

**Introduction to System Dynamics Modeling**  
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**Lecture - 18.2**  
**Formulation Non Linear Relationship Table Functions: Part II**

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REFERENCE POINTS

Capacity defined as  
Normal rate of output

CU must pass thro (1,1)

~~and so~~  
When SP = 1, then  
Shipment = DP = Capacity

Alternately

Capacity defined as  
More possible output

implies utilisation is less than 1 in  
normal scenarios

then  $SP = \frac{DP}{\text{Normal Cap. Util.} \times \text{Cap}}$

then utilisation will pass thro  
the reference point  
(1, Normal  
Capacity utilisation) &  
Saturate at 1.

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I will just go there. Now, there are two ways to do it; one is we can define our capacity as our normal rate of output. Defined as normal rate of output; when I say normal rate of output what we mean is that, if this is the speed at which you typically work, we can take that as the capacity. So, if suppose the pressure is higher, we are able to do little more also, we are able to work little faster. So, we can define a capacity as a normal rate of output. So, my schedule pressure becomes very large, then I am able to produce our work faster and produce little more than what I do normally, ok.

So, in this case what is it capacity utilization must pass through 1 comma 1, yeah must pass through 1 comma 1; that means, it is not saturating at 1 comma 1, we were able to produce at a slightly higher than what is the capacity? When I ask what is the capacity, they might say ok, capacity is to produce a 50 burgers in 1 hour let us say, some a 100 burgers in 1 hour. But if there is lot of pressure, they may be able to produce 110 burgers or 120 burgers.

So, suddenly it does, it sound likes as if they are producing more than their capacity. In fact, they are not; when they defined capacity as normal rate, normally what can you do, I can do 100. But if really if I have to, there is lot of pressure, lot of orders; we can speed things up, we can cut the breaks, we can make it little more better, I can produce little more maybe 110, 120, 125 whatever, some slightly larger number, right. So, that is what we mean define, the capacity as here.

So, here if, so that is mean when schedule pressure is 1; then shipment must equal your desired production must equal your capacity. So, this will be your reference point, right. So, the schedule pressure is just 1; that means, if this is our normal rate of output is 100 and if demand is also 100, I must be able to achieve it. If it is something more, I can whatever it (Refer Time: 03:12) put a little more effort and be able to achieve a little more than that is also. So, that is what is defining here. So, this is one approach right, this is one way; but we can also define alternatively also.

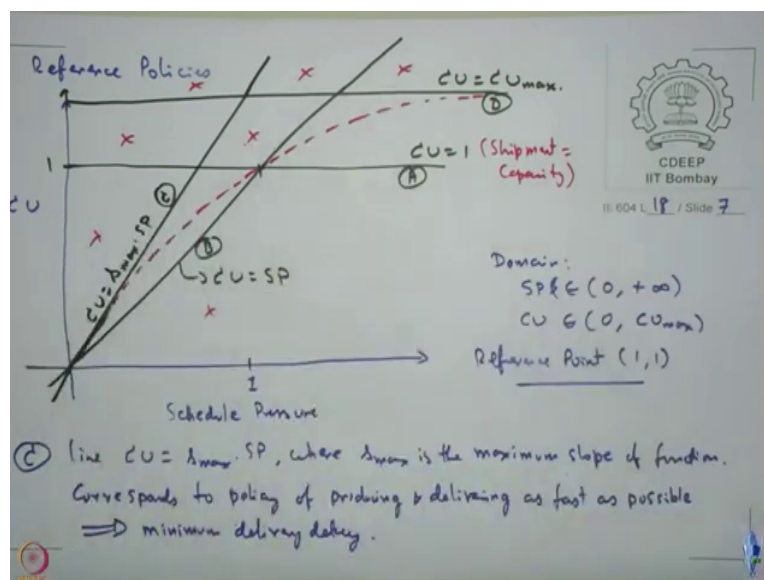
Alternately, we can define the capacity as a maximum possible output; alternately capacity defined as max possible output, this implies that capacity utilization is less than 1 in normal scenarios, right. Then your schedule pressure is defined as desired production divided by normal capacity utilization multiplied by your capacity. Then utilization will pass through the reference point 1 comma normal capacity utilization and saturate at 1, read it carefully.

So, here what we are saying is, suppose we define capacity as the maximum possible output, this maximum burgers I can ever produce right. That means, your reference point cannot pass that 1 comma 1, schedule pressure is there, regularly how much can you make. So, that will

be much lower than that actual capacity we are defining and the maximum saturation will occur at value 1, ok.

So, we will take this one, what has written in blue we will do it for this one. So, that capacity as defined as the normal rate of output, so that means, capacity utilization must pass through 1 comma 1; that means, the schedule pressure is 1, that means I must be able to produce it. So, schedule pressure is more than 1; that means, I may be able to produce little more than what you can do or what is defined normal capacity.

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Next we have to define some reference policies, right. So, let us go ahead and do that. Let us define some reference policies. The schedule pressure and capacity utilization, this is your reference policies right; both are dimensionless this is what we have. Now we can define some reference lines. Let us define one line. When shipment is equal to the capacity then

what should be the line? See, it should be 1; when shipment is equal to capacity, C U should be 1.

So, let us just check that, let us put that as 1 and let us draw a straight line. So, in this straight line C U is equal to 1 and this corresponds to the policy that, shipment equal to capacity, right. So, this is line A that we have. What else policy can we have? What if I am able to shift all the, whatever desired production I am able to make it right; what will be that line? Whatever the desired production I am able to ship it? That means shipment is equal to desired production right that must it must be in a 45 degree line.

So, let us go ahead and draw the 45 degree line. Hope it looks 45 degree is, but yeah go with it. So, this case C U is equal to the schedule pressure, right. If capacity utilization equal to schedule pressure; that means, whatever I am is my demand, I am able to keep shipping that same amount. So, let us this be curve B, curve A; but yeah. Now, comes the more interesting lines. What else could be the reference policies here? When we do many models, we typically assume this saying that if schedule pressure is less than capacity, then do schedule pressure or do capacity. I can use a min or max function and just simply model it, correct or if then else construct we saw it last time. But in this case we want to make it more realistic saying that, you do not get exact such shifts, as we approach closer to the full capacity, I may not be able to produce at the same speed one.

And two, if the order is very less, I am able to get it even faster; because I am at a very low capacity, right. So, suppose the schedule pressure was hardly anything. So, I may be able to do it things much faster, then what it is, right. So, how much faster can I go? Suppose my schedule pressure is low; that means, my demand is very low as compared to capacity, capacity is say 100, my demand is say for 10, right. So, I (Refer Time: 09:49) value will be somewhere here.

So, in that case how much early can I go? How much sooner can I finish it, right? So, I must be able to do it much faster, right. So, there the utilization could be much higher; but I can only go to up to some limit, there has to be some processing delay, there must be some

heating delay or some processing delay, I cannot speed up much lower than that particular value.

So, for now just bear with me. So, let us draw one more line called, let us call this line  $C_U$  is equal to  $S_{\max}$  multiplied by  $S_P$ . Let us call this curve  $C$ , let us see what it means. Let us do only for curve  $C$ ; curve  $A$  and  $B$  is kind of obvious. So, we are just going for curve  $C$ . So, the line  $C_U$  is equal to  $S_{\max}$  multiplied by  $S_P$ , where  $S_{\max}$  is the maximum slope of function. It will correspond to policy of producing and delivering as fast as possible; as fast as possible means minimum delivery delay.

Just read it ones, so what you are saying is, when the demand is only 10 and where capacity is 100, then I can actually work much faster and produce things at a much higher rate; though the target delivery delay may be one week, I am able to deliver it within 4 days. I am even able to deliver in 1 day, may be it takes some time to actually produce it; but I may not need to wait until the 5th day or 6th day to deliver it, I might deliver it much faster. But there is some limit  $S_{\max}$ , I do not know what it is; but it is some limit. Let us see go ahead and try to find out how to derive this  $S_{\max}$ .

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For (C) : Shipments = Capacity  $\times$  CU  
 $= C \times \lambda_{max} \times SP$   
 $= C \times \lambda_{max} \times \frac{DP}{C}$   
 $= \lambda_{max} \cdot DP$   
 (max possible Shipments  $\downarrow$   
 shipment rate)  
 $\lambda_{max} = \frac{B}{DD^*}$   
 Delivery Delay  $DD = \frac{B}{\text{Shipments}}$   
 Min. Del. Delay  $DD_{min} = \frac{B}{\text{shipment}^*} = \frac{B}{\lambda_{max} \cdot \frac{B}{DD^*}} = \frac{DD^*}{\lambda_{max}}$   
 $\lambda_{max} = \frac{DD^*}{DD_{min}}$

So, let us try to compute for curve C; for C itself we are continuing that. Let us go to our basic equations; shipments is defined as capacity into capacity utilization, correct. So, now on that curve C, it is nothing but C into S max into schedule pressure, which is C into S max. What is schedule pressure? Schedule pressure is nothing but desired production divided by capacity; which means, it is S max times my desired production. What is desired production? Backlog divided by target delivery delay D star. So, recall your. So, suppose this is your shipments, what does it represent? This represents the max possible shipment rate, correct.

Now, if you recall the delivery delay equation. So, let us then call it shipments star say. So, this delivery delay was defined as backlog divided by shipments correct. So, suppose then to get the minimum delivery delay, I can substitute B by shipment star; which is nothing but B

by  $S_{\max}$  into  $B$  by  $D D^*$ , which is nothing but desired delivery delay by  $S_{\max}$  or  $S_{\max}$  is nothing but your  $D D^*$  divided by  $D D_{\min}$ , right.

So, you can compute the slope, because you know both the parameters;  $D D^*$  is your reference delivery delay, what the company is promising ok, this is the delivery delay I am promising, it will take a week to produce.  $D D_{\min}$  is you know, what is the minimum time it is going to take what is the minimum total cycle time it is going to take to produce where there is absolutely no delays. So, that we cannot go lower than that, there are physical limits to it. So, based on that we can define ok, that is the maximum speed at which I can ever ever produce, push comes to show that is the maximum I can ever do, can easily be computed and we can get those values or let us go back to the sheet. So, now, we have three values, right. Can I come up with the one more curve?

Suppose, my schedule production is much higher than my or my desired production is much higher than my capacity, right. So that means, the value here, so this is the  $S P$  and this thing, so this point has to be 1. But can I keep on producing on this line  $B$  forever; no right, I cannot keep on producing on this line or even this line forever. So, I need to saturate at some point; let us for modeling sake let us simply define another line. Say,  $C U$  is equal to  $C U_{\max}$ , let us call it line  $D$ . It is possible in reality also to get your  $C U_{\max}$ , because the maximum utilization ever possible.

We have physical limits you know, this is the maximum overtime people can work, this is the maximum people I can employ, even if there is physical the machines can only work so much time. So, based on that, you can say how much maximum you can produce; even if you can produce say at normal rate 100, if the order is for 10,000 units, you clearly cannot make it. Maybe you can go up to 110 or 120 or 125, 130, 150 whatever. So, that is the maximum you can never do right, somewhere it will saturate, right. So, now, we have these nice reference policies, ok.

Now, step 4 in our list of steps or guidelines, you are suppose to find out what is the what happens under extreme conditions. What happens if the schedule pressure is 0 0? It has to be 0. So, one extreme condition can backlog be negative, you can say that does not occur in this

particular scenario. So, that should not occur say anything less than 0, I should not be able to ship anything, right. So, if let me schedule shipment has to be 0; if no orders come in, backlog has to be 0, desired pressure has to be zero, that means shipment has to be 0.

And when schedule pressure is very large or desired production is very large; that means, it is say saturate at  $C_{max}$ , at some point, at some value here it should reach  $C_{max}$ . Those are only two possible limiting conditions, you have anything 0 or lesser it should be 0, anything more than at a large value  $C_{max}$   $C_U$  will take only a maximum value of  $C_{max}$  whatever be the value, up to infinity suppose it has to take  $C_{max}$ . So, that is the extreme condition that we have thought.

And domain you can, may be quickly write it here. So, domain  $S_P$  has to be between 0 to plus infinity it can go there; but capacity utilization typically between 0 to  $C_U_{max}$  that is the domain of the variable, we need to ensure that this what happens. So, you define that.

Now is the fun. Now we need to define. So, we need to find some possible shapes for the curves; that is what kind of, because we are not drawn that, the entire idea of this is to figure out what is the shape of the curve that we are interested in. But before doing that, can you identify the feasible and infeasible regions here.

Naturally because of various lines, we are getting various zones and regions; one obvious is anything above this line is infeasible, right. So, I do not need to bother about that. So, let me just put a infeasible, infeasible, infeasible area;  $C_U$  can never take any values beyond that, so that is gone. What about this region here? Why not, because it is to the left of  $S_{max}$ , so I can never produce it. So, this also becomes my infeasible region, infeasible region. What about this?

Student: Feasible.

It is possible it is feasible, because I am going to get this between  $S_{max}$  and the regular capacity utilization; say we are going to assume that people do not work much more slower than they have to. And suppose it target is 1 week and there is no pressure; we hope it will



finish in 1 week; it does not get unnecessarily delayed. So, this is fine. So, this region also company will not like to operate here; when there is no schedule pressure, you are going lesser than that deliberately, so this also is infeasible. What about this region and this region? Feasible or infeasible; what about this, what about this? Not feasible, anyone else? Ok, we will come to that.

See what are the other reference point that we had? the curve passes through 1 comma 1 this was the reference point. Curve passes through 1 comma 1, it can certainly not pass from here right; then this curve has to come from in this zone and pass through here. Then it has to do some real gymnastics, if it has to jump here; it will be like some here and then goes up like that. If it is going to pass through 1 comma 1; that means, as I approach this capacity, I am unable to work at this  $S_{\max}$  space or I am unable to work at the minimum delay. Because I need to take time right, because normal capacity I am using my target delivery delay.

So, if anything is going beyond that, my pressure is going beyond that; then my delivery delay will get delayed, where the total delay will be more than the my target. That means, I cannot go faster than my schedule pressure; because that itself, maintaining itself is difficult. So, maintaining a region earlier than that is going to be even more difficult or not possible at all. So, that sense this region also became infeasible.

So, now I am left with only two domains here where it can be feasible, right. So, logic dictates that at low values I may be operate up to  $S_{\max}$ . So, let us just arbitrarily put that blue should I, I will try red again, it is easier to see; maybe up to a point I can operate at the  $S_{\max}$ , right. Beyond that it is very difficult to maintain the minimum delay; because natural delay is more orders come in, then I cannot operate at very minimum delay, then I start moving away from that.

Then I pass through 1 and then I keep going and saturate at some particular point. So, this can be a typical curve that we cannot play with. Let us then try to understand the physics or the reality behind these two lines here, these are the key lines that is here. This is when I am operating at the schedule pressure at a regular delay. This when I do not have much work, then I can operate very fast; I can achieve the minimum delay. Suppose it takes average of 1

week to do things, if I am operating here; that means, I am going to finish the job in 4 days, right.

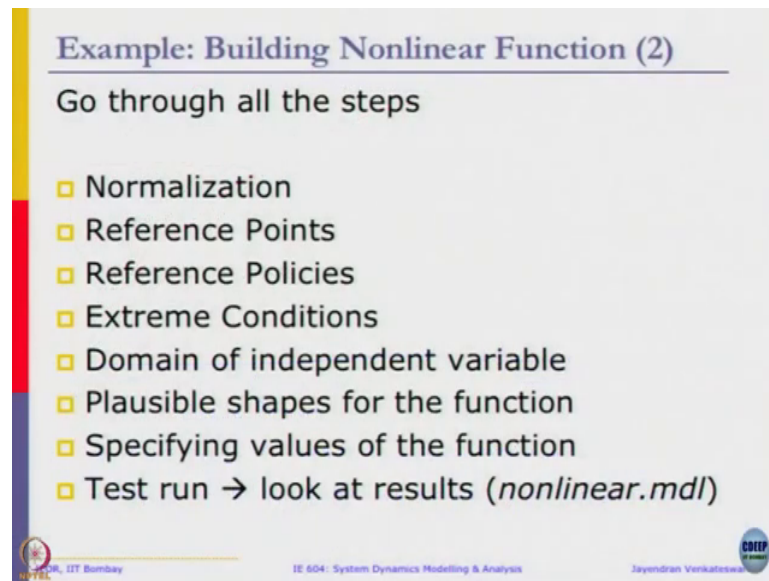
As more and more work comes in, it will going to be very difficult to maintain that 4 days delay; because I am only promising people 1 week, so I am going to start working only at the 1 week delay time. So, as my schedule pressure goes higher or may decide production or the backlog becomes larger; right my schedule pressure becomes higher means what, desired production means this goes higher, which means backlog should have gone higher, everything else are constant.

So, this backlog becomes larger, I cannot work at this minimum delay. So, we have to work at this at a standard ok; this is the schedule pressure as a normal capacity I am able to produce say 100 burgers, ok. Let me produce that, because order is 100, I can produce 100 that is the speed at which and it will take a so much time, so I workout that much amount of time to achieve that.

Yes, whether we stop very early at 10 percent, 20 percent, 30 percent that depends on our speed and if actual region. So, you can actually come like this and what you are saying is it can go much tangentially, and it can you know saturate flatter or it can go very steep and saturate much quicker to that or it can take much longer time to saturate, so all those. So, within this feasible region, we can have many infinitely many combinations of curves; but we are not going to get the different shape of the curve, the shape continues remain the same that is the idea.

So, at this point we have to make some assumptions, so that we can you know enter the values and start building the model. Suppose we say our normal capacity is 80 percent of the maximum capacity, ok. So, then if capacity utilization is 1, then  $C_{max}$  will be about 1.25, right; because normal capacity is 80 percent of the maximum ever possible, then the max possible. And then you define that 80 percent as 1; that means, capacity max has to become 1.25 the simple numbers we can have. And then we can divide it into different intervals to identify and draw what is the curves that we want. So, let me just go back to our slides.

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**Example: Building Nonlinear Function (2)**

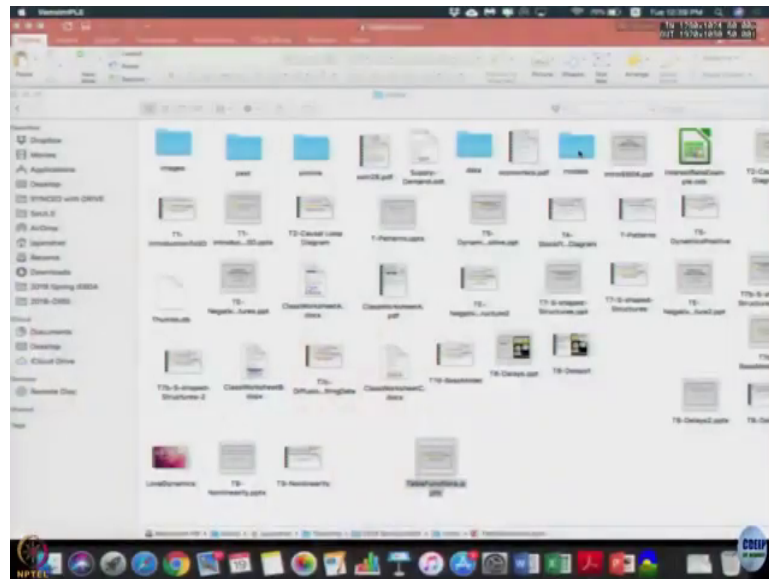
Go through all the steps

- Normalization
- Reference Points
- Reference Policies
- Extreme Conditions
- Domain of independent variable
- Plausible shapes for the function
- Specifying values of the function
- Test run → look at results (*nonlinear.mdl*)

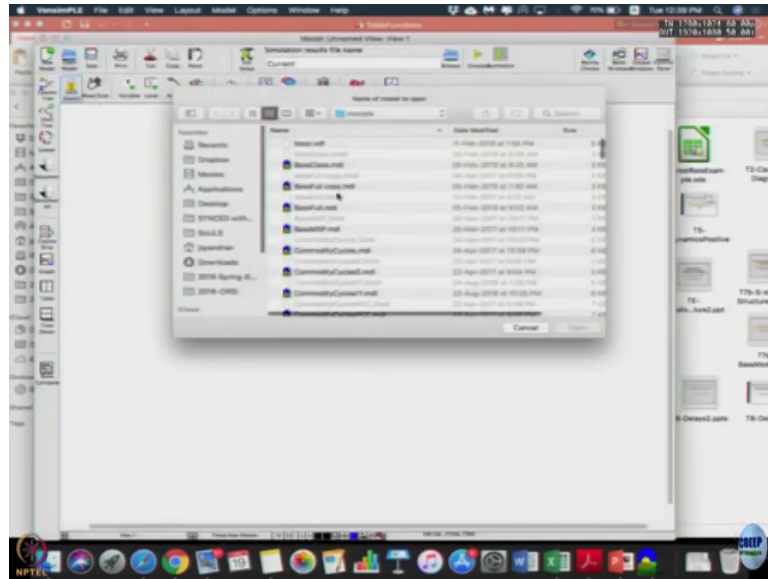
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So, these are steps we did; we did normalization, we identified reference points, reference policies, extreme condition, domain, possible shapes, we have drawn, we had drawn one shape. Now we need to specify the values of the function; that is up to us, how long we want to do it to saturate it. I have entered one such combination in our VENSIM; let us just quickly see that model.

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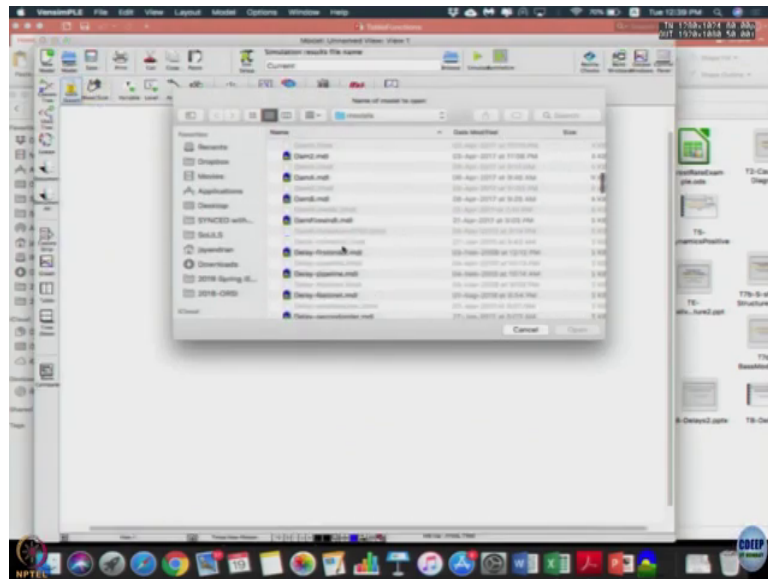


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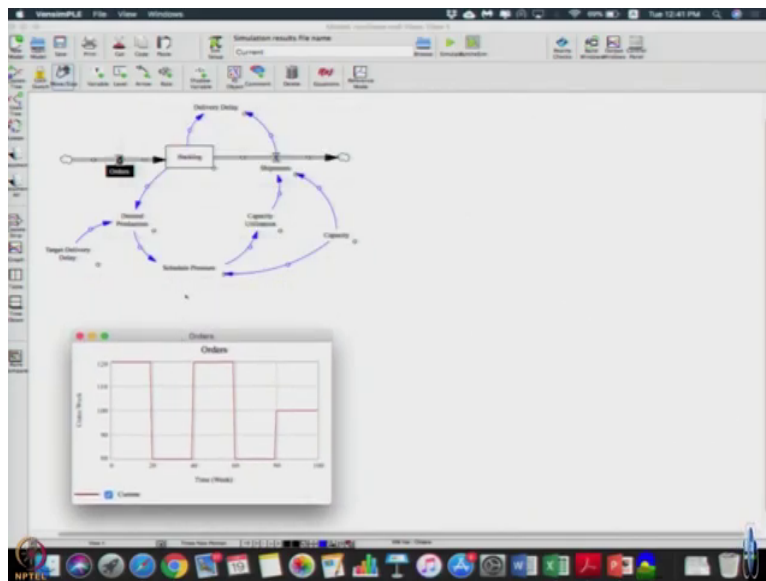
Since you now to enter the table function, we have just directly see that function.

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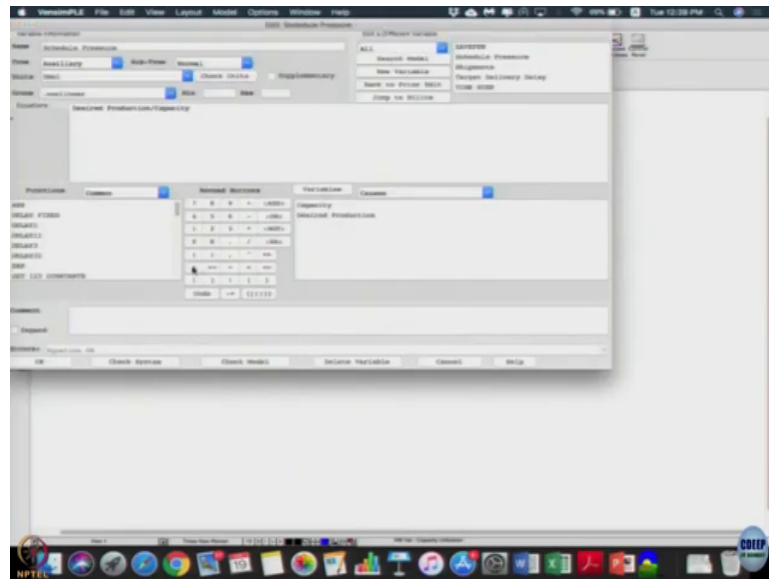
Non linear.

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So, model I am not sure this is the final one.

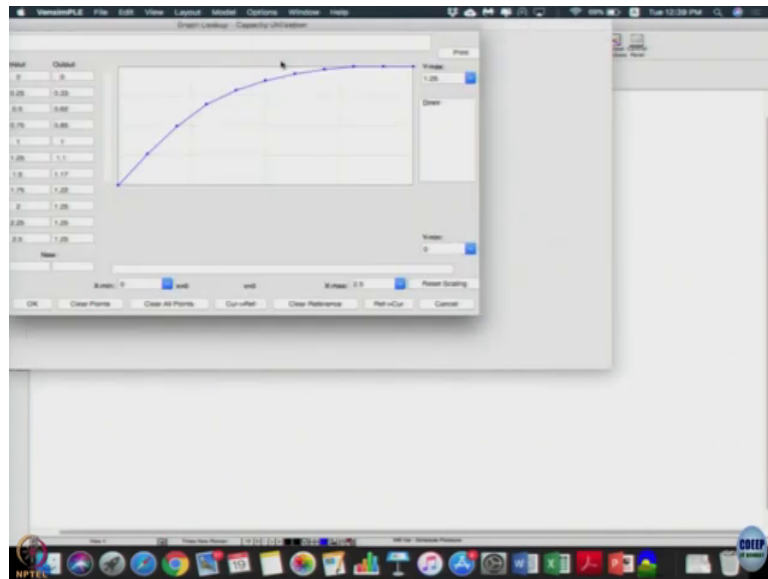
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Schedule pressure is defined, capacity utilization as graph, ok.



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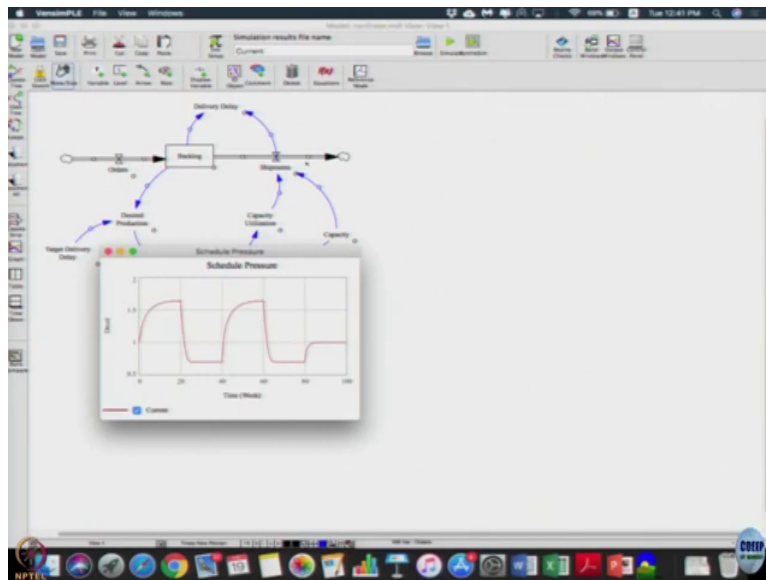
So, here you can observe the input schedule pressure goes increments of 0.25 and goes all the way to 2.5. And your output goes from 0.33, 0.62, 0.85, 1 it is just some arbitrary numbers that has been put; but idea is it saturates at a max value of 1.25, we put two some extra values, so that if schedule pressure goes beyond 2.5 also, it has to be, C max has to be 1.25 there is a care has been taken. One extra, a few redundant values are also there, you can show that the curve actually flattens out at that particular capacity utilization.

How we know it is operating S max? It is because the normal input is 25, but here it is 33 percent output. So, you are operating at a higher speed than you are able to work on it; at 1 comma 1 it meets, at 0.25 you are operating at slightly higher than that, 0.5 you are operating slightly higher, even 0.75 you are operating slightly higher, only then you cut into 1.

After that you are operating at a lower speed 1.25 or operating at only at 1.1; 110 percent of your capacity, though your pressure is to work at 125 percent. When pressure becomes 1.5, still it goes only little more, little more etcetera and then saturates, right. So, these values are closer to your S max and here it cuts your 1 comma 1 point, starts at 0, 0 and then it saturates at 1.25.

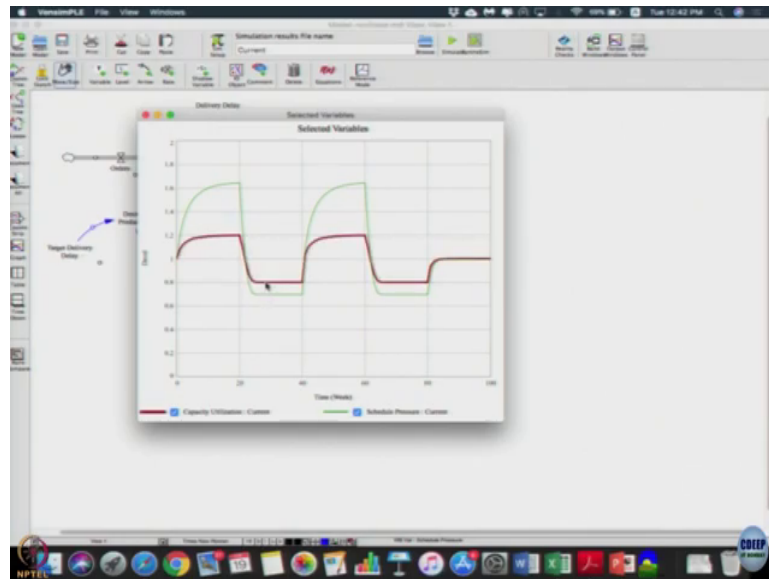
So, if you simulate it; I am not even sure what is the order equation, looks like we are doing some step input function, we can visualize it, this is orders that has come.

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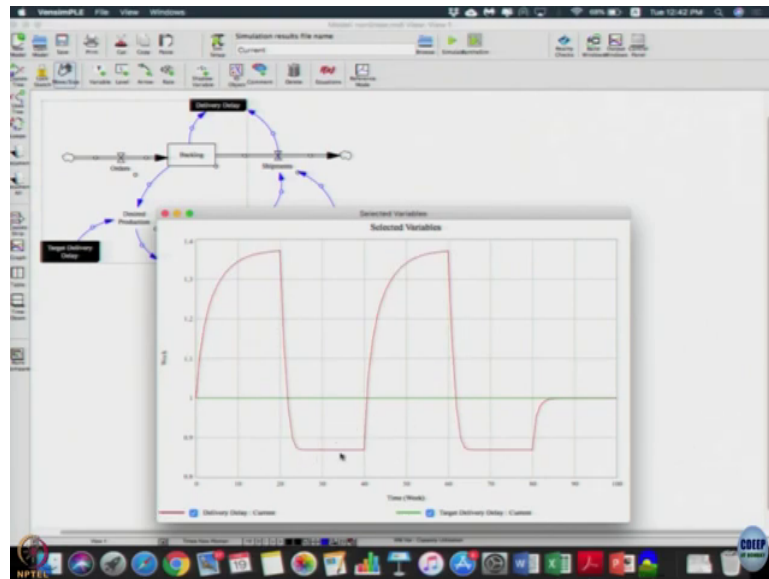
Based on the orders, this is the schedule pressure that has occurred; sometimes it goes beyond 1.5 and then sometimes it goes close to 0.6.

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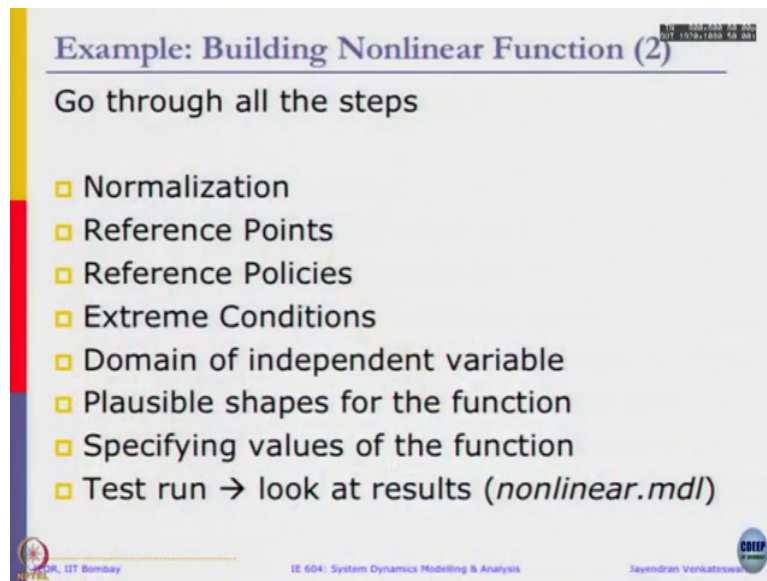
You can see both together. You can see the capacity utilization, the red line starts at 1 and then goes up to 1.2 and then again falls down to 0.8 and fluctuates there; where schedule pressure goes all the way to one point more than 1.6 and falls much below. When schedule pressure falls below still I am capacity utilization at 0.8, I am able to work faster. So, I am operating not on that line, a little away from the line, closer to  $S_{max} X_{max}$ . If you model delivery delay against the target delivery delay.

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Consider target delivery delay always 1, which is 1 week which is given. When the schedule pressure was high, when orders was more, in that time my average delay increased; but then average delay then fell down, I am able to produce much faster. So, this we are approaching the minimum delivery delay, the right here.

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**Example: Building Nonlinear Function (2)**

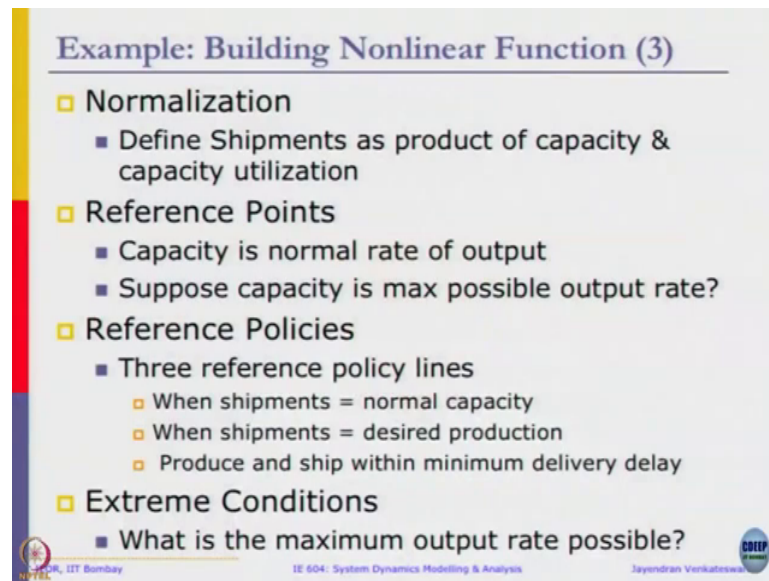
Go through all the steps

- Normalization
- Reference Points
- Reference Policies
- Extreme Conditions
- Domain of independent variable
- Plausible shapes for the function
- Specifying values of the function
- Test run → look at results (*nonlinear.mdl*)

Logo of IIT Bombay is visible on the left side of the slide. Logos of IIT Bombay and CDRI are visible at the bottom of the slide.

So, building the model we observed how schedule capacity utilization change for schedule pressure, how the orders affected change in schedule pressure, what are the ranges etcetera we just saw.

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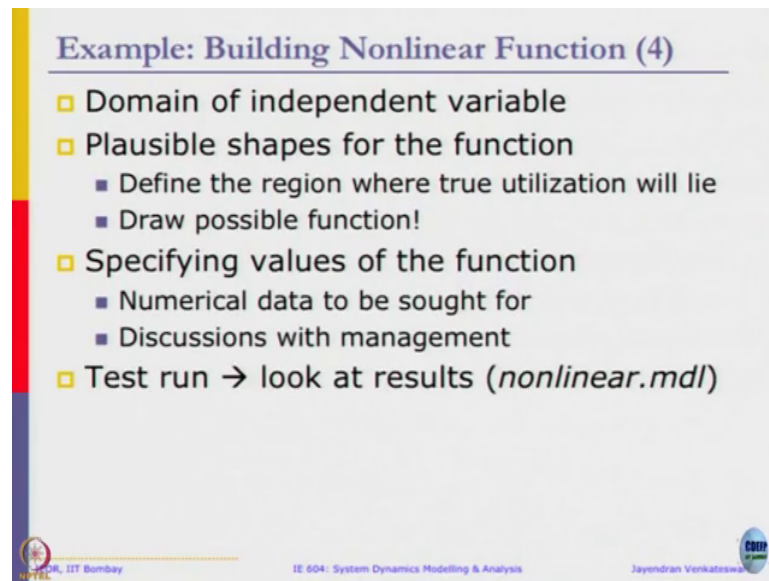
**Example: Building Nonlinear Function (3)**

- Normalization
  - Define Shipments as product of capacity & capacity utilization
- Reference Points
  - Capacity is normal rate of output
  - Suppose capacity is max possible output rate?
- Reference Policies
  - Three reference policy lines
    - When shipments = normal capacity
    - When shipments = desired production
    - Produce and ship within minimum delivery delay
- Extreme Conditions
  - What is the maximum output rate possible?

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These are steps that we did; how do you normalize the reference points and the extreme conditions we checked.

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**Example: Building Nonlinear Function (4)**

- Domain of independent variable
- Plausible shapes for the function
  - Define the region where true utilization will lie
  - Draw possible function!
- Specifying values of the function
  - Numerical data to be sought for
  - Discussions with management
- Test run → look at results (*nonlinear.mdl*)

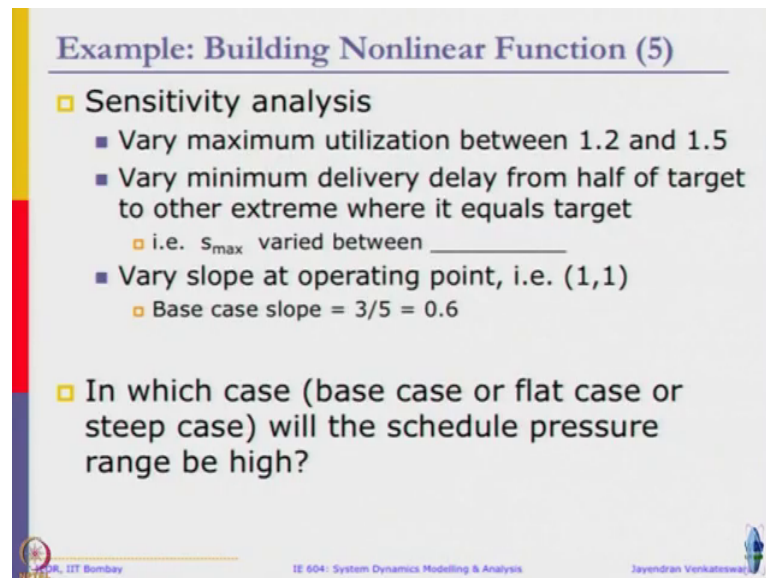
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We draw the functions and did a test run.

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**Example: Building Nonlinear Function (5)**

- Sensitivity analysis
  - Vary maximum utilization between 1.2 and 1.5
  - Vary minimum delivery delay from half of target to other extreme where it equals target
    - i.e.  $S_{\max}$  varied between \_\_\_\_\_
  - Vary slope at operating point, i.e. (1,1)
    - Base case slope =  $3/5 = 0.6$
- In which case (base case or flat case or steep case) will the schedule pressure range be high?

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You can do sensitivity analysis to figure out vary the maximum capacity utilization between 1.2 to 1.5; vary the minimum delivery delay from half of target to other extreme where it equals target. You have minimum delivery delay; you have to figure out  $S_{\max}$ , right. So, we can change, suppose  $S_{\max}$ ; suppose the minimum delivery delay could be just half a week, then what happens.

Then you can vary the slope at their operating point, the base case slope is 0.6 at the operating point, but you can try to change it more flatter or more steeper to see how it fluctuates your production rate is going to, shipment rate fluctuates based on the shape of the curve. And then you can try to work on this question; base case flat and steep case will schedule pressure be high. So, you can try to, I will upload some of the sample models, you can take a look at it and run it.



Thank you.