Managerial Economics
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Lecture – 10

Now, next we will move into the optimization technique. Now what is optimization technique, if you know till now we have talked about. The relationship between economic variables, and we have understood that how to find out the relationship between two variables; that is through the slope or through the calculus method. Next we will come to the optimization technique, and optimization technique is what. We optimize in such a manner that, we are achieving the desire result or the desire relationship between the economic variable.

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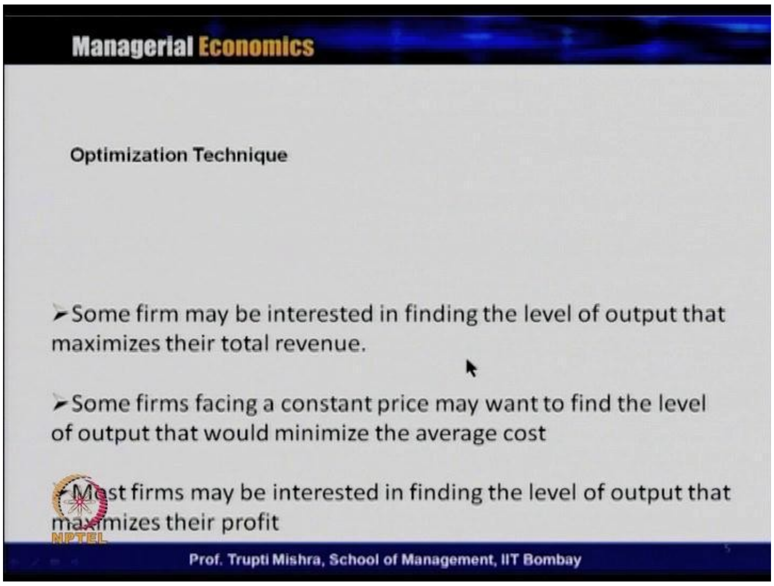

So, basically it is a technique of managerial decision making, maximizing or minimizing function, generally this optimization technique is used either for maximizing or for minimizing the function, and it is a technique of finding the value of independent variable which maximizes or minimizes the value of the dependent variable. So, basically we need to maximize the value of independent variable or minimize the value of dependent variable in order to, understand that how particularly those two variables are related. So, generally what are the cases where these optimization techniques are being used; like sometimes some firm may be interested in finding the level of output that maximizes their total revenue. So, it is basically finding the maximum level of output, which maximize their total revenue or the level of output maximize their total revenue. Some firms facing a constant price may want to find the level of output that would minimize the average cost.

So, may be a case, where the firm facing a constant price, or may they are finding a level of output which will minimize their average cost. And if you look most of the firm they are always interested to find out, what should be the level of output which maximizes their profit. So, if you look at this is the basic objective, basic aim of any firm, in order to understand the level of output which maximizes their profit. So, we will see how this optimization technique or what are the thumb rules, or what are the different approach or different methods to use this optimization technique in order to, for solving this managerial decision problem. So, we will take a function here, basically what we maximize; either we maximize the total revenue, or we minimize the cost, because the basic objective is to maximize the profit, and maximization of profit can take place; either by maximization of the total revenue or the minimization of the total cost, because profit is one, it is just the difference between the total revenue and total cost.

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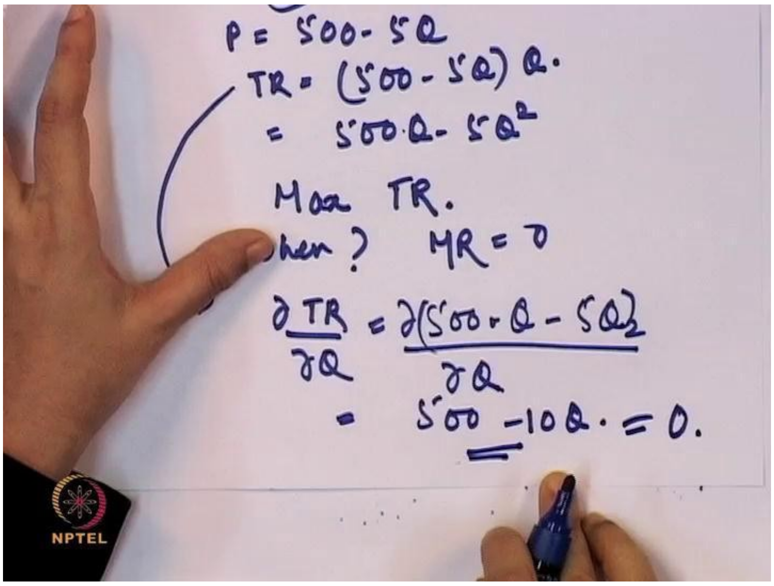

Now, what is total revenue, total revenue if you know, then total revenue this is p Q, p is the price and Q is the quantity demanded. Suppose we take that p is equal to 500 minus 5 Q. Now what is total revenue, total revenue is 500 minus 5 Q multiplied by Q, so that comes to 500 Q minus 5 Q square. So, total revenue is p Q, if the value of p is 500 and total revenue is 500, 5 Q multiplied by Q, that comes to 500 Q 5 Q square. Now, what is the role of optimization technique here or what is the role of, or how we can use this optimization technique over here, in order to maximize this total revenue. So, here the optimization problem is, maximization of total revenue; total revenue is p Q. Now, when this total revenue is maximum, total revenue is maximum, when marginal revenue is equal to 0.

So, the optimization problem here is to maximize the total revenue, and when total revenue is maximum, total revenue is maximum when marginal revenue is equal to 0. Now, let us find out marginal revenue. So, from this total revenue function, if you take the first order derivative, then we get the marginal revenue function. So, first order derivative the total revenue function with respect to Q; that will give us the marginal revenue function. So, if you take this, then this is 500 Q minus 5 Q square. We have to take the del of this with respect to q, so that comes to 500 minus 10 Q. Now, what is the thumb rule, the thumb rule is when marginal revenue is equal to 0, total revenue is maximum. So, 500 minus 10 Q has to be 0, if the total revenue has to be maximum. Now, let us see what is the value of Q, when we set marginal revenue is equal to 0, and why we are setting marginal revenue is equal to 0, because the basic economic principle says that, if the total revenue is maximum, then marginal revenue is equal to 0.

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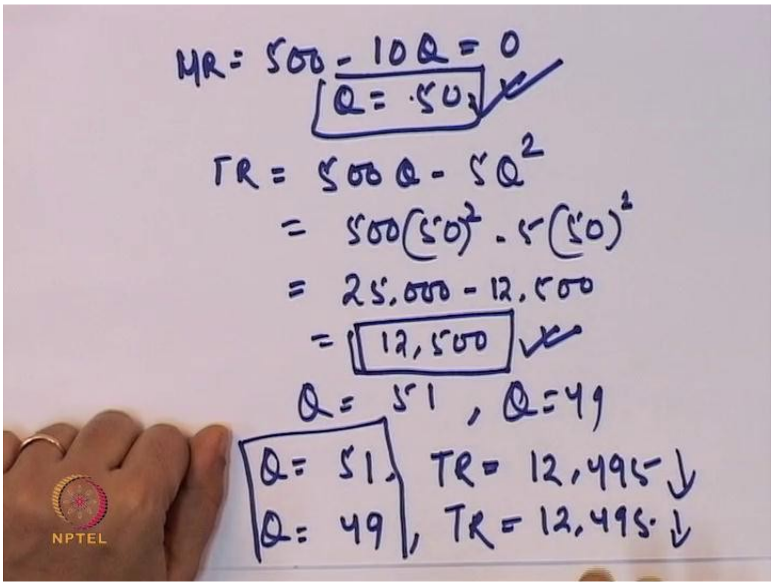

So, what is our marginal revenue, marginal revenue is 500 minus 10 Q, which has to be equal to 0, so this is our marginal revenue. Now, if you solve this then it comes to Q is equal to 50. So, what is this Q, when the level of output is equal to 50 units, the total revenue is maximized. There is maximization of total revenue when Q is equal to 50. Now we will find out what are the values of the total revenue. So, our total revenue is500 Q minus 5 Q square. So, if you're putting the value of Q as 50; that is 550 square by 5 again 50 to the square. So, this comes to 25000 minus 12500, so it comes to 12500. So, the value of the total revenue is 12500. This is the maximum total revenue for the firm. And to achieve this, the Q has to be at least 50 units in order to maximize the total revenue.

Now, how we can check this, that this is the maximum amount of the total revenue, when the value of Q is equal to 50, we know the total revenue is 12500, but how to check that this 12500, is the maximum total revenue for the firm, which is facing a demand function; like p is equal to, if you remember this is 500 minus 5 Q, how to check this. We will take two different value of Q in order to check this. We will take Q is equal to 51 and we will take Q is equal to 49. So, if you take Q is equal to 51 and putting the value in the T R; total revenue equation, that is 500 Q minus 5 Q square, we get a value of total revenue which is 12495. Suppose we assume that Q is the level of output, if it is not 50, if you produce below this also still we can maximize the total revenue. So, let us assume Q is equal to 49, putting the value of Q as 49 in the total revenue function; we get a value that is 12495. So, we have 2 values; one if 51 and second one is 49. So, one is on a higher side when the level of output increases, whether it has any change in the total revenue, and second when the level of output decreases whether it has any change in the total revenue, and here we found that whether the Q increasing or whether the Q decreasing, the total revenue is decreasing.

If you look at total revenue is decreasing, because this is if what are the total revenue when Q is equal to 50. So, it can be concluded that 50 is that level of output, where the total revenue is maximum any level of output, either more than 50 or less than 50 is showing a decreasing total revenue. So, we can conclude that 50, when the level of output is 50, the total revenue is always maximum; particularly when the demand function is this, and when the total revenue function is this. Now, we will take in the case of the cost minimization, because the first case is revenue maximization, through revenue maximization the firm can increase the profit, and the second one when we can minimize the cost, again the difference between the revenue and cost is more and that leads to increase in the profit, which is in line with the basic objective of a firm, that is maximization of the profit.

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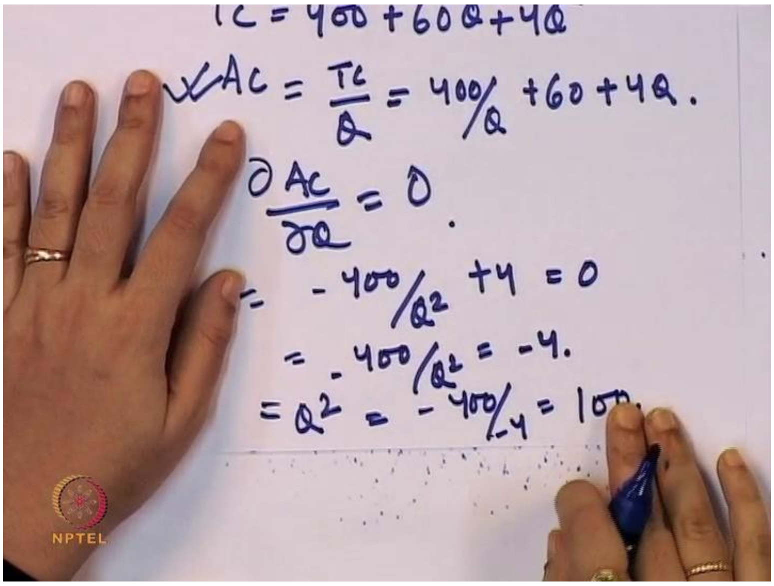

So, let us take a case of the cost minimization, in which situation generally there is minimization of cost, particularly when the firm is planning to setup a new production unit. They want to know, what is the minimum average cost through which they can setup a new production unit? When they are planning to expand their skill of production, they are looking for the minimum average cost through which they can expand the scale of production, or planning to raise the price of the product how it is effect the demand. So, these are the case, where the technique of optimizing output is require, by minimizing the average cost. So, here what is the optimization problem, the optimization problem is the cost minimization. Let us take a total cost function, what is total cost function here. Suppose this is 400 plus 60 Q plus 4 Q square. The cost minimization is not with respect to the total cost, rather with respect to the average cost. How to find average cost from here, total cost divided by Q will give us the average cost.

So, what is average cost, this is 400 divided by Q, plus 60 plus 4 Q, this is our average cost. So, we need to minimize the cost, in order to find out the difference to be more between the total revenue or the total cost. So, the first case what we are doing, we are trying to maximize the total revenue in order to maximize the profit. Now what we will try to do, we will try to minimize the cost, so that the difference between the revenue and cost is higher which leads to a higher profit. So, in this case the minimization is not related to total cost, rather the minimization is related to the average cost. Now what is average cost over here, average cost is the total cost divided by the unit of output; that is T C divided by Q, which is 400 by Q plus 60 plus 4 q.

Now, what is the rule of minimization, the rule of minimization is, the derivative must be equal to 0, if you remember in the previous case in order to maximize the T R, the rule was that marginal revenue has to be equal to 0. So, in this case minimization case we always take a thumb rule for this, that this first order derivative with respect to the average cost it has to be equal to 0. So, we need to find out, the derivative of average cost with respect to Q, and that must be equal to 0. Now we will find out what is the derivative of average cost with respect to q. So, that comes to minus 400 by Q square plus 4 which is equal to 0. So, this is again 400 Q square, which is equal to minus 4; that leads to Q square, if you simplify again, then Q square is equal to minus 400 by minus 4, which is equal to 100. So, if Q square is equal to 100, so we need to find out the level of output here.

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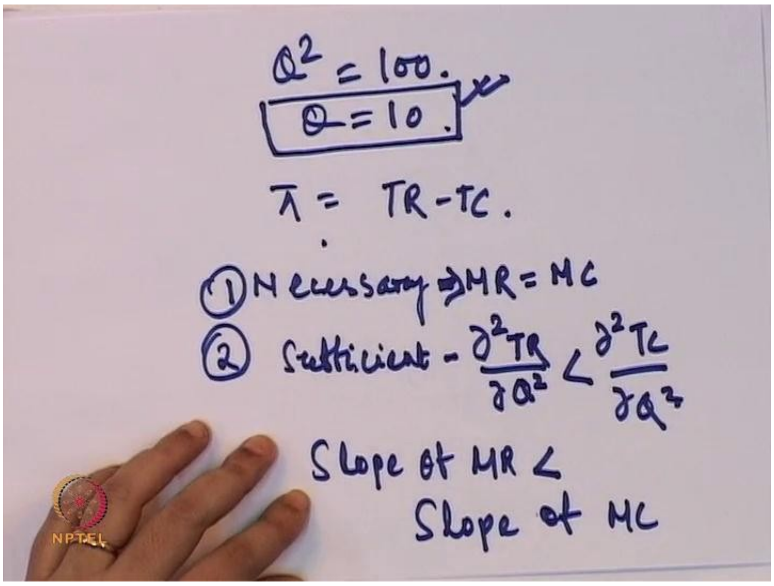

So, if Q square is equal to 100, then Q is equal to 10. So, when the unit of output or when the level of output is 10, this is the optimum level of output where the cost is minimum. So, at this level of output, the firm minimizes the cost, and if there guiding principle is on the basis of minimization of cost, the firm should follow a level of output that is equal to 10 units in order to minimize the cost. So, what we checked over here, optimization technique is used, either to maximize the revenue or to minimize the cost. So, we took a optimization problem which maximize the total revenue and there the thumb rule was to, maximizing the total revenue when marginal revenue is equal to 0.

And we took the second one second optimization technique which was the minimization of the cost, and here the optimization of problem is to minimize the average cost of production in order to maximize the profit. And here the thumb rule to minimize the cost, was to minimize the level of output, and for that is first order derivative of average cost with respect to Q has to be 0. Following that we got the level of output, and we say that this is the level of output, what the firm should follow in order to minimize the cost. If you look at all the business firm, they have a common objective. The common objective for all business firm is to maximize the profit. So, if you look at indirectly in the last two cases, last two optimization problem also we are trying to do. So, we are trying to in one case, we are trying to maximize the revenue, so that profit can be more, because their difference between the total revenue and total cost would be more, and the second case we minimize the cost.

So, that again the difference between the total revenue and total cost can be more which will maximize the profit. Now, we will take a problem where we will maximize the profit, rather than maximizing the total revenue or minimizing the cost. Let us see how we can do this by taking a profit function. And the basic need for this is if you look at the goal or the objective of the firm is to always to maximize the profit. So, now take a profit function and what is profit function; that is pi is equal to total revenue minus total cost. There are two conditions to maximize the profit; one is the necessary or the first order condition, which says that marginal revenue should be equal to the marginal cost. This is the first condition for the profit maximization, and second condition for the profit maximization, is the sufficient condition or the second order condition which says that; the second order derivative that is del square T R and del Q square should be less than del square T C and del Q square.

So, essentially it means, the slope of the marginal revenue function has to be less than the slope of the marginal cost function. So, for profit maximization, there are two conditions; one is necessary and the first order condition; that is marginal revenue equal to marginal cost. Second one is the sufficient condition or the second order condition where it says that; the second order derivative of the total revenue function should be less than the second order derivative of the total cost function, or on in other word the slope of the marginal revenue should be less than the slope of the marginal cost. So, let’s take a profit function in order to understand, that how the profit is maximized, and how the first order and second order condition gets fulfilled when the profit gets maximized.

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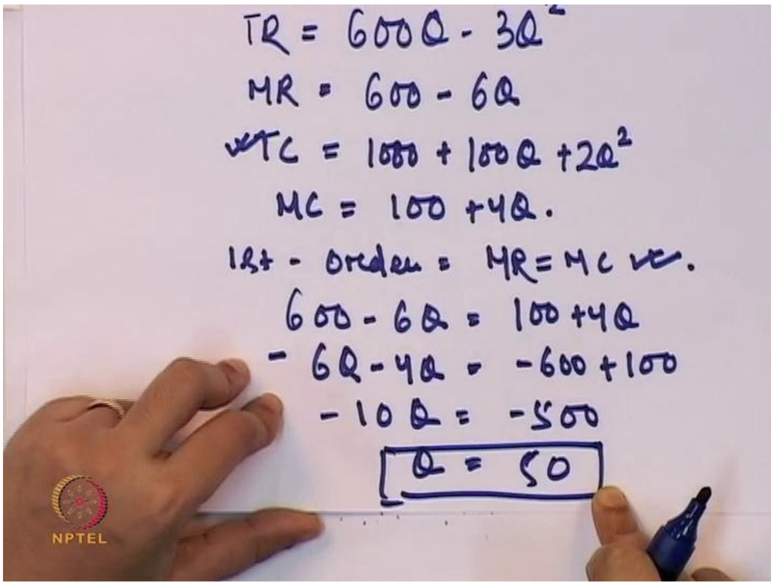

So, we will take a function that is total revenue, which is equal to 600 Q minus 3 Q square. So, what is marginal revenue, marginal revenue is first order derivative of this. So, this comes to 600 minus 6 Q. Then we will take a total cost function, total cost function is 1000 plus 100 Q plus 2 Q square, what is marginal cost function. The first order derivative of the total cost function. So, that comes to 100 plus 4 Q. Now, what is the first order or the necessary condition? The marginal revenue should be equal to marginal cost. This is the first order condition or necessity condition what is our marginal revenue that is 600 minus 6 Q is equal to, what is our marginal cost, 100 plus 4 q. So, if you simplify this is 6 Q minus 4 Q is minus 600 plus 100. So, minus 10 Q is equal to minus 500 and Q is equal to 50. So, the outcome of the first order condition is, we found out the level of output; that is Q is equal to 50. Now what is the second order condition, second order condition is that. The second order derivative of the total revenue function has to be less than the second order derivative of the total cost function.

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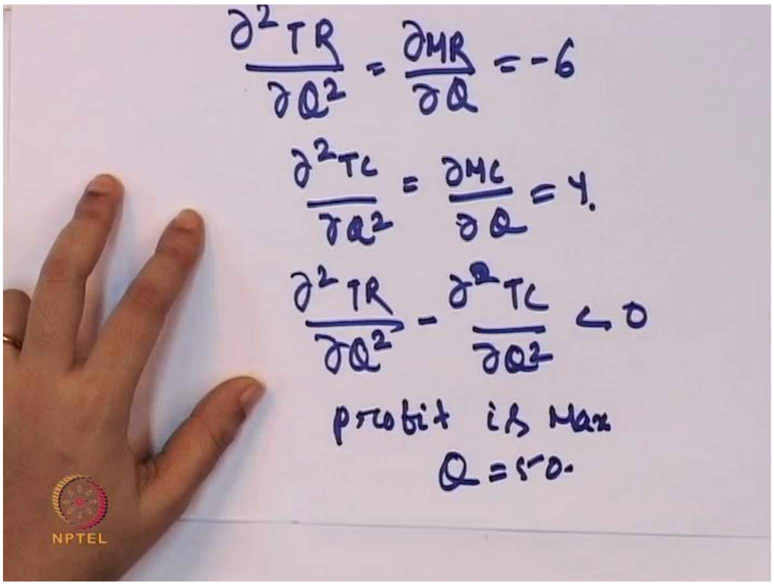

Now, let us see whether, particularly in this functional form whether we are fulfilling the second order condition or not. So, second order condition is del square T R, del Q square equal to del M R with respect to q. So, this is minus 6 del square T C del Q square. So, this is del M C with respect to del Q which is equal to 4 So, if you look at, this is less than this, and if the sum of both of this is also less than 0. So, this is del square T R del Q square minus del square T C, del Q square there is also less than 0. So, we know that, the second order condition gets fulfilled. So, we know that, profit is maximum when the necessary conditions get fulfilled; that is Q is equal to 50. Again we can do a random checking the way we did it for the other optimization problem; that you take any level of output which is more than 50 or less than 50. In order to understand, whether this is the level of output, which actually maximize the profit or not.

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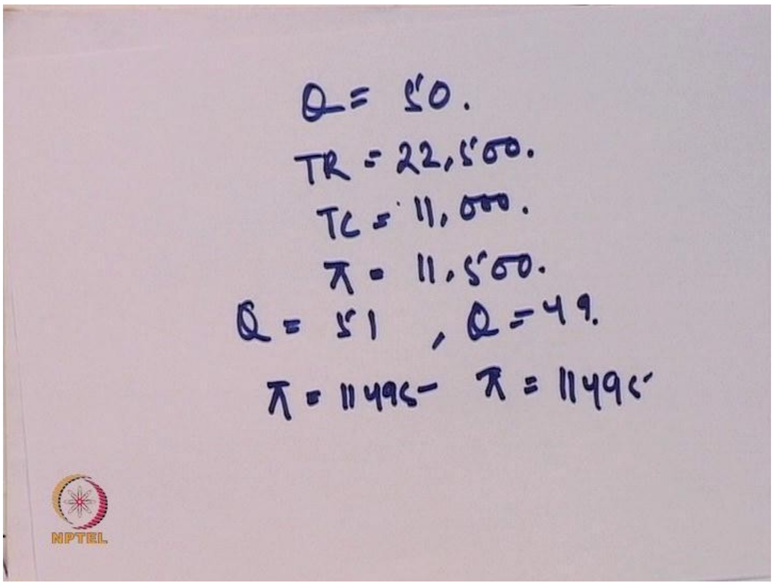

So, taking Q is equal to 50, we get the total revenue which is equal to 22,500 putting the value of Q, we get the total cost which is equal to 11,000. So, in this case, the profit is 11500. Suppose, you take a value Q is equal to 51 and Q is equal to 49. In the first case, the profit is equal to 11,495 and second case the profit is equal to again 11,495. So, we can conclude here that since the first order condition gets fulfilled, the profit is maximum when Q is equal to 50, because when we increase the level of output from 50 to 51, the profit is less; and when we decrease the level of output from 50 to 49 still the profit is less. So, we can say that, Q is that level of output which maximizes the profit. So, till now we are taking the optimization problem, and we are maximizing the revenue or profit or minimizing the cost without the constraints. So, next class, we will introduce the constraints, and then we will see how to use this optimization technique, in order to solve for the profit maximization or for the cost minimization.