Managerial Economics
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Lecture - 37
Theory of Production (Contd…) - I

In continuation to our session on theory of production and cost, we are going to cover few more concepts today, in today's session. So, if you remember in the last couple of session we had just discussing about the different type of production analysis. We started with short run production analysis, and we discussed through the law of diminishing return. And then again we started the return to scale that is the long run analysis of production and there we check that how the scale differ with respect to change in the input, and proportionately the change in the output.

Then we discussed the case of producer equilibrium or so called the least cost input combination, with the help of the two concept; that is isoquant and the isocost, and through which how they reach the how the firms or how the producer they reach the equilibrium.

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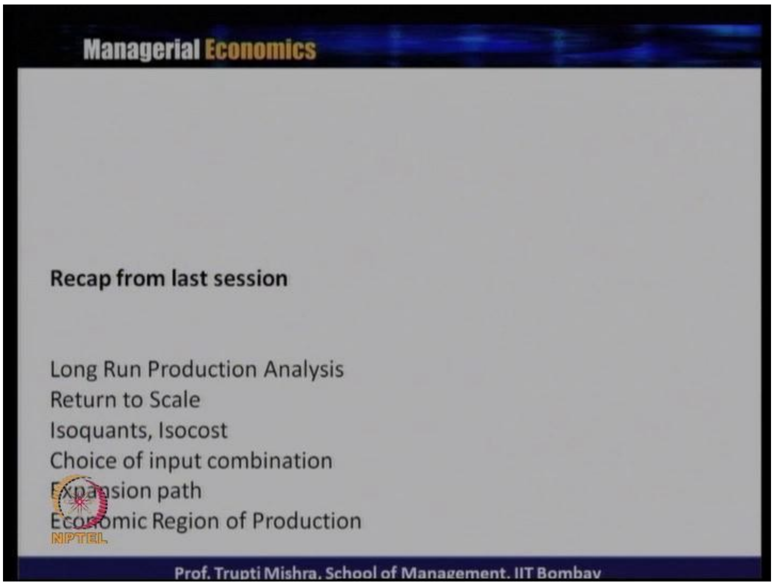

Then we discuss about the expansion path and economic region of production which talks about, basically which one is the feasible region where the two inputs can be substituted one to another. And that is the efficient region because by, producing or because by using less of input the producer is producing the desirable output.

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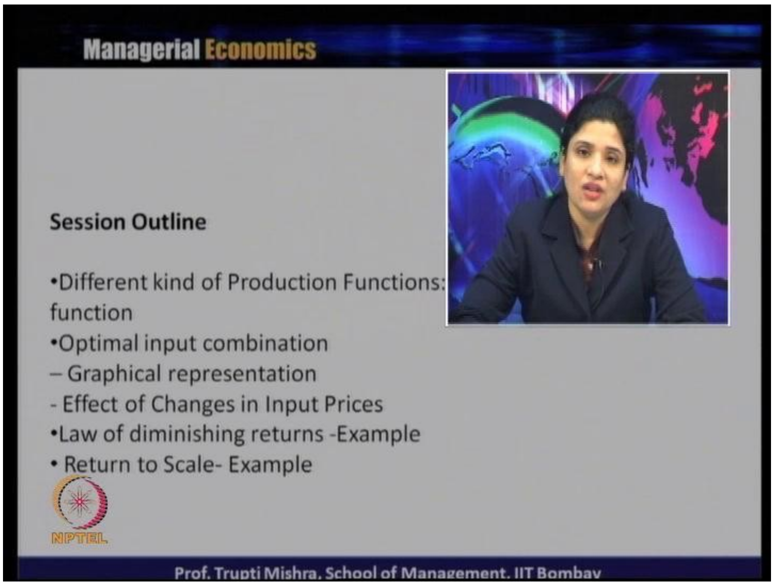

So, in today's session we will see, what are the different kind of production function? Mainly, and mainly we talk about the Cobb-Douglas production function which is used more in economic analysis. Then again we will continue our discussion optimal input combination through a graphical representation, how the graphical representation in case of a maximization of output and minimization of cost.

And again we will see that when there is a change in the input price whether, it is the input price of the capital or input price of the labour, how it changes. Then we will talk about numerical examples related to the law of diminishing return and the return to scale. How generally this, the firm uses this law of diminishing return and returns to scale empirically. Whether it is really works that marginal product gets decreases and then it reaches the negative. And whether there is a evidence of increasing decreasing and constant return to scale.

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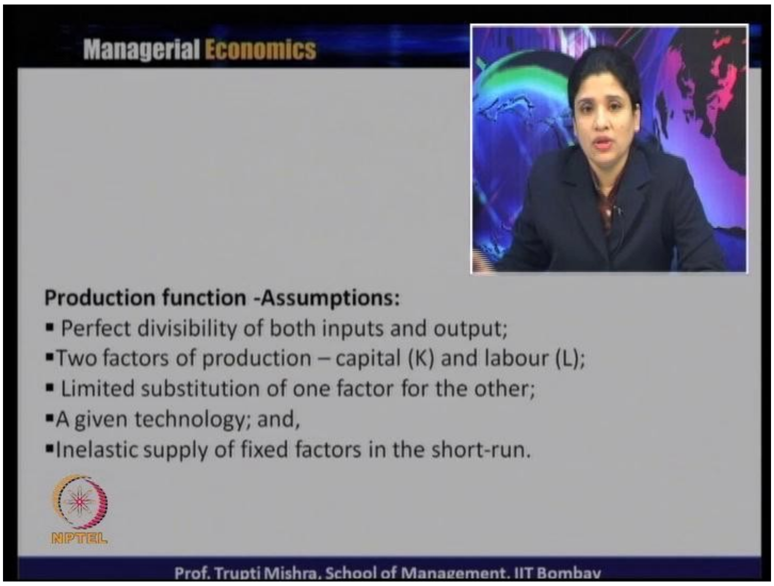

So, to start this today's discussion, we will see that, all the production function they are based on the assumption. It is not that, we can just formulate a production function just taking a functional form, which talks about the relationship between input and output. Rather the production function they, in order to formulate the production function we need to assume certain thing. And what are the general assumptions over here. There is perfect divisibility of both inputs and outputs. So, inputs are divisible and output also divisible.

Two factor of production generally we use, you do not use more than two factors like, if you look at there are number of factor of production, like labour, capital, time, raw material, technology and entrepreneurship. But for all these analyses whether it is short run whether it is long run, we generally use only capital and labour as the input, not any other inputs in the production process.

Then we are assuming that both the factor inputs, that is labour and capital, they are substitute to each other, but they are in a limiting sense. There is no unlimited substitution or they are no closely they are not perfectly substitute to each other. Like if you remember the, if it is perfectly substitute, then the output can be produced either with the help of capital or with the help of labour. But in this case we are assuming that, certain amount of both the inputs are necessary in the production process, the production cannot be run only on the basis of the input.

Or only on the basis of the capital then technology is given technology cannot change may be at least in the short run in the long run it can be changed. And also we assume that there is a inelastic supply of fixed factor in the short run. And that is the reason the short run, there are few factors those are considered as fixed. And in specific sense when we are taking the case of two inputs, here generally capital is fixed and there is inelastic supply. And whenever there is a increase in output or whenever there is a need to increase the production output, generally the labor gets changed in order to increase the output, because when there is inelastic supply of fixed factor in the short run.

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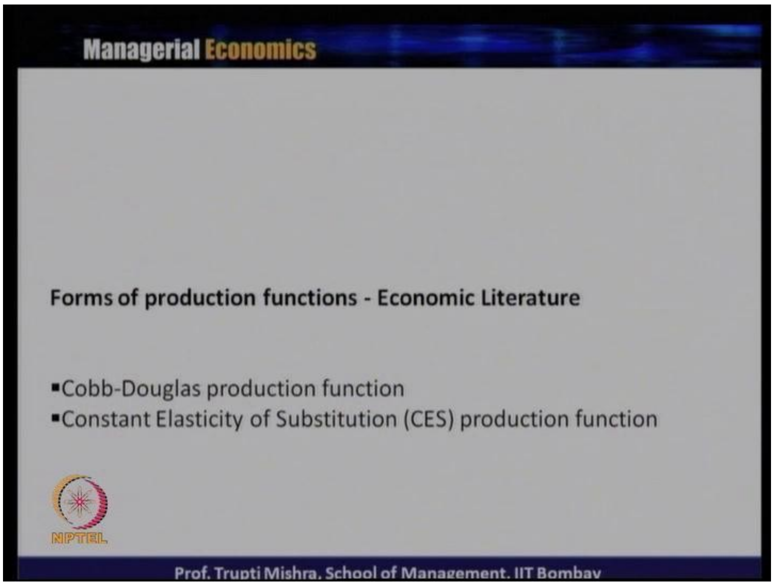

So, in that context, when we in the economic literature there are two main type of production function are used. One, Cobb-Douglas production function and second one is the cost and elasticity of substitution production function that is CES production function. We mainly use, typically in economic literature, either Cobb-Douglas production function or cost and elasticity substitution, popularly known as CES production function.

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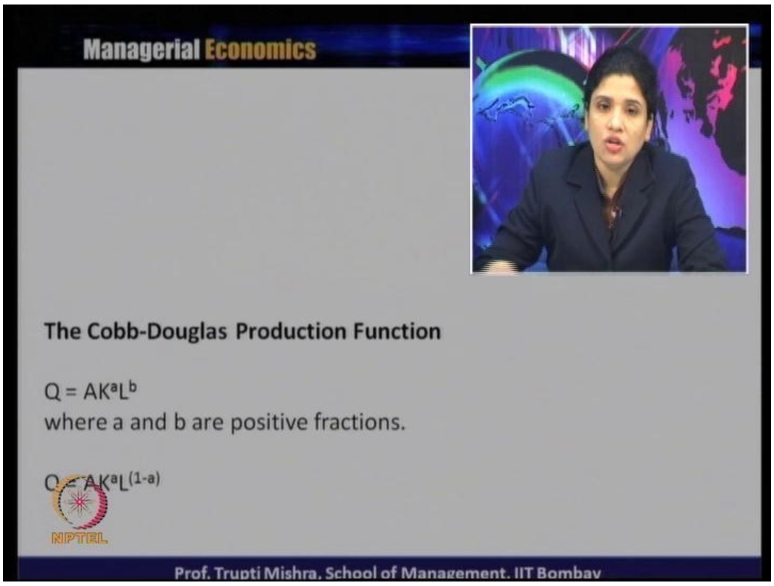

Today we will focus more on the Cobb-Douglas production function, because this is mostly used in case of the economic analysis. Cobb-Douglas production function takes the form of Q that is output, which is a function which is A K to the power a and L to the power b, where a and b they are the positive fractions. And K and L is the, K is the capital and L is the labour over here. So, Q is the A K to power a and alternatively we can take this as L is to the power 1 minus a, because a plus b has to be equal to 1. So, if a plus b is equal to 1 then alternatively we can formulate this production function as Q is equal to capital A K to the power a and L to the power 1 minus a.

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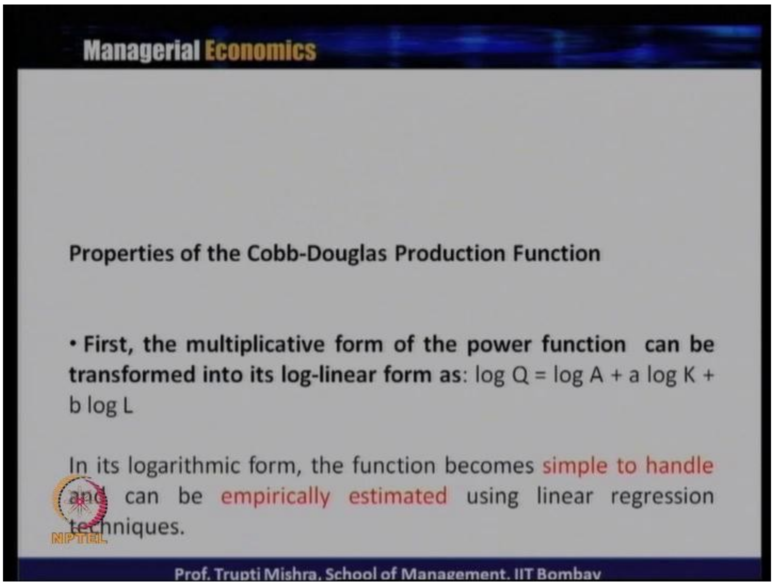

Now what are the properties of Cobb-Douglas production function. Firstly, the multiplicative form of power function can be transformed into a log linear form, like log Q is equal to log A plus small a log K and b log L. so in logarithmic form the function becomes simple to handle and can be empirically estimated using linear regression technique. So, the first property is Cobb-Douglas production function, can be transformed into a log linear form. And why it is generally, what is the benefit if it is getting transferred into a log linear form? It becomes simple to handle, and when we are doing a empirical analysis using the Cobb Douglas production function then this is easy to handle. And using linear regression technique we can empirically estimate the Cobb Douglas production function.

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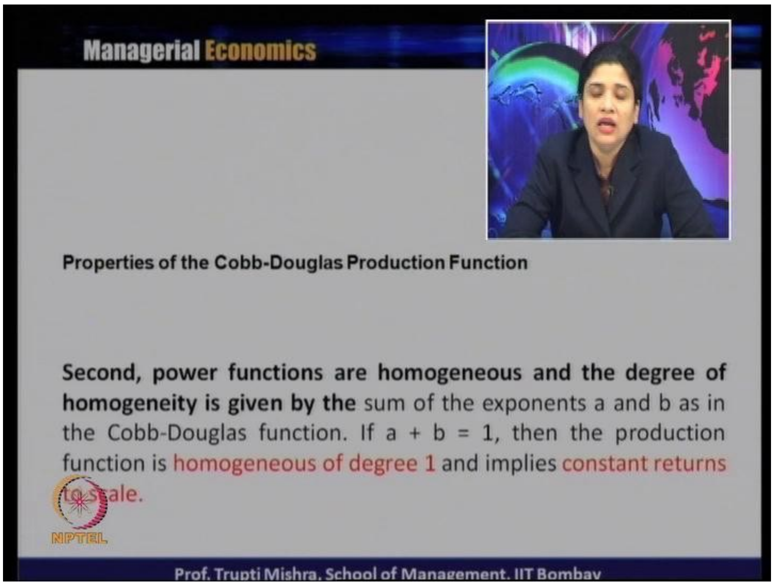

Then secondly, the second property of Cobb-Douglas production function is that, power function are homogenous, and the degree of homogeneity is given by the sum of exponent, a plus b as in the Cobb Douglas function. So, if a plus b is equal to 1, the production function is homogenous degree 1 and implies a constant return to scale. So, the power functions are homogenous. And the degree of homogeneity is given by the sum of exponents of a and b, as in the Cobb-Douglas function.

So, if a plus b is equal to 1 then this is the production function is homogenous of degree 1, and implies constant return to scale. If a plus b is greater than 1 then it implies a increasing return to scale. And if a plus b is less than 1 again it implies a decreasing return to scale. So, it depends up on the value of the exponent in the Cobb-Douglas production function that is a and b, that determines of what kind of production function it is, and what kind of scale it is bearing on.

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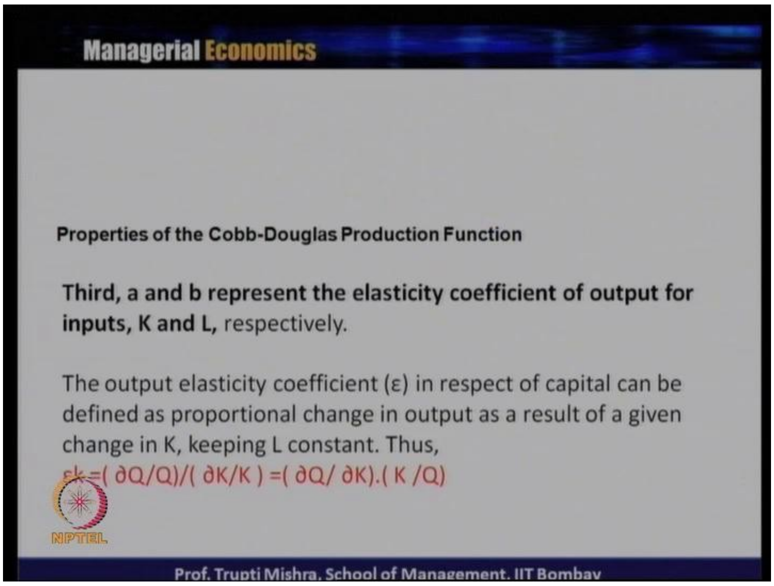

Thirdly, the a and b represents the elasticity co efficient for output for input, K and L, respectively. So, the output elasticity coefficient E in respect to capital can be defined as the proportional change in the output, as a result of given change in K, keeping L constant. So, if you are keeping L constant, and if you are trying to find out what is the elasticity coefficient of output for input, with respect to capital only then this is del Q by Q that is the change in the output, with respect to change in the capital So, del K by K. And if you simplify this then this del Q by del K multiplied by K by Q. So, this is nothing but the elasticity coefficient with respect to input K, keeping L as the constant.

So, partial elasticity of this production function which is a dependent on capital and labour, keeping L as the fixed the elasticity coefficient with respect to capital is del Q by del K, multiplied K by Q.

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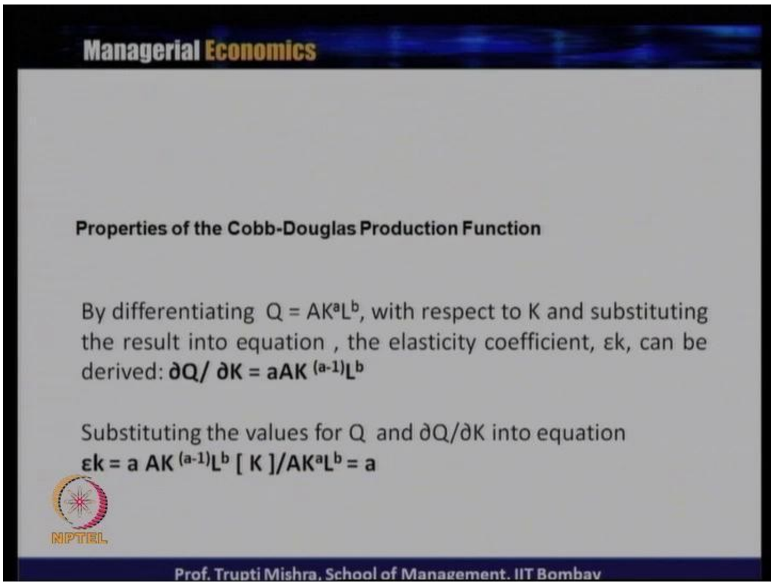

So, taking the specific production function that is Q is equal to A K to the power a L to power b, with respect to K. And substituting the result into equation, the elasticity of coefficient, E K, can be derived as del Q by del K that is a then capital A K to power a minus 1 and L b. So, substituting the value of Q and del Q by del K in the equation elasticity of coefficient with respect to capital says that a A K by a minus 1 L b, and in the important bracketed to have K divided A K a L by b, and when we have simplify this we get it equal to a. So, elasticity coefficient with respect to capital keeping labour as constant the value is equal to a.

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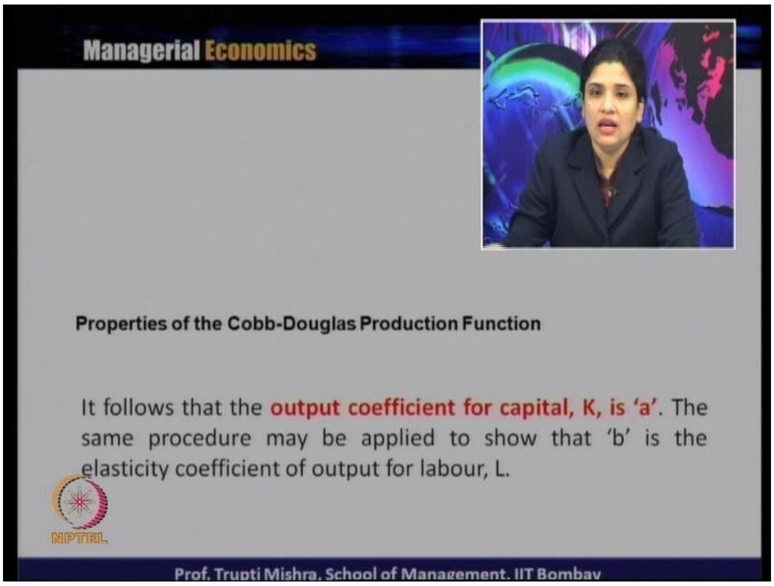

Similarly, when we find out for the b, the same procedure we can follow. And we can find out the output coefficient with respect to, labour capital is constant, and the value of output coefficient with respect to labour is coming out to be b. So, elasticity coefficient of, capital, elasticity coefficient for capital keeping labour as constant it is a. The same procedure can be applied to find out the elasticity of coefficient with respect to labour. And the elasticity coefficient of output for labour is coming to the L. And the value of it will come as the b.

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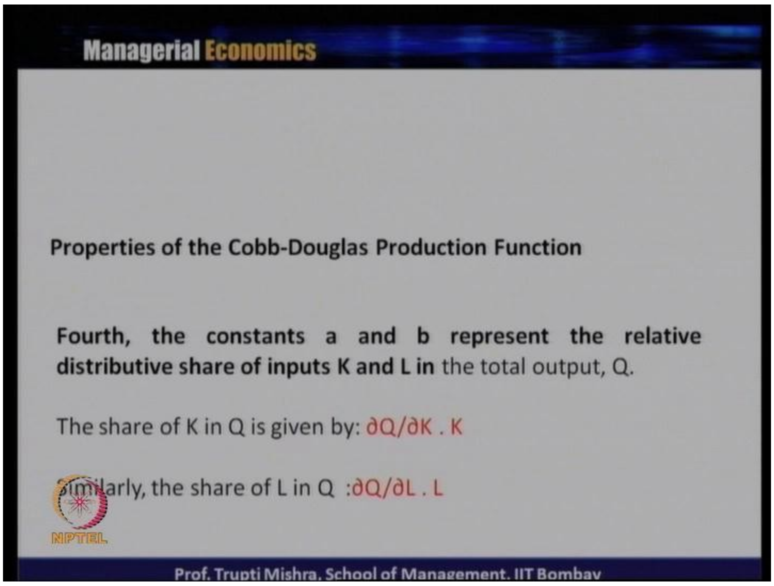

So, properties of Cobb-Douglas production function in continuation with this, we have the fourth property. And here the constant a and b represent the relative distributive share of input K and L in the total output Q. So, fourth property talk about the constant a and b what it represent, so basically constant a and b associated with input capital and labour, represent the relative distributive share in the input K and L in the total output, Q. So, the share of K in Q is given by del Q by del K multiplied by K. And similarly the share of L in Q is del by del L multiplied by L.

So, del Q by del K multiplied by K is the share of K in Q. And share of L in Q is the del Q by del L multiplied by L. So, the, if you look at this del Q by del K multiplied by K, the first part is talks about the change in the Q with respect to change in the K, multiplied by the actual amount of K. And the share of L in Q is that del Q by del L that is the change in the output with respect to labour and multiplied by L.

So, four properties, talks about the constant of a, b associated with labour and capital. And they generally represent that relative distributive share about K and L in the total output.

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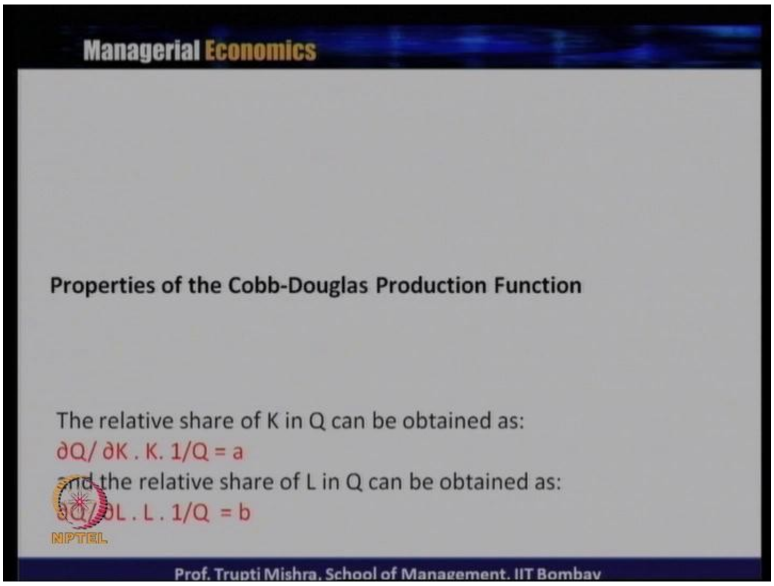

So, in continuation with the fourth property, the relative share of K and Q can be obtained as, del Q by del K multiplied by K multiplied by 1 by Q which comes to a; and the relative share of L in Q can be obtained as del Q by del L multiplied L 1 by Q that comes to b.

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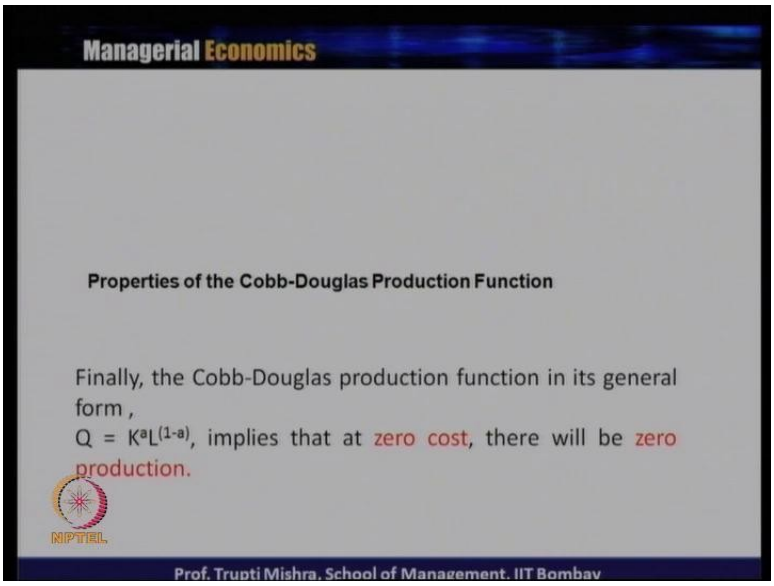

Finally, the Cobb-Douglas production function it is general form, that is Q is equal to K to power a L to the power 1 minus a, implies that at zero cost, there will be zero production, because the value of intercept is or the value of constant is missing here. So, if in the general form if it is Q is equal to K to the power a and L to the power 1 minus a. It implies that at zero cost, there will be zero production because the capital a value is missing over here.

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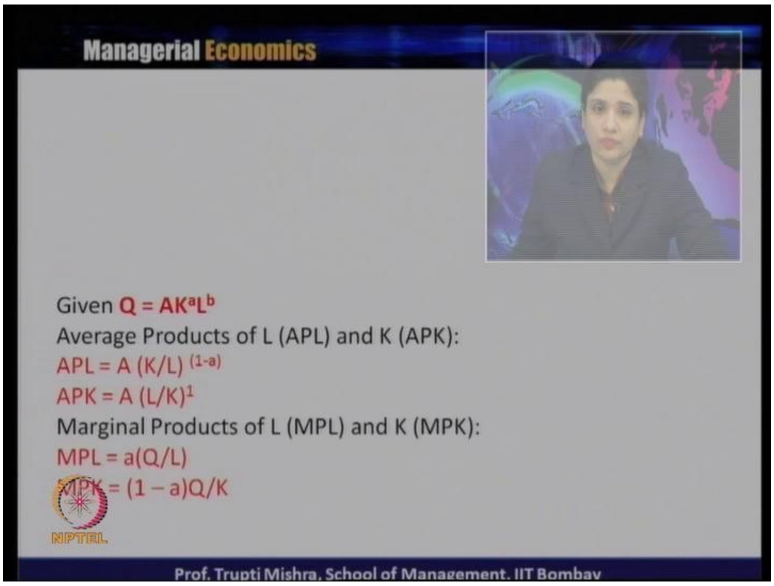

So, given these Cobb-Douglas production function, if the production function is A K to the power a L to the power b. The average product of L is APL and K is APK. So, APL is A K by L 1 minus a and APK is A L K by 1. And similarly we can find out the marginal product for capital and marginal product for labour, MPL is a Q by L and MPK is 1 minus Q by K. So, considering this is a considering this as a Cobb-Douglas production function, accordingly the value of the average product for labour, average product for capital, marginal product for labour and marginal product of capital will change.

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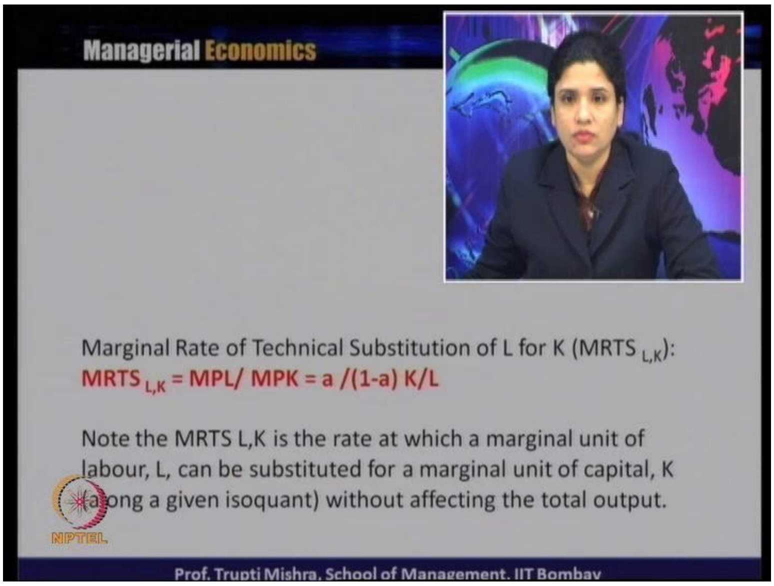

Similarly, when we are finding the marginal rate of technical substitution of L for K, then taking the specifically this Cobb-Douglas production function. So, as we know there is marginal rate of technical substitution is the, slope of the isoquant. And how the slope of the isoquant can be represent? This is the ratio of the marginal product of both the inputs. So, in this case the ratio of the marginal product of capital and labour, so this is, when we are finding out a marginal rate of technical substitution, specifically for the Cobb-Douglas production function, then marginal rate of technical substitution for, L for K, is MPL by MPK that is a by 1 minus K by L.

Here, we have to note that the marginal rate of technical substitution L by K is the rate, at which, marginal rate of L can be substituted for marginal unit capital K along a given isoquant without affecting the total output. So, it is like rate of substitution between two inputs without and the even if the input level is changing or rather amount of getting used as from labor and capital changing, still it has to be in the same isoquant, So, the level of output is not changing. Similarly if you take a serious production function or any other form of production function, we can, in the similar way, we can derive the basic concept using in the production analysis like average product marginal product and the marginal rate of technical substitution, both for L for K and K for L.

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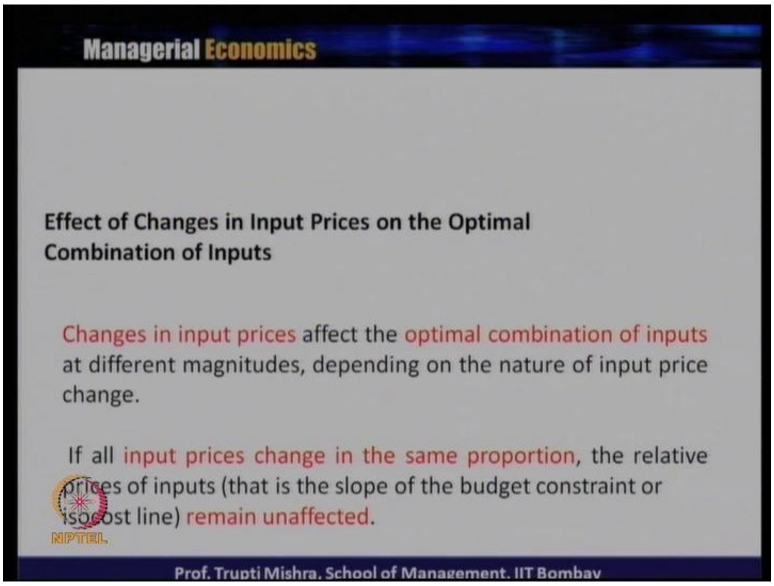

Now, if you remember in the last class we talk about the least cost input combination. And least cost input combination is one where the slope of the isocost is equal to the slope of the isoquant. And this is the point at which the producer or the firm maximizes the output, looking at the given constant.

So, today we are going to spend, may be another, spend in detail that how the equilibrium conditions are derived, how we can say that the slope of isocost has to be equal to the slope of isoquant, we will see. Then we will look at the graphical representation and then we will come to the point where the input price is changes, and how it generally affects the least cost input combination and how the effects are being captured. So, we will first see that how the equilibrium conditions are derived, or may be, how the precondition for the least cost inputs are derived. And then we will look at into the graphical representation, both for the maximization case and for the minimization case.

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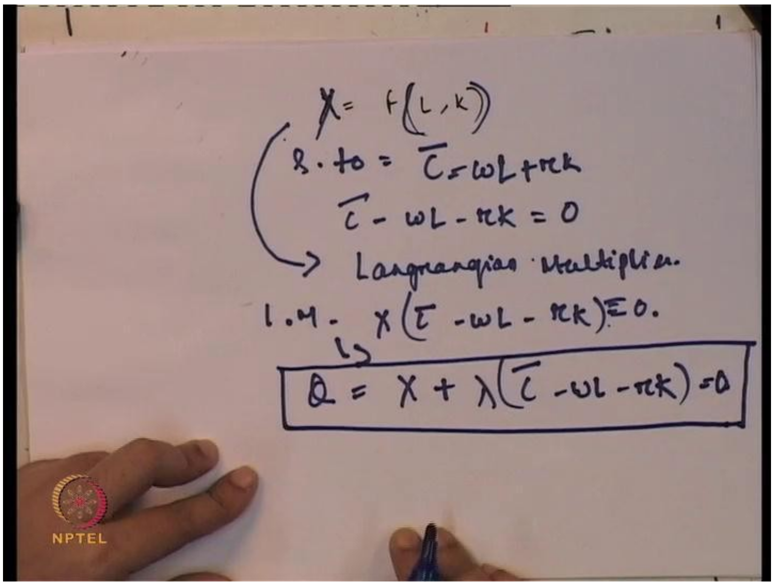

So, let us look at into the equilibrium condition, how the equilibrium conditions are derived. So, this is the, if this is the production function that is for X, for X is the function of labor and capital. In this case, how we can find out, what is the equilibrium condition? Now here there is a constant that is subject to C bar that is w L plus r K.

So, if you remember this is your isocost. Now this, if you can change this constant into this then this is C bar minus w L minus r K which has to be equal to 0. Now we will, whenever we need to maximize something minimize something with a with respect to a constant, in this case we need to use a Lagrangian multiplier. Generally known this as a Lagrangian multiplier method. And here what is the Lagrangian multiplier. Lagrangian multiplier, the Lagrangian multiplier here is X C bar minus w L plus r K which is equal to 0.

Now what is this Lagrangian multiplier? Generally this is the undefined constant or undefined constant which generally use to maximize or minimize a function. Because, if there is a constant associated with this, if there is a constant associated with this, we cannot directly maximize the production function. And that is the reason we need to take the help of the Lagrangian multiplier method. And these are the, the Lagrangian multiplier is the undefined constant which generally use the, to maximize or minimize a function. So, once we get the Lagrangian multiplier method then we will get the composite function. Composite function is X plus lambda C bar minus w L minus r K which has to be equal to 0.

So, this is the composite function, using the lagrangian multiplier method. now what is the next job, next job we need to maximize it; and we will see what should be the first order condition, and what should be the second order condition, in order to maximize in order to minimize. So, given this as the composite function, what should be the first order condition? If you, if you remember, all the first order condition if it is a maximization or may be it is a minimization, the partial derivative has to be equal to 0. So, here we will take the partial derivative with respect to the undefined constant, and we will set then equal to 0 in order to find out the first order condition.

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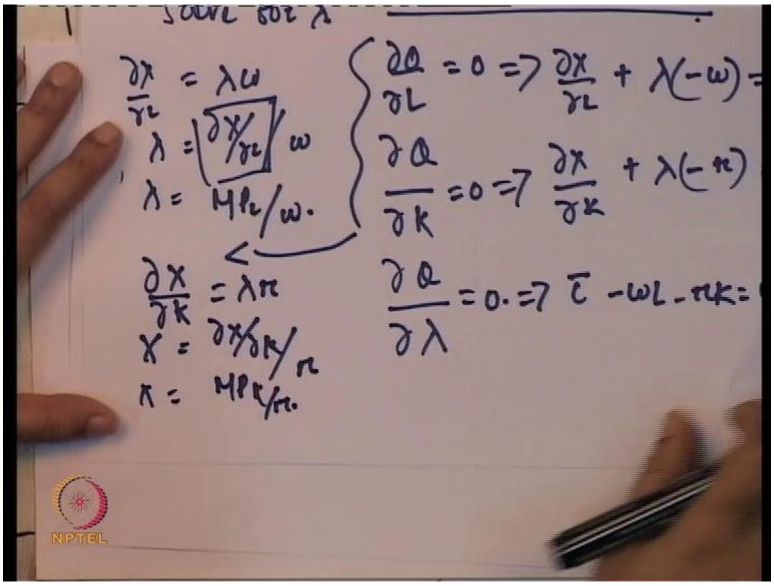

So, del Q by del L which is to be equal to 0; del Q by del K which has to be equal to 0; and del Q by del lambda which has equal to be 0. So, this implies del X by del L plus lambda minus w equal to 0; then this implies that del X by del K plus del lambda minus r is equal to 0; and this implies c minus w L minus r K which has to be equal to 0. So, from equation, first two, if you solve for lambda then this comes to del X by del L is equal to del w or then this equals to del X by del L by w. And this leads to lambda, this is our marginal product for labour by w. Similarly from equation two, if you find, if you solve for value of lambda then this is del X by del K is lambda r or X is equal to del X by del K by r, and this is since this is MPK by r.

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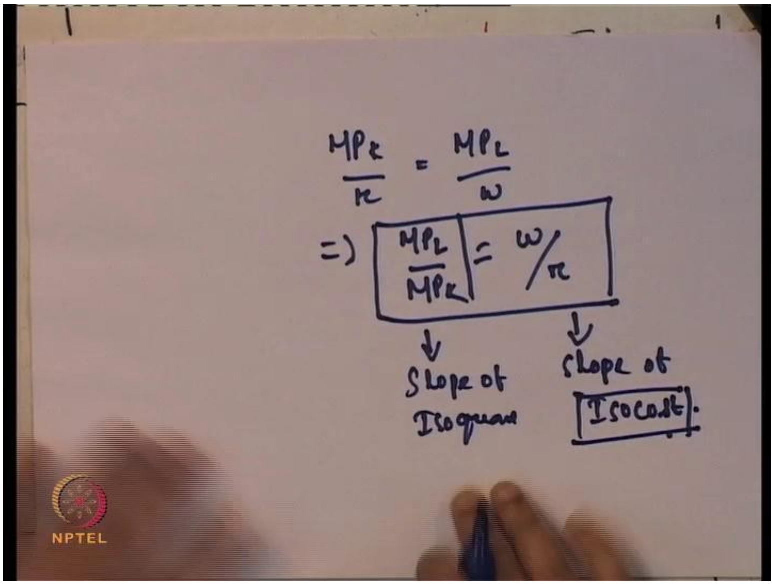

After solving for both this lambda, then this comes from to MPK by r which is equal to MPL by w leads to MPL by MPK equal to w by r. This is the first order condition for the least input combination because, this represents the slope of isoquant, this represents the slope of isocost.

Since the constant is given in the form of isocost, and the output is given; in this case we can say that the first order condition has to be the point at which slope of the isocost has to be equal to the slope of the isoquant. So, the ratio of marginal product of labor and capital gives us the slope of the isoquant; and the ratio of input prices gives us that is w and r that gives us the slope of the, slope of the isocost. So, first order condition for least cost input combination says that, at the point of equilibrium or at the point of least cost input combination the slope of the isocost has to be equal to the slope of the isoquant.

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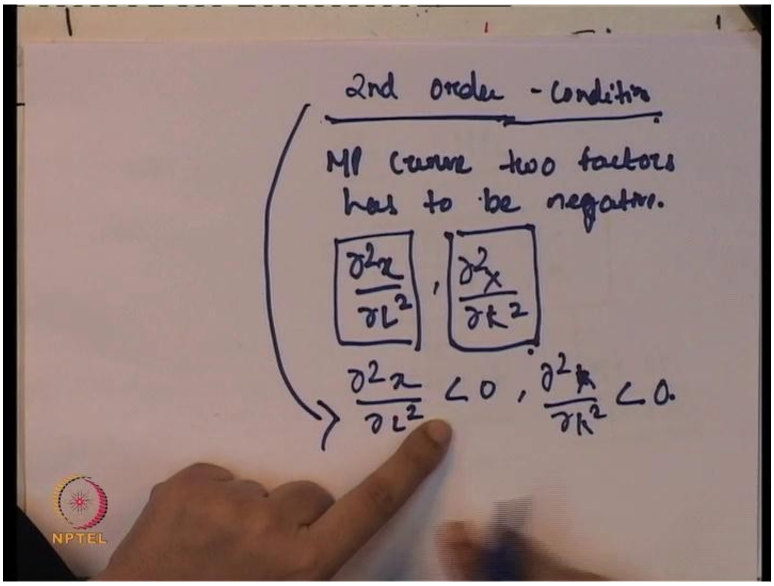

Next we will see, what is the second order condition for this maxima or minima with respect to the least input combination. This requires the marginal product curve for both the factors has to be negative. So, del square X del L square and del square X del K square. So, this is what, in order to find out the slope we need to know the take the second order derivative with respect to labour, and secondary derivative with respect to capital. So, this has to be 0 that is del square X by del L square, has to be less than 0; and del square X and del K square has to be less than 0. So, second order condition for the least input combination requires the marginal product for both the factors that is capital and labour.

The marginal product curve for both the factor has to be negative. And how we will find out the marginal product curve for both the factors from negative? We need to take the second order derivative for the, with respect to capital and with respect to labour of the composite function. And that gives us del square X by del L square for the second order derivative for the labour; del square X by del K square the second order derivative for the capital.

And second order condition says that the it has to be negative, and that is the reason the second order derivative the del square X del L square has to be less than 0; del square X del K square has to be less than 0. Next, we will see how graphically, we look at both the input and the maximization case and the minimization case, in case of the least cost input. So, what is the essential difference between the maximization and the minimization case. In case of maximization case, the cost is given and if taking the cost as constant and the isocost line is the constant, the producer has to maximize the output. Whereas in case of minimization case the output is fixed, and looking at the fixed output, what is the challenge for the producer, the challenge for the firm is to minimize the cost.

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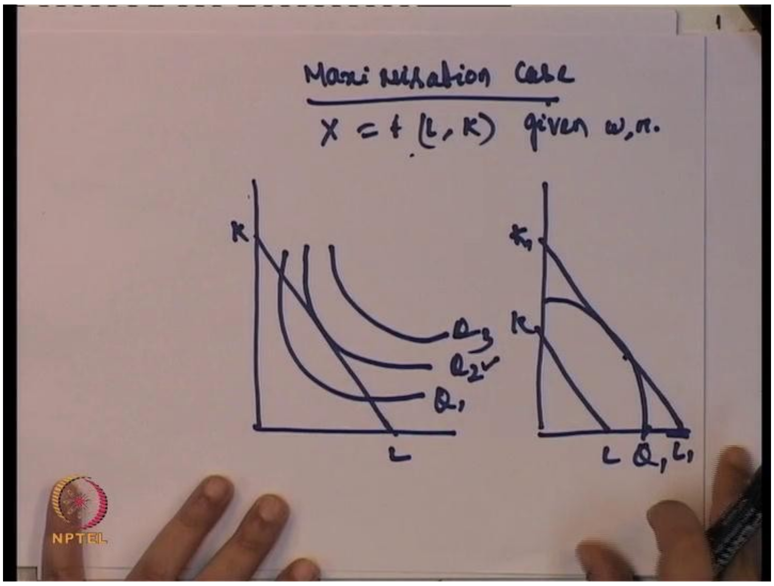

So, let us look at the maximization case first. So, maximize with respect, maximization of X which is a function of labour and capital, with respect to the input prices that is w and r. So, if you look at, there are two graphs; graph one is, where there are three isoquant and the isocost is given; and graph two is where the isoquant is there in a different shape, and there are two isocost line. So, in case of maximization case, what happens? This isocost is given, and with this isocost the challenge of the producer is to get the maximum level of output, and looking at this the consumer will always pick up a combination in Q 2 level of output.

Because the Q 2 level of output can be achieved with the isocost K and L which is given. But in case of, second case if you look at, the isoquant is taking a shape of concave which is not possible because, in case of concave it is not following the basic rule of the production analysis, like if you look at basic rule of a isoquant. Because even if, with the same isoquant they are able to achieve the combination it is not giving the same level of, same level of output across all these stages. May be the input combination are different because they are when they are moving from one point to another point.

They are using more of the inputs of both, but they are producing the same level of output which is not the cost efficient or which is not the input efficient. That is the reason in case of maximization case, the output level is, can be achieved with the maximum output level can be achieved with the isocost line given or in term of the cost is given. Next, we will see the minimization case where the output is given. The challenge for the producer is to minimize the cost of production or minimize the input prices with respect to the given level of output.

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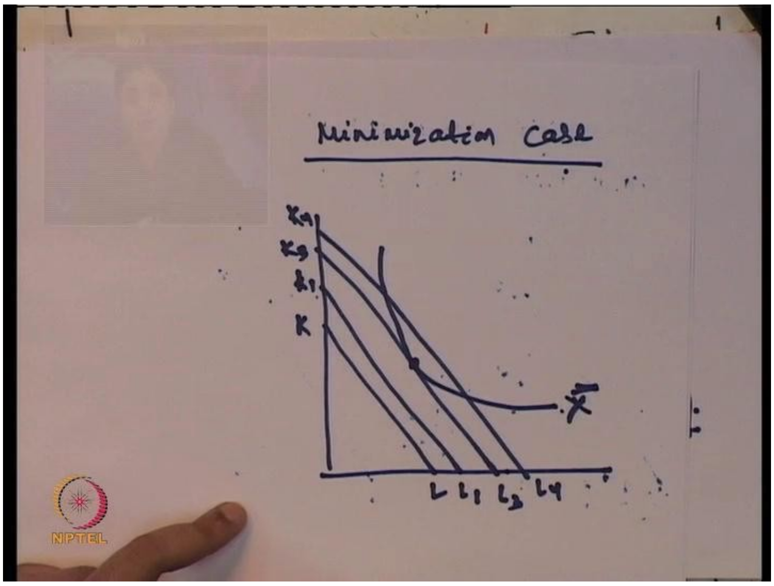

So, these are all is cost. These are the points which talk about the cost of production. If you are taking any of these combination of input prices. And looking at, this if you look, at if the X bar is the output that is given then in this case the producer will always look for this that, to produce this level of output which one can be the minimum cost. So, in this case to achieve this level of output, K 3, L 3, is the minimum is cost or the minimum cost of production that is the reason they will choose this point as the least cost input combination. Because to produce the input level of output this is the minimum possible cost. So, in case of the minimization case, the challenge of the producer, the challenge of the firm is to minimize. Or they will always look for the combination which gives us the least cost to the producer least cost to the firm for a given level of output.