Introduction to System Dynamics Modeling

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Lecture – 17.1
Modeling Non-Linear Relations
Modeling Non-Linear Relations: Introduction

So, in today's class we will take a look at how to Model Non-Linear Relations. So, we have been looking mainly at linear model still now, though the behaviour could be non-linear, the relations we have been pretty much using at linear models. Simply define a process is linear if process response is proportional to the input stimulus given right, like savings bank account that we modeled or some concepts like you know putting 10 percent more effort may get 10 percent more results coming in. So, those kind of things are expected to be linear. And we can model it as a and linear systems are extensively studied.

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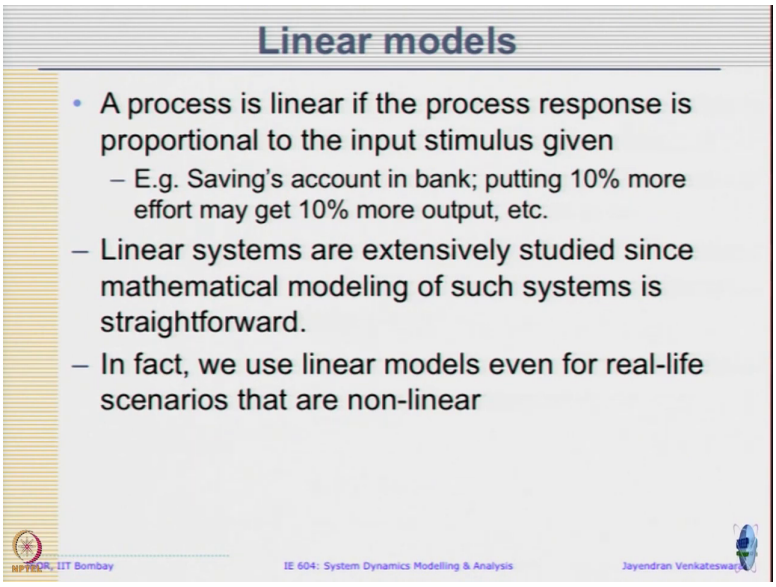

Since mathematical modeling of it is quite straight forward. In fact, so, much so, that many times even non-linearitys in the actual system is linearized. So, that we can effectively analyze it ah, though it may not be an accurate model it does give valuable insight. So, many times we end up approximating real life scenarios into linear system so, that we can analyze it.

So, today's class we take a look at how to model simple to little more complicated non-linear relations between any two variables ok. But, non-linear relations are fundamental to many systems. Can you think of some examples where the relation between the two variables could be non-linear? When we purchase things what we do? When we purchase vegetables or purchase anything bargain or even if whatever when you buy clothes or something you do not

expect the same price right, you do not say one whatever you always bargain; meaning this relation is non-linear it is not linear right when people start giving offers saying ok.

If you buy one pant it is or one jeans its 1000, but if you buy 3 you get it at a price of 2. So, there is a non-linear relationship right there which are using in everyday decision making right. So, if I am going to model that it has to be non-linear relation that has to be captured. So, do like that can you think of some more examples.

We always want volume discount as we increase in volume we always say that right you know instead of buying whatever 1 k g if I am buying 5 k gs. So, give me a discount or instead of buying 1 set I am buying 3 sets. So, can I reduce the price? So that means, a relation between the price and quantity is non-linear right. So, like that can you think of some other scenarios where you will end up such kind of relations?

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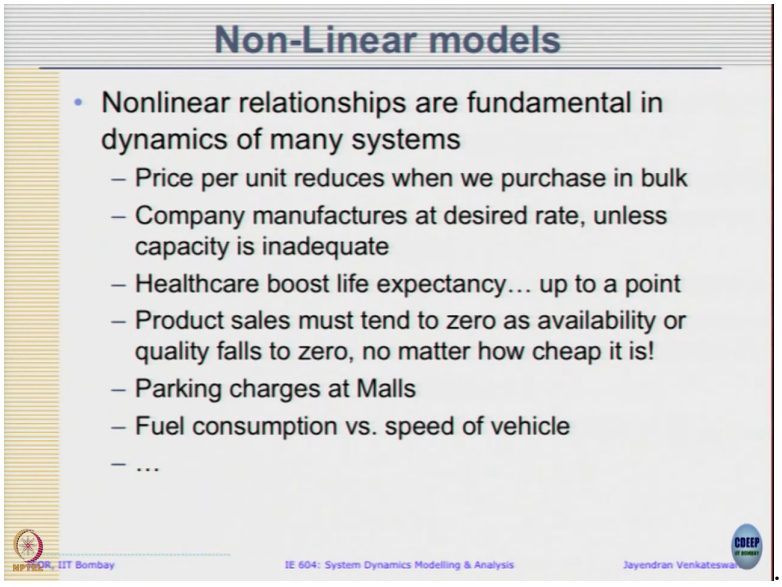

Price per unit reduces when we purchase in bulk, company can manufacture desired rate unless capacity is inadequate, it is a non-linear relationship. When you assume you can produce whatever you are assuming is well within your capacity, but exceeds your capacity then it cannot product at the same rate or I cannot keep selling as much as I can. I can only sell as far up to the inventory or stock I have. Once they exhausted I cannot sell anymore right that relationship right. There is a non-linear relationship or healthcare boost life expectancy, but up to a point the healthcare spending is or it only it will go only up to a point it cannot extend life infinitely.

Product sale must tend to zero as availability or quality falls to zero, no matter how cheap it is. So, it is not just only the price that drives it, the inventory also should be available just because you reduce the price, but does not mean, but if inventory is zero does not mean that you can keep selling right, but that is a non-linear relation right.

There in reality or parking charges at malls is not linear it goes in jumps, it jumps in there are breakpoints and you get for so, many duration that many hours of there is only hours you get charged rounded to the nearest breakpoint, fuel consumption speed of vehicle again non-linear relationships.

Student: (Refer Time: 04:15). .

Yeah. Like that there are many such scenarios which we may want to capture in our simulation models. So, that it can better represent the real system. When we look at linear systems what we are trying to do is linearize the model, so, that we can analyze it better, but we when we move into simulation modeling, we tend to make it more closer to reality because we are not doing any mathematical analysis we are actually doing the simulation of it.

So, it can handle non-linearitys in simulation because of that we want to capture reality as close as possible which will result in capturing all these non-linearitys more explicitly so that we can better represent the real processes.

So, let us look at some simple examples. First understand how to you know use these or represent these non-linearitys in our simulation models ah. But as soon as you start doing this non-linearitys builds some functions you will hit the limits of using linear systems analysis what are methods that we have used and for some scenarios simulation to be the only result to analyze it or if you simplify the model we can apply we can analyze it analytically.

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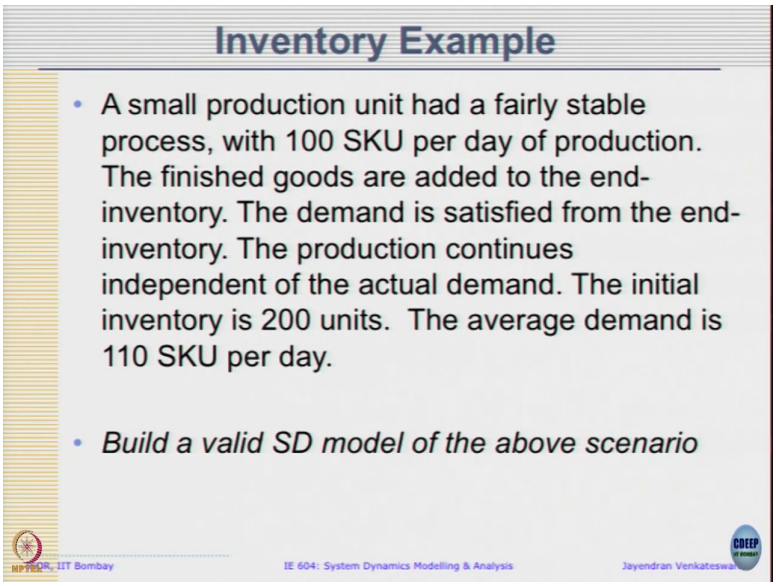

So, let us take a simple inventory example. A small production unit has a fairly stable process with 1000 with 100 SKU per unit per day of production. So, it makes 100 SKUs per day. The finished goods are added to the end inventory. The demand is satisfied from the end inventory.

The production continues independent of the actual demand. So, whatever the demand they keep reducing things at this constant rate of 100. The initial inventory is 200 units. The

average demand is 110 SKU per day. We need to build a valid SD model of the above scenario.

So, how many stocks are here in the system? Finished goods inventory that is the only stock in the system. So, what will be the flows that affect this stock? Production rate and sales rate will be the two flows that affects the stock production rate will add to the stock and sales rate will reduce the stock.

So, in this what we are going to do is, have the demand as a third variable. So, we will have production rate affecting this inventory and the inventory is reduced by sales rate, a latest model demand separately as 110 units which affects the sales rate.

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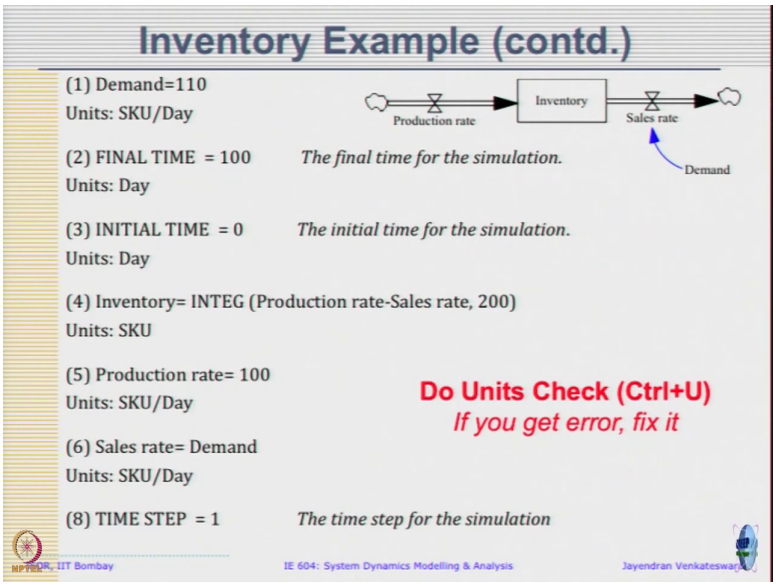

So, I want you all to model the system. So, what you see here is the documentation as provided by Vensim. So, when you build a model and you click the document all button in Vensim, it represents what are that underlying equations along with the variable names units etcetera are all captured in this slide. You may or may not have tried it before, but does not matter, just use your common sense rate this and create the appropriate variables within your model and build the model.

So, in this what is the stock here which row number is a stock?

Student: (Refer Time: 07:34).

4; how did you identify it? One for example, you see the word INTEG, whenever there is a INTEG; that means, whatever its variable name next to it is a stock, that is it and whatever affects through INTEG production, rate sales rate then that must be the flows. Then the equation of flows are given, the units for each is also given, initial time final time goes from the model settings, time unit is also specified and whichever is not the rate that must be an auxiliary variable right. So, you can build this model.

Once you finish it, do units check; if you get an error fix it. If you observe for each variable name is written then just say output directly from Vensim and it is sorted alphabetically, that is why you get demand first then final time initial time inventory etcetera. If I ask you to document probably you will write the inventory first and then the flow rates then the other variable, it is just sorting it alphabetically. Final time, initial time step comes in your model setting, others come in your model; kindly become familiar with such interfaces so that you can understand what the model is presented.

When you simulate it what happens to the inventory, at what point does it cross 0? At time 20 and just keeps falling which is intuitive because demand is 110 and production is 100. So, minus 10 is removed from at every time step and initially I have 200 is the stock. So, it takes 20 time units for it to get 0 right.

So, with this simple model if you take it and show it to your production manager suppose he is a consultant he will say what nonsense how can inventory be negative you can explain no this is just approximation, but he is not going to listen this does not represent reality make me a model realistic first it is not that capturing my actual system that we have.

This is a simple example. Imagine if say demand itself is randomly varying and production was even if its constant some period it will be negative some period will positive, still we will not cannot comprehend whether what happens if that inventory is actually negative, what does it even mean? You cannot convince such things to managers.

As a analyst you can think of invent negative inventory is called backlog. Then you should have an explicit variable called backlog. Then how will you respond to that, why did not you have model backlog separately? And you may say I do not represent backlog at all if there is no inventory my this thing is lost. So, how did you assume there is backlog?

You go ask for [FL], if you say [FL] you do not measure it as a negative inventory right. He does not wait you that you will come back and have [FL] again that demand is lost. So, whether you are assuming as lost demand or not it has become more explicit. Idea is to make the model more clear. So, in this case it is quite simple suppose a demand is lost right.

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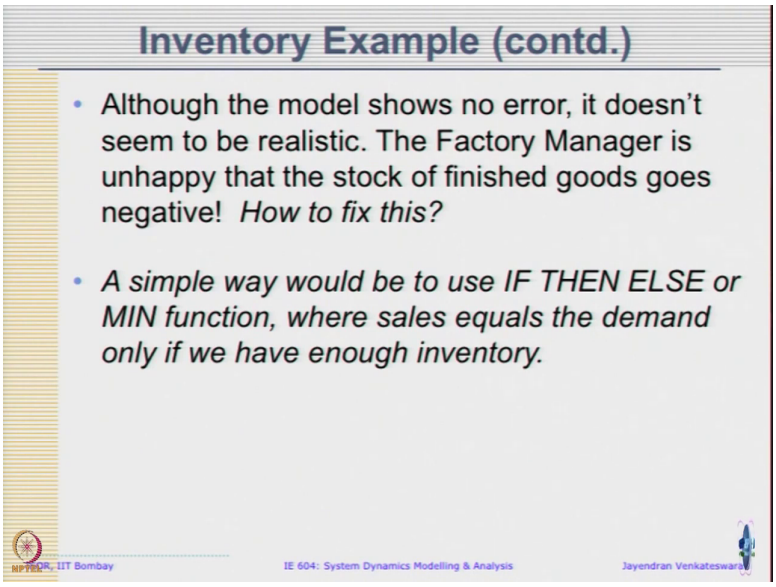

So, how we have to make it, clear, inventory should not be negative let us say, although model shows no error, it does not seems to be realistic. Say factory manager is unhappy if the stock of finished goods goes negative. So, how do we fix it? Logically how will you fix it, how really you want to do? If demand is there and inventory is sufficient meet the demand then I give it.

If demand is not there I mean demand is more than inventory then I give whatever inventory available to meet the demand if demand is less than inventory then I give the full demand right. So, this is a simple if then else statement. A simple way to fix it is use an if then else construct or a min function where sales equals the demand only if you have enough inventory. Can you model this? I mean I am sure you can model this, model it.

So, now sales rate will be a function of what? Inventory and demand. So, why do not you have a link from that and complete the model, then simulate it then after time 20 your inventory should be 0, but your sales rate should still be there should be at least 100 units sales rate. Sales rate need to follow a fall from. So, the previous model if you simulate it you will find there sales rate is continued to 110, production continues to be 100, inventory became negative.

If we if sales is a function of inventory and demand then inventory has to fall to 0, sales rate will be initially 110, after sometime it will become 100 which will balance the production rate. Why do not you simulate it and see if that happens; model it and simulate it. Go to sales rate; if you want to know how if then else works you just click if then else it will give the help you can read it and model it.

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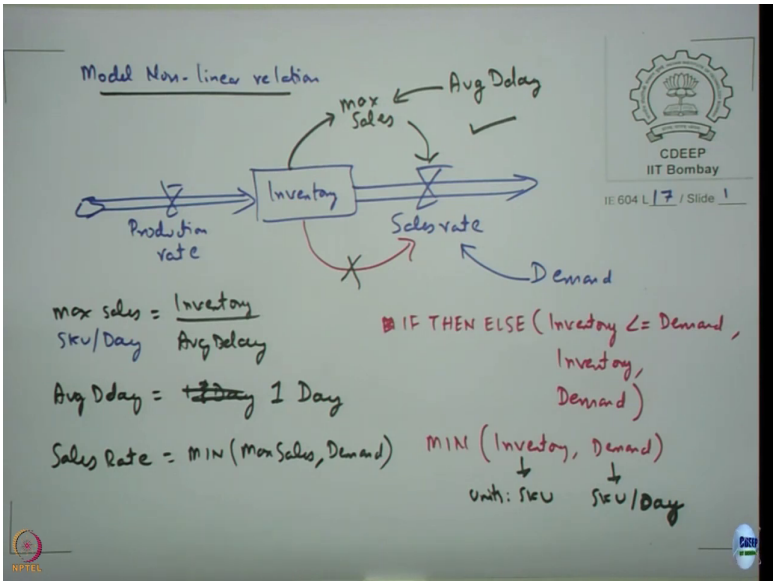

If you do this and sales rate can either have two functions: one it could be just if then else, I think it should be inventory less than or equal to demand then inventory else demand or you could use some min function. The min (Refer Time: 13:46) inventory or demand, it is for sales rate.

See as soon as you introduce this model became non-linear right, you do not need to have explicitly 2 power x or x into y; even a simple min or max function makes your model non-linear. So, does it work, it is simulated, did you do units check? Units are not ok. What does that error say? Inventory units is SKU, demand is SKU per day. How do you fix this?

So, to do that, so, this is a very common phenomenon, we are because many times when we use these terms analytically many times we end up multiplying or dividing by 1. So, we do not usually represent it explicitly and it does not affect analytical result, but simulation validity it will affect.

So, to make it proper let us define from inventory another variable called as max sales then defined something called as average delay. So, our equations we will do max sales is equal to; obviously, its inventory divided by average delay. The units of max sales is SKU by day. Simply keep average delay equal to 1 per day, I am just writing the units right there and now your sales rate equation is equal to minimum of this max sales possible or max sales comma demand. Now units match. So, it becomes more apparent. It is just good programming or modeling practice to not multiply inside the equation any constant numbers then it becomes very difficult.

So, now we have brought the average delay as a constant right. So, you are going to just divide it and we do not need to have defined max sales itself, but making it explicit gives us (Refer Time: 16:41). Suppose average delay to move inventory is 3 days then all you have to do is come and change this into 3; that means, though I have inventory, only one third I can sell at any time period.

So, those gets captured if I you know make all my parameters explicit and there is a physical meaning to each of it, like, this average delay could be the average time taken to move inventory right or instead of delay you can call it a average fraction of inventory I would like to sell.

So, I might have policy where I can only sell 90 percent of the inventory at any point in time, some 10 percent I am not willing to sell or I just cannot get it all the way to the end. So, I may just put some limit or I might want to model say some fraction is being lost or there are defects in the system etcetera. So, usable inventory could be only 90 percent there will be variety of scenario. So, it is good to explicitly bring out the parameters.

Why do not you add this component? So, we will be removing this line, this causal link rather and including the one in the top. Hopefully, when you run it you should get the same behaviour, but without the units error check units error, when you check the units it should not give you any more errors, it should say all units are ok. Please check that. Oh Sorry, it is not average delay is 1 day sorry not 1 per day. Having shown this please you use all these non-linear constructs like if then else min whereas, there is a min there will be a max with caution.

It is very tempting when you start modeling systems, if this thing had happened this is the behaviour this is the behaviour etcetera, ensure model does not become too fuzzy; because once you start putting this its little more difficult to do sensitivity analysis of the models when you start doing that, but where it is unavoidable we have to use it. So, we will just we just figured out how to use it, let us just keep it there ok. We did there were errors and we improved the model to ensure that there are no unit error ok.