

Introduction to System Dynamics Modeling
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Lecture No – 23.2
Testing System Dynamics Models
Testing System Dynamics Models: Example 2

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Example 2: Infectious Diseases

- *Total population* of region is initially 10000 citizens. New *infections* make citizens of *susceptible population* become part of *infected population*, which initially has just 1 person.
- *Number of infections* = *infection ratio* * *contact rate* * *susceptible population* * *infected fraction*.
 - *Contact rate* amounts to 20 contacts/ week & *infection ratio* is 75% per contact. The *infected fraction* equals *infected population* over sum of all other subpopulations.
- If citizens from *infected population* die, they enter statistics of *deceased population*, else they *recover*.
- The *recovering* could be modeled as $(1 - \text{fatality ratio}) * \text{infected population} / \text{recovery time}$.
- Suppose that the average *recovery time* and the average *decease time* are both 2.5 days. The *fatality ratio* is 90%.
- Only *recovered* and *infected* population can spread disease to *susceptible population*

Example from Small System Dynamics Models for Big Issues by Erik Pryor

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IE 604: System Dynamics Modeling & Analysis

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Let us look at example 2, this is model on infectious diseases; total population region is initially 10000 citizens, 10000 that yeah, new infections make citizens of susceptible population become part of infected population which initially has just 1 person ok. So, then it is getting closer to the existing model that we already seen.

The number of infections is infection ratio and contact and susceptible population infected population. Again the model is quite straight forward there and the values of contact rate is

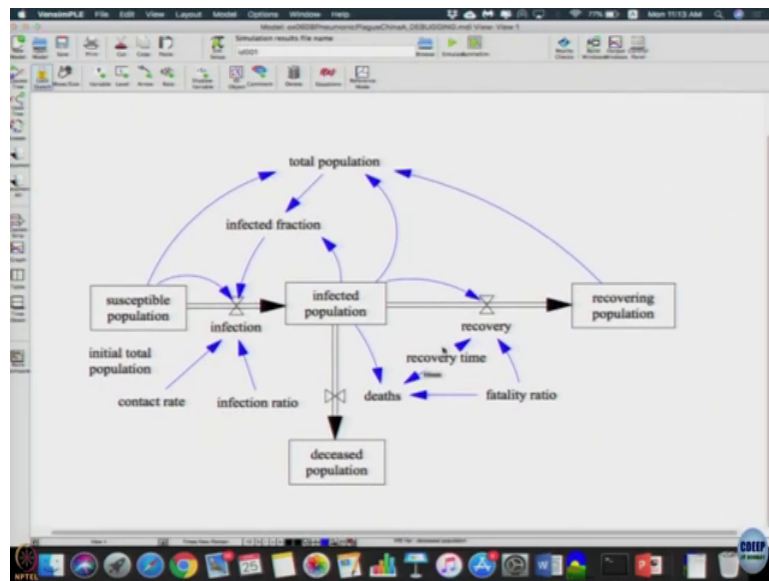
given and the infection rate values also given and the fractured infected equals infected population over some of all other sub populations.

Let us see; what are other subpopulations right now it is only susceptible infected. If citizens so infected population die; they enter statistics of deceased population else they recover ok; now it is interesting, now we have deceased population and recovered population. So, we have susceptible, infected deceased and recovered; the recovering could be modeled as $1 - \text{fatality ratio}$ into infected population divided by the recovery time. So, how long it takes to recover affects that?

Suppose average recovery time and average decease time; there is average time, it take to die are both 2 and a half days, the fatality ratio is 90 percent. Only recovered and infected population can spread disease to susceptible population ok. So; that means, once people die; they do not spread the disease. So, assume they do not spread the disease further. So, only the susceptible and infected population spread the disease and even recovered also I mean recovered infected spread the disease to the susceptible population.

So, let us look at what the model says and see whether we can spot the where as in the model. Let us open; yeah this is non disease population; so you can take it ok.

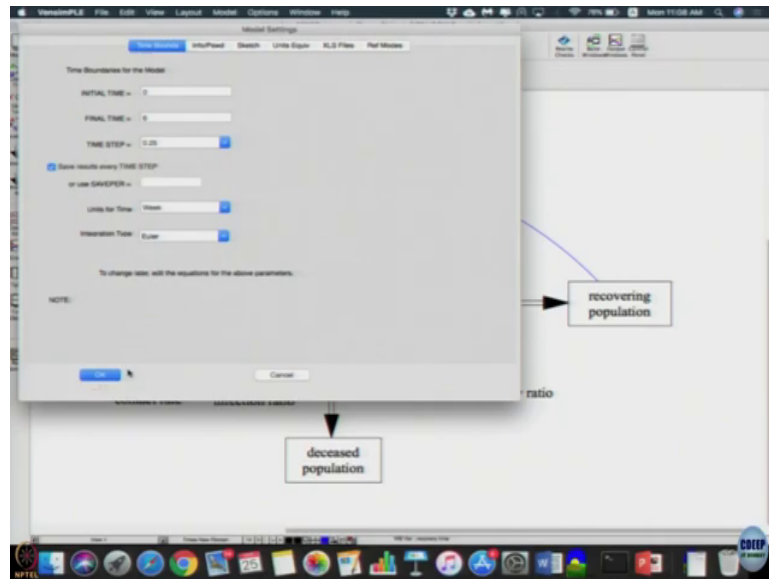
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So, the susceptible population, infected population, recovering population, deceased population; find susceptible the flow seems to be make some logical sense, infection moves them here, recover moves them here and or here either one of the places. The infection is given as a product of contact rate, infection ratio, susceptible population, infected fraction.

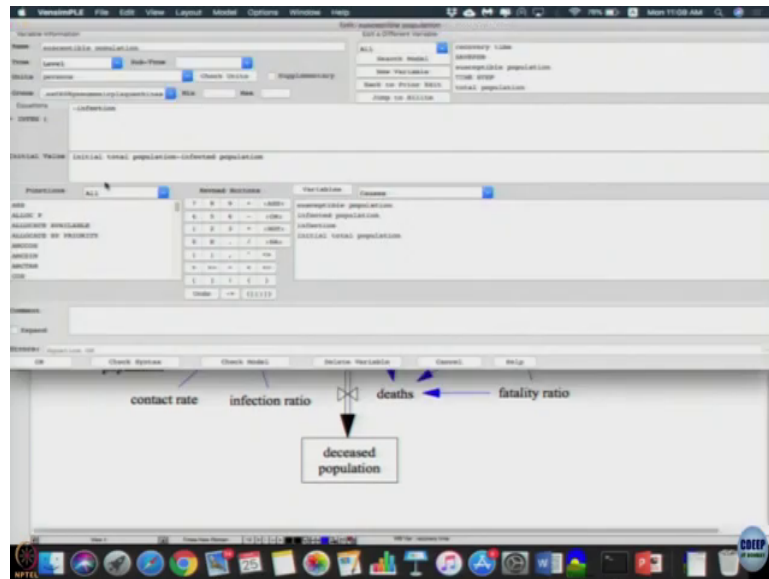
So, they are all seems to be connected there; infected fraction as given as a relation between this and this infected population total population; so that seems to be fine. Recovery dependent on infected population time and fatality ratio. So, that seems to be connected fine that is also given as the similar relation that seems to be fine ah.

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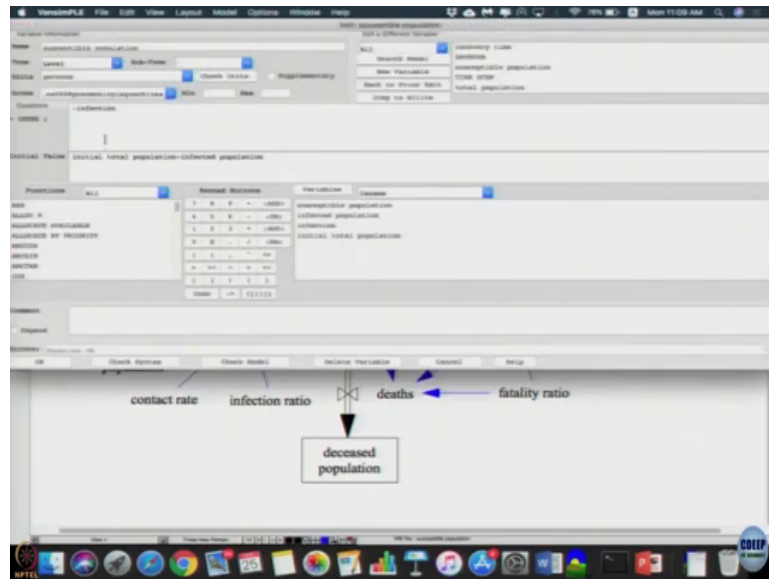
Let us quickly look at model settings. So, final time 0.25 times step week is time units is week ok.

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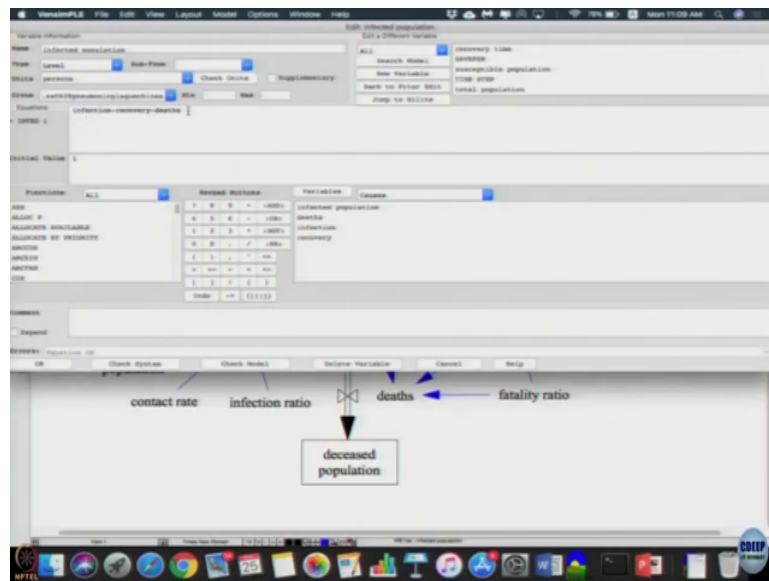
Quickly look at the equations; susceptibility population is minus infection which is fine because there is only outflow there is no inflow here.

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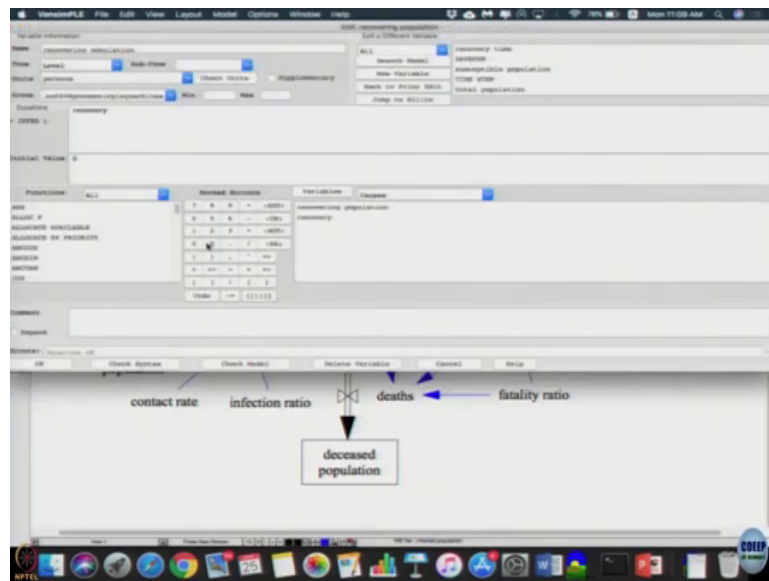
So, initial population; initial total population is minus infected initial value; initial total population is 10000 which is fine.

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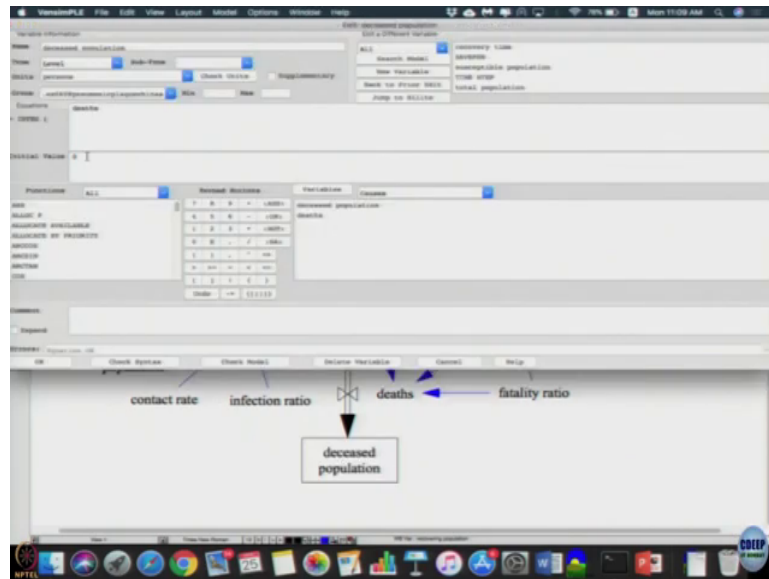
Initial infected is 1; that is fine, the equation the infected is infection minus recovery minus death that is also fine; flows are fine.

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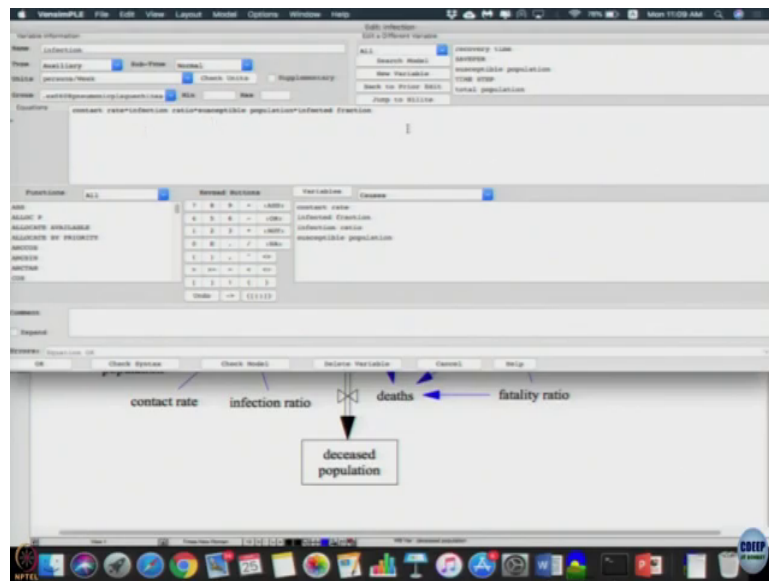
Recovering population initial is 0; adds recovery which is fine only inflows they were.

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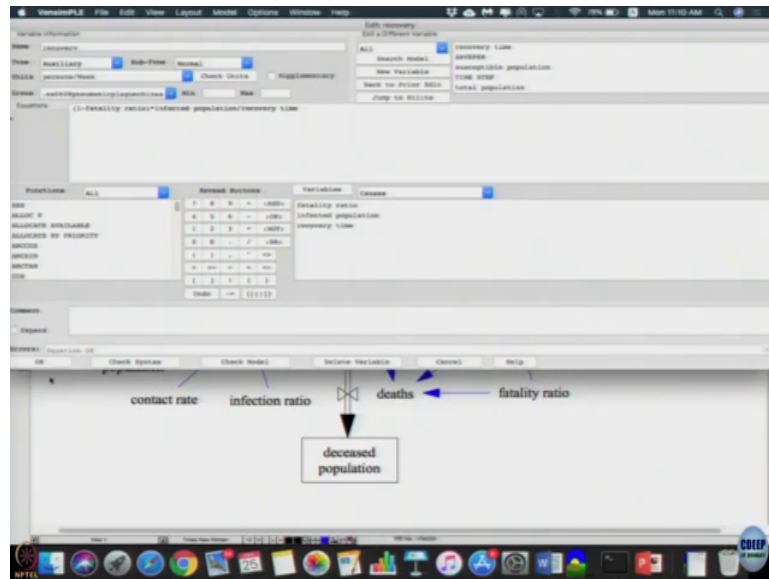
Diseased population again 0; just fine and deaths for plus which is fine.

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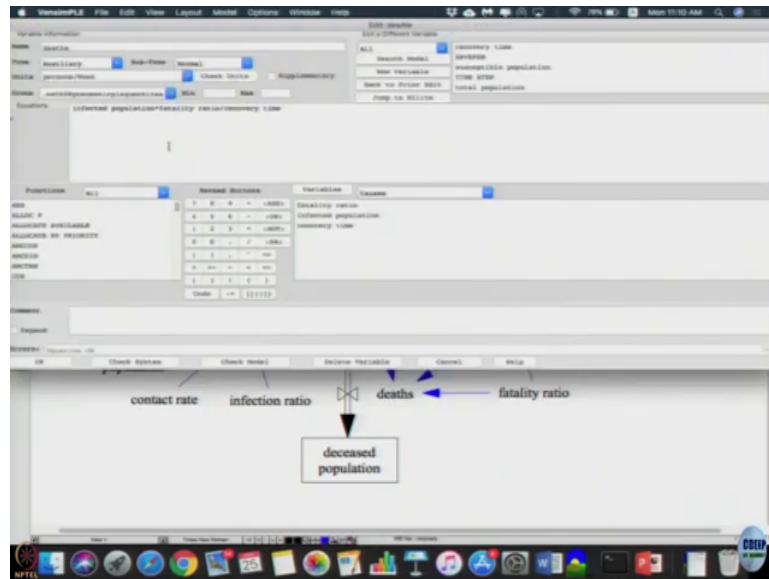
Infection is; it is a product of all which is fine where we should be yeah.

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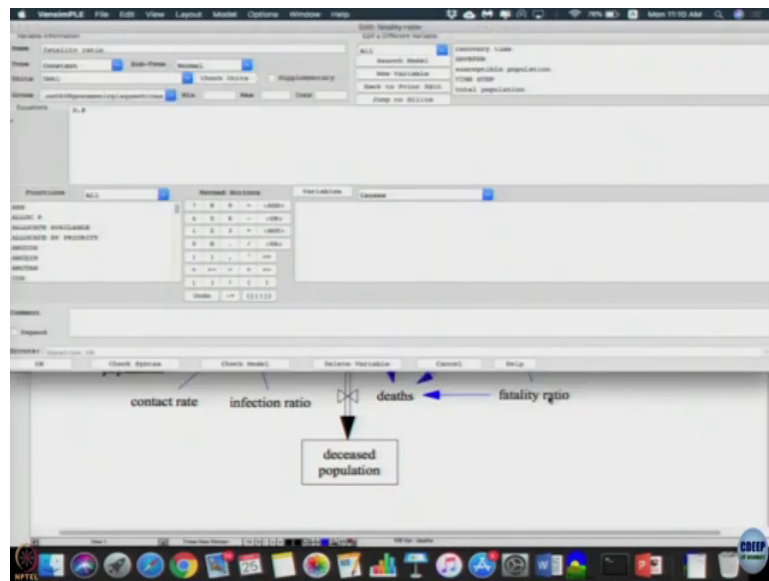
Recovery rate is 1 minus fatality ratio infected population as recovery time that has a description given that is also fine.

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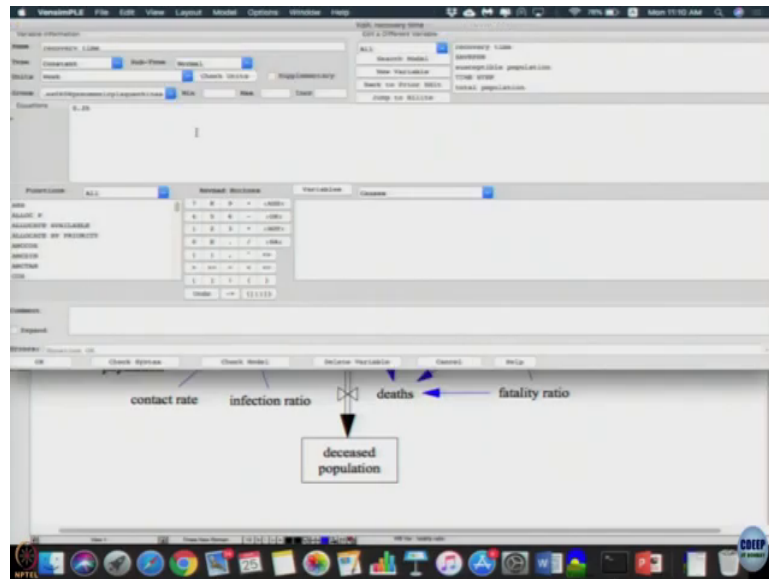
Deaths is infected in fatality ratio; so those who did not die recovered that is how it is model. So, that is a 1 minus fatality ratio is there.

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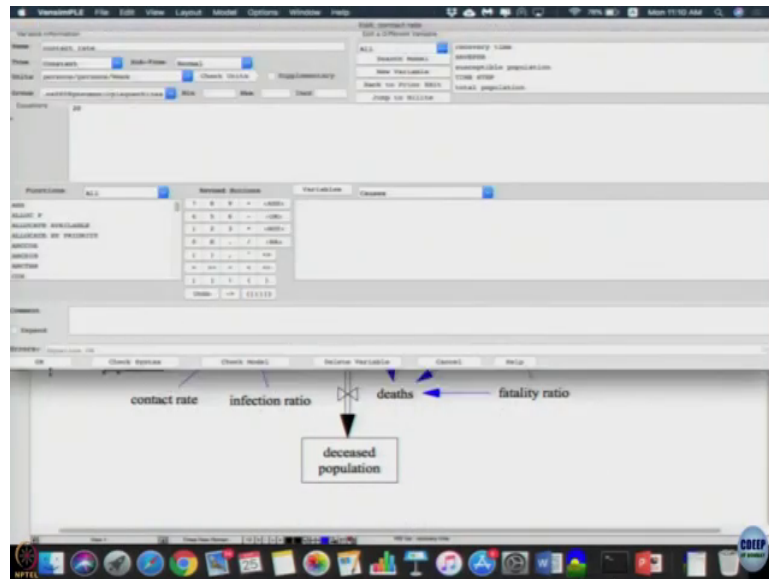
Fatality ratio as good as 0.9 which is fine ok.

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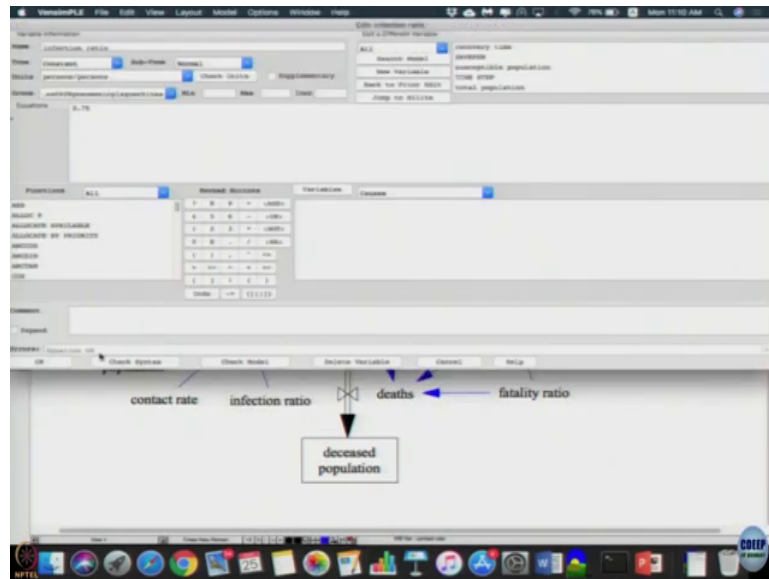
Recovery time was actually given as 2 and a half days it covered into weeks; so 2 by 7, 0.35 which is fine.

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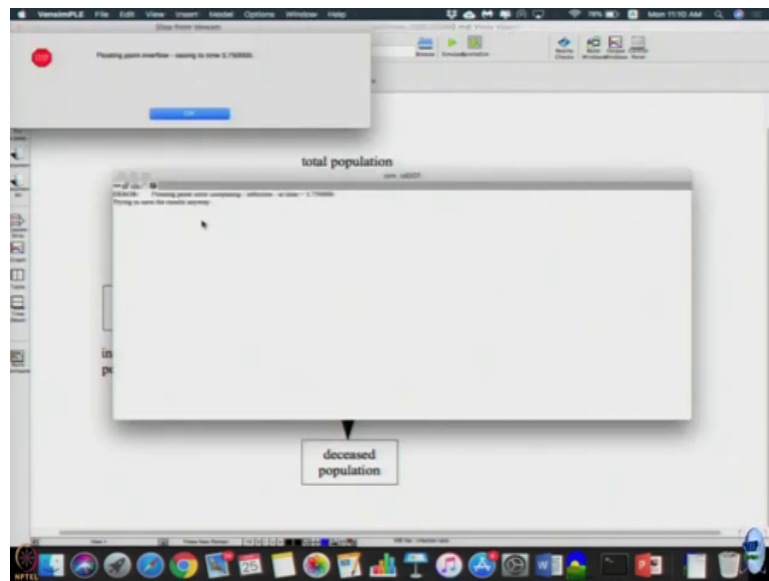
Contact rate as given as 20 per person per week which is fine.

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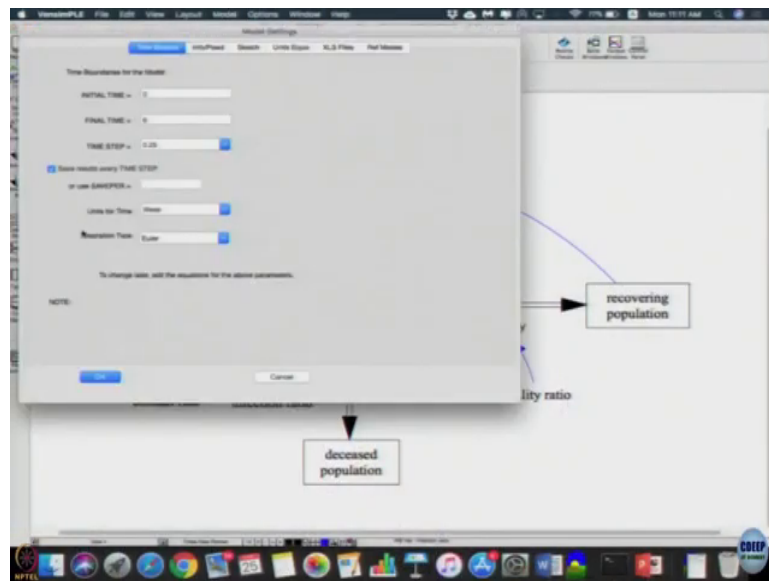
Infection ratios 0.75, which is fine which is what has given ok; model seem generally let us run it ok.

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Again, I am getting floating point error; 3.75, but model seems structurally hm. Let us see, the recovery time was point; recovery time was 0.35 per week right ok.

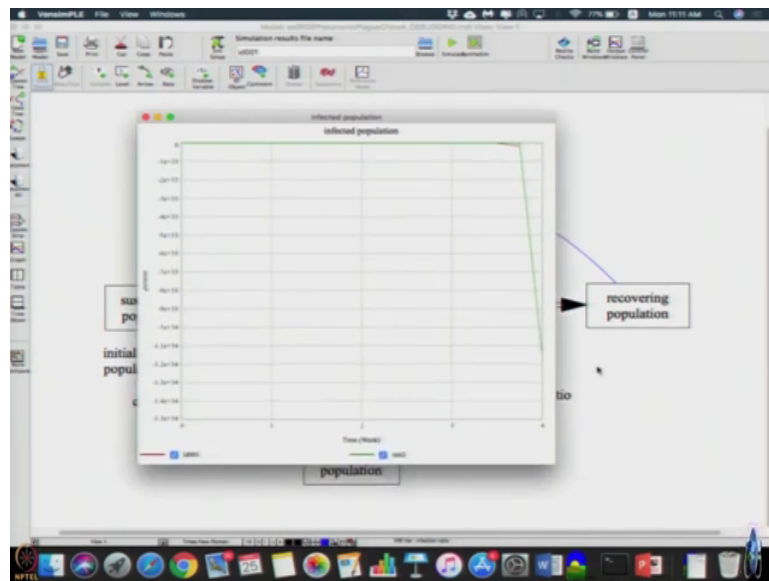
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Let us go to model settings; yeah, let us look at a time step; here time step is 0.25 where in our case the smallest time step was 0.35, recovery time. Here it was 20 into 0.75. So, here the time step the smallest is 0.35, while here the time step uses 0.25.

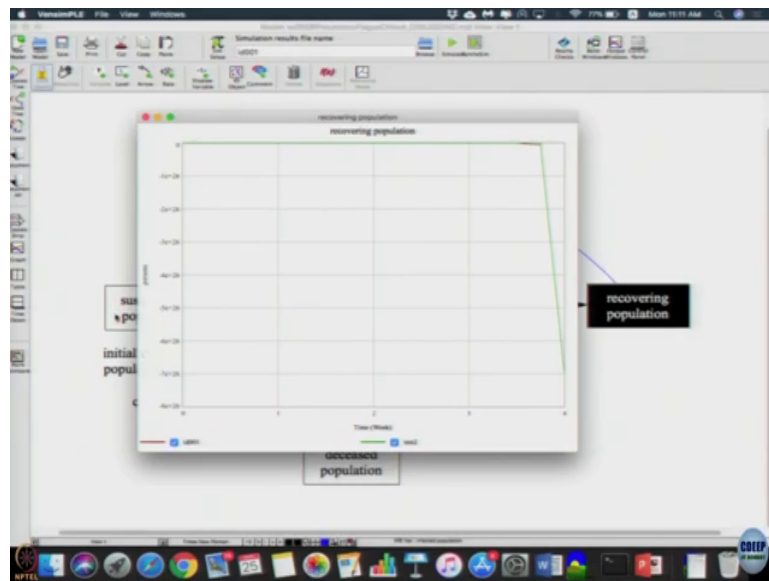
So, finally, we want time step which much lower than the smallest delay right. So, that could be a cause for an unusual dynamics; we can actually look at the dynamics.

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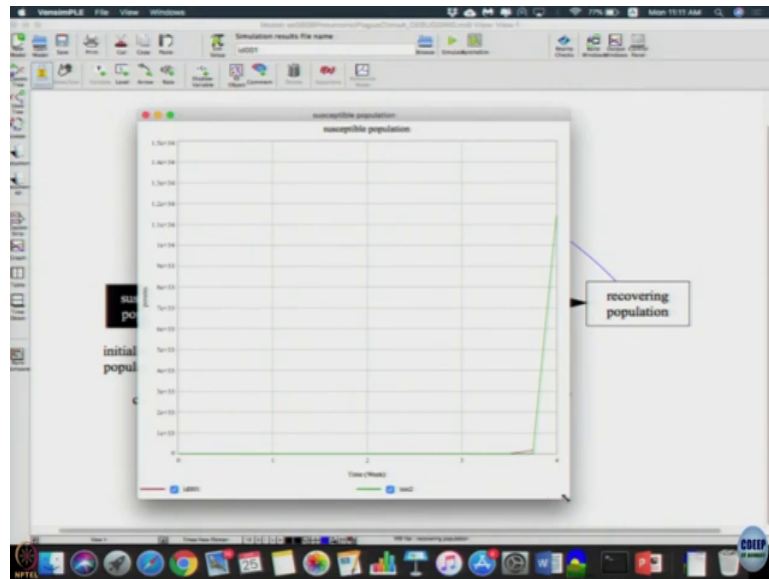
Let us look at infected population, it is just going negative large.

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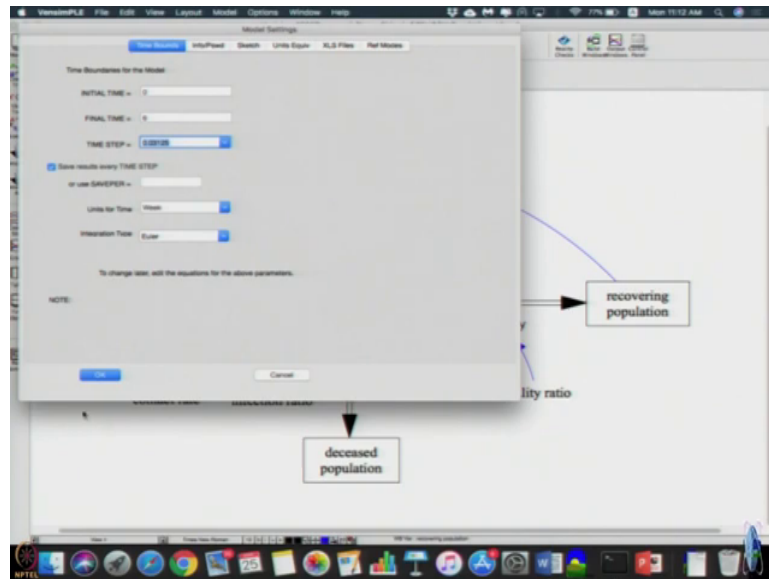
Infected population or recovered population.

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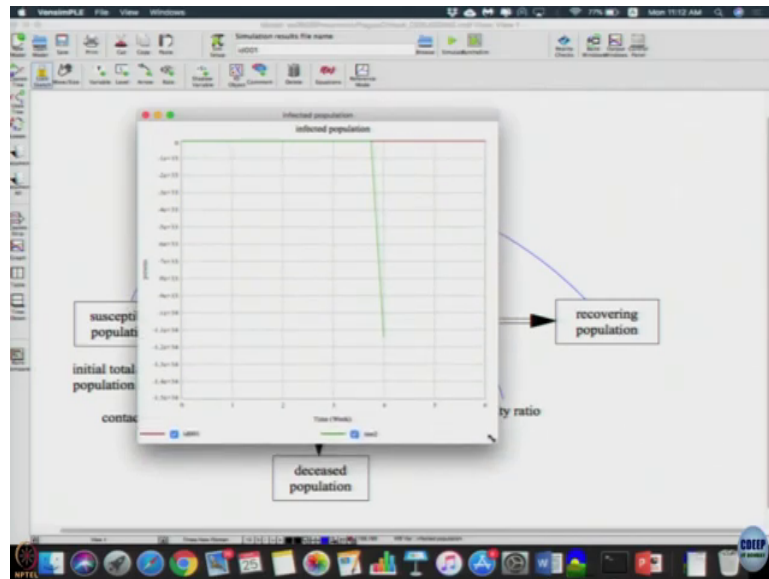
Everything just crashes and become negative, susceptible population is increasing which also does not make any sense, there is no inflows at; it is increasing right.

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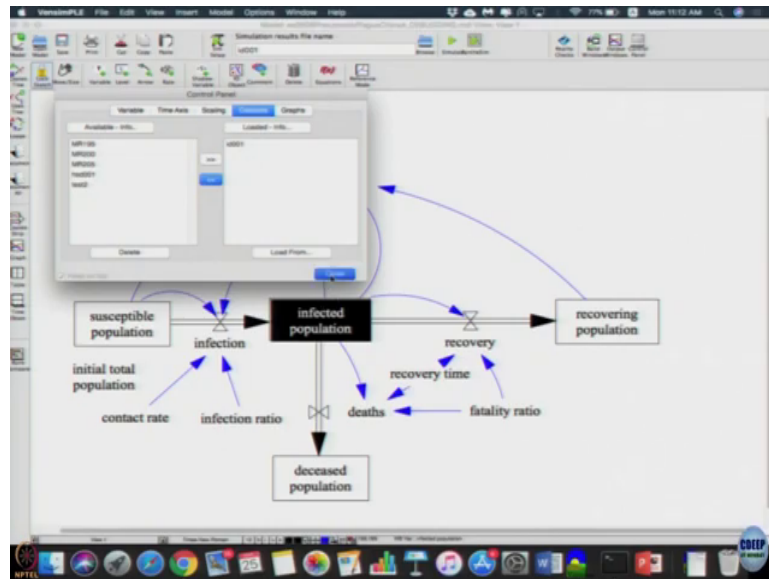
So, let us see whether time step is only issue here other seems ok; switched up 0.25. Let us go say 0.0315, let us try that; let us click ok, let us run it; wait error stopped at least.

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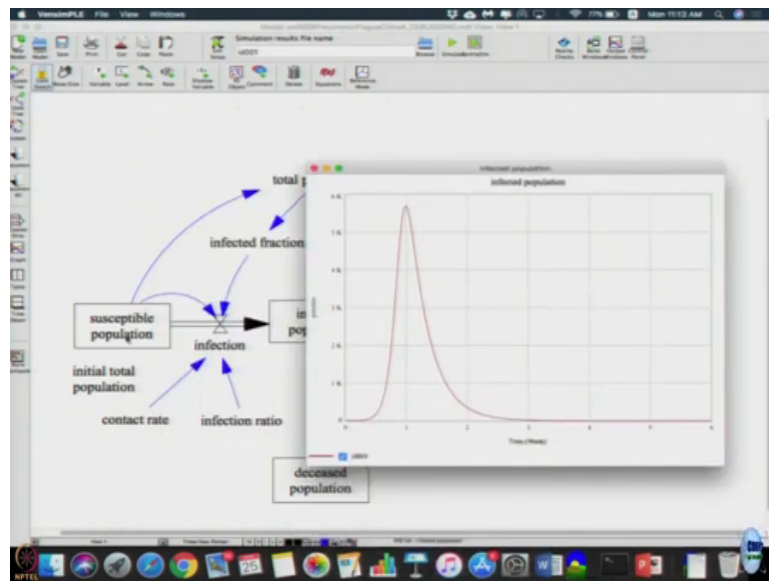
Let us look at infected population, let me run it again. Just to let me get rid of the data set.

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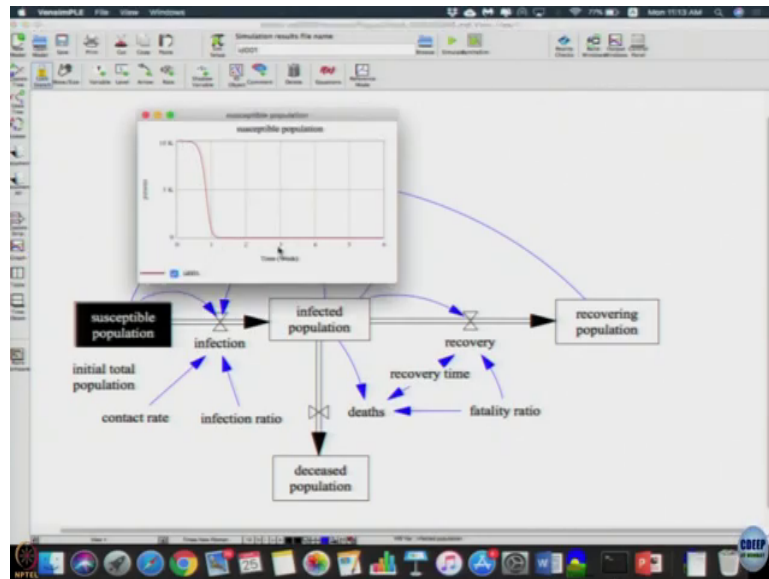
Many some other data set; it has 2 which is loaded we just delete that which has 2; id 001 is what we need to see, id 001 ok.

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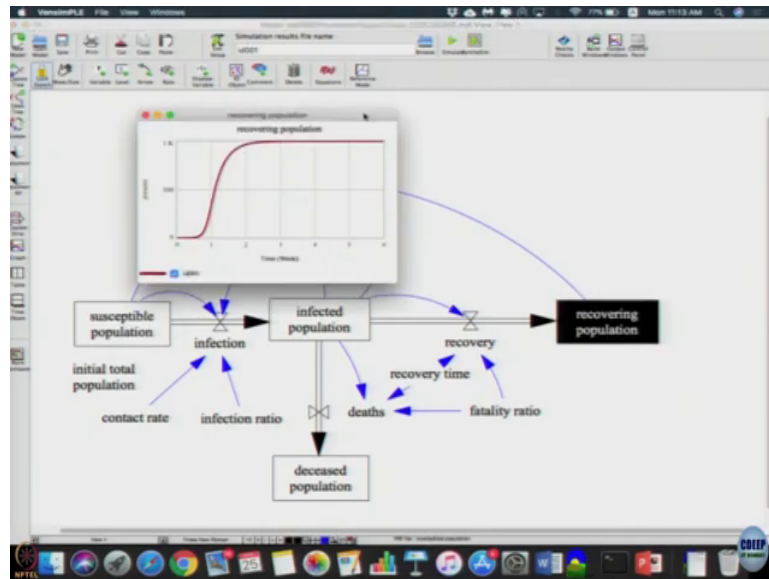
So, susceptible seems reasonable; I mean infected seems reasonable.

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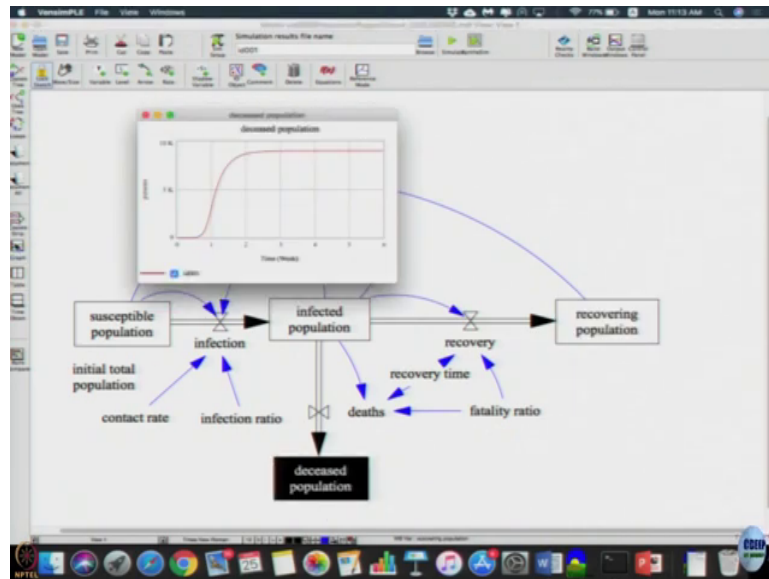
Susceptible as to decrease and went to 0; that is also reasonable.

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Recovering graph showing classical shaped growth that is a dynamics experts from this model.

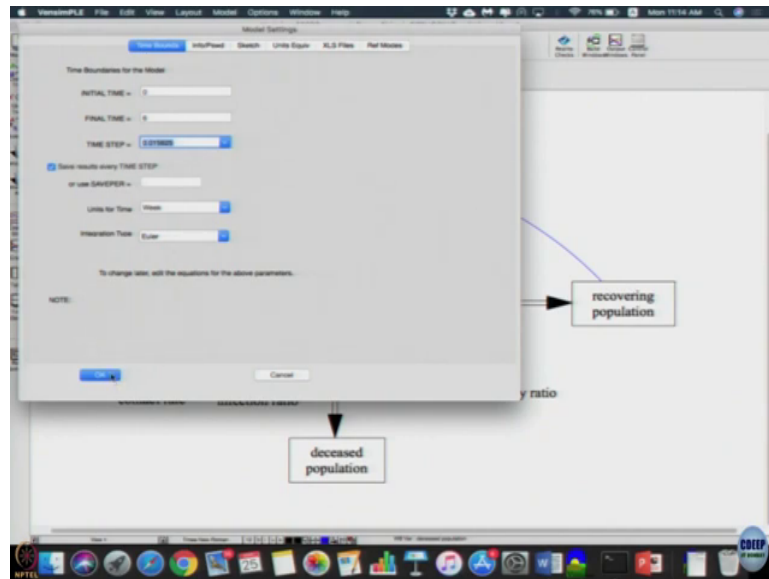
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This is population again shows a shape growth, but 9000 people die and 1000 people recover that is because fatality ratio is 0.9, will that makes sense yeah.

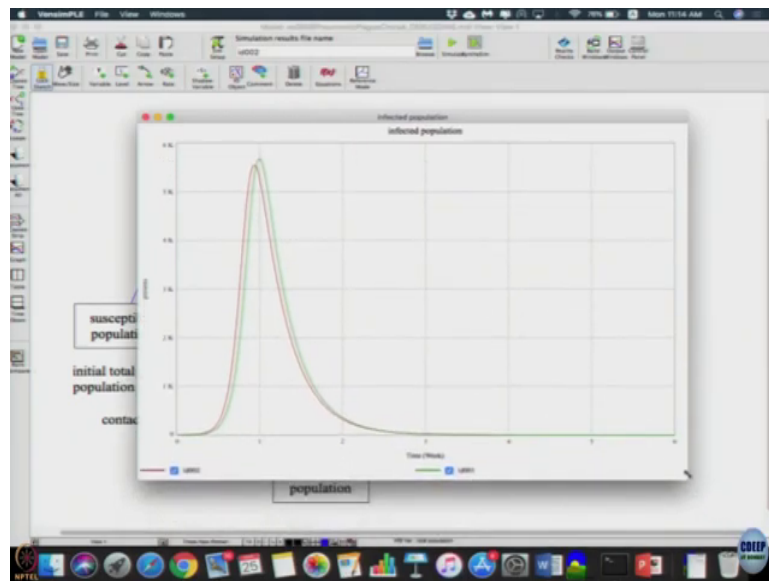
So, this is the overall dynamics and this is only thing that seems to be error in this particular scenario ok. Questions in this case when this case the main error was caused by incorrect simulation time step so, one way to check it is let us create a id 002.

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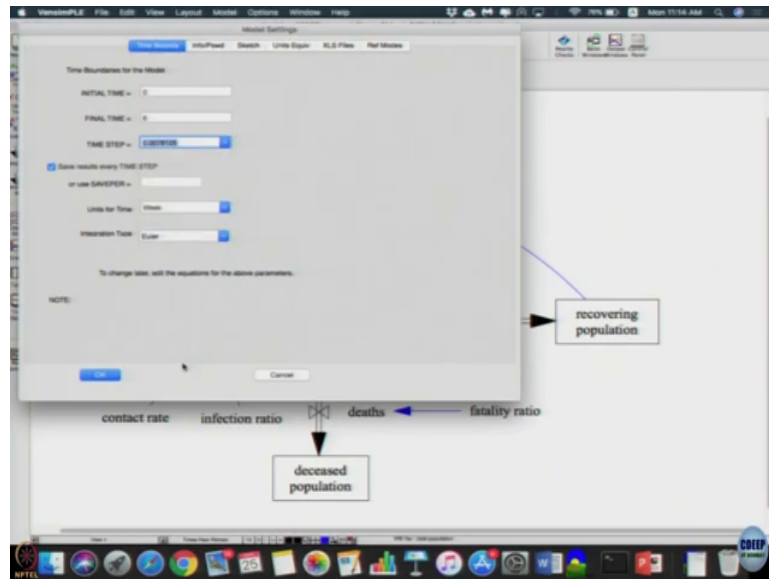
So, in this case I am going to change the simulation time step further down; 0.0156, we do not know what time step we choose; let us choose another time step to see whether there is any difference in the curves. So, I reduce the time step further let us stimulate again; let us go for the infected population.

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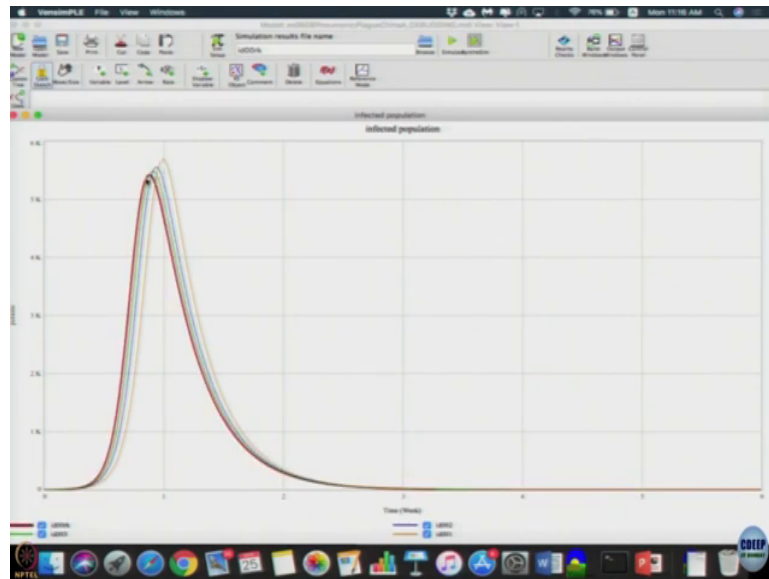
There is a difference; that means, still we need to choose a more appropriate time step such a way that we are getting a curve where it should not keep changing. Whenever there is a change occur; that means, you have to keep reducing; until you get a time step where the changes almost nil. So, let us do one more; let us give an id 3.

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Let us look at settings that is further reduce it to smallest one possible at least in Euler method, let us run it.

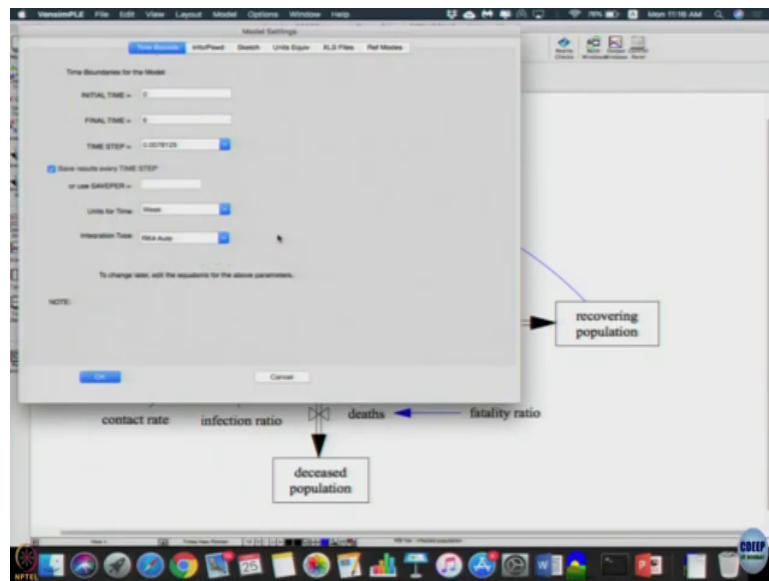
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So, id 3; so farther small changes in the model. The first stimulation time step; the second it was from point what was it? From 0.03 to 0.125; that is huge jump, there is further jump when we further reduce the time step. So, these kind of population dynamics or rather the infectious disease model, you should remember that the origins of the model came from; not from the system dynamics community, but same from the epidemics modeling community who is differential equations to model.

So, if you want higher accuracy to represent rate it better; then its good area to go for other integration types.

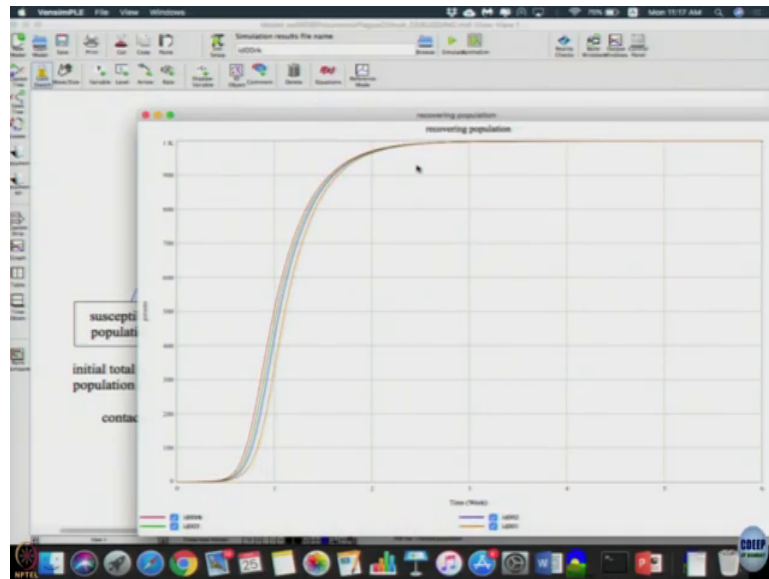
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So, instead of Euler method a more powerful integration tool would be a Runge Kutta method. Those who did not because a numerical methods will know that Runge Kutta's have a accuracy. So, let us just change it to r k 4; call it r k simulated let us see what happens; that gives us the most accurate results just, I will it seems to sell improve based on the earlier ones.

So, in this particular case we may want to prefer r k 4 methods and check whether different time scales also makes difference else will give the lowest time step where results does not change significant.

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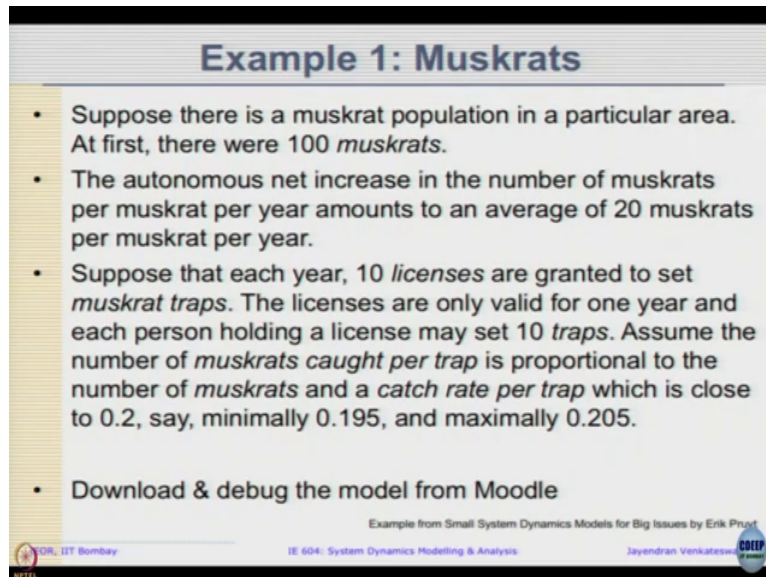


Let us look at the recovery population; these are things we relate to observe, in the sense where it; its steady state values is exactly the same right. Steady state values what we are interested in, it does not matter what we use; it all converts to the same 1000, as you can see right here, but the transient shape of the curve also does not change. So, the actual dynamics does not change.

But we are interested in highly more accurate results, the trajectory changes a bit there is no large chain deviations in the dynamics; in this large deviation then it is a major error in your logic. So, you need to choose an appropriate time step, but once you get kind of similar results and then if you more and more accurate results; it is you have better to have a smaller time step, but a better method of integration.

So, starting point is similar, ending point is similar only the transient line which change.

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Example 1: Muskrats

- Suppose there is a muskrat population in a particular area. At first, there were 100 *muskrats*.
- The autonomous net increase in the number of muskrats per muskrat per year amounts to an average of 20 muskrats per muskrat per year.
- Suppose that each year, 10 *licenses* are granted to set *muskrat traps*. The licenses are only valid for one year and each person holding a license may set 10 *traps*. Assume the number of *muskrats caught per trap* is proportional to the number of *muskrats* and a *catch rate per trap* which is close to 0.2, say, minimally 0.195, and maximally 0.205.
- Download & debug the model from Moodle

Example from Small System Dynamics Models for Big Issues by Erik Png

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So, we saw two things one first one was a small error in the flow equation stock flow equation. Second one had a floating point error because of time steps selection. Let us look at third scenario and see what kind of errors it can have.