Bandit Algorithm (Online Machine Learning)
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Lecture – 47
Regret Analysis of SLB-III

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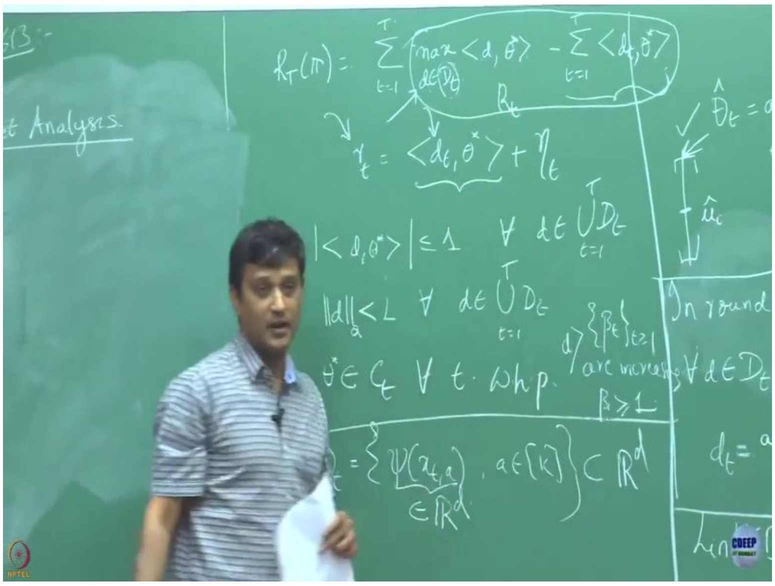

So, maybe I need to mention this here again I am going to assume specifically that my beta t the sequence they are increasing with beta 1 first element is going to be greater than or equals to 1. So, I am going to construct a confidence set right using C_t to define the C_t I need Beta t's, I am just saying that that Beta t's are all increasing with beta 1 being greater than or equals to 1 fine. So, let us now do Regret Analysis for this algorithm.

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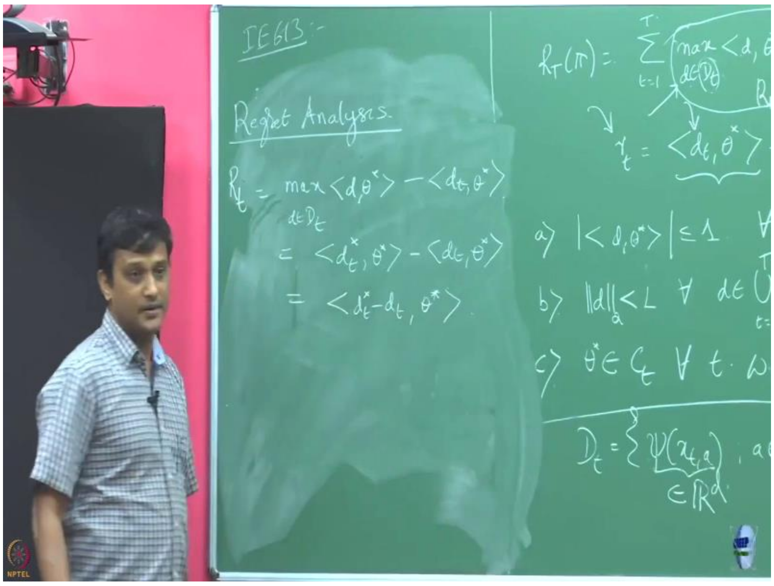

So, what is the regret it is going to be in round t? The regret in a particular round t is going to be this much right. So, let me write it. So, this is nothing but max so regret in round t. So, I am just going to use this notation R_t to denote this quantity R_t and R_T is nothing, but summation of all R_t 's is that fine.

So, R_t is nothing, but max over d theta star minus whatever you played d_t into theta star. So, this is nothing but d_t star theta star minus d_t theta star. So, I am just denoting d_t star to be what? The one maximizes this. Notice that this d_t star is different from this d_t that you have played in that round t and that is this d_t here ok.

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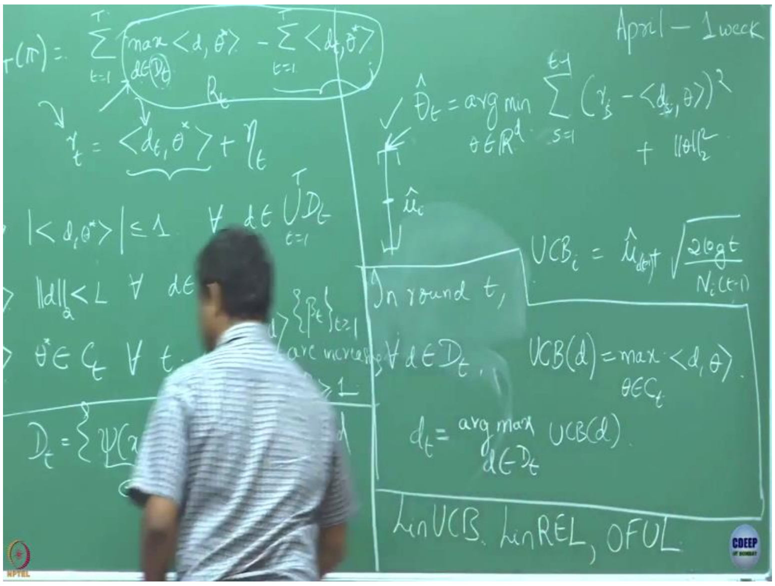

So, now, this is nothing but d_t here d_t right it is just this. So, now, let us focus this what is this quantity? If this is nothing but I am just going to put it as this is nothing but I am just going to replace this UCB value again by this quantity here what I have written here.

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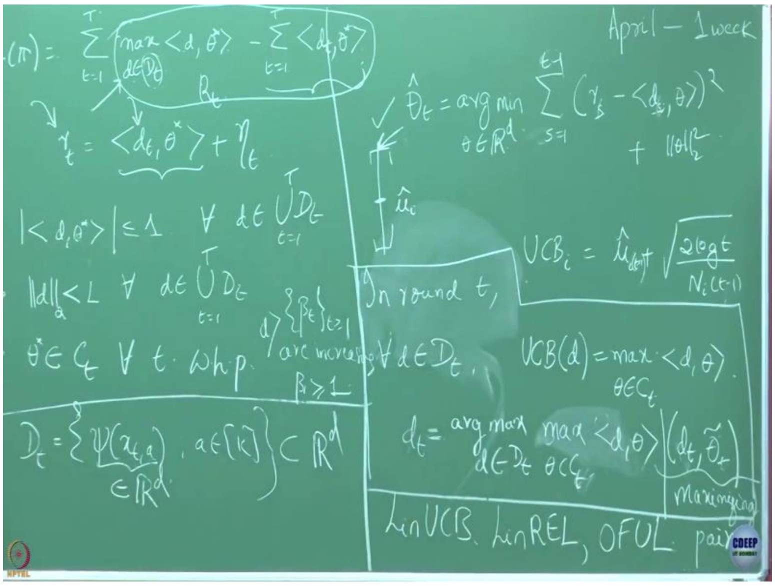

So, this is nothing but arg max of; max of theta belongs to C_t times d theta t right I have just replaced this UCB quantity, by this quantity. What it is basically doing? It is maximizing this product treating both them as variables over theta as well as d. What such program is called? This is an optimization problem, right. Is it a linear optimization problem?

Student: No.

No, right. What is such optimization problem? It is a bilinear optimization problem because both are variables for you; you are trying to optimize both over theta as well as d its a bilinear optimization problem. So, now, let us denote by theta d t and theta. So, for I have to make this t minus 1 right ok, I am going to use what is that the way we have defined C_t should be yeah I think C_t only you are going to use all the quantities this is fine. I am now going to denote it as t and tilde.

So, whatever the pair d and theta that maximizes this quantity I am going to denote it as d_t and theta t tilde, is that fine? So, this d_t or which maximizes this are already written as at d_t and now the theta which maximizes that this quantity; I am going to write it as theta t tilde ok. So, this is the maximizing pair. This theta t tilde does it belong to C_t? It belongs to C_t right because that is where it is chosen from ok. So, now, let us try to understand this.

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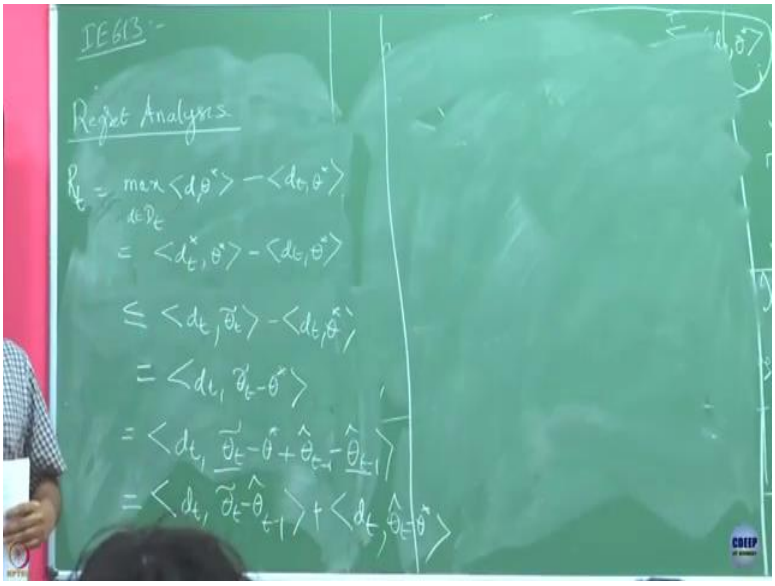

So, before I write this is it true that if I write it as d_t times theta tilde minus d_t theta star. Is this correct? What I have done is this is an inner product between d_t star and theta star right, I am saying that this is dominated by d_t theta t star theta t tilde, is this true? Why is this?

I have already said that this pair d_t thetas t is the maximization over all possible ones and notice that I have already used the fact that this guy C_t contains the theta star; this guy contains theta star. So, because of this, this pair should be dominated by this pair convinced fine. So, now this is nothing but I can write it as theta tilde minus theta star. Yeah.

Student: That is true (Refer Time: 07:31) theta star belongs to C_t.

That I have assumed right with high probability right as of now I am assumed.

Student: (Refer Time: 07:37) probability.

Yeah that I have assumed. So, we have to later show that it is indeed possible, I should be able to construct such a set where theta star belong to C_t with high probability ok. So, I have assumed that that is why I am I am safe here ok. We are going to do one more manipulation here what we will do is to this we will just add and subtract. So, I simply I have added and subtracted theta hat t minus 1. What is this theta hat t minus 1? This is the estimate I am going to get about theta star in round t minus 1 ok.

So, now let us simplify this; this is nothing, but. So, you will see that why I am doing all this manipulations here to get it in some certain required format. So, what I will now do is. So, I am going to club this and this and write it as theta t and theta t tilde minus theta t minus 1 hat plus d_t times theta t minus 1 hat minus theta star. Is this correct? Its the same thing right this is I have just taken this minus this and then this minus this and plugged it here both of them are getting multiplied with d_t ok. So, now, let us focus on this term here.

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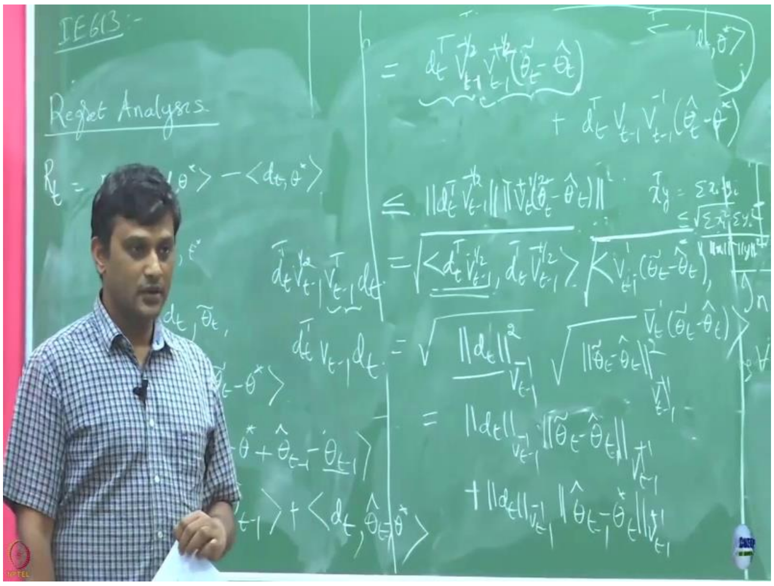

So, the first term here can I write it as this is nothing but by definition this is theta transpose and what I will do is I will deliberately get this term here and similarly the other term, this is fine right. I have just did nothing I think I have just inserted a matrix and multiplied by its inverse, so nothing changes.

So, now what I will do is. So, now, this is nothing but inner product of these two vectors d_t transpose V_{t-1} is an another row vector this matrix let us say assume its inverse exist and then try this column matrix. So, this matrix into this column matrix is another column matrix. So, this is nothing but product of these two these two vectors, I could also do the same thing for the other, but let me only just continue with this guy. So, by the way what is V_{t-1}?

Student: (Refer Time: 12:11).

So, it is nothing but.

Student: Lambda i plus.

We have say lambda i plus summation.

Student: d_s.

d_s transpose d_s.

Student: Basically.

So, now I am going to apply.

Student: Cauchy.

Cauchy Schwarz now we are going to apply Schwarz. So, apply Cauchy Schwarz what you are going to get? d_t transpose V_{t-1} norm of this times norm of. So, just let me say this we need a square root or no?

Student: Norm is enough.

Its norm is enough right and this is a l_2 norm. So, is this; so, similarly I can do it for the other also V_{t-1}.

Student: Times V_t that was the.

Yeah now this is fine right what is the issue?

Student: It is actually (Refer Time: 13:59) basically you want to write it (Refer Time: 13:43).

No, that we will come that is a next step as of now it is just like this is one vector, this is another vector and this is the norm of that vector this is the norm of the second vector.

Student: This V_t transpose V_t minus t 0 vector so.

Yeah. So, if you want to do it the other way just if you get make it a column vector if you want, but it does not matter right it is a row vector or a column vector the norm is still going to be the same its norm is going to be. So, now, what is this? This is nothing but d_t inner product wait a minute.

Student: (Refer Time: 14:25).

So, this is nothing, but inner product of d_t with V_{t-1} with itself right. I mean so right now I am messing up with columns and row vector, but this is one vector this is another vector and similarly this is also inner product between V_t inverse theta t minus theta t hat times V_t inverse theta t theta t. Is this correct? So, norm of a x is nothing, but inner product of x with itself right I am just.

Student: Square root square root.

So, now, I need to have a?

Student: Square.

Square.

Student: Square of the norm.

Do I need it?

Student: We can avoid it.

No, I do not need it right like this is already squared sum why you need it, why you need a square root here?

Student: (Refer Time: 15:52) is a square root of the (Refer Time: 15:55).

Yeah that is here ok. So, I think I should have squared here also right; I should have squared here.

Student: Then that should also be a square root.

Yeah.

Student: Are greater than 1, then this square root is a l_1.

Yeah that is fine let us see just let us see. So, let us understand this if we have a inner product between x transpose and y. So, this is nothing but what? Summation x_i y_i and what now we are saying is this is upper bounded by?

Student: Root of sigma.

Sigma x_i square and sigma y_i square right.

Student: Yes.

And what is this giving us? This is a giving us if I just say like this.

Student: Its a square root.

This is just the square sum yeah then I need to have a square of whole of this right.

Student: Yes.

So, what is this quantity? This is now nothing, but x transpose x right.

Student: Yes.

So, this is this is up square root of this, right?

Student: Yes now the first term is like be the square norm of x.

Yeah this is like.

Student: Norm of x.

Norm of x.

Student: Square root norm already do the normally.

Yeah squared and norm of y squared right.

Student: Yes.

So, let us now let us go with this we need to have a square root here for this consistent upper bounding let us take this. So, now, I am going to write this as this quantity here can I write it as d_t with?

Student: Square t squared.

Again it need to have a?

Student: Square.

Square ok. And then what is this other quantity? This is going to be theta tilde, theta hat with respect to V_{t-1}. So, this is t minus 1 and inverse. Is this correct?

Student: Squared.

Squared yes. Till this point it is correct we have just like by definition the norm of a d_t with respect to V_{t-1} is exactly this definition. So, now we know that this is nothing but simply d_t V_{t-1} and this quantity is again nothing but norm of theta t, theta hat V_{t-1} minus 1.

So, you notice that this one I got it for the first term ok. So, I can do a same thing everything remains same right except that my theta t tilde is replaced by theta t hat. So, if I do the same business here I am going to get this extra term.

Student: Sir.

Yeah.

Student: So, if you open that inner product involving d_t transpose V_{t-1}.

Yeah.

Student: So, that is going to be like V_t transpose V_{t-1} times V_{t-1} transpose times d_t.

Yeah.

Student: So, that is now this norm would be with respect to V_{t-1} times V_{t-1} transpose right.

So, what are you saying this, these are not correct same?

Student: Sir I do not think so.

Let us see this. So, what is this?

Student: (Refer Time: 20:05).

So, this is what by definition is this is nothing but inner product between these two right.

Student: Yes, yes, yes,

So, then?

Student: It open this.

Let us open this.

Student: It is a row vectors. So, it will open as d_t transpose.

d_t transpose.

Student: V_{t-1}.

And then?

Student: V_{t-1} transpose.

V_{t-1} you want to you are just taking a transpose of this right first or ok.

Student: This is already a row vector.

This is already a row vector and then I want to take the transpose of this.

Student: Yes.

So, I am going to get V_{t-1} transpose.

Student: d_t.

d_t.

Student: Yes.

And what is this by definition? This is a same quantity right.

Student: So, we could, so we did not have a V_{t-1} transpose when we defined this norm with respect to this.

So ok fine let me write this. So, this is nothing but d_t times V_{t-1} and.

Student: V_t transpose V_{t-1} d_t.

So, this is what we have defined it right fine. So, there is a this there is a difference between these two. So, let me see.

Student: V_{t-1} is symmetric that we know but.

Yeah.

Student: So, what do we?

So, let me see suppose if I do this nothing changes right I mean I will do that also similarly later there. So, this is also half and this is also half, then this is half half and then this is I just keep it V_{t-1} only here.

Student: Yeah.

So, now everything is fine?

Student: Yes.

Yeah fine. So, that is why.

Student: This because.

This is going to be going to be the same. So, because of that if you just going to. So, what is this? This is half and this is another half with transpose, but that will give you back V_t. So, is this fine?

Student: People say that seven different V_t only therefore, we could define that root.

We could define that.

Student: Root because eigen type of.

Yeah this is psd right. So, we all the eigenvalues are going to positive. So, I am which line you are talking about where otherwise what we can have done this is fine right. So, usually this kind of we are going to define this norm with respect to a matrix if this matrix happens to be a positive semi definite or positive definite.

Student: (Refer Time: 22:48).

So, positive definite; so, in this case we already know that this my matrix V_t is already positive definite. So, is this now all fine? Ok. So, now, let us. So, now, let us focus on this parts. Now, let us come to the big assumption I have made where I have assumed that sorry I this one I said theta minus 1 right these are all theta minus t minus 1 why I have not written t minus 1 ok. So, this is all t minus 1.

What I know ok. So, I should be bit careful here what I want to do is I want to take the minus on its side and I want to take the plus on this side ok. So, this is minus minus this is plus this is going to be minus this is going to be plus and this is going to be plus here. Is that fine? I have just and I have just made I have made this minus and this is plus.

Because of that this is going to be everything is fine I have just like I have just replaced V_t by V_t inverse and this by V_t just V_t with without inverting it.

Student: (Refer Time: 24:38).

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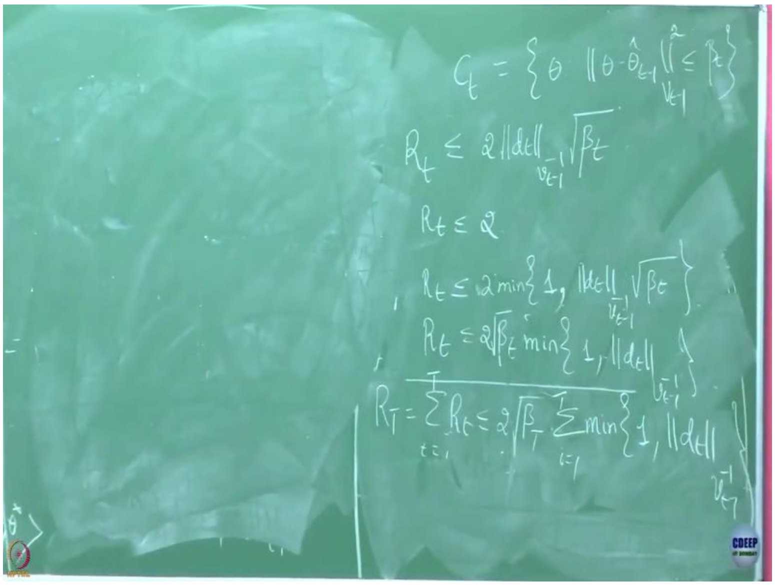

So, now, let us go back and see what our assumption says we have assumed that what is my C_t my the C_t kind of I have assumed that it is going to be of this form right theta such that theta minus theta minus 1 hat is going to be upper bounded by what? Beta t.

Now, can I replace upper bound this quantity by beta t? Because I know that this quantity is upper bounded by beta t right. Can I also upper bound this quantity by beta t? See notice that the difference between this and this is this is theta t tilde.

Student: Still in C_t.

But it is still in C_t right theta t is still in C_t. So, I should be able to still bound that this quantity is also by both this and this quantity by beta t right. So, now, I that is why I am going to write it as 2 times norm of d_t V_{t-1} inverse times beta t. So, we had I think earlier we had this square right the way we have defined then I am going to upper bound it by what? Square root by beta t.

So, now what is this? Now finally, what you have arrived is R_t is upper bounded by this quantity right after all this manipulation manipulations and applying our exploiting or assumptions we are able to show R_t is upper bounded by 2 times this d_t norm with respect to V_t inverse and square root of beta t.

So, now, let us see I want to now make another manipulation, I know that R_t is upper bounded by 2 also why is that? I know that by my first assumption I know that this mean quantities there at most one right absolute value of this is for each d is upper bounded by 1.

So, the claim is then this quantity can be at most 2 right if this is one and this is minus 1 then I am going to get that upper bound right. So, in any wrong that is why the regret can be either at most 2 or upper bounded by this quantity. Is that correct? So, R_t is upper bounded by min of and further I have assume that this quantity beta t are increasing and they are at least 1. So, that is what I will also pull that out.

So, this should be beta t square root of min of 1 times. I mean just I assume that if I replace this, so 1 by upper bounded by beta t then I will just pull out that square root of beta t then I am going to get it. So, finally, I have been able to reduce my R_t to this format ok.

So, now, now it all boils down to I know that I have already a known assume that there is some sequence (Refer Time: 28:52) now the quantity is how to bound this quantity. This depends all the observations I have made so far right. So, how to bound this quantity? So, now, let us focus on this ok.

Now, the claim is this quantity here. So, what is now R_t? R_t is nothing but summation of this quantities right. Now, what is this R_t is nothing, but summation of R_t t going from 1 t. So, this should be nothing but upper bounded by 2 times summation square root of Beta t and what I will do is further I will also.

Student: (Refer Time: 29:38).

Make it square root Beta t Beta capital T because I know that Beta t sequence is increasing right I will take the largest value of that and then I am going to make it as minimum then it is going to be sum I equals to 1 to capital minimum of 1 norm of d_t V_{t-1}. Is this correct?

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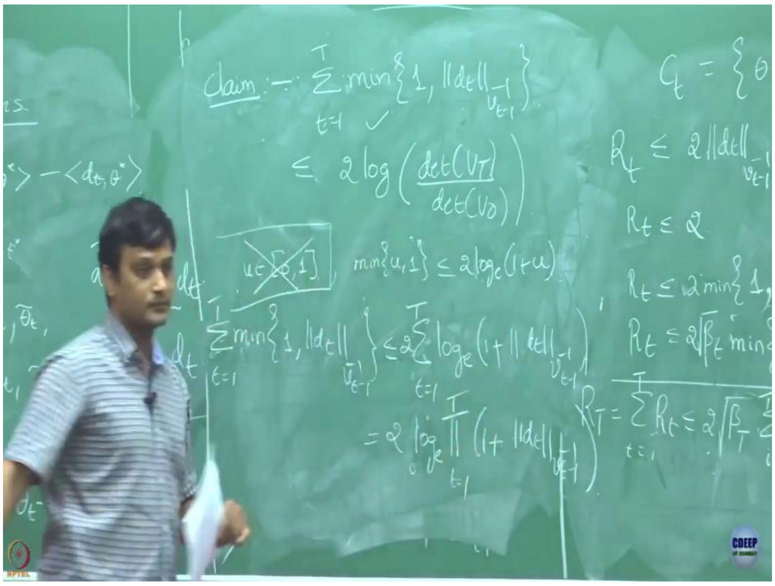

Now, claim we are now going to argue that summation i equals to 1 to t min of 1 comma d_t V_{t-1} inverse, it is going to depend on my i matrix is like this, this is going to be log of determinant of V_t divided by determinant of V_0. So, now, what we are basically connecting is basically how this norm of this depends on the determinant of some of my vectors I have observed so far.

So, let us quickly try to get this how why this is true. So, then we are actually now going to use some linear algebra and matrix algebra here. So, first I am going to use this property that is always upper bounded by 2 log this is log to the base e here actually is 1 plus u is this inequality correct? And this is this equality holds. So, if you are going to take u between 0 1, then minimum of this 2 is going to satisfies this ok.

So, now let us try to apply this inequality to this quantity ok. So, I am going to take one term here. So, this is sorry this is t here, so then min of 1 d_t V_{t-1}. What is this quantity is going to look like? So, first before I could apply this I should argue that this norm is going to be between 0, 1 right if I can use that then I can use this quantity and bound it as 2 times log 1 plus norm of d_t V_{t-1} ok.

This question is suppose if u is already between 0, 1 right. So, then why not this min of u u comma 1 is just u right. So, why is this? Ok just let me just check this what I am doing is correct.

Student: So, I think u can be from (Refer Time: 33:17) in that (Refer Time: 33:20).

Yeah. So, let us say I think maybe let us let us let us try to ignore this part. So, let us say u is positive let us try to verify that this holds always ok. So, let us take the simplest case when u equals to 1 this is 1.

Student: (Refer Time: 33:37).

This is anyway this then is this holds now.

Student: Taking u is greater than 1, then the u is 1

Yeah. So, then this and in that case it is going to be always greater than 1 right.

Student: Equivalent 2 is.

So, that is fine. So, log you just check what is a log 2 to the base e.

Student: It is going to be 1.38 like equivalent 2.

After 2 multiplied I mean this quantity right 2 times.

Student: (Refer Time: 34:12).

This for u equals to 1. So, that is greater than 1, so that is fine. So, I think maybe I do not need this actually ok. Now, then I really do not need to worry about what is this quantity right this is some positive quantity I know ok; this positive quantity with this, this, this holds ok. Now, what I am interested in? This going to t 1 to T; so, this is going to be t 1 to T.

So, now if I can just write it as 2 times log of e product 1 plus norm of d_t V_{t-1} this and where this is now t equals to 1 to capital T ok. So, just sum of log I have used this log of product, so fine. So, let us take this inequality let us keep it like this.

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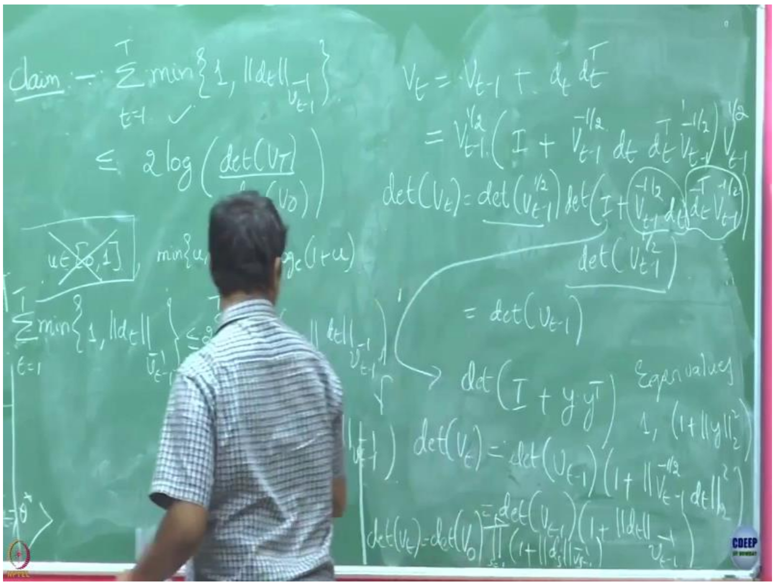

Now, I am going to write my V_t, I can write my V_t to be iteratively like this right plus d_t transpose d_t.

Student: Offers d_t d_t transpose.

d_t d_t transpose, if I do this. So, just see whether this is correct. So, I have taken V_t to the power half on the left side and then again I have taken V_t to the power half again on the right side. So, if you just multiply this it is just the numerator right it is just this. So, if I have a matrix C which is nothing but product of 2 matrices a and b what is determinant of C? It is determinant of a and b right.

So, I will let us just let us use this determinant of V_t is equals to determinant of V_{t-1} half times determinant of this entire quantity and then determinant of V_{t-1} half.

So, this you can pull out and now this is nothing, but determinant of V_t now let us focus on this part. This matrix if you just look into this, this has first a identity matrix i plus now look into this. So, this part it is a vector, this part is also a vector.

So, now, it is just nothing, but outer product of two vectors. So, let us focus on this, this is of the form determinant of I plus y y transpose where y is the things I have put it in circle ok. So, do you know what should be the determinant of such a matrix? Ok.

So, what should be the eigenvalues of such a matrix? So, you can just go and compute. So, the eigenvalues of such matrix are 1 and so eigenvalues here are [noise. What would be the rank of this matrix? 1 2. So, eigenvalues are going to be 1 and other eigenvalue is simply going to be 1 plus norm of y 2. So, it will have only these two eigenvalues and these are the two values. So, if I know these are the eigenvalues what is the determinant of this quantity?

Student: Product of.

It is just going to be the product of these two co values.

Student: (Refer Time: 39:26) multiplicity has to be (Refer Time: 39:29).

That is why I am saying how what is the rank of this.

Student: This might be (Refer Time: 39:35) t minus (Refer Time: 39:37).

Yeah.

Student: 1 eigenvalue.

1 eigenvalues will be this and how many 1s will be there?

Student: d minus.

d minus 1 rest are all 1 1 1 1

Student: Similarly.

So, only this is the only these are only two distinct ones this one will be repeated ok. So, you can check this. Now, determinant of such V_t is nothing but determinant of V_{t-1} times what is this now? This is nothing but 1 plus this quantity here norm of V_{t-1} half times d_t square. But we know that this is nothing but what? By definition this is nothing but 1 plus norm of d_t V_{t-1} inverse. So, right this quantity here it is nothing but this ok.

So, now yeah let us let us let us try to quickly wind this up I just. So, I have this now what we have done? We have iteratively written determinants of V_t in terms of determinants of V_{t-1} and into this quantity. So, go back and check that this indeed correct like these are all eigenvalues of this matrix. So, right now we are just writing.

So, if you just compute this is going to give you determinants of V_t is equals to determinant of V_0 times product of these guys right, agreed I have just written iteratively I have now replaced determinant of V_{t-1} by the same equation. So, it will give me finally, determinant of V_t equals determinant of V_0 plus product of these three terms. Product of these two terms all this terms it should be s here that is going from s 1 to t fine.

So, now let us go back I do not know maybe we will just rub this. So, now, if you look into this what I have? The product of this is nothing, but determinant of V_t divided by determinant of V_0 right. So, if I am going to have a product from t equals to 1 to capital T this is going to give me what?

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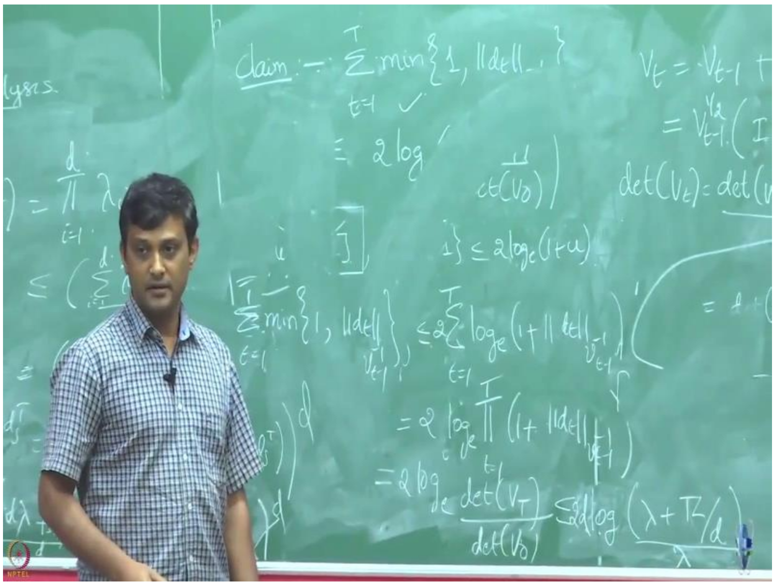

Student: (Refer Time: 42:47).

What was this? This was like min of 1 comma norm of d_t times V_{t-1} inverse. So, this is nothing but 2 times log of e.

Student: Determinant of.

Determinant of V_T divided by.

Student: Determinant (Refer Time: 43:16).

What is going to be determinant of V_0?

Student: Lambda.

Lambda multiplied by how many times?

Student: Lambda to require d.

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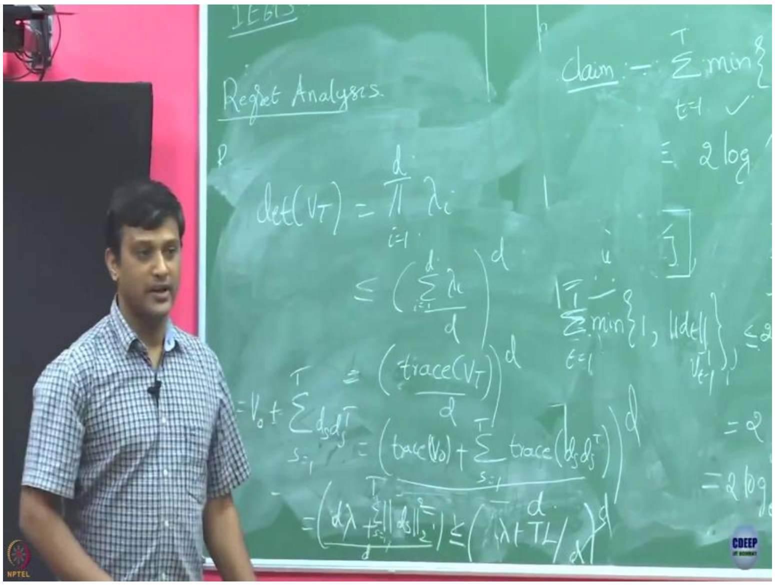


d right whatever it is fine. So, I have this quantity now. So, finally, now let us try to find out what is the determinant of V_t is going to be. So, I can bound this last term then we are more or less done ok. What is going to be determinant of V_t? Determinant of V_t.

So, what is going to be the rank of this matrix V_t? It is going to be full rank right this is a positive definite matrix ok. And in that case what is its eigenvalues? all of them are going to be positive and if I am going to multiply all of their eigenvalues what I am going to get? the determinant right.

So, this is nothing but this is going to be a determinant. So, whatever lambda is here are the eigenvalues of this matrix V_t. So, how many of you know GP, AP inequality? Geometric Progression, Automatic Progression. Can you apply GP AP and tell me what should be the bound for this? So, what is the AP? Arithmetic Progression of this is nothing but.

Student: Minimal number of elements d only (Refer Time: 44:57).

d or?

Student: d (Refer Time: 55:00).

d. So, now, let us do another round of simplification i equals to 1 to d. What is summation of lambda I? What is the sum of eigenvalues can you connect it to the trace of a matrix?

Student: (Refer Time: 45:15).

It is going to be what? Trace just going to be trace right. So, this is going to be. So, now, what is V_t? V_t is nothing but V_0 plus other quantities right. So, I know that V_t is nothing but V_0 plus.

Student: Sigma s equal to 1 2 3.

What is this each one of them is a matrix, how they will look like? So, if you are going to look at their diagonal values.

Student: Squares.

There are squares right like each component squared the diagonals where each component is squared. Is it through the trace of this is nothing, but the trace of each one of them? So, this is nothing but trace of V_0 plus summation of trace of d_s d_s transpose for s equals to 1 to t this whole thing divided by d. What is trace of d_s d_s transpose?

Student: (Refer Time: 46:47) norm of.

Norm of.

Student: (Refer Time: 46:51).

Norm of d_s right squared norm. So, this is going to be upper bound right. So, sorry this is nothing but trace of V_0.

Student: V_0 we can write as.

What is?

Student: d lambda.

d lambda right because there are d elements getting added lambda times. Now, this is nothing but norm of?

Student: d.

d_s.

Student: Squared

Squared this is sum right one to d this is divided by d whole to the power d.

Student: s equal to t.

S equals to?

Student: 1 to t capital T.

Yeah right this is s equals to 1 to capital T followed. So, now, now let us appeal to our second assumption. What was our second assumption?

Student: Norm of (Refer Time: 47:56).

Norm of.

Student: (Refer Time: 4700:59).

s is less than or equals to l right. So, let us apply here what this gave me? This gave me d lambda T times L whole divided by d to the power d.

Student: (Refer Time: 48:16).

Yeah first d get cancelled. So, now, finally, what we have shown is this quantity determinant of V_t is upper bounded by this quantity here.

Student: Sigma (Refer Time: 48:35).

Yeah. So, this square here.

Student: Yeah.

Because that is norm square; so, fine I do not know which portion I need to erase. So, let us continue to erase here. So, now, what is this I have finally, able to bound as this is 2 times.

Student: (Refer Time: 49:06).

Log V_t is this quantity right if I take a logarithmic of this is going to be 2 d times lambda plus T L by d.

Student: Log log.

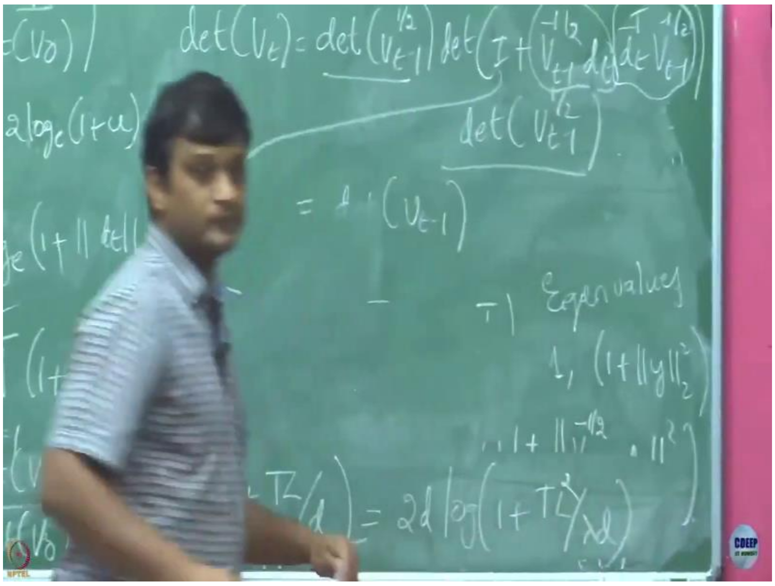
So, did I miss now what is this denominator is going to give me? So, what was the determinant of V_0?

Student: Lambda to the power d

Lambda to the power d right. So, where how does that come now? So, that d will come out of this and there is a lambda here right.

Student: Log log.

(Refer Slide Time: 50:20)



Inside the log only right it is already inside the log. So, this is going to be 2 d.

Student: Delta.

Log 1 plus T L by lambda d this is what we have got. So.

Student: Suppose T L square.

T l square.

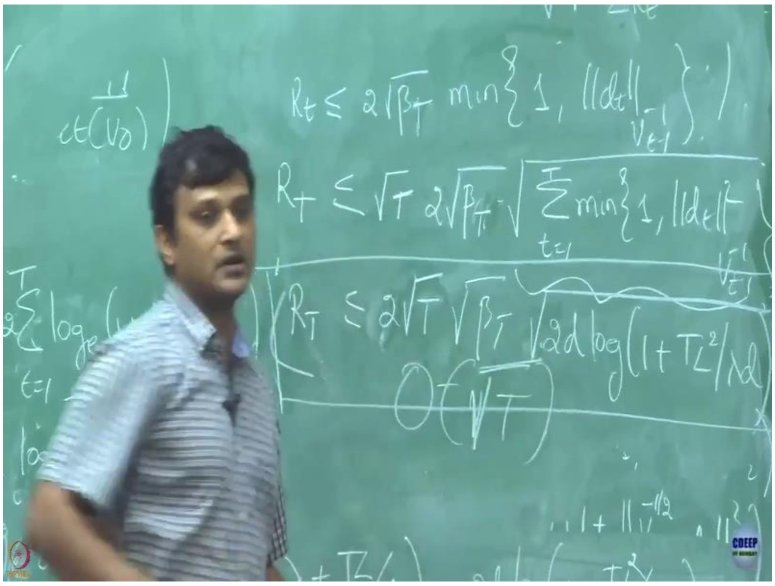
Student: (Refer Time: 50:36).

No the assumption is the square norm is L we just said just check whether we said on the squared norm or.

Student: Just the norm.

Just the norm right if it is let me see what I wanted actually yeah. So, this is an L square ok, so fine. So, let us finish this now can somebody tell me where was the what was the bound I had?

(Refer Slide Time: 51:13)



So, if I remember correctly I had R_t is upper bounded by summation 2 beta t summation t equals to capital T min of 1 comma d t.

Student: V_{t-1} V_{t-1}.

V_{t-1} inverse right.

Student: Root Beta t root Beta t.

Root Beta t. Now, what I have shown finally? This quantity here is bounded by this quantity right. So, this is now 2 d square root of Beta t times log of 1 plus t l square by lambda d.

Student: (Refer Time: 52:15) this line (Refer Time: 52:18).

Oh yeah. So, there is a. So, before I think I have made a small mistake. So, anyway. So, let me see if what is that we had said this d_t is equals to R_t right what I wanted to basically use is. So, the bound we had on R_t what is the bound we gave on R_t.

Student: Beta (Refer Time: 52:53).

There is some issue. So, I want to actually do this 2 summation R_t square and I wanted to bound R_t square I do not know what is that R_t.

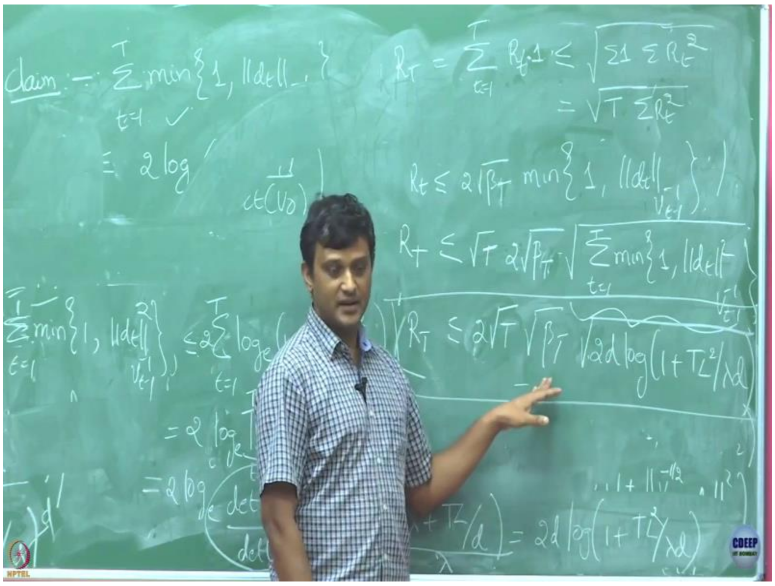
Student: Sir actually because if the (Refer Time: 53:10) summation on like we know one it is d_t square.

This is d_t square here.

Student: Yeah whatever (Refer Time: 53:24) whatever you have written it is correct, but we got (Refer Time: 53:31).

Just a minute hold on. So, fine as of now let us take for time being we have done with this bounds. So, let us conclude this let us say till this point we are correct right, if I am going to bound this minimum of these two quantities this is the bound I am going to get ok. So, let us take another 2 minutes and finish this otherwise we will have a discontinuity.

(Refer Slide Time: 54:00)



So, what is this R_t? R_t is my R_t or t equals to 1 to capital T this right. And what I showed earlier? This R_t is.

Student: (Refer Time: 54:24).

What this is we said this is upper bounded by 2 times square root of Beta t times what is that min of?

Student: 1 comma.

1 comma d_t V_{t-1} this whole quantity.

Student: Oh.

So, now, let us find. So, I know that this quantity here.

Student: Sir, your analysis is not correct.

No our analysis is correct I think this part was just like I did not put the things correctly. So, this part here I am going to treat them as like this if I am going to treat this is it true that now I am going to get the summation of 1 and summation of R_t square ok. So, then this is nothing but square root of T times summation of R_t square and now whatever we got here from this to this point. So, we had this as a just for d_t or d_t square?

Student: d_t.

I think that is my mistake we had it as d_t square can you check this term.

Student: It is a assumption.

No when we actually could this is product is equals to this determinant right.

Student: Yes?

Was that norm of d_t or d_t square?

Student: (Refer Time: 56:28).

Can you just verify that I think that is supposed to be d_t square?

Student: (Refer Time: 56:36).

It should be square just check that that is why it is messing up it should be d_t square.

Student: (Refer Time: 56:54) it is d_t square.

What is that?

Student: (Refer Time: 57:00).

Yeah here also yes.

Student: So, whatever claim we (Refer Time: 57:03).

Yes. So, see that is why this whole of this works if this is d_t square I think this is the mistake we have made.

Student: (Refer Time: 57:11).

This is actually this now. Is that fine? So, just verify that let us let us quickly wind it up. So, now, we have.

Student: (Refer Time: 57:26).

R_t is upper bounded by what?

Student: (Refer Time: 57:29).

So, this is why because we have only bound on this d_t square I cannot directly apply whatever the values I have that is why I am trying to get this quantity in terms of R_t square ok. So, now, if I have this in R_t square now what is this? This is now square root T square and then still it is going to be 2 times.

Student: (Refer Time: 58:59).

Square root beta I am now going to I have made it the largest possible right, I have made it beta capital T and now this is going to be square root of.

Summation t equals to on 1 to t minimum of 1 comma norm of d_t square d_{t-1} inverse. So, is this correct now? So, now, now this is nothing but square root T T square root t this is beta T. Now, on this I have a bound now which is this quantity here, this quantity is what? 2 d log 1 plus T lambda square by lambda d. So, what finally, I am able to show is my regret bound R_t is upper bounded like this.

So, if you see this right now I have not told anything about how the square root beta T looks like we will talk about it next time, but at least if you assume this beta t is fixed you will see that this regret bound goes like square root t and also there is one more T here, but that is inside the log term. So, the dominant term here is going to be square root T this beta T term here they them self grow like in d square root d this beta term they also grow in d.

So, later when you plug in that value this is become square root d, but there is another square root d term here because of that it becomes like d square root T. So, right now let us just only worry about now in terms of. So, now, we have a bound which goes like square root T. So, now, later when we plug in the value of beta T, then we will worry how it depends on the other parameters ok.

You see that already like this kind level that this analysis is different from how we did the analysis for our standard multi armed banded right. So, now, it is using lot of linear algebra and matrix manipulations, its because our assumption that my reward function is linear. So, because of all this linear algebras kicked in ok.

So, let us stop here.