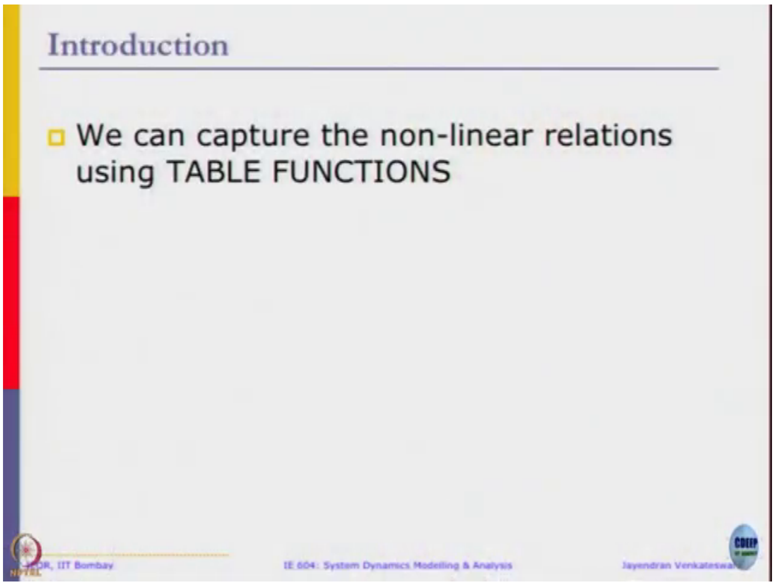
Introduction to System Dynamics Modeling
Prof. Jayendran Venkateswaran
Department of Industrial Engineering and Operations Research
Indian Institute of Technology, Bombay

Lecture - 18.1
Formulation Non Linear Relationship Table Functions
Formulation Non Linear Relationship Table Functions: Part I

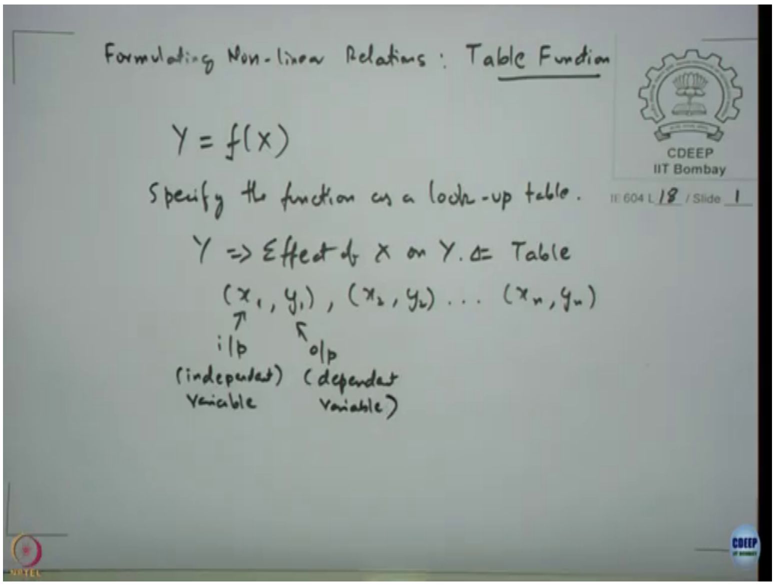
So, today's class we will take a look at get to look at Formulating Non-linear Relationships, specifically how to build these Table Functions. We saw yesterday that, we can capture various non-linear relationships using table functions. What you mean by that?

(Refer Slide Time: 00:33)



So, let us see yeah.

(Refer Slide Time: 00:43)



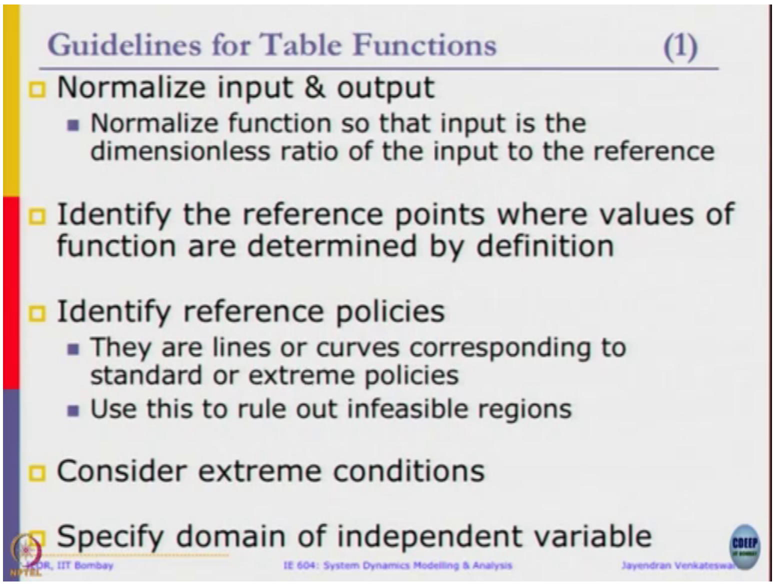
Formulating non-linear relations, we will going to look specifically at table function ok. See what you want to do is many functions are of the from Y equal to f of X right, some function of X Y is some particular function of X; but we do not know the exact analytical expression for this function of X.

So, we specify the function as a lookup table is what you want to do it is just say that Y is got as an effect of X on Y which you are just specifying as a table just specify as a table. So, this effect we want to specify as a table.

So, what we mean by a table is that, we want give these pairs of x 1 comma y 1 x 2 comma y 2 and so on where x is your input and y is your output, we want to specify the set of values and then use it in our simulations. So, instead of specifying the function directly we are specifying some combination you know when if this x is the effect of x on y. And we allow simulation to either interpolate between these values between x 1 and y 2 or extrapolate if that is beyond less than x 1 or greater than x 2. So, the x 1 is your independent variable this is your dependent variable.

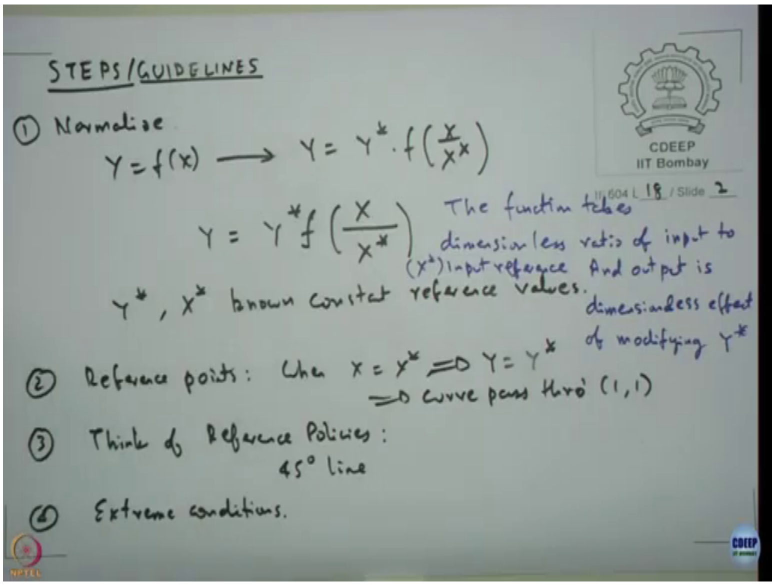
So, this is what you are going to do. But when we specify this you know given some arbitrary description, if you want to figure out how to go about doing that, so, we will learn it through an example today, but there are certain steps that we can follow to come up with possible shapes for this kind of lookup functions that should be increasing function decreasing. There are only so, many possible shapes we can give the function. So, we will kind of follow a pretty laid down procedure which will help guide up thinking process using which we can come up with various possible shapes for the function. So, I will just kind of read out the guideline.

(Refer Slide Time: 03:43)



So, first step let us start working with normalized input and output in yesterday's example that, we saw if you have done an unit check you would have find an units error because you have got an input of density and output was some dimensionless fraction. So, it becomes very difficult when suppose if we want to apply it for some other scenario then it becomes quite difficult. So, one way to overcome that is we start normalizing input to the output. So, normalize the function. So, input the dimensional ratio of the input to the reference function. So, what we mean by that?

(Refer Slide Time: 04:23)



So, steps to build a table function also a rather guidelines steps or guidelines first one is we want to do is normalize. So, instead of Y equal to f of X let us use. So, instead of Y equal to f of X let us use Y equal to Y star into function of X divided by X star it did not come up properly. So, let me rewrite it here properly Y equal to Y star function of X divided by X star.

So, so in this case we are assuming your Y star and your X star are known, constants and they serve as our reference values.

Typically, we may assume that you know when the value is X star then the output should be Y star. Suppose you know the reference value, in that case then we normalize it saying that ok. Now let us take a an once you normalize this is the actual value of X divided by reference value of X. So, we are normalizing it on output we then multiply with the reference value of Y to get the actual value of Y. So, this is the typical way we are going to normalize so, the input to the function. So, now, the input to the function became a dimensionless ratio then output of the function also be a dimensionless quantity right.

So, step 2 before we yes while we have constructed the normalization we are going to need to have something called as reference points right. So, for example, we may say that you know when X is equal to X star it implies that Y must be equal to Y star pretty much what you are doing is you are saying that this curve passes through 1 comma 1; that is this curve which is a dimensionless ratio which is going to pass through 1 comma 1 only then we will get X if then X equal to X star Y equal to Y star. So, these could be reference points does 0 comma 0 occur when X is 0 does Y become 0 or Y become 1. Some reference values will be known whatever the function that we are taking we were talking about right.

But remember this function let us see, the function takes dimensionless ratio of input to input reference and output is dimensionless effect of modifying your output reference Y star input reference is X star input to X star. So, the function X will take the dimensionless ratio of the input X star to the ratio of the input and output is dimensionless effect of modifying your Y star.

So, the input and output is a kind of scaling right. So, to get the true value on Y we need to multiply from the with the reference value of Y star. And before we give input to the function we are taking that the ratio between the X and X star where X star is the reference for the input ok. These are some example for the reference points.

Once you have the reference points, suppose we have a few points, then we can think of what we call as reference policies. Suppose for example, here we say the reference point is when X is X star then Y equal to Y star; this implies that the curve passes through 1 comma 1. Then think of a reference policy, then again think of you know is there a meaning for say 45 degree line that is for 1 percentage change in your X value, then from the reference does Y also change correspondingly same 1 percent.

So, then it is a 45 degree line if it changes higher then I am going to have a line with higher slope or if you change lower then I am going to have a line with a lower slope right. So, those kind of policies is what we are going to think about this is just an example it could be a 45 degree line or 45 degree line may not make any sense. So, this is over then next we will start thinking about reference policies.

And we will use these reference policies to identify possible infeasible region when X is very low can Y take a say value 1; if that is never going to occur then we know these regions are actually infeasible to us. So, we can use it to narrow down the possible shapes for the function or possible values the function can take.

So, along with reference policy we also need to think about what happens in the extremes when X takes a really low value say like minus infinity then is there value that Y should take or is it when X is goes less than 0 what happens that 0 what happens at plus infinity what happens. So, that whatever values that X is going to change because we actual simulation what is going to happen we do not know, but the output of Y it should make some sense right. So, then we think of these extreme conditions.

So, though we are starting here, there is hidden step which I did not mention and normalize. While we normalize first you have to identify our Y star and X star and start identifying what are those ratios first right. So, that will be like hidden steps 0 then we normalize we will learn it through an example.

So, let us go back to our slides step 1 is to normalize step 2 is identify reference points or values of function determined by definition. Identify some reference policy these lines or curves corresponding to standard or extreme policies that you are aware of we can use this to identify infeasible regions then we consider extreme conditions.

Student: (Refer Time: 11:53) from reality and.

From reality from theory from stated reasonable expectations yeah.

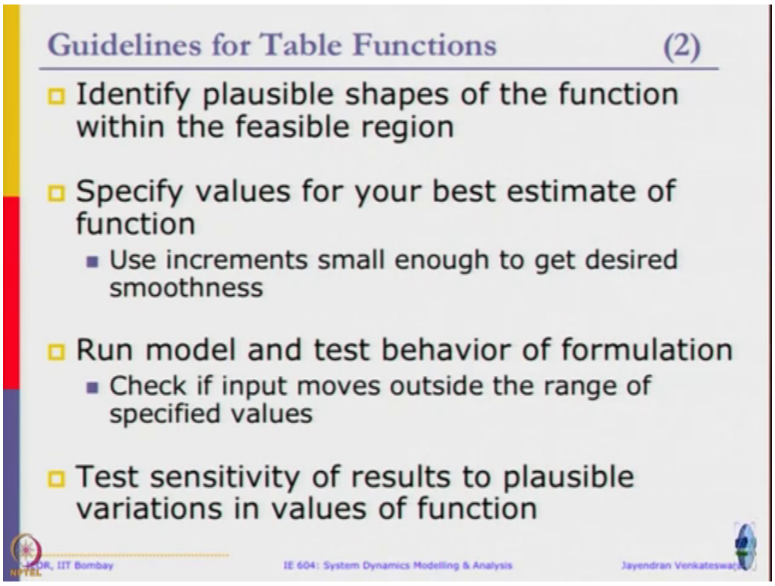
Student: What we have what we can (Refer Time: 12:07).

Correct.

Yeah, if it is based on some assumption that assumption has to be based on something right, why are you assuming that, is it something reasonable to assume is it supported with some theory or something then we go with it.

Then next guideline specify domain of the independent variable we told X is a independent variable. So, what is its domain is going to take value between 0 to 1 minus infinity to plus infinity or 0 to infinity what is the range so, that we can operate within that. So, what happens if it goes outside is clearer, once you do that then we start to identify possible shapes of the function within the feasible region.

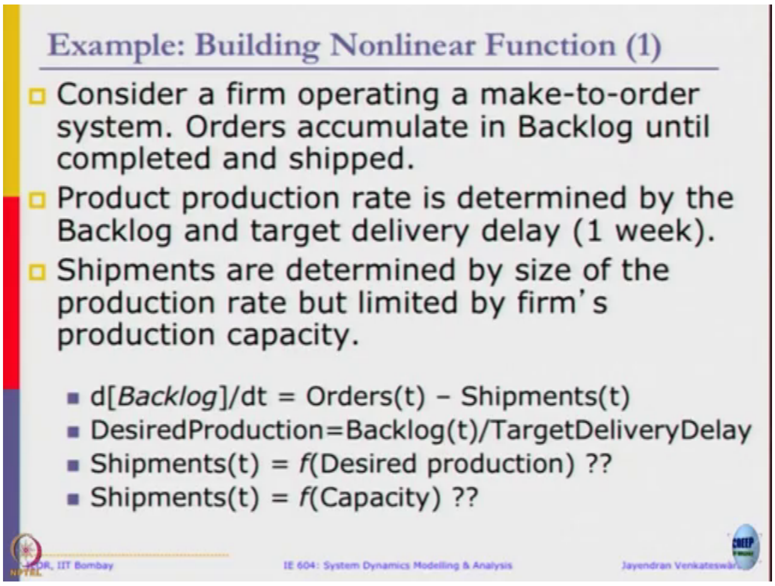
(Refer Slide Time: 12:51)



Then we specify values for our best estimate of the function use increments small enough to draw the shapes get the desired smoothness. Then based on that run your model and test behaviours for the various formulations instead of couple of different curves we can try to simulate and see whether that behaviour changes drastically based on that actual curve values. Many times we are interested in the actual behaviour of system then I will be particularly interested in the last final value.

So, the to see the patterns of behaviour it may be very it may not be that sensitive to the shape of the curve. Check if input moves the outside the range etcetera so, that we can identify errors and test sensitive results to possible variations in the shapes as well as the values of the function. These are general guidelines to build a table function its too much to remember everything right away. So, let us learn it through an example or let us take up some simple scenario and try to go ahead and how we can think it through and build some possible shapes.

(Refer Slide Time: 13:57)



Let us consider an example. Let us consider a firm operating make to order system orders accumulate in backlog until they are completed and shipped. So, as soon as order comes they go into backlog and then product production rate is determined by the backlog and target delivery delay. So, as soon as order comes it is put in backlog, then people to take look at the backlog and decide ok, but based on target delivery delay and the backlog they are going to determine the production rate.

Shipments are determined by size of the production rate, but limited by firms production capacity ok, it is quite obvious to see that there is only one stock backlog. So, as soon as order comes it goes into backlog right as long as you see the backlog exists then people start to work and based on the target delivery target delivery delay they decide to work faster or slower.

Then once they complete the work, they have to ship it the shipment is determined by size of production rate, but limited by a firms production capacity. So, desired production rate can be defined as backlog divided by target delivery delay. Now, but shipment is a function of production desired production function of capacity that part is not clear.

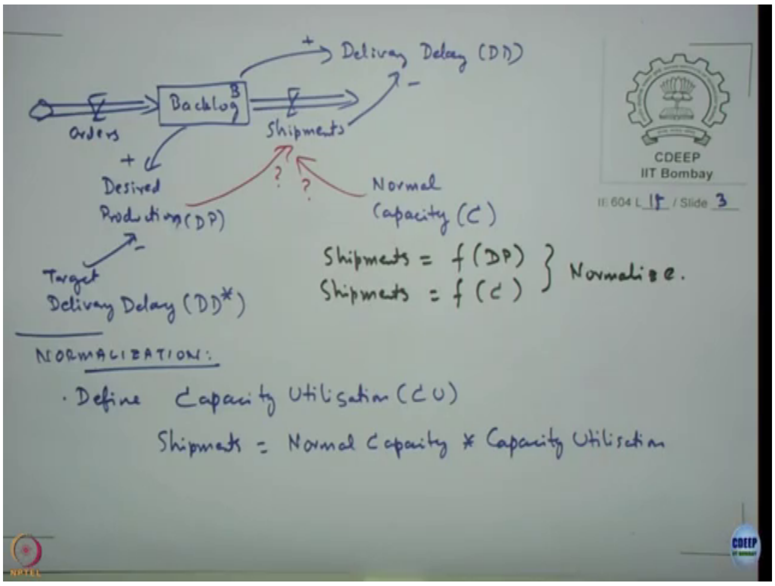
So, one other thing you might all have encountered is where you must all have visited McDonalds, KFC, Pizza Hut right they are all make to order right they do not before make things beforehand that is exactly what is happening. In the firm order comes as soon as this order is there, then they start producing I am sure they have some bench marks when you order this it should take so much time. So, based on that they start to work, but their pace is limited by their capacity right. And then they are going to and as soon as order is delivered then you take it and leave right that is pretty much the system that we are trying to model here.

But now what is the shipment rate that is going to be governed by the production speed or by their they are based on desired production desired production is based on customer orders right. During rush order there is lot of orders, but also it is also defined by their capacity if there is only some 3 people working you know how long it is going to take or else if there is only 1 person is going to work then that is the time period it is going to take correct. So, it is so, that is a kind of real life analogy you can think of.

And if you have been looking at system there even the order is very visible. They usually type it in the computer and gets visible in the room in the back end they put kind of post it kind of things also there put tags they put and that is when they usually see that is literally the backlog that you are seeing there. And for each activity they usually have benchmarks saying what is going to be target delay you know once order comes its a fast food joint.

So, they have to ensure it is fast. So, maybe instead of in this example it says a week as a time unit, but it could be a 5 minutes or 1 hour or whatever it is pizzas 40 minutes is are supposed to reach you in 40 minutes right. So, it defined by the capacity as well as the desired production so, that is what we are going to do. And we are going to follow the same steps that we saw in defining this.

(Refer Slide Time: 17:37)



First let us draw this example. So, let us get a stock flow visualization of the model. So, let us go with the you are very familiar with this. So, let us stick to that orders come in shipments reduce the backlog, based on the backlog we have already told that we have something called as desired production, desired production is also influenced by target delivery delay.

Then we have the normal capacity, there is a reason I am calling it normal capacity this is a kind of model we have right now. Let us call this desired production target delivery delay D star the Delivery Delay DD star. Normal capacity as you can just keep it as C if you want to measure the actual delivery delay then how will we do it how will I compute it target delivery delay is an input given what is the target; I mean a the delivery delay how will I measure it?

What is it?

Student: (Refer Time: 19:31).

Yeah. So, in this how will I calculate it analytically or simulate it?

Student: Backlog divided by shipment.

Yeah, backlog divided by shipment should give you the delivery delay right. So, we can just keep it as Delivery Delay DD; backlog divided by shipments here desired production divided by target delivery delay. So, let us put a minus there this is a setting that we have for a backlog then our we can just call it B for backlogs.

So, in this say suppose the backlog you are measuring in terms of say say bottles or something. So, if that is a unit of measure, then desired production is bottles per week so, what they are producing. So, then what should be unit for capacity? How do you measure capacity?

The same units as production, so, capacity also can be bottles per week, assuming bottles or widgets or SKUs or burgers or pizzas whatever boxes or cartons. Whatever the unit you are taking from backlog the same unit is backlog by say cartons per time capacity also will be say cartons per time the same units gets followed.

Now, we want to do the first step of normalizing it right. So, somehow these equations are known what we do not know is, if we know for a fixed capacity and fixed production values then we can compare it, but that is going to keep changing that is the production is going to keep changing right. Now, what do you want normalize? This is what we want to normalize. So, we have a shipments is a function of desired production, shipments is a function of capacity this is what we want to normalize ok. Let us see how we go about trying to do this normalization.

Let us go in the reverse. So, let us define a new variable called as say capacity utilization. Let us define let us define a variable called a capacity utilization many of you will come across capacity utilization. So, what we are saying is how much of the capacity has actually been utilized. So, once you define that capacity utilization, then your shipment is nothing, but a product of the normal capacity multiplied by the utilization of the capacity correct.

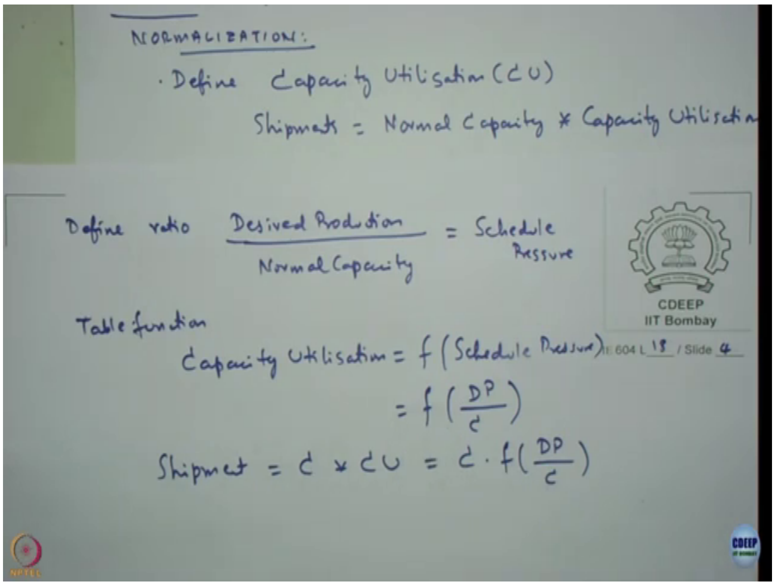
How much of the capacity I am using I am using 80 percent of the capacity. That means, the shipment should be 80 percent of the capacity into the actual capacity right. So, that should give me your shipments. So, based on that let us define shipments as normal capacity multiplied by capacity utilization yeah.

So, this normal capacity we already know, capacity utilization is what we do not know, but you already got something which is a dimensionless effect of the outcome that is the reference value Y star. So, this is your outcome of your function f of whatever it is right. So, this is outcome of the function which is dimensional effect of the outcome on the reference value right. So, this is the dimensionless output.

Now, let us see the input side. On the side of the input we have desired production ok, at any point we will be able to compare the desired production versus the capacity on hand correct. Suppose I have orders for 1000 and I have capacity to do 1000 then I am say yeah I am good or if I have capacity to do 1000 and my desired production is only 500. Then I know I am over capacitated right I have enough bandwidth to complete that order correct.

So; that means, what we are doing we are actually comparing the desired production with the normal capacity and is since both are the same units. Suppose you take a ratio of it then we know as compared to that normal capacity what is my desired production is it above that or below that or how far much below or how far much above I can make the comparison correct and then relate that with my capacity utilization.

(Refer Slide Time: 25:07)



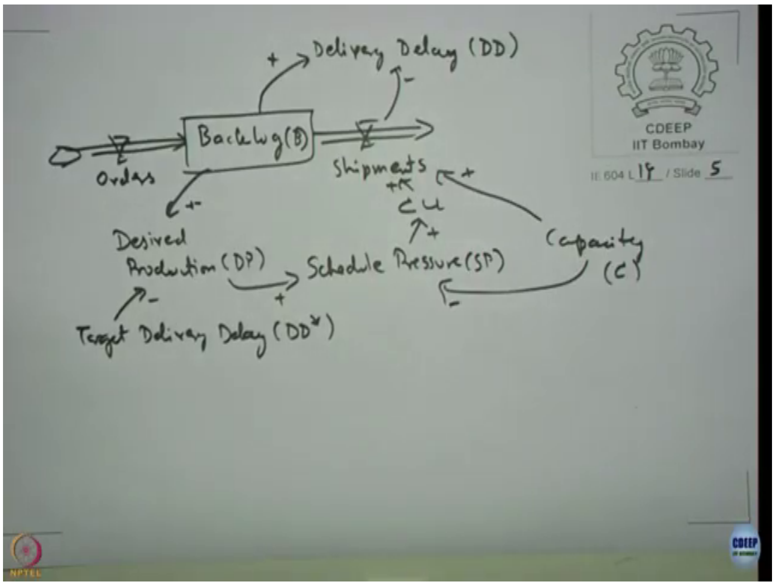
So, now let us define ratio of desired production, divided by normal capacity so this ratio we can come up with some name. So, why both are different? Suppose desired production is lower than my normal capacity then we say ok, they must be able produce the order in the as per the target delivery delay. If the desired production is much larger than the capacity then we do not know they will produce as per capacity, but it never typically works like that.

And it is because as we get busy and busy things how I can say your marginal improvement comes down right. So, this is the total activity that you need to do within the given time based on this capacity. So, let us just call this ratio by a new term called as schedule pressure.

Now, finally, the normalized function or the table function is going to be defined table function, will be defined for this term that capacity utilization is a function of schedule pressure is nothing but a function of desired production by capacity. And we look at it when we want the final shipment is nothing but normal capacity into capacity utilization that is capacity into function of desired production by its capacity is what we are going to build.

So, the table function we are going to build is what should be the capacity utilization as scheduled pressure changes over the normal capacity. When schedule pressure is very low to the capacity then what is what should be the value of utilization? The schedule pressure is comparable to capacity then what is the utilization if scheduled pressure is higher than capacity then what should be the utilization right. So, those are things that we are going think about. So, now, both input and output are now became dimensionless.

(Refer Slide Time: 28:11)



Now, let us modify our SD model to represent this. Desired production target delivery delay, then we are defining a new variable called as schedule pressure is a function of that as well as the capacity, then we have a capacity utilization. So, lack of space I am just writing it like this plus minus plus plus plus. So, your C let us call it SP, yeah then we already have a delivery delay also ok.

So, this is how our model is going to look, only changes are here schedule pressure and c u has been added and casual links are made so, that we can get all the functions that we just wrote in the previous slide.

The step 2 is to identify the reference points right. So, that is our step 2 now there are couple of ways to define these reference points and policies.