Managerial Economics
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Lecture -11

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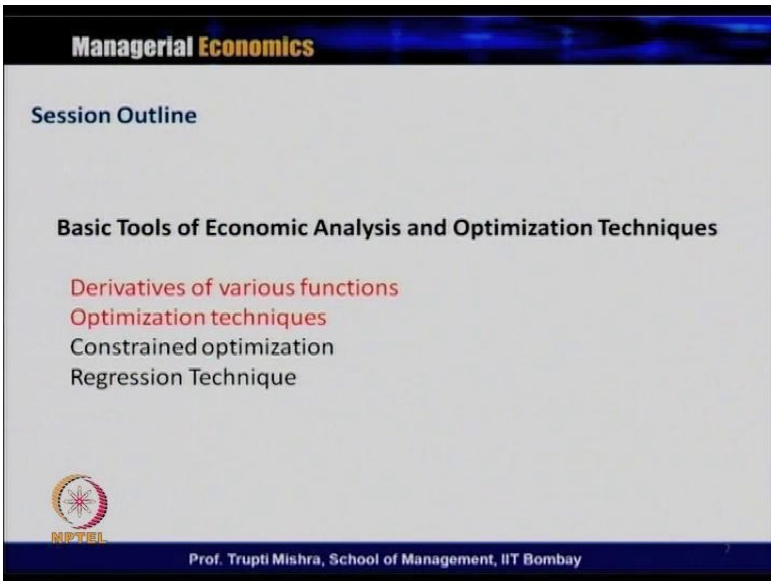

So, we will continue our discussion on relationship between different economic variables, given a quantification or through different methods graph or the mathematical equation. So if you remember in the last class, we have started discussion about the derivative of various function, how to solve the various function. Then we introduce the optimization technique, where we did two type of optimization; one is the maximization of revenue or maximization of profit, and second one is the minimization of the cost. So, whenever we are doing this optimization technique, either it is a maximization or it is a minimization problem, we did not consider the case of a constrained, and we just optimize the maximization of a profit function or we just optimize the minimization of a cost function.

So, today I will discuss the optimization technique with a constrained, either in the form of the income or in the form of the cost, when it comes to cost, and then it comes to the revenue, either its maximization or it is a minimization of the cost.

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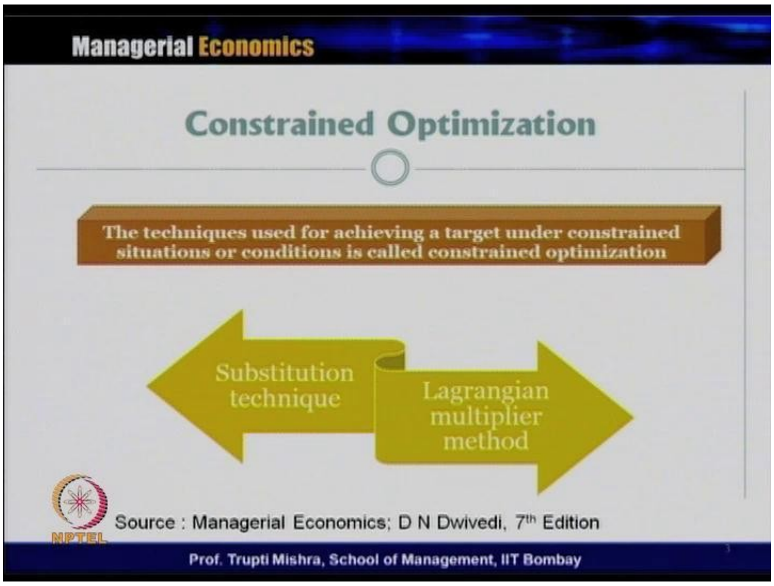

So, in case of constrained optimization, this is a technique used for achieving a target under constrained situation or condition, is called constrained optimization. So, may be the motivation for optimization is remains same. It is achieving a target, either to maximize the profit or to minimize the cost, and but here the difference is that there is a constrained along with the objective function, and how to do this constrained optimization? We generally discuss two type of technology, we will talk about the substitution technique, and later on we will take the Lagrangian multiplier method.

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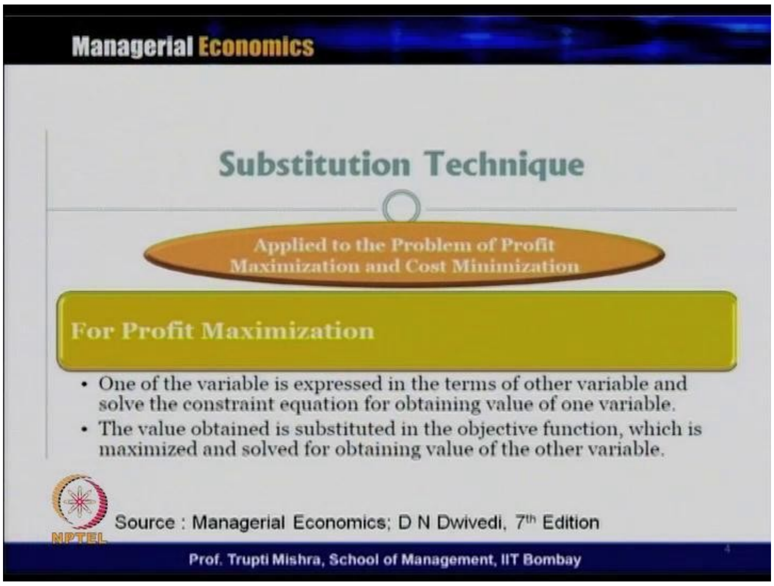

So, taking the substitution technique, it can be applied to the problem of profit maximization or it can be for the cost minimization. For a profit maximization, one of the variable expressed in term of the other variables, and solve the constraint equation for obtaining the value of one variable. Suppose, there are two variable x and y, so the best way for solving it through the substitution technique, is to represent one variable with the other variable, and then you solve for that variable, and finally you substitute the value of one variable, in term of what you have solved to the other variable. And here the value is obtained is substituted in the objective function, which is maximized, or solve for obtaining the value of the other variables. So, whatever the value is obtained by substituted, it will be again substituted back in the objective function, which is maximized and solve for obtaining the value for the other variable.

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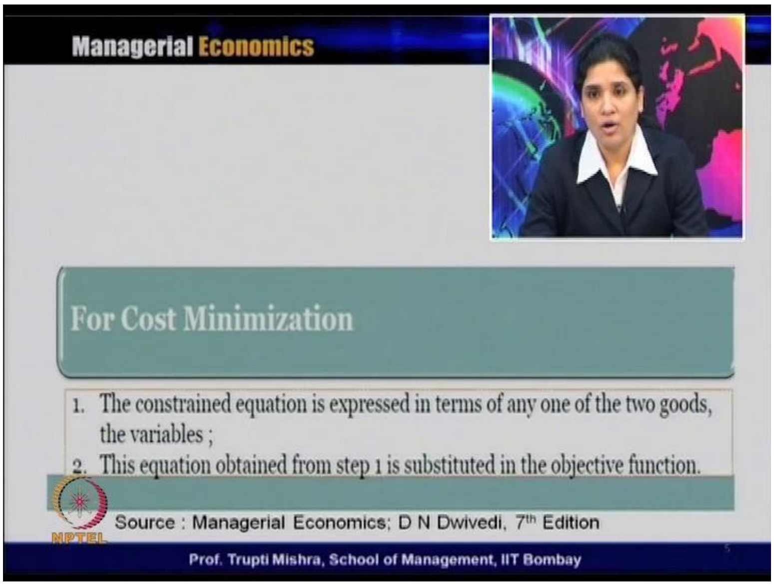

So, we will see how we use this substitution technique in case of a profit optimization problem, and in case of a cost minimization. And how this is different for cost minimization; may be the method again remains same. The constrained equation is expressed in term of any one of these two goods of the variables, and the equation is obtained from step one, is substituted in the objective function. So, whether it is a cost function, whether it is a profit function, the basic rule for this substitution technique, is that we expressed one variable in term of the other variables. We get the value of one variable, and finally again substitute back to the objective function. So, we will just take an example, how generally we do the constrained optimization, along with the constrained with the objective function, whether the objective function is a profit maximization or whether the objective function is the cost minimization?

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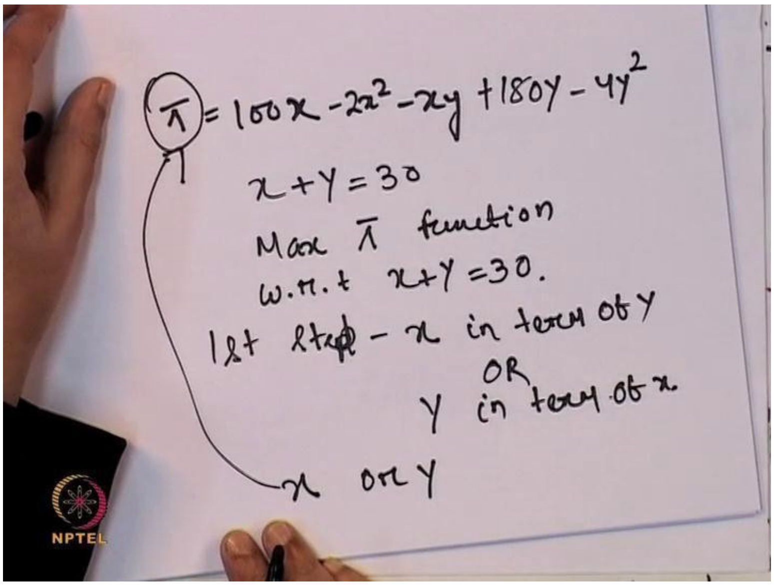

So, we will take a case of profit maximization first; and in case of profit maximization, we will maximize the profit. So, here profit is equal to 100 x minus 2 x square minus x y plus 180 y minus 4 y square. This is the profit function, and the profit, here the optimization problem is the profit maximization. Since we are saying that this is the case of a constrained optimization, there is also a constraint attached to this, and the constraint is in the form of x plus y is equal to 30. So, now what is the optimization problem? The optimization problem is, maximization of profit function, with respect to the constraint, that is x plus y which is equal to 30. Now, how we will do this, the first step is, we will express x in term of y or we can express y in term of x. And after the getting the value of x or y, again we will substitute this value of x and y in the profit function. So, now we will see, let us substitute the value of x and y, before it converting into another term, or may be the profit maximization problem.

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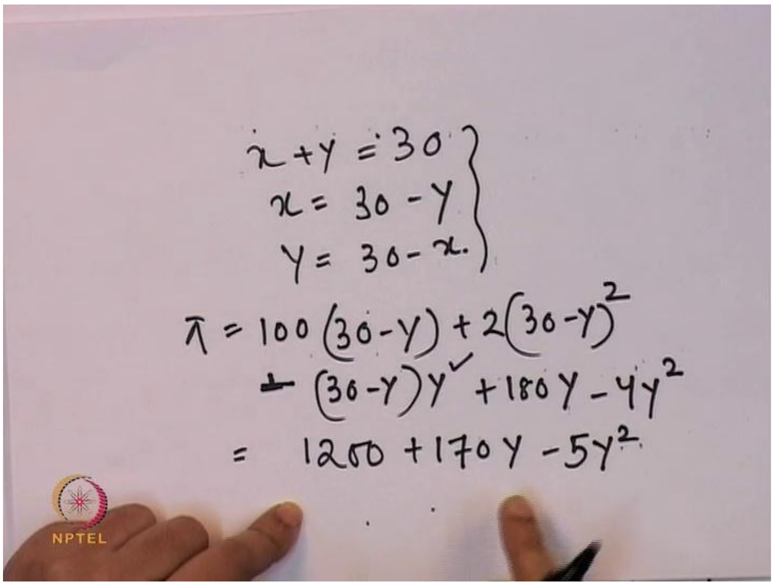

So, suppose x plus y is equal to 30. So, this can be written as x is equal to 30 minus y. So, in this case x, we are representing in term of y, or y can be 30 minus x. So, substituting the value of x and y in the profit equation what we can get; pi is equal to 100, it has hundred x, so x we are representing in term of y. So, this is 30 minus y, plus 2 ,30 minus y square, because it was 2x square, minus 30 minus y , because it was x y, plus 180 y minus 4y square. So, if you look at the profit function now, all the terms in term of y, there is no x over here in the case of the profit function.

Now again, if you will simplify this, then this comes to 1200 plus 170 y minus 5 y square. So, what we did, the first step is this, where we represent x in term of y. Now substituting the value of x in the form of y in the profit equation, which gives the profit equation, which is equal to 1200 plus 170 y minus 5y square. Now, to find out the value of y what we have to do. We have to take the derivative of pi with respect to y, and which we need to set equal to 0. So, if you are taking this, then this comes as the first order derivative, because for any maximization minimization rule in order to get the value, always the first order derivative has to be equal to 0.

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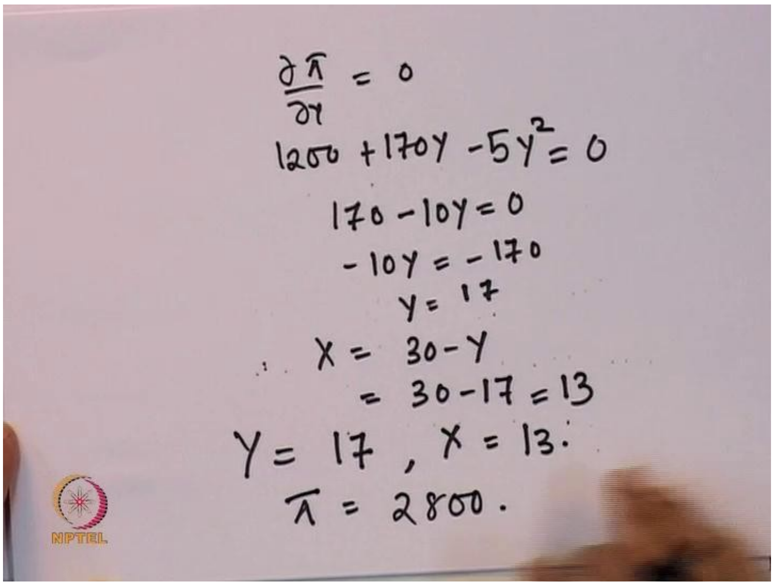

So, dell pi by dell y is equal to 0, which is like 1200 plus 170 y minus 5 y square, which is equal to 0. Now solving this, this will give you 170 minus 10 y which is equal to 0, or may be minus 10 y is equal to minus 170, and y is equal to 17. Now, what is our x, x is equal to 30 minus y. So, this is equal to 30 minus 17, which is equal to 13. So, we get a value y which is equal to 17. We get a value of x which is equal to 13. Now, putting the value of y and x in our profit equation, we get profit which is equal to 2800. So, here how we maximize the profit with respect to a cost constrained, and with respect to a value of x and y. The first step is always to represent one variable in term of the other variable. So, in this case what we did, we represented x in term of y.

And after represent the value of one variable in term of the other variables; then we put the value in the objective function. So, if you remember in the previous slide what I have showing that, we represent the profit function only in term of variable y. Then after getting the profit function, we took the first order derivative equal to 0, in order to get the value of y, and through that we got the value of y which is equal to 17, and from there we got the value of x, which is equal to 13. By putting with a value of x and y in the original profit function, we get a profit which is equal to 2800. So, by substitution technique following the two steps we got the profit, we got the value of x, and we got the value of y. Now, we will see through substitution technique, how we can do a cost minimization problem.

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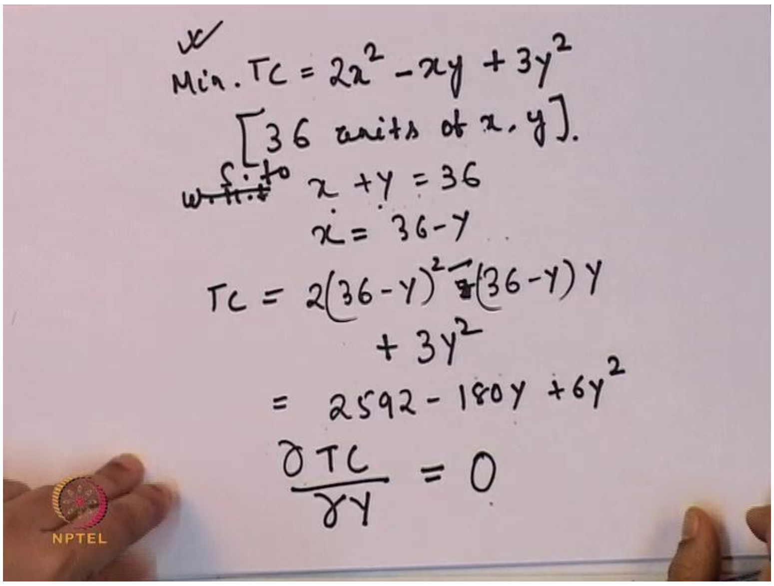

So, here what is the optimization problem, the optimization problem is to minimization of the total cost. Now, what is total cost, let us take total cost is equal to 2 x square minus x y plus 3y square. Now, the firm here what is the constrained. The firm has to get a 36 units of x and y as the combine order, now what is the optimum combination. Optimum combination is to, what is what should be the minimum cost to produce this 36 unit of x and y. So, in this case, what should be the constrained? The constrained is again, if you look at x plus y is equal to 36. So, optimization problem is to minimize the total cost with respect to, or may be subject to, x plus y is equal to 36.

Now, following the substitution technique, what is the first step. The first stage we have to represent one variable in term of the other variable. So, x is equal to 36 minus y, because if you remember the first step substitution technique is always representing one variable in term of the other variable. So, here x is equal to 36 minus y. Now, putting the value of x in the cost equation 2 x square. So, this is 2 36 minus y square, x and y. So, this is 36 minus y, y plus 3 y square. So, this is 3 y square. So, if you again simplify this, this comes to 2592 minus 180 y plus 6 y square. So, after putting the value of x in the cost function in term of y, we get a total cost function which is equal to 2592 180 y plus 6 y square. Now, in order to get the value of y; and in order to get the optimum combination or the optimum cost, what we have to do? We have to take the first order derivative of total cost function, with respect to y and we have to set it equal to 0, in order to get the value of y.

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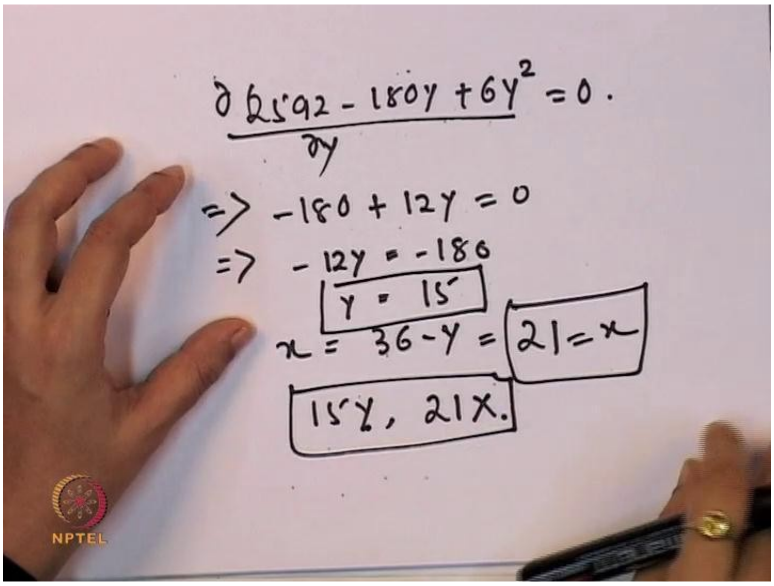

So, now we have to take a derivative, the first order derivative of 2592 minus 180 y plus 6 y square, and this has to be equal to 0. So, if you do this, then we get the value 180 plus 12 y; which is equal to 0, which you further simplify, then it is minus 12 y is equal to minus 180, and y is equal to 15. And if is y is equal to 15; then x is equal to 36 minus y, which is equal to 21. So, y is equal to 15, x is equal to 21. Now this is the optimum combination, the firm should produce 15 unit of y and 21 unit of x, and this is the optimum combination for the firm. Now, what is the next best task for us? The next best task for us is to, whether producing this combination, the firm is incurring the minimum cost of production, or what should be the minimum cost to produce this combination. So, for that what we need to do, we need to put the value of y, we need to put the value of x in the cost equation, and we need to find out the minimum cost. So, what was our cost equation?

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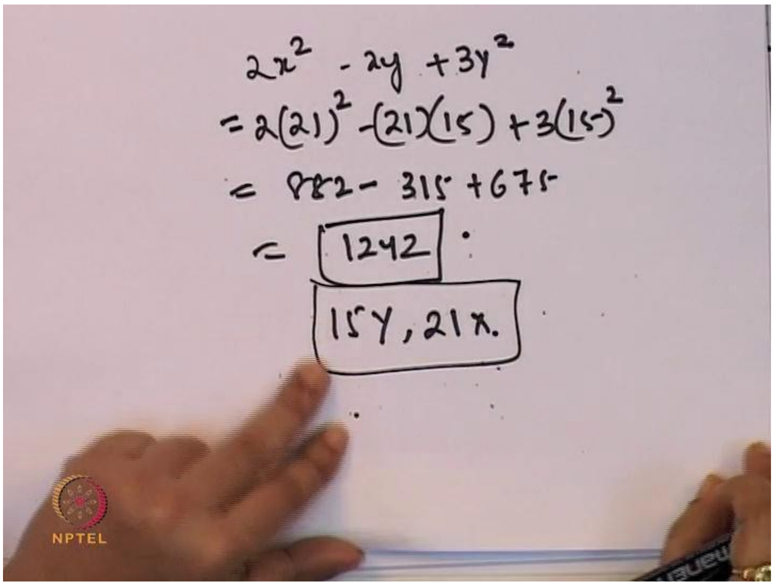

The cost equation is 2 x square minus x y plus 3 y square. So, putting the value of x is equal to 21, and y is equal to 15, this comes to 882 minus 315 plus 675, which is equal to 1242. So, this is the minimum cost what the firm incurs, in order to produce 15 unit of y and 21 unit of x, so what is the optimization problem here. The optimization problem here is to, minimize the cost with a constrained, that at any cost the firm has to produce 36 unit of both the goods; that is x and y. So, this is the optimum combination for the firm, and this is the minimum cost to produce the optimum combination of the firm. Next, we will see the second method to for this constrained optimization, and that is the Lagrangian multiplier method.

So, apart from substitution technique, the most popular or may be the most commonly used technique to do a constrained optimization is always a Lagrangian multiplier method. So, what is Lagrangian multiplier method, it is again one of kind of method to solve the constrained optimization, and it involves combining of both the objective function and the constrained equation, and solving by using the partial derivative methods. Basically, it takes the partial derivative with respect to both the variables, and then it gets the value of x and y, and by getting the value of x and y, it maximizes the profit or minimizes the cost. So, we will see how it works for the Lagrangian method.

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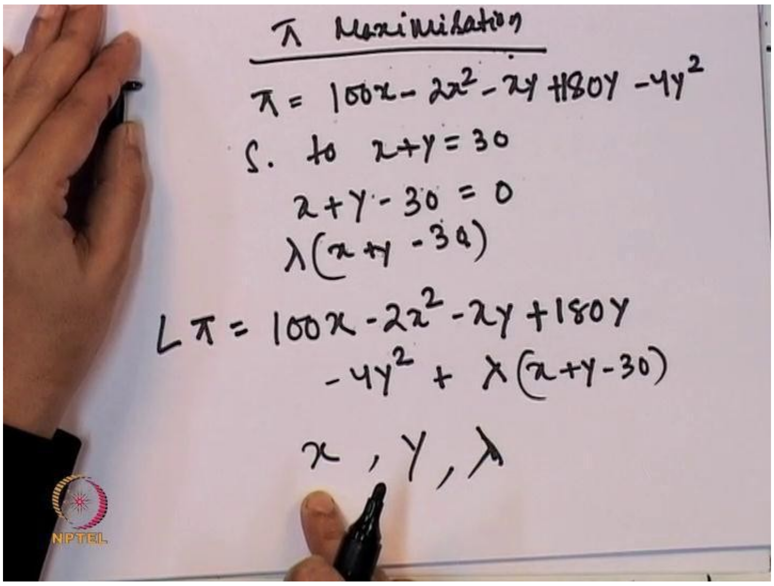

Let us take a profit maximization case. Suppose, the profit equation is, hundred x minus 2 x square minus x y, plus 180 y minus 4 y square, again subject to x plus y is equal to 30.The same profit equation what we took for the substitution technique, and the same constrained what we take for y using the substitution technique method. So, x plus y is 30; that is constrained, and profit is what we take for the substitution method. Now, how it is different from the other method. In case of other method we are substituting the value of x and x for y or y for x, here we will not do that; rather we will use a partial derivative method, to solve this profit maximization problem. In this case what we do, so x plus y is equal to 30. So, we will find another variable here; that is x plus y minus 30 is equal to 0, and the lambda x plus y minus 30. Now, we will reframe the objective function using the, adding a Lagrangian multiplier over here. And what is the Lagrangian multiplier here; that is lambda x plus y minus 30, this is the another term what we are getting here.

So, what is our new profit function? New profit function is 100 x minus 2 x square, minus x y plus 180 y, minus 4 y square. This is our original profit function, along with that we add a Lagrangian multiplier; that is lambda x plus y minus 30. So, if you look at, now the constrained also we have added in the objective function. So, this is our Lagrangian function. Lagrangian function comes from the constrained and what we add in the objective function, in order to maximize the profit.

Now, this is the profit function now. Now we have to find out the value of unknown server here, what are the unknown server here; the unknown is x, the unknown in y, and the unknown is lambda. So, we need to solve for the value of x, we need to solve for the value of y, we need to solve for the value of lambda. Now, how we will do that; we will take a partial derivative of the profit function with respect to x, we will take a partial derivative with respect to profit with respect to y, and we will take a partial derivative with respect to lambda, which is the Lagrangian function or which one is the Lagrangian multiplier.

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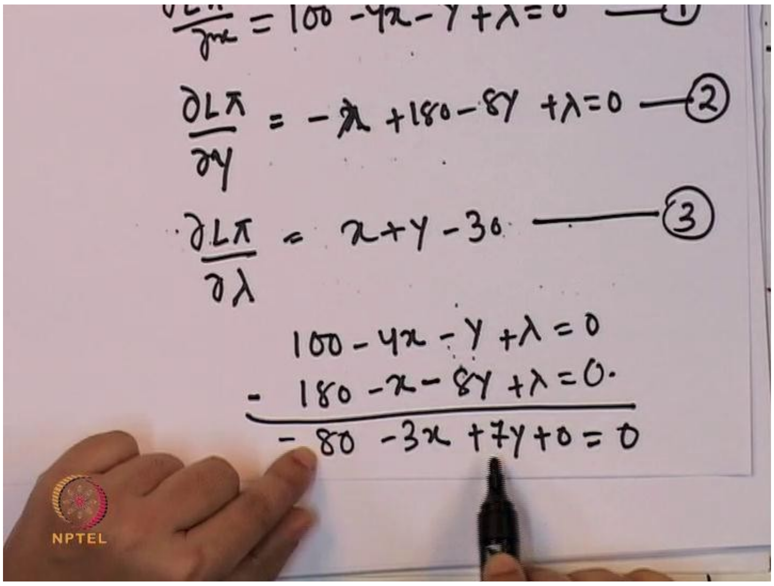

So, we will take the first one; that is, l may be first order derivative of this with respect to x. So, this we will get as 100 minus 4 x, minus y, plus lambda which is equal to 0, and let us call it the equation one. Similarly, for the second one, we have to take the derivative with respect to y. So, this comes as lambda, sorry this is x plus 180, minus 8 y plus lambda which is equal to 0, and this is equation two. The third unknown is, with respect to lambda, so this is dell l pi with respect to lambda; that gives you x plus y minus 30, and this is equation three. Now if you make a summation and if you can make a two equation, then it comes to 100 minus 4 x minus y plus lambda, is equal to 0, and if you add the second 2 equation; this is 180 minus x minus 8 y plus lambda is equal to 0.

So, if you do a subtraction from 1 to 2 over here, then you would get minus 80, minus 3 x plus 7 y plus 0, which is equal to 0. So, if you look at what we did over here. We basically in order to get the value of x and y, we got a got two equation, may be two joint equation in order to get the value of the unknown. So, in the first case, this is 100 minus 4 x minus y plus lambda, and second case it is 180 minus x minus 80 y plus lambda which is equal to 0. If you subtract the second one from the first one, then we get minus 80 minus 3 x plus 7 y plus 0, which is equal to 0. Now, if you look at the equation three; x plus y minus 30, this is our equation three, we will multiply 3 with this equation.

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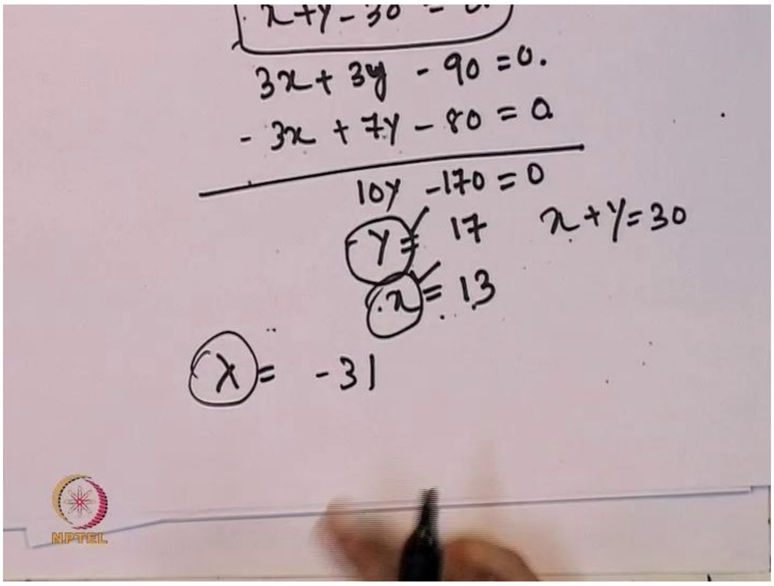

So, this is x plus y minus 30, which is equal to 0. So, if you multiply 3 in equation three, this comes to 3 x plus 3 y minus 90 equal to 0, and what was our previous equation when we did for this, this is minus 3 x plus 7 y and minus 80. And if you take this again, this comes to minus 3 x plus 7 y minus 80 which is equal to 0. From these two equations, if you sum it, then it comes to 10 y minus 170 which is equal to 0 and y is equal to 17. So, we got the first unknown value; that is y is equal to 17. Now we can get the value of x from here, because x plus y is equal to 30. So, from that we can get the value x, which is equal to 13. And now we can get a value from the. This is our first unknown, this is our second unknown; our third unknown is lambda. So, from the value of x and y, we can get the value of lambda, and lambda will come to minus 31. So, we know the value of all these three unknown, once you put in the value in the profit equation, we get the profit, and we can see whether the profit is maximum or not. Similarly, using this Lagrangian method; we can also solve a cost minimization problem, where the optimization problem is to minimize the cost. So, let us look at to the cost minimization problem.

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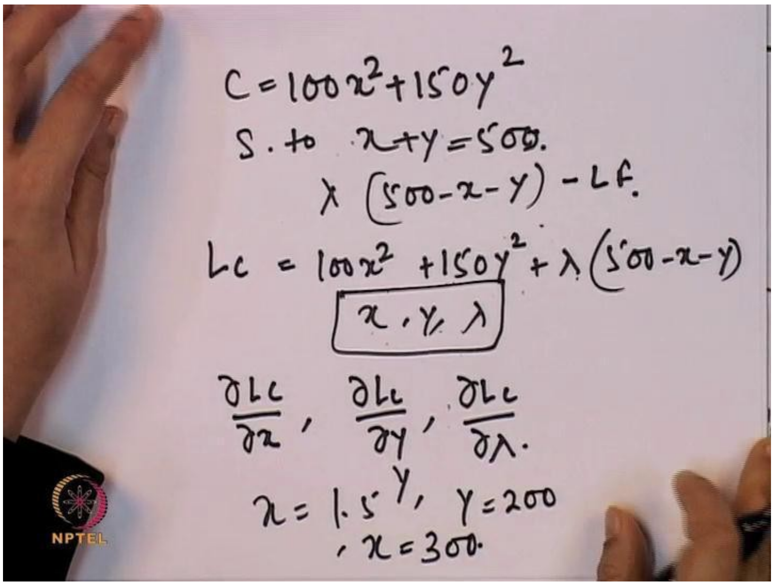

Now, the cost function over here is, 100 x square plus 150 y square, and this is subject to x plus y, which is equal to 500. Now, Lagrangian method; what is the first step. The first step is to get the Lagrangian function from the constrained equation, then again form a cost function adding the Lagrangian multiplier or the Lagrangian function. So, in this case, the Lagrangian function is lambda 500 minus x minus y. So, this is our Lagrangian function. Taking this, what is our Lagrangian cost function; that is, 100 x square plus 150 y square, plus lambda 500 minus x minus y. So, if you look at, now we have again three unknowns; that is x y and lambda. In order to find that what we will do, we will follow the same, may be formula what we did for the profit maximization. We will find out dell L c with respect to x, we will find out dell L c with respect to y, and we will find out the dell L c with respect to lambda. And after getting the equation, again we can get the value of x as 1.5 y.

So, that comes to y is equal to 200 and x is equal to 300, and from there we can get the value of the cost, and we can get the value of the Lagrangian multiplier. So, what is the essential difference between the substitution technique and the Lagrangian technique? We used both the methods to solve the constrained optimization problem, and what is a constrained optimization problem. A constrained optimization problem is one, where we maximize the profit function or minimize the cost function, with a constrained that is in the form of the other variable. So, in case of substitution technique what we do. We substitute the, we represent the value of one variable in term of the other variables, and then we substitute that value in term of the other variables in the objective function, whether it is a profit function or whether it is a cost function. Then we solve for it, and when get one value we represent that in term of others and get the final value of the other variable also.

And in case of Lagrangian multiplier, Lagrangian multiplier method, we form a Lagrangian function on the basis of a constrained, at that objective function and solve the objective function through the partial derivative method, in order to know the value of unknowns, and what are the unknowns over here. Unknowns always in term of the variable, two variables those are in the objective function, here typically in the case of the x and y. So, we started our discussion for this typical topic. We started our discussion with the relationship between different variables, whether its linear, co linear may be non-linear or curvilinear.

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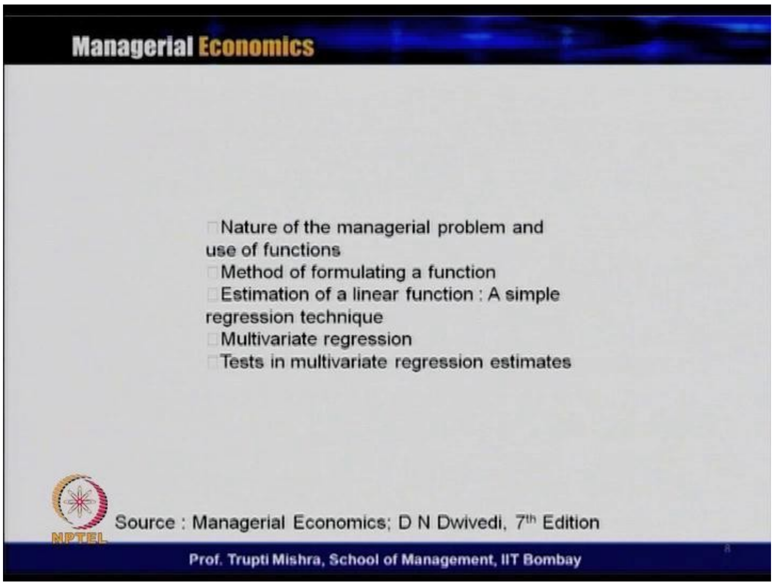

Then we discussed different function attached to different kind of functions; linear, non-linear, and the curvilinear. Then we saw. Then I think we discussed the method of how to solve the different functions, and then we talked about the optimization problem, where we maximize the profit and minimize the cost. And in today's class, we have talked about the optimization with a constrained, using both the method; that is Lagrangian method and the substitution technique.