level; and that is the reason we bring this d by 2 variables in to our average cost function. And d by 2 is nothing but the average inventory held between the initial and terminal period, and it is assumed that the demand is spread evenly. Now, to find out the optimal inventory level held at any point of time generally we do the first order derivative that is for the average cost has to be equal to 0.

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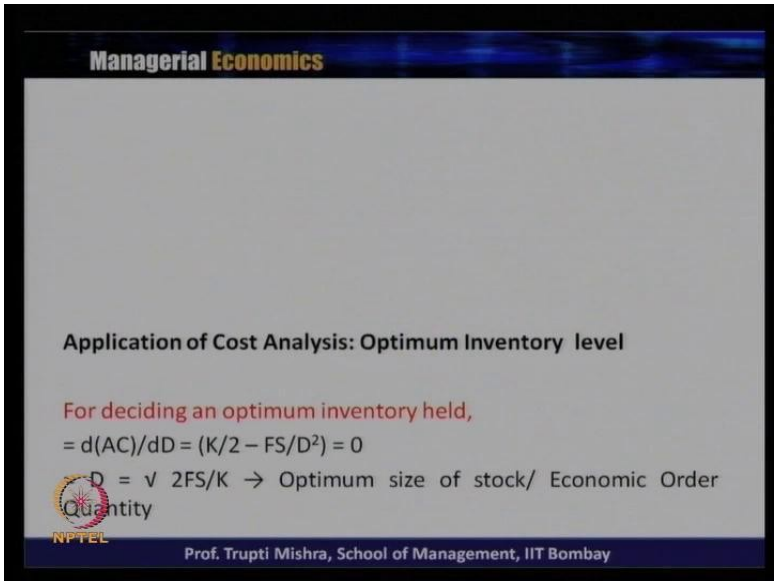

So, d A C with respect to D, so if you take a first order derivative of average cost with respect to capital D, then it comes to $\frac{d(AC)}{dD}=\left(\frac{K}{2}-\frac{FS}{D^2}\right)=0$. If you simplify this for the capital D that is root $D=\frac{\sqrt{2FS}}{K}$, and this gives us the optimum size of stock or the economic order quantity, so popularly this is known as the economic order quantity, and this is the optimum size of stock.

So, D how how do you find it generally we take the average cost that is the first order derivative, we take the first order derivative of the average cost with respect to the capital D equalize it to 0. And then we solve it for D, capital D and that gives us the optimum size of the stock, or that gives us the economic order quantity, so that D comes to be $D=\frac{\sqrt{2FS}}{K}$.

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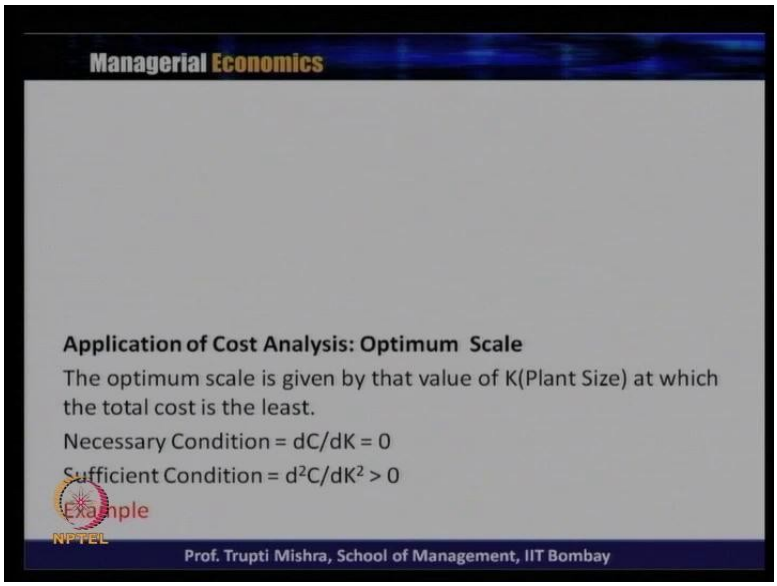

Now, we will look at the application of cost analysis in case of the optimum scale, in the first case we talk about the optimum output, how this cost analysis is used in the optimum output, then we check that how this cost analysis is used in case of the optimum inventory level. Then we will say to identify the plant size, to identify the scale of operation that is the, what should be the optimum scale, what should be the optimum size, what should be the optimum, optimum level where the firm should operate? And will see the use of cost analysis is being there, so the optimum scale is give by the K, that is plant size at which the total cost is less.

So, if you look at there is a significant difference here, from the application of cost analysis here in the last two cases that is optimum output and optimum inventory. Because in previous both these cases, we identify that level of output or the level of stock, where the average cost was minimum, but here we are saying that the optimum scale was given by the value of K or by the plant size at which the total cost is the least.

Now, what is the necessary condition for this optimum scale, the necessary condition for this optimum scale is $\frac{dC}{dK}=0$; that is also known as the first order condition that is $\frac{dC}{dK}=0$, so first order derivative essentially has to be equal to 0. And sufficient condition is $\frac{d^2C}{(dK)^2}=0$ that is the second order condition, where it has to be greater than 0. Now, will take an example to understand that how to use this cost analysis in order to identify the optimum scale.

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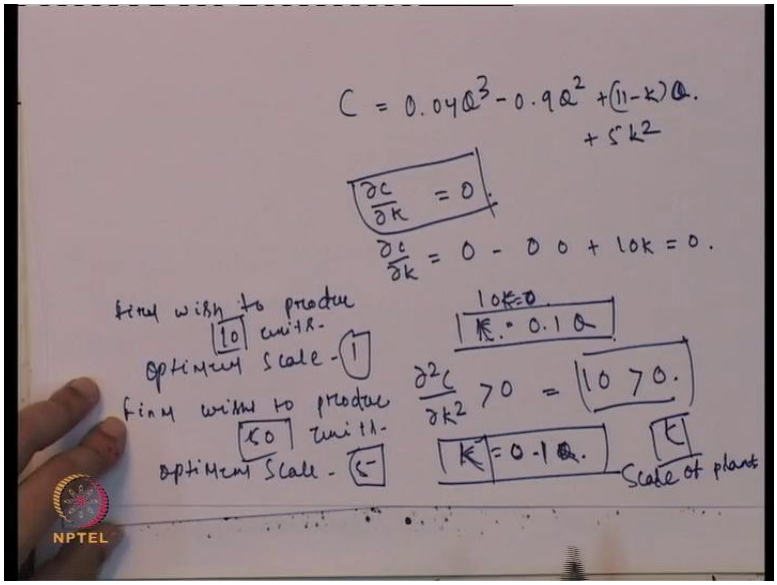

So, let us take a cost function that is C minus C is equal to $C=0.04Q^3-0.9Q^2+(11-K)Q+5K^2$ now what is the first order condition, the first order condition is $\frac{dC}{dK}=0$. So, if you do that, then d c by d k will come to the first term will be 0, the second term will be 0, the third term will be 0, and the fourth term will be 10 k, this is has to be equal to 0, so 10 k is equal so 10 k is equal 10 k is equal to 0 and K is equal to... So, if you look at K is equal to 0.1 Q, so first condition from that we find out that k is equal to 0, and what is our first order condition, first order condition is $\frac{dC}{dK}=0$.

Now, we will look at the second order condition, and the second order condition says that $\frac{d^2C}{(dK)^2}>0$has to be greater than 0, so in this case if you take this, then the first order derivative the second order derivative then we get to equal to 10 which is greater than 0. So, if $K=0.1Q$ that is from the first order condition, this is from the second order condition, so we can say that both the conditions are getting fulfilled that is the first order, that is $\frac{dC}{dK}=0$.

The first order derivative has to be equal to 0, and the second order derivative has to be greater than 0, so in the in this example whatever we are looking at now we are getting both these, condition get fulfilled and we can say that the K is equal to 0.1 Q. And how do you interpret this if the firm is to produce, if the firm is to produce 10 units, optimum scale is 1, and if the firm is to produce firms is to produce 50 units, the optimum scale has to be has to be 5, because $K = 0.1Q$.

And what is K, K is the scale of the plant or size of the plant, so this is how the application of the cost analysis, in case of the optimum level of the scale, identifying the optimum scales. So, if you look at the the cost analysis whether it is average cost, whether it is marginal cost, whether it is total cost, in all these cases generally, it is getting used when the business decision has to be made irrespective of, whether this is related to output, whether this is related to inventory or whether this is related to the scale.

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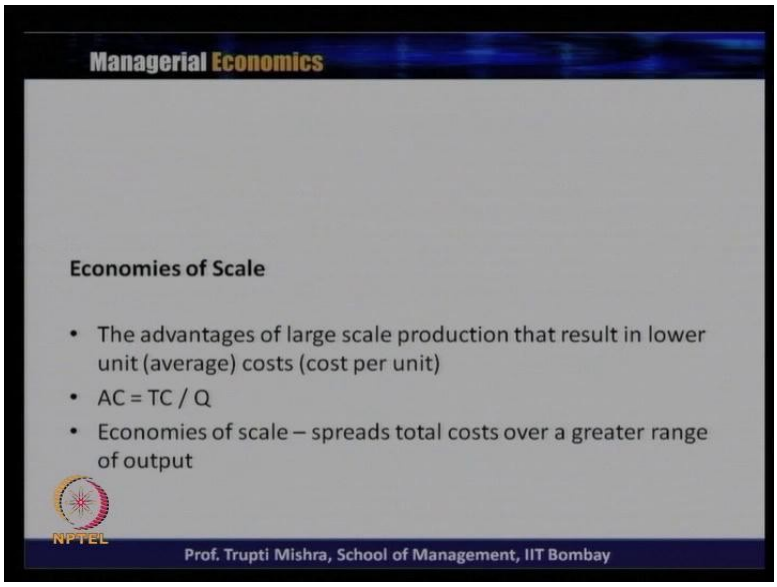

Next, will move to the topic that is economies of scale, and I think we have just the introduced the concept of economies of scale in our previous session, when we are curious enough to know that why the long run average cost curve initially decreases, then reaches the minimum. And beyond that if still the scale of operation of the firm is going on or still the production is going on, then the firm generally, the average cost generally increases. So, let us understand the economies of scale in details, what are the sources, and what are the types of economies of scale, and how generally the firm gets advantage out of it.

So, how do you define the economies of scale, this is the advantage of the large scale production that results in lower unit cost, or you can say that is the advantage of large scale production that results in lower average cost per unit. And mathematically how do you find that the average cost is TC by Q, and economies of scale if you look at it is total cost over a greater range of output, because whatever the total cost that is initially decreasing, then it reaches minimum and increases.

That means that the total cost is not at any point of time generally it spreads over the greater range of output. And that is why we experience decreasing cost we experience constant, average cost and also we experience the increasing average cost.

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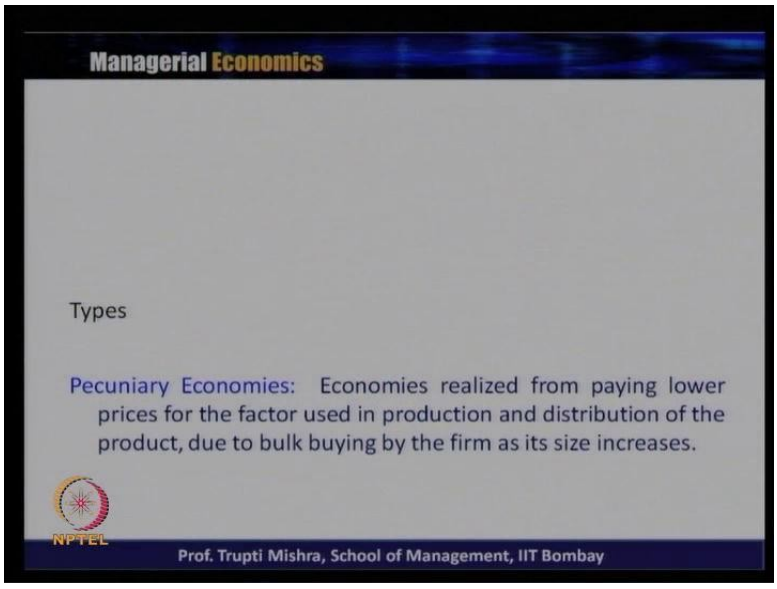

There are two types of economies of scale, one is pecuniary, and this type of economy generally realize from paying lower prices for the factor used in production, and distribution of the product due to bulk buying by the firm as size increases. So, here it is about paying a lower input price, whenever we pay a lower input price, we generally experience the pecuniary economies of scale or we get advantage, in term of pecuniary economies of scale.

So, when the price is being paid less for the input using the inputs, we generally experience the pecuniary economies of scale; that we discuss in a later point of time after we discuss about the real economies of scale. But for the time being the understanding for pecuniary economies of scale, is it is not about using less quantity of input rather it is the less price what we pay for the input.

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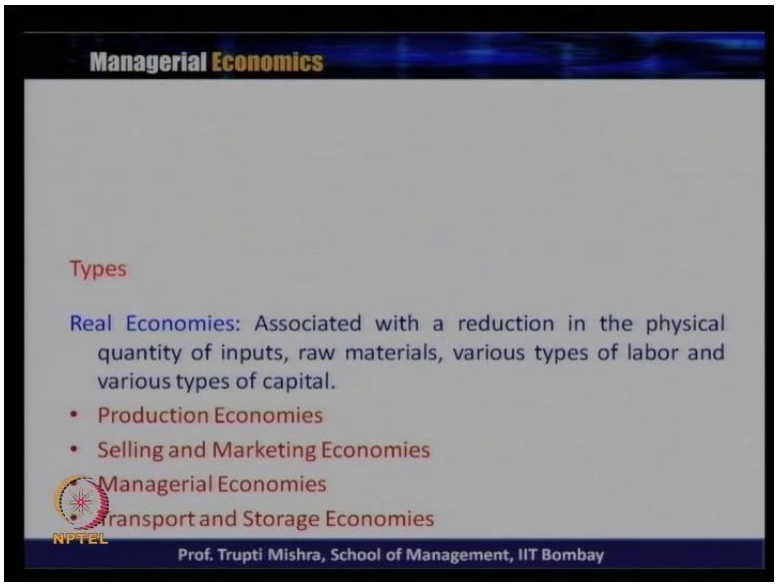

Then the second time is second type is economies of scale is real economy, so real economy is associated with a reduction in the physical quantity of inputs, raw materials, various types of labor and various types of capital. So, pecuniary economy if you remember what we discussed just now, they are not using less input rather they are paying a lower price for the inputs, when the scale of operations increases, and that is how they are getting the advantages or that is how they are generating the economies of scale.

However, in case of real economies it is the reduction in the physical quantity of inputs, raw materials, various type of labor and various type of capital. So, there will be reduction in the quantity of input, there will be reduction in the raw materials, there will be reduction in the labor, there will be reduction in the capital; and if this happens when the scale of operation increases, then only you we can say that the firm is enjoying the economies of scale.

So, we will discuss this in detail this real economies, and there are generally four type of or four category of the real type of the real economies or the real economies is divided into four category. One is production economies, second is the selling selling and marketing

economies, third one is the managerial economies, and the fourth one is the transport and storage economies.

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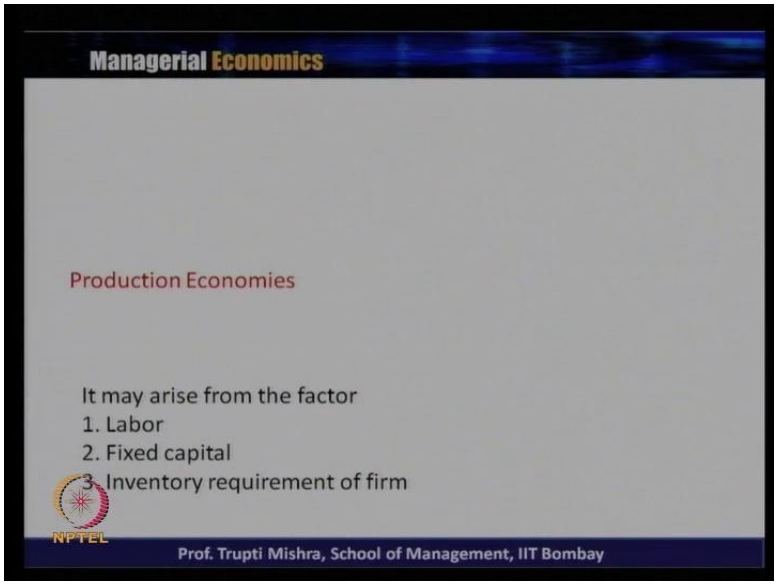

So, we start our discussion with the production economies that is under real economies, how generally there is a production economies of scale, and production economy generally it may arise from the factor that is either from labor, or from the fixed capital, or from the inventory requirement of firm. So, the advantage under production economies of scale, either it comes from labor or from the fixed capital or it comes from the inventory requirement of the firm. Now, when you say that production economies comes specifically from the labor, let us identify the factor, what happens with respect to the labor input that the generally the firm gets the advantage out of it.

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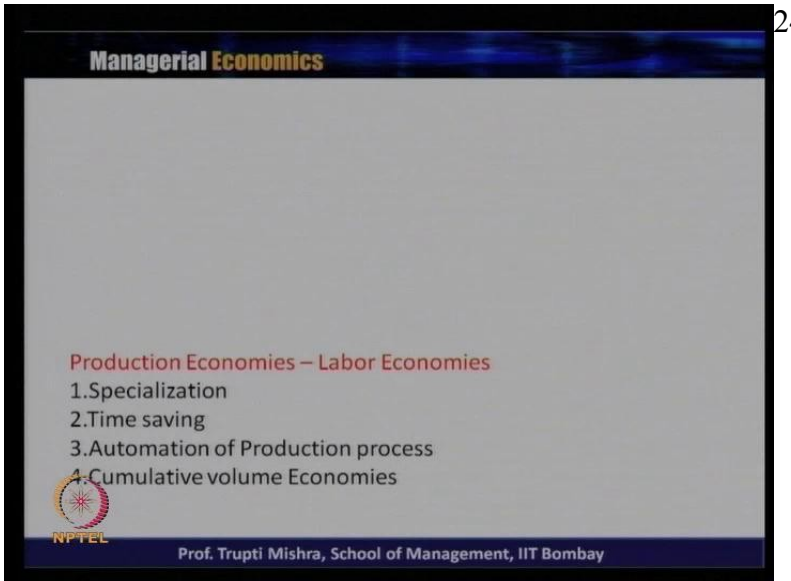

So, when you categorize again in to the labor economy, these are the factor because of what, generally the firm gets some cost advantage or firm, generally may be use less of the labor; in order to produce the same level of output when the scale of operations increases. So, the first factor here is specialization, second is time saving, third one is the automation of production process, and fourth one is the cumulative volume economies.

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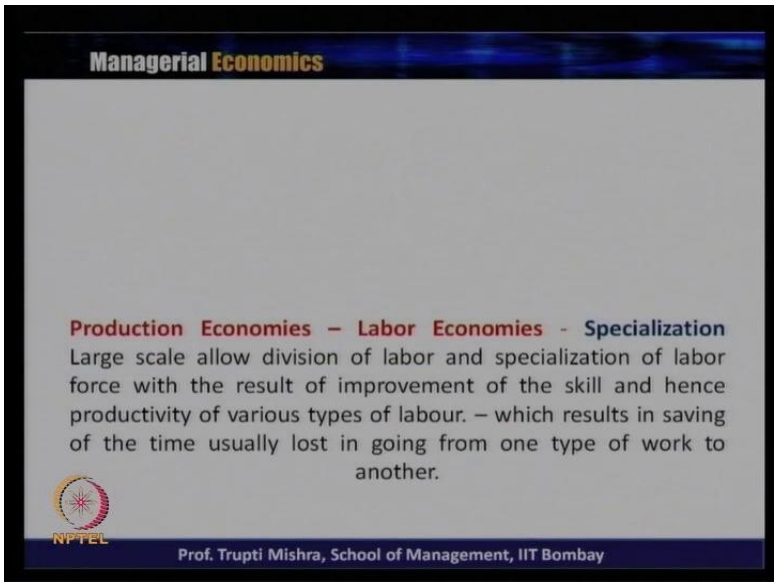

So, will discuss the specialization first and if you look at the sub category, this is part of real economy which again comes under the sub category of production economy, again comes in the sub category of labor economy, and then it comes to the specialization. So, large scale allow division of labor generally, because the number of labors are more that allows the division of labor, and that leads to the specialization of labor force with the result of improvement of the skill. And hence the productivity of the various type of labor, which results the saving in the time usually lost in going from one type of work to another.

So, if you look at in case of large scale, the pool of labor force is more, so each the pool of labor force is divided into the different group, and they have assigned a specific task; and that is how the division of labor on the basis of their skill, on the basis of their expertise. And in

this case what is the advantage, the advantage is that if use the laborer skill is fit into the work, they will do a better work or they will perform in a better way, which result in the improvement of the skill, result in the productivity of the labor. Because they are doing a work which is which is more suitable to them, according to their skill; and that leads to more productivity, and the decrease in the time require to do the work. And that is the reason if you look at, in case of large scale since the division of labor is there that leads to specialization of the work of the labor force and finally, that leads to the efficiency of the, or the productivity of the labor.

And there is less of time and that is bring the cost differential to the firm, and the average cost generally comes down. So, the first factor what we have analyzed is the production economies, in case of labor economies, specialization is the specialization of labor of the division of labor is one, generally where we the firm or the producer in general they get some advantages.

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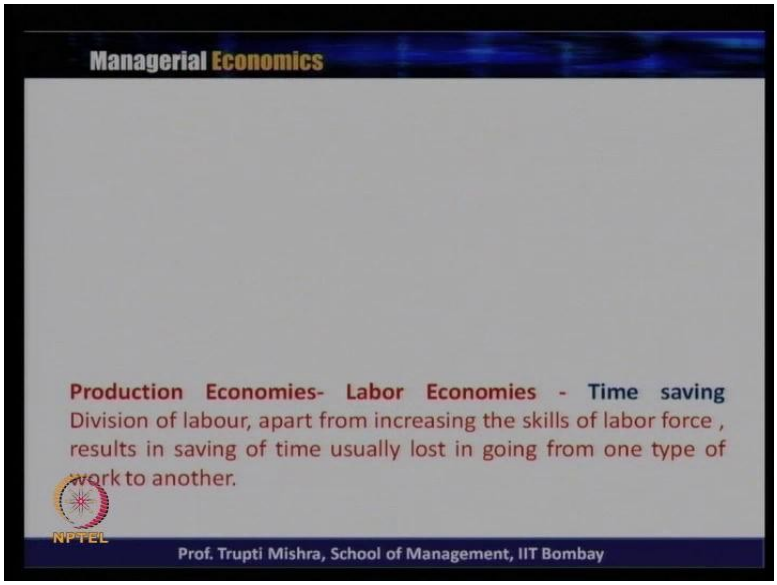

Then we talk about the second factor under labor economies that is time saving. So, division labor apart from increasing the skill of the labor force, results in saving of time usually lost in going from one time to one type of work to another type of work. And the time again brings some productivity, because there is no loss of time the same time is getting utilized for the productivity of the labor. So, if you look at division it is related the division of labor in time saving, in case of small scale firm how it how they do not get a advantage of time saving, because the same group of people either they have to work here or they have to work there.

Because, there is not a different group to specifically work on project 1, project two 2 or project 3, because it is a small operation, but in case of large operation it is possible to divide them, according to their skill, according to their expertise. And in that case they get the advantage in term of productivity, and in term of the time saving which brings the economies of scale to the firm.