Managerial Economics
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Lecture - 36
Theory of Production (Contd…) - II

As we know, that capital and labour they are closely substitute to each other. So, whenever the producer changes the production process from one level to another level, generally they do changes with a input combination. And when the change in the input combination takes place when the producer is increasing amount of one input, he has to reduce the amount of the other input.

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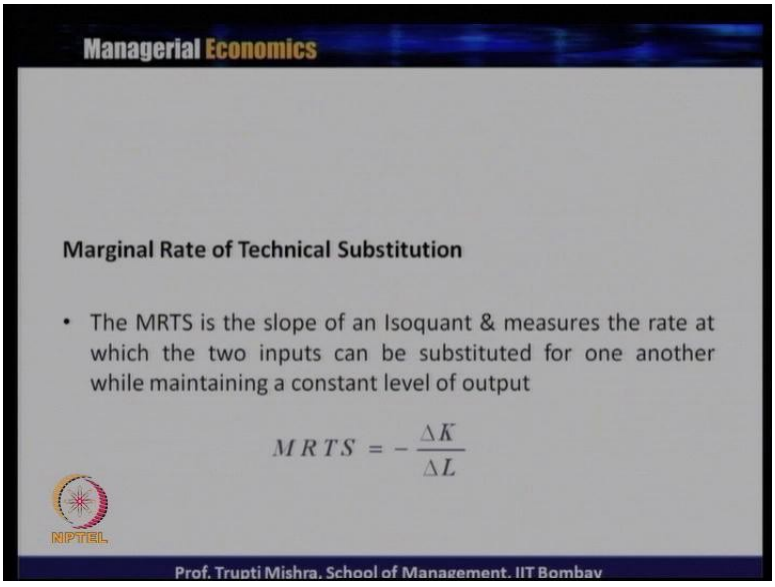

So, marginal rate of technical substitution is 1, this is the rate at which two inputs can be substituted for one another while maintaining a constant level of output. And this is also the slope of the isoquant. So, if you remember the concept of marginal rate of substitution, what we use in case of consumer theory; the counter part of this marginal rate of substitution is, marginal rate of technical substitution in case of production analysis.

So, marginal rate of technical substitution is nothing but the slope of isoquants and it is the rate at which the two inputs can be substituted of for one another while maintaining a constant level of output. So, marginal rate of technical substitution, the change in the K with respect to change in the L, and why this is negative? Because whenever we have to increase the amount of one input, we have to reduce the amount of the other input. So, if you look at

the graph now, how to find out the marginal rate of technical substitution that is from the slope of the isoquant.

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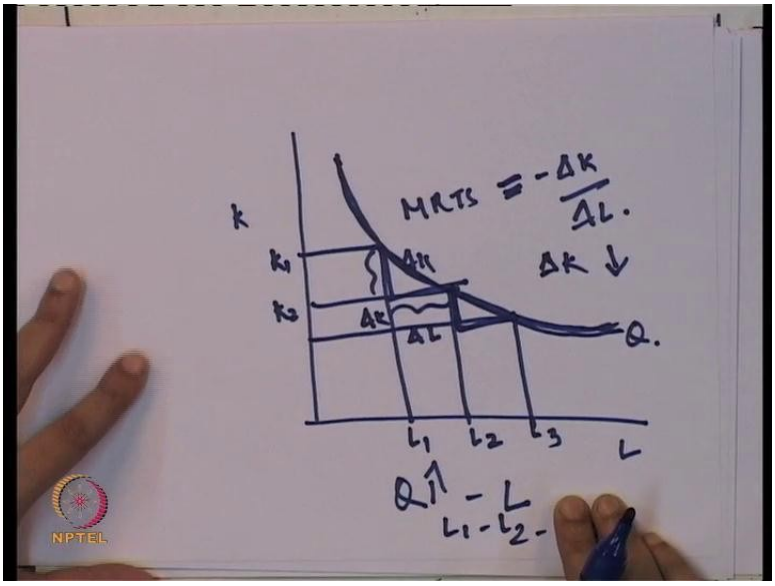

So, in the X axis we take L, in the Y axis we can take K. This is our indifference curve. And the marginal rate of technical substitution is nothing but the slope, that is change in the K, and this is the change in the L. So, marginal rate of technical substitution is equal to minus del K by del L. This is leads to one more properties of the isoquant, which leads to the fact that, marginal, this isoquant is always downward sloping, because whenever we have to increase one input suppose from L 1 to L 2 then there is a decrease from the other input that is K 1 to K 2. Or whenever you are increasing from K 2 to K 1, you have to decrease the labour amount that is used from L 2 to L 1. So, you cannot increase the quantity of or amount of one input, without keeping the fixed, the other has to be decreased then only you can increase it. So, marginal rate of technical substitution is the slope of the isoquant.

And if you look at, this slope goes on decreasing when you, goes on producing may be increasing, producing more by increasing one of the input. So, if you are increasing the quantity, just by changing L, initially from L 1 to L 2 and again L 2 to L 3. The amount what the producer ready to sacrifice to increase this L that goes on decreasing. So, the change in the K goes on decreasing, and that is the reason if you look at, the marginal rate of technical substitution is decreasing, and the isoquant follows a follows a shape of a convex isoquant, because the slope is decreasing. The producer till the time they are going on adding the going

on, going on adding amount of one input in order to increase the output generally the rate at which its get exchanged, the producer is no readier to sacrifice the other input in order to increase the one input on a constant basis.

That is the reason the marginal rate of technical substitution or the slope goes on decreasing, and the isoquant follows a convex shape, now in which case generally the isoquant follows a different kind of shape.

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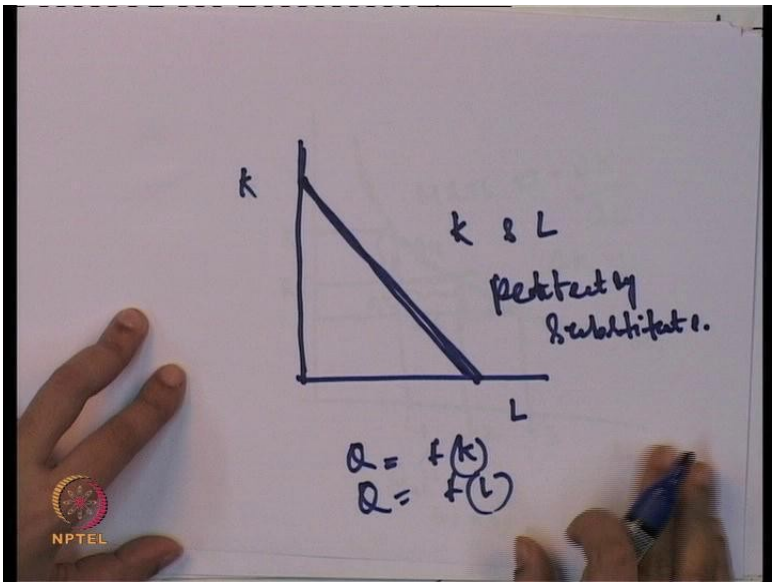

So, if both the inputs they are closely substitute, like we are taking labour and capital here, if both the inputs are closely substitute then isoquant will be a, downward sloping line which touches both the axis, because Q can be produced only with the help of capital, and Q can be produced only with the help of labour. So, indifference curve is takes this shape, if K and L they are perfectly substitute. So, in case of perfectly substitute inputs, the isoquant follows a straight line and it touches both the axis, because output can be produced with the help of either capital or labour.

Now let us understand if capital and labour both are complimentary to each other, you cannot produce output at least by some amount of the other, other inputs. So, it is not cannot be produced only with the help of capital or only with the help of the labour. And both capital and labour they are perfectly complimentary to each other.

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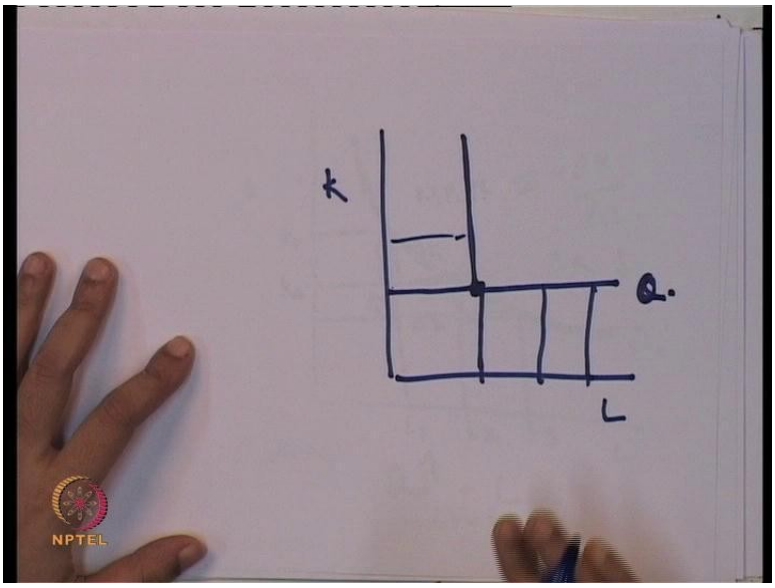

So, in this case the isoquant follows a L shaped curve. And why it follows a L shaped curve because if you look at it is a point rather than a L shaped curve because at this point the level of input is such, that it inverse the combination of capital and labour. Apart from it, whether you use more of labour, or you use more of capital, you cannot produce more amount of output, because this is exactly, perfectly complimentary to each other. Like if you take the example of a monitor and a keyboard, you cannot use only a monitor because the output is nil, you cannot only use the keyboard because the output is nil. So, monitor and keyboard, they are perfectly complimentary input, you cannot produce any level of output if you use only input, or only input, that is there in the form of monitor or in the form of the keyboard.

But interestingly when you have more of the other input also still cannot increase the output. In order to increase the output, you need to increase proportionately both the inputs, like if you have two key boards and one monitor still the output level is not going to increase. In order to increase the output level at least you have to two monitors and two keyboards then only the total output will increase. So, perfectly complimentary, in case of perfectly complimentary inputs, the isoquant, the isoqunat takes the shape of the L, L shaped isoquant. Because it is perfectly complimentary to each other and the equal units of inputs are equal in order to increase the output.

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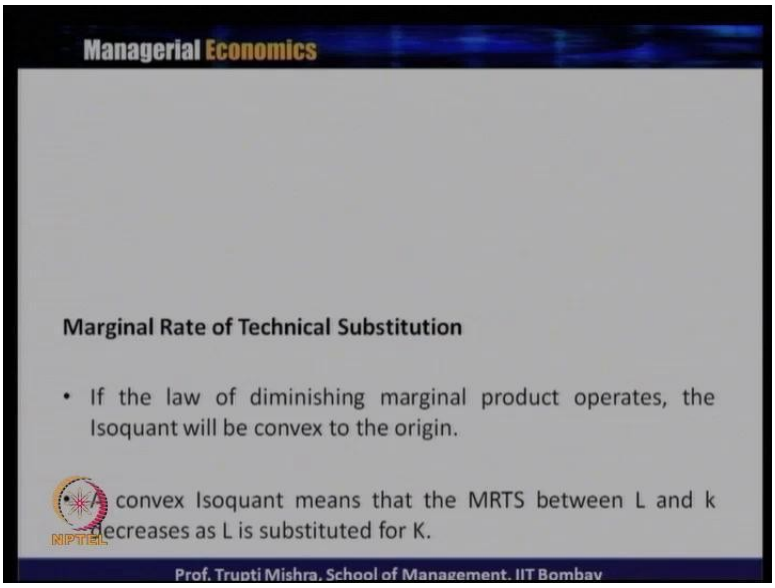

Then next, we will see some, more the points on the marginal rate of technical substitution. And if the law of diminishing marginal product operates, the isoquant will be convex to the origin as we just explained in case of the graphical. A convex isoquant means, that the marginal rate of technical substitution between L and K decreases, as L is substituted for K, what we have already explained through the graph. If you go on substituting the capital for labor, eventually the slope decreases and the isoquant follows a, that is the reason the isoquant follows a convex shape.

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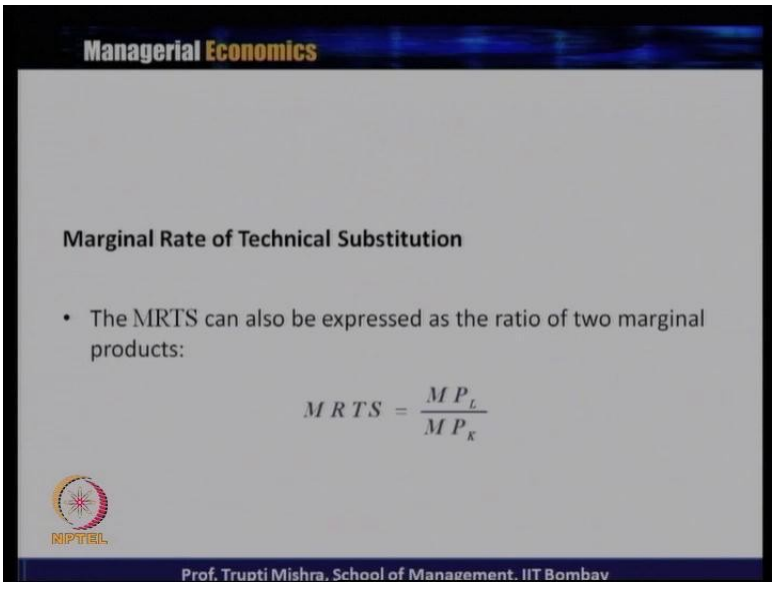

The marginal rate of technical substitution can also be expressed as the ratio of 2 marginal products that is ratio of marginal product of labour and capital.

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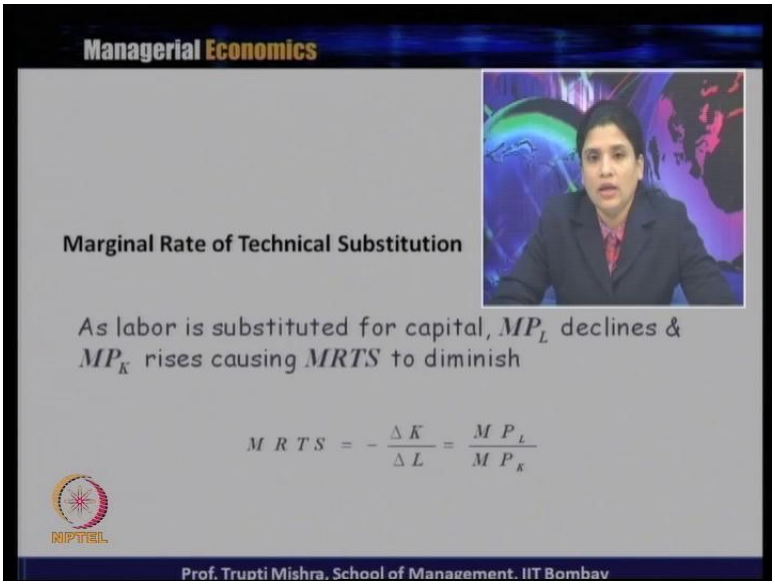

As labour is substituted for capital, generally the marginal product for labour declines, and marginal product for capital increases causing the marginal rate of technical substitution to diminish. So, marginal rate of technical substitution is the change in the K with respect to change in the L or this is just the ratio of marginal product of labour and capital.

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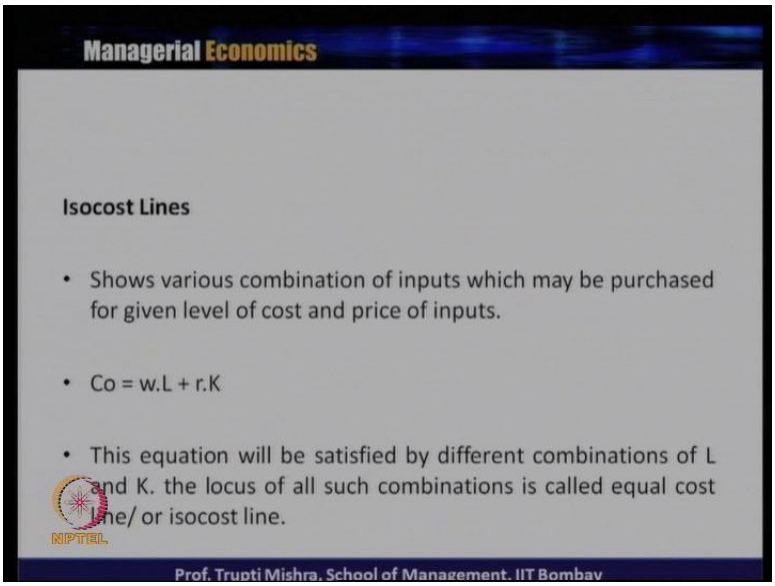

Now we will introduce the constraint over here. Producer can increase the output, by changing any level of inputs. But what is the constant over here, the constant over here is that, whatever the cost of the input, whether the firm can buy the inputs or not, there is always a constant in term of the fund available or the money available to the firm or the industry, that is the reason we introduce the concept of isocost line which is one way the budget constant for the firm, and that restrict them to produce any level of output, by using any level of the inputs. So, isocost lines show a various combination of inputs which may be purchased for given level of cost and the price of inputs.

And generally isocost lines takes the form of C 0 which is equal to w L plus r K where L is the labour and K is the capital, w is the cost of L that is typically the wages and salary, and r is the interest that we generally pay for taking the capital. So, C 0 is a combination of w L plus r K nothing but the, nothing but the price associated or the cost associated with the inputs. The equation will be satisfied by different combination of labour and capital. And locus of such combination is called the equal cost line or the isocost line.

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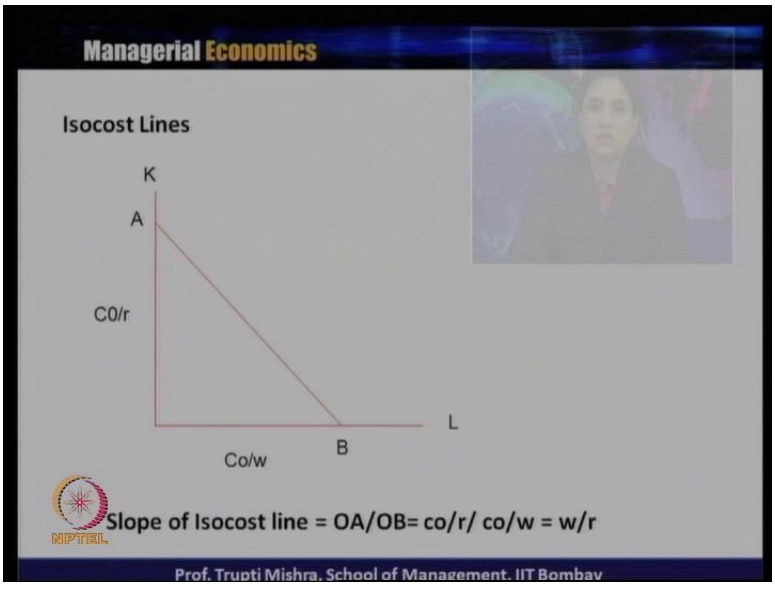

So, C 0 is equals to w L plus r K. assuming that the firm is spending entire fund is only on the, only on the labour or firm is producing the output only with the help of the labour, then this becomes 0.

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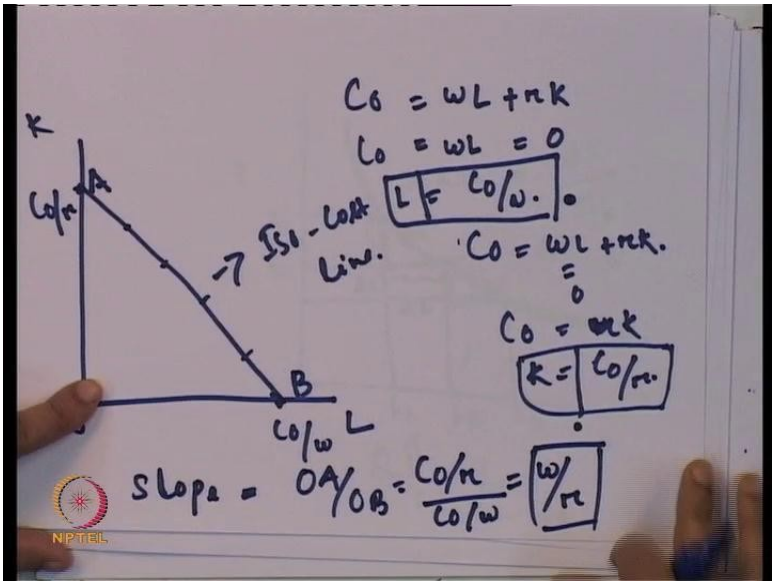

So, C 0 is equal to w L. And if you solve for L then this is C 0 by w. Similarly if the firm is producing the output only by using capital, then this becomes 0. So, C 0 is equal to just r K, and K is equal to C 0 by r. So, if you plot this now, with the help of this, we got two extreme points. So, if the firm is just spending the entire money on, just using labour or just using the capital. So, in this case, the firm is producing the output Q only with the help of labour, in this case the firm is producing the output only with the help of capital.

So, that is the reason we get a value here that is C 0 r and here C 0 w. If you join this two point we get the isocost line. And in case of isocost line point A and point B are two extreme where the entire output is just, or the entire money available to the firm is just getting spent on the labour. Here the entire money available to the firm is just getting spent on capital. In between we have different combination of labour and capital, or the firm is just using different combination of labour and capital, to produce the output level Q. And what will be the slope of the isocost line over here.

The slope will be OA by OB which is C 0 r by C 0 w which leads to w by r. Now, what is this w by r, this is nothing but the input price; w is the price for labour, r is the price for capital. The slope of the isocost line is the ratio of the input prices, because w is the input price for labour and r is the input price for the capital.

So, we, we know isoquant which gives the level of production with the different combination of capital and labour. Isocost which is the budget constant of the firm, because firm cannot go

on producing the output by changing the input level, because there is, always a budget constraint. They cannot just go on adding the input, because they also have to bear the cost of inputs. So, that is represented in term of the isocost line. So, with the help of isocost and isoquant, let us see how to get into the optimal input combination.

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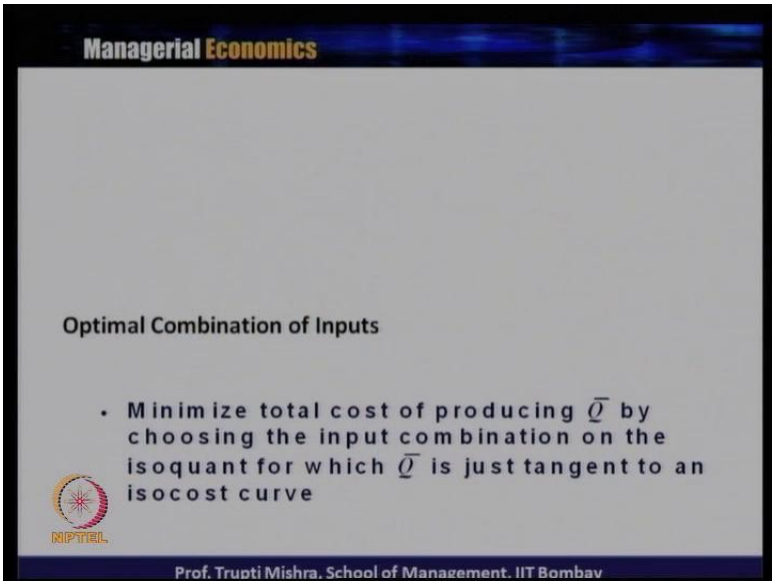

Now what is a optimal input combination? This minimizes the total cost of production Q by choosing the input combination on the isoquant for which Q is just tangent to an isocost curve. So, Q is level of output which is given, and the optimal combination of inputs generally minimize the total cost of producing Q. And how they generally do this, this optimal combination of input? By choosing the input combination on the isoquant, for which the isoquant is just tangent to the isocost curve. So, Q star, Q bar is given level of output, and optimal combination of input will help to minimize the total cost of producing this Q bar. And how to achieve that picking up a combination or choosing a combination on the isoquant, where is tangent to the isocost curve.

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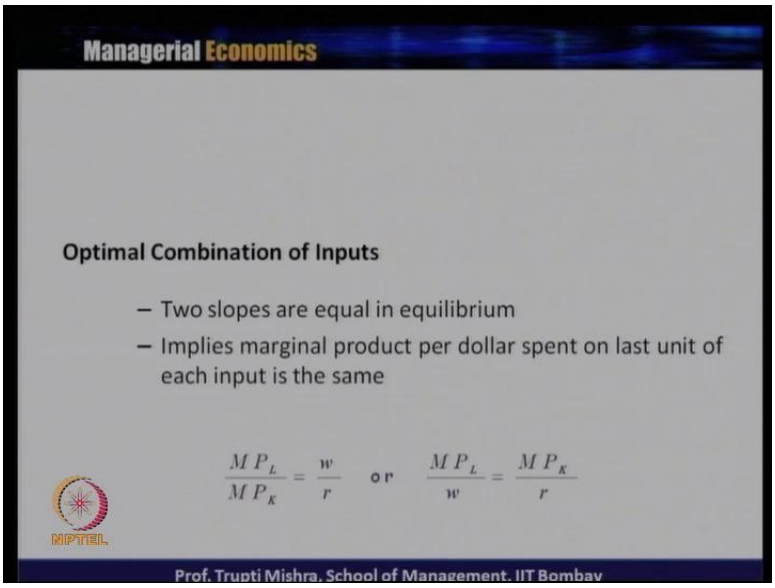

So, the conditions for optimal combination of the inputs are, two slopes are equal in equilibrium, means the slope of the isocost and the slope of the isoquant, which implies marginal product per dollar spent on the last unit of each input is same. So, what is the slope of the isoquant? That is ratio of marginal product of labour and marginal product of capital. And what is the slope of the isocost? That is the ratio of input price that is w and r. So, if you simplify this, then this is M P L by w and M P K by r the first left hand side gives us the ratio of marginal product and input price for labour; and right hand side gives us the ratio of marginal product and input price for the capital. And if the equality is maintained, then in this case we can say this is the optimal combination of input to produce a given level of output.

So, the, what are the conditions for this optimal combination of input? Two slopes are in equilibrium. So, basically the ratio of the marginal product of both capital and labour, should be equal to the input prices associated with the capital and the labour.

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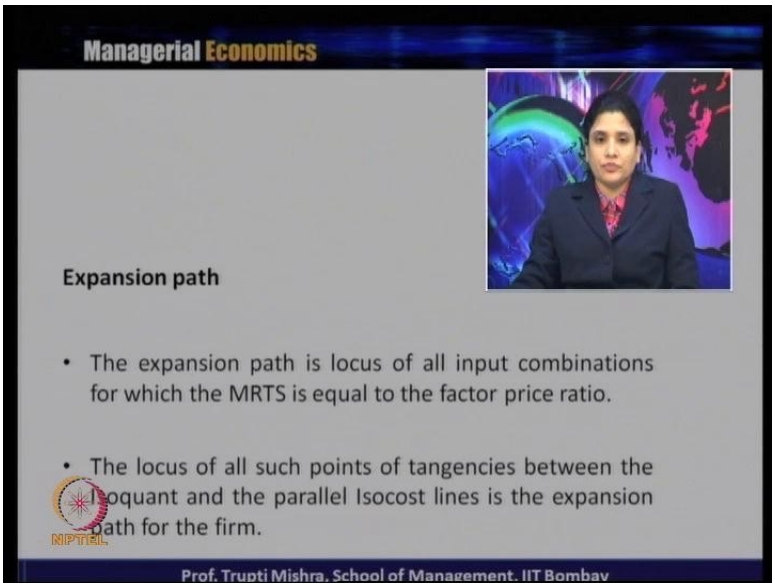

So, now, we will see this graphical representation of this optimal combination of input.

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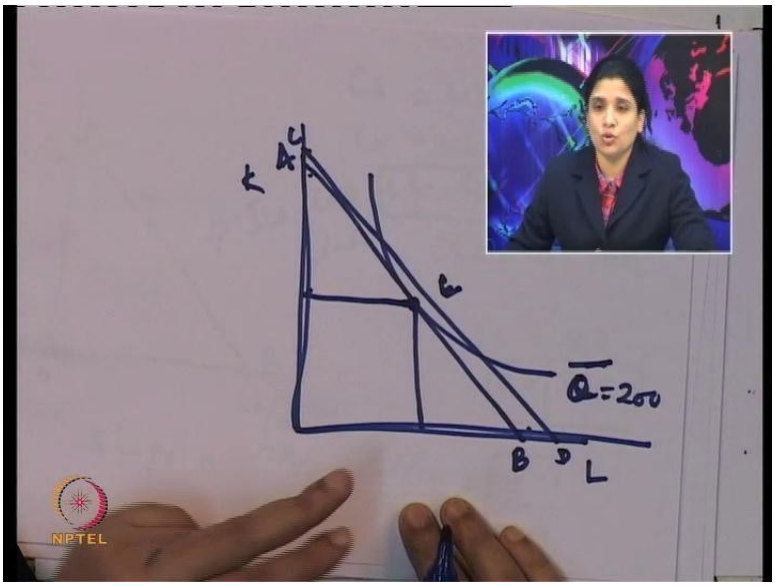

So, Q is 200 suppose this is given. And how to find out the optimal combination of input over here; may be choosing a point, which is just tangent to the isocost line, if the isocost line is A, B, and there is one more isocost line that is C, D. Why C D will not be chosen? Even if it is at the same isoquant crossing the isoquant still it is not tangent to the isoquant. And it is that is the reason it is not going to be chosen as the optimal combination of the input. So, if Q bar is the isoquant which produce 200 unit of level, level of output; and A B is the isocost, in this case point E will be chosen or this will be chosen as the optimal combination of the inputs.

Because at this point the slope of the isocost is just equal to slope of the isoquant, or we can say that the, slopes are equal, or may be the isocost is just tangent to the isoquant.

Then we will understand the concept of the expansion path. And what is expansion path? Basically this is the optimal combination, input combination for the different level of output, and with the different isocost line.

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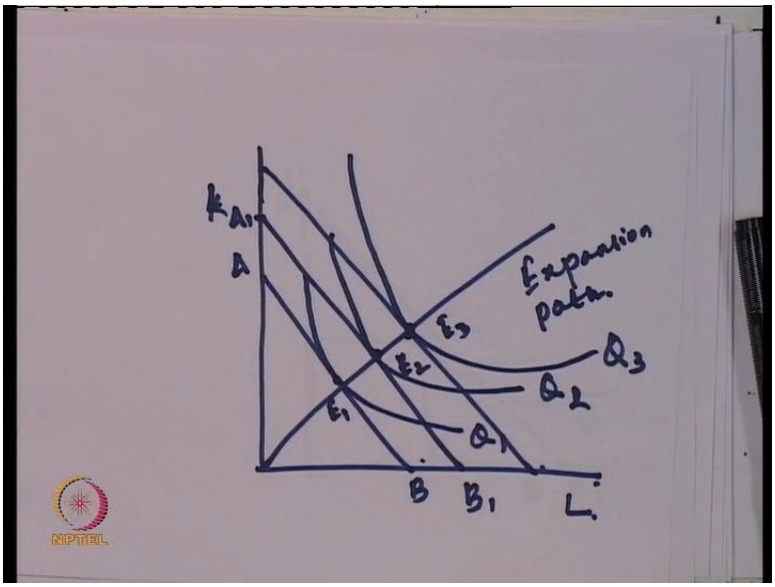

We will take K over here we will take L over here. So, we have Q 1, then we have one input. So, with the increase in the budget constant that is may be A 1 B 1, the producer will always try to get a higher level of output. And the optimal level input combination will be again the same level or the same condition at the at this point where both the slopes are the in equilibrium.

Now, suppose there is again increase in the budget constant, the producer will always try to produce at a higher level of output. And the producer equilibrium, also this point can be called as the producer equilibrium. And if join this, the three point then this is the case of the producer expansion path.

So, Q 1, Q 2, Q 3 is the different level of isoquant. And in order to produce this Q 1, Q 2, Q 3 the producer has to take the help of the different isocost whatever the budget constant given by the firm. And identifying the isocost and the corresponding isoquant, we have three

different level of the optimal combination of input or we call it is a producer equilibrium point. Joining these three points it will gives us the expansion path.

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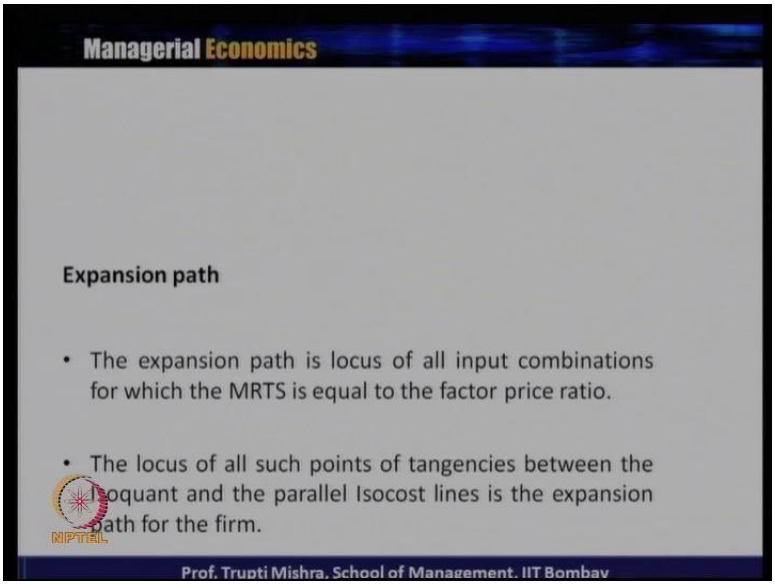

So, expansion path is the locus of all input combination, for which the marginal rate of technical substitution is equal to the factor price ratio. So, if you look at in the graph also, at each point the slopes are in equilibrium, there is a equality in the slope which also implies that marginal rate of technical substitution is the slope of the isoquant. And factor price ratio is the slope of the isocost. So, expansion path is the locus of all input combination, for which the marginal rate of technical substitution is equal to the factor price ratio. The locus of all such points of the tangents between the isoquant and parallel isocost line is the expansion path for the firm.

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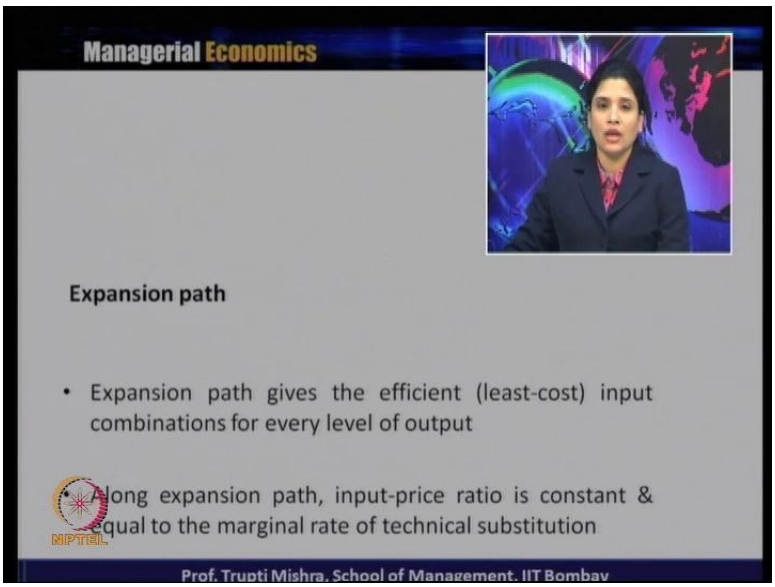

Then expansion path gives a efficient that is the least cost input combination for every level of output, because this is the locus of all optimal combination of input, at the point where the slope of isocost is just equal to the slope of the isoquant. And along the expansion path, the input price ratio is constant and equal to the marginal rate of the technical substitution.

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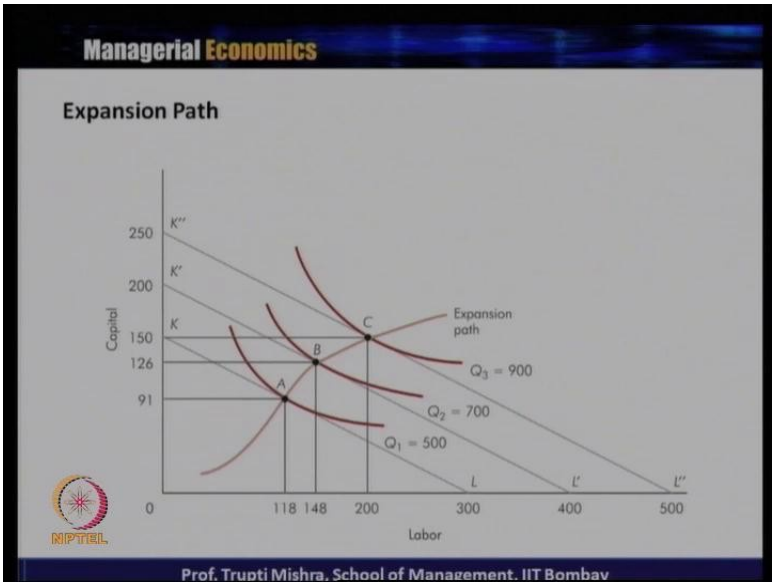

So, this is a typical expansion path, where may be this Q 1 is 500, Q 2 is 700, Q 3 is 900. And Q 1, Q 2 and Q 3 are different isoquants with a different level of outputs. K L is 1 isocost, K dash L dash is another isocost, K double dash and L double dash is another isocost; and A, B, C are three different points which talks about three different level of output, with three optimal input combination. And if you join these three points you get a expansion path which is the locus of the least input combination. So, this expansion path takes the shape on the basis of the relationship between both the inputs the labour and the capital.

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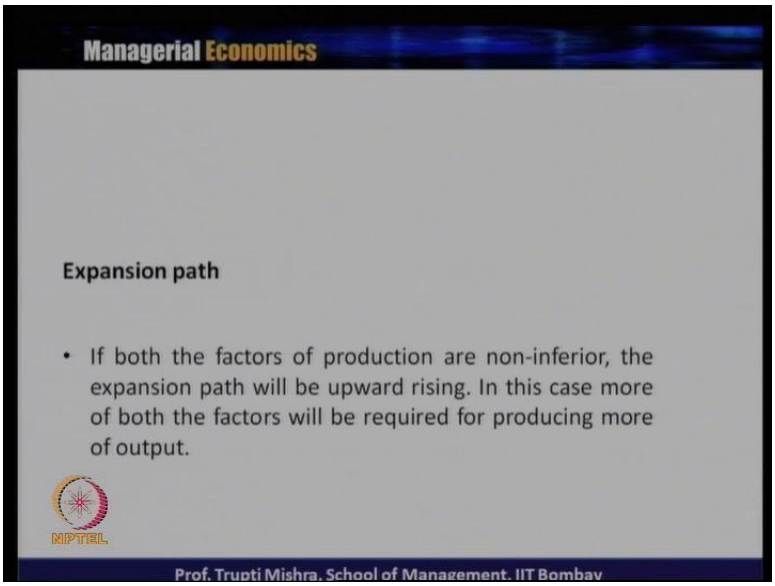

How both the inputs they are related to each other? Whether they are substitute, whether they are complimentary, or whether they are, may be perfectly substitute to each other. And also if both the inputs are not inferior, the expansion path will be in upward rising. In this case more of both the factor will be required for producing the more output.

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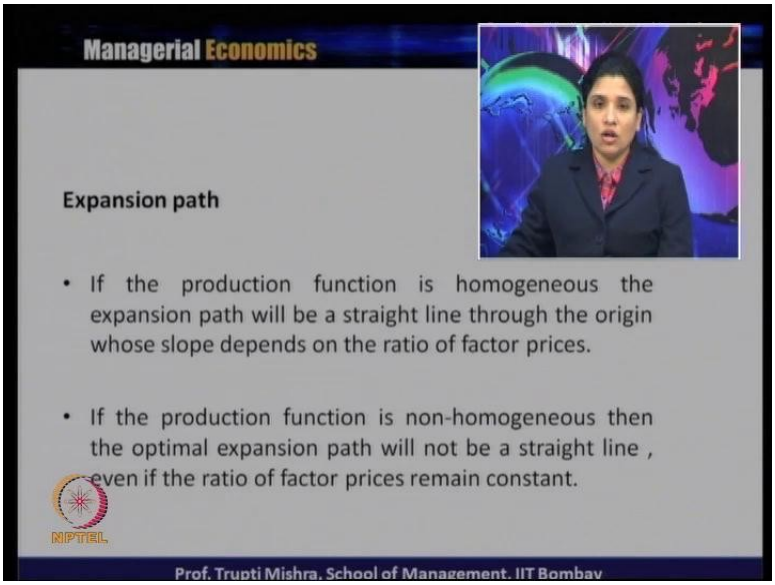

But if it is homogenous, if the production function is homogenous; the expansion path will be a straight line through the origin whose slope depends on the ratio of the factor price. So, if it is non inferior, it is upward sloping; if both the input they are non-inferior then it is upward sloping; if the production function is homogenous then expansion path will be straight line, through the origin whose slope depends up on the ratio of factor prices.

And if the production function is non homogenous then the optimal expansion path will not be a straight line. Because even if the ratio of factor prices remain constant, in case of non-homogenous, it is not a straight line, it is a somehow like a zigzag, even if the ratio of factor price remain constant. Because it is not, it is not homogenous. So, whenever the input there changing, it is not changing by the fixed proportion, rather it is not changing by the different proportion.

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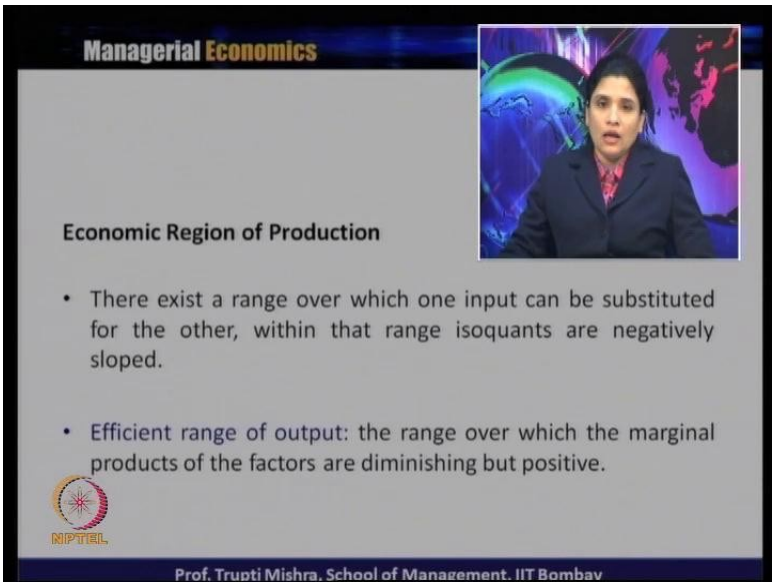

Then we will talk about the economic region of production. How this economic region of production comes in to picture over here, because if you remember two inputs they are closely substitute to each other, they, the producer goes substituting one input for the another input. In order to, may be change the input combination, or sometimes just to see where what is the availability of the resources, availability of the inputs are there. But the question comes here that, how long one input can be substituted to the another inputs.

Because, if it is closely substitute, then only it goes on to the extreme which is x axis or the y axis; otherwise it is just there is a limit with which the inputs can be substituted to one to another. And the typical region is generally called as the economic region of production. So, there exist a range over which one input can be substituted for the other within the range of isoquant that are negatively sloped. And this is also the efficient range of output, because this is the range over which the marginal product of factors are positive, and it is not negative.

Once the marginal products for the factors are not negative, they cannot be substituted to one to, one input to another input. We will understand this relationship between the input substitution, and we will identify the economic reason of production in a graphical manner.

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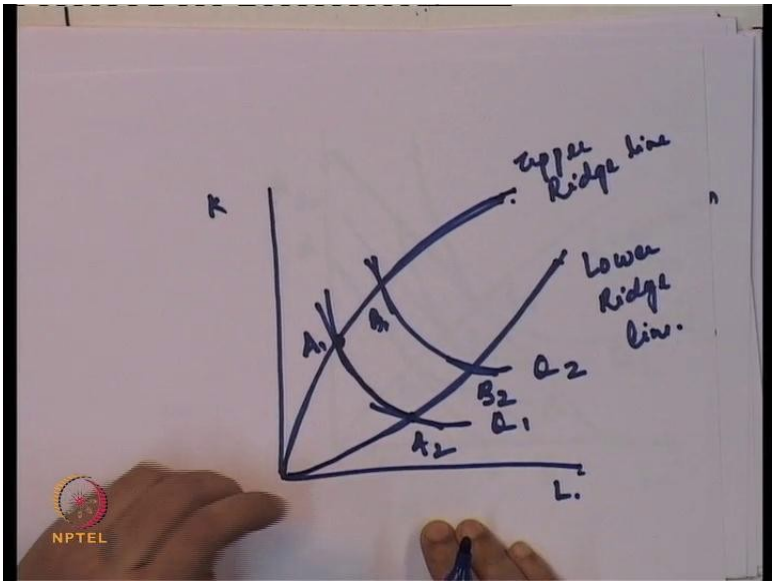

So, we have 2 isoquant, Q 1 and Q 2. And on the basis of, we have two concept here one is upper ridge line and one is the lower ridge line. Production does not takes place when the marginal product of the factor is negative. The locus of points isoquants where marginal products is 0, this is generally known as the ridge line.

So, whether it's upper ridge line or whether it's lower ridge line, the locus of all this, the ridge line is the locus of the points where the marginal products are 0. So, in this case if you look at, the upper ridge line is the point is where the marginal product of capital is 0.

And the lower ridge line is one where the marginal product of labour ridge is 0. And within this ridge line, these are the efficient range where the one input can be substituted for the other input. So, production techniques are technically efficient, if you look at the technically

efficient, if inputs are substituted for one another where the marginal product is not negative. It is between the range where it is positive.

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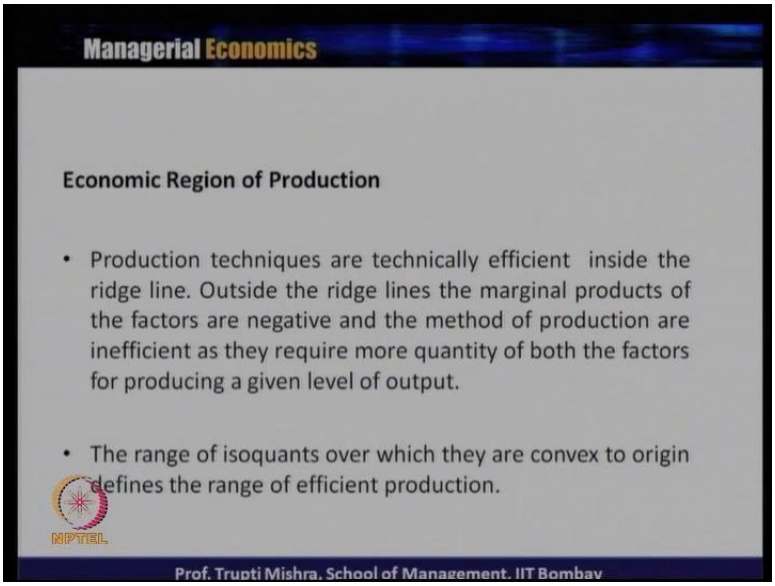

So, production techniques are technically efficient in techniques inside the ridge line. Outside the ridge the components of the factors are negative, and the methods of production are inefficient. As they require more of both the factors for producing the given level of output. The range of isoquant over which they are convex to origin, defines the range of efficient production.

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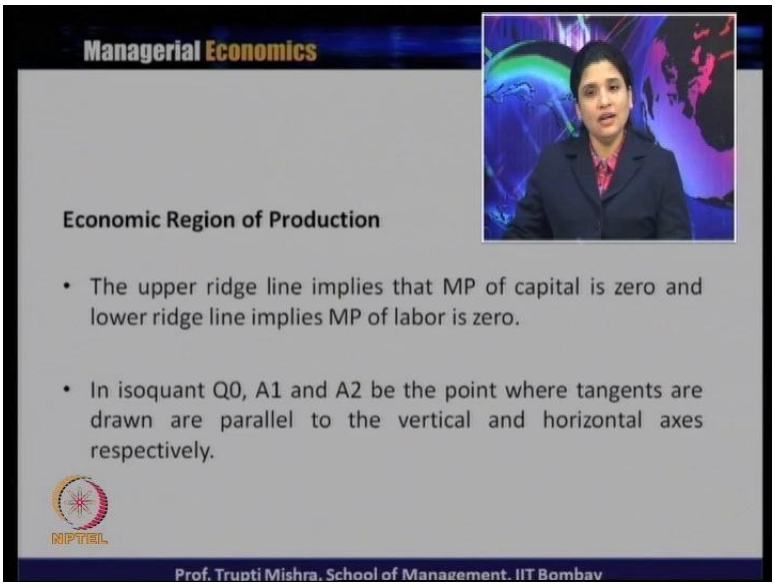

So, in this case, the figure, the upper ridge line the marginal product of the capital is equal to 0. And in case of lower ridge line, the marginal product of labour is equal to 0

Beyond upper ridge line, marginal product of the capital is negative; beyond lower ridge line, marginal product of labour is negative. For isoquant Q1 and Q 0, A 1 and A 2 is the range where inputs can be substituted to one another. Similarly for isoquant two, the between point B 1 and B 2, input can be substituted to one to another. And this range is generally known as the efficient range of production, because beyond this point if you look at you are using more of input, but still we are getting the same level of output.

So, generally this is known as the efficient range of production at the different input level. Like suppose, if you introduce one more level of output then this is Q 3; and again, you get a point may be this is C 1 and C 2, where the input substitution can takes place. And this can be called as the efficient range of production.

So, the basis of economic region of production or the efficient range of production, is a range, where the input substitution can be done efficiently or may be usage of input combination can be done effectively. And beyond which, marginal product of capital or marginal product of labour goes in a negative direction. So, in this case, even if you are choosing a point beyond this, you are using more of the input, but still you are producing the same level of output. So economic region of production which talks about efficient range of production where two inputs can be substituted for one another; and they produce a efficient level of output.

So, in the next class we will take some numerical, we will try to some examples of the short run and long run production analysis. And we will see what are the different kind of production function generally gets in used of the economic analysis or the economic theory. And these are few of the session references, that is generally used for preparation for this session.