Introduction to System Dynamics Modeling
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Lecture – 14.3
Delays
Modeling Delays: Higher Order Material Delay

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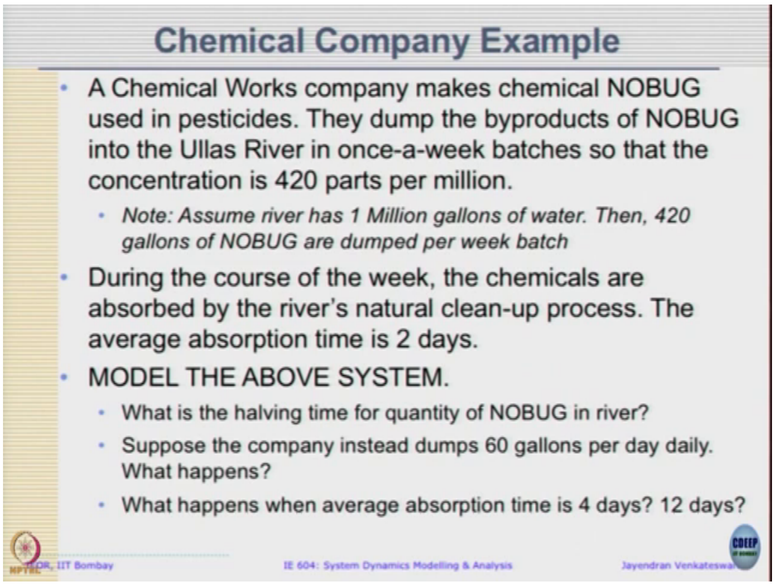

Unfortunately, none of the models will explicitly come and say; hey I am a Delay model or something, it will be a general description and we just model it; it just so happens that it will be some form of delay or the other which you can recognize. If it explicitly says delay, good for you if not cell there will be delay existing; especially this example. The chemical company makes chemical called NOBUG using pesticides, they dumped the byproducts in the say river once a week in batches such that the concentration is about 420 parts per million.

So, we can assume that the river has 1 million gallons of water; that means, 420 gallons of NOBUG or dumped per week into the river. During the course of the week, the chemicals are absorbed by the rivers natural cleanup process; the average absorption rate is 2 days; model the above system. Go ahead, how will you model this? The model is will be quite simple, but description will be quite elaborate. Since, I just covered first order material delay logic suggests that it should be similar to that ok.

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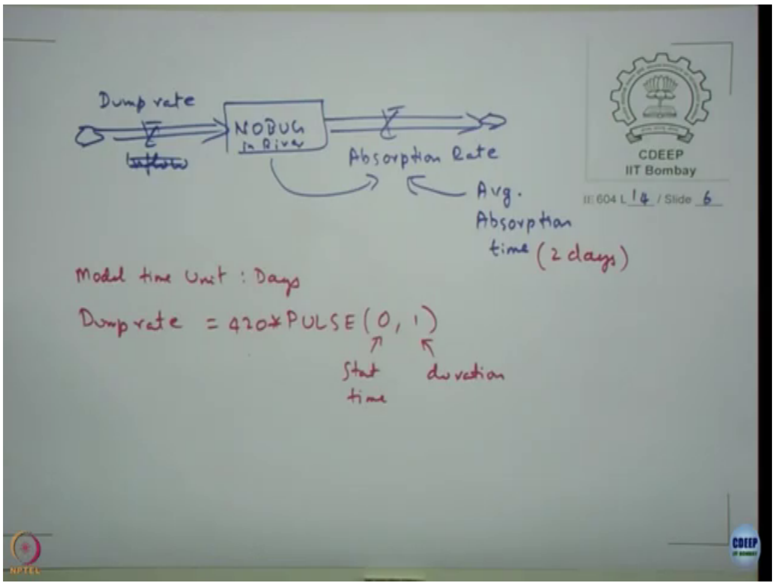

You got the model this one or at least you are thinking of a model like this; not a drawn it. So, it is very simple again when I teach I may use words like inflows and outflows and start, but you can use some proper rates like you know dump rates which it represents what is actually happening absorption rate is what they have told; absorb the cleanest natural reverse natural cleanup process. So, it will clean up whatever is in the stock; it will clean up with an average absorption time. So, I will just write the equations in red.

So, you get the average absorption time is given as 2 days right. So, ensure your time unit in the model; in your model time unit is selected as days ensure that. So, absorption rate is nothing, but NOBUG and river divided by absorption time because its rate stock over time it has to be. So, NOBUG in river divided by absorption time is what you write here; initial value you can put it as 0 in stock say initial value is 0.

So, now to kind of simulate let us take a very simple case; let us start with the dump rate as just a pulse quantity; we already seen pulse before. So, pulse has two inputs, one is the start time, other is the duration. So, a time 0; there is 1 pulse pulse; 1 unit occurs at time 1, but what we already know is it dumps 420 gallons right. So, I can. So, I just multiply this by 420. So, dump rate is 420 multiplied by pulse 0, 1.

So, you can make this in vensim. So, you have all the information; you need the units of dump rate will be gallons per day NOBUG units is gallons there is amount of NOBUG in the river gallons absorption rate again gallons per day comes from here gallons comes from here. Weekly dumping is happening, but we will just take it as only one beginning of this week it happens. So, let us just observe that then we will model the next part.

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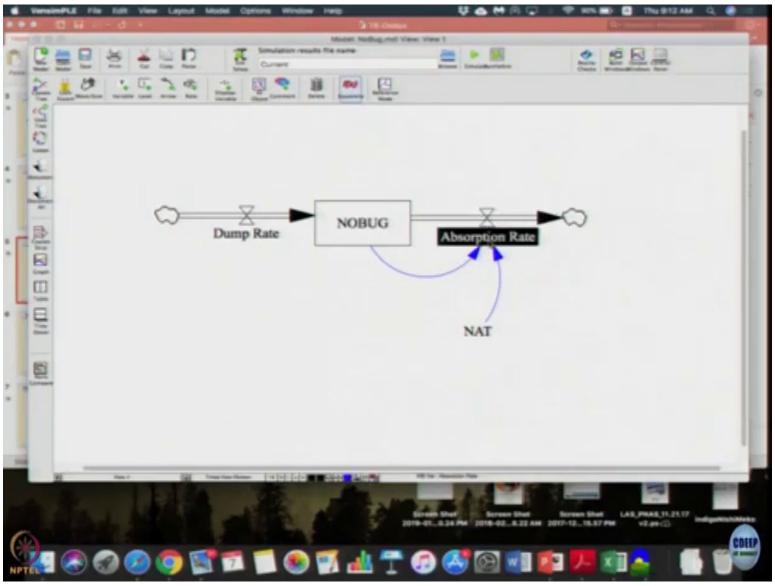

This is how the model looks NAT Natural Absorption Time I just came up with some acronym looks like it.

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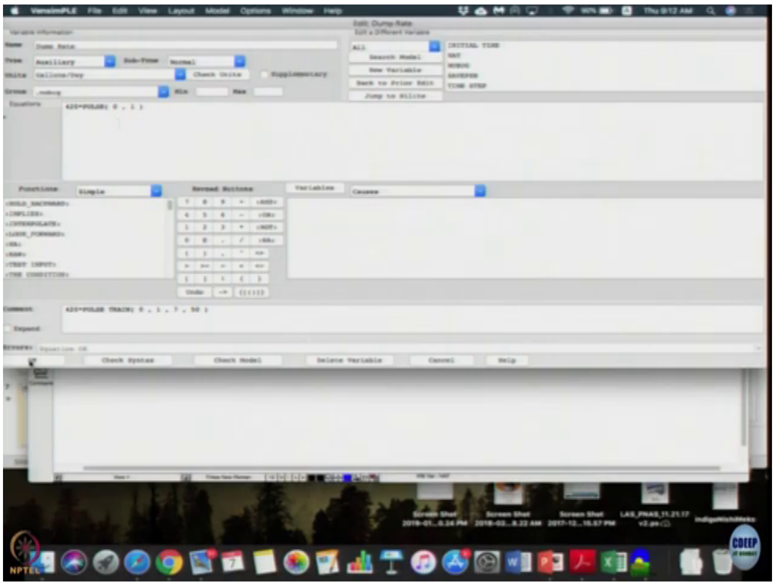

Just look at the curve absorption rate is NOBUG by NAT; NAT is given as 2, this is easy 1 dump rate is 420 into pulse of 0 comma 1 just the pulse input.

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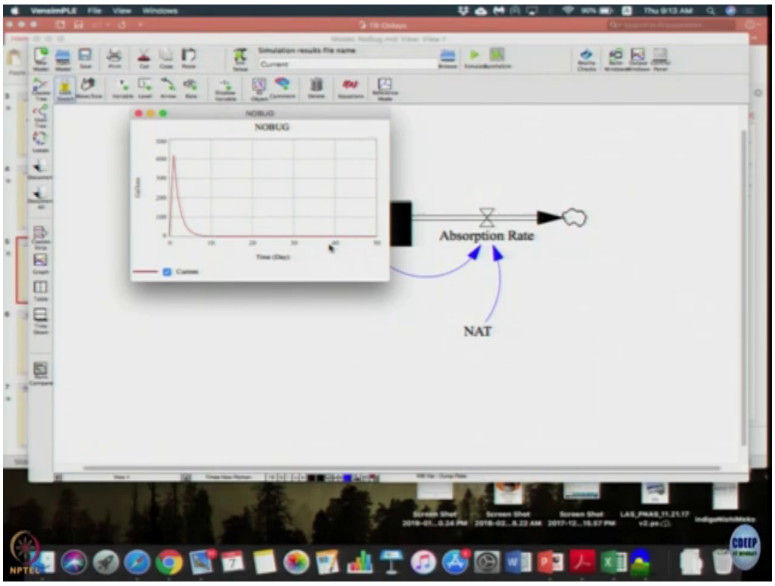

So, if I run the model; we can see that NOBUG increases and then as exponential decay. So, the halving time would be about 70 percent of your average delay; so 0.7 into D, D is 2 so 0.7 times 2 is or 1.4, 1.5 about week day and a half time; it will come into half the quantity right any questions on this so far? Right, now what do you think will happen if we dump every week? It will go up, it will come and again it will go up come down go up, come down is that what we expect? Let us see what happens.

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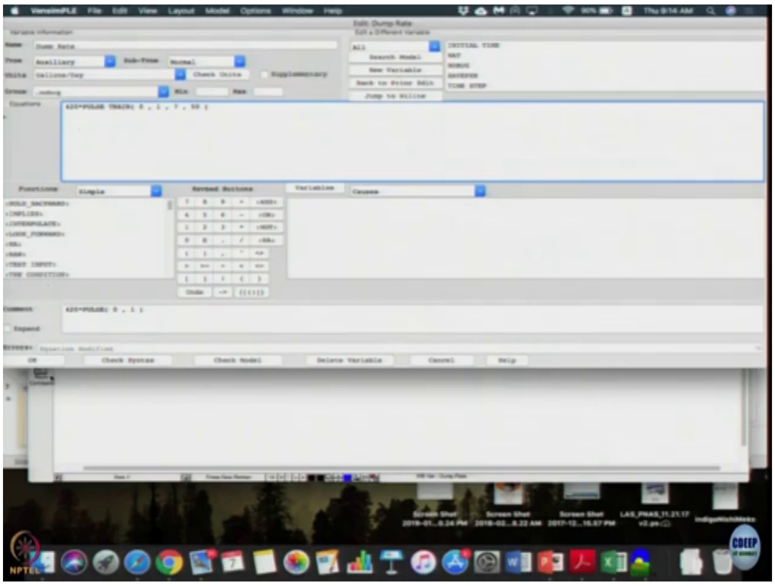

So, dump rate ups what did I do? I will make that guy here there is another function called as pulse strain; what pulse strain does is as the name suggests it gives a pulse input periodically. So, in this case it gives a pulse input starting at time 0 of a duration of 1 and it that repeats every 7 days that is once a week, every 7 days it repeats and the end time is 50; after 50 time minutes pass no more dumping happens there is after ok.

So, that is a very simple model that we have. So, it is called pulse strain; you can look it up in the help file. So, every week 420 gallons is dumped is it ok. So, let us observe what happens I am going to run the model. So, we go to NOBUG.

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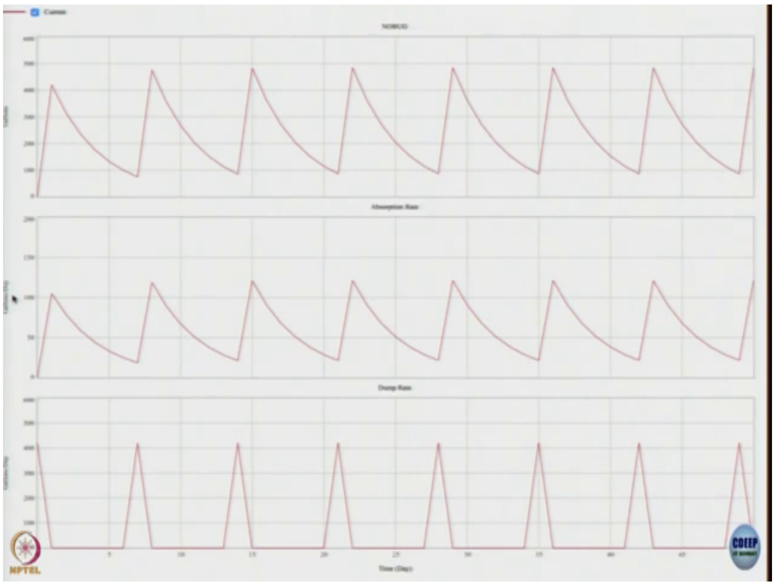

Let me do causes trip, it nicely occurs as we see here the top graph is the stock; stock increases up to 400 and the day of dumping 420 and then gradually comes down absorption rate also happens and so nicely fits our model right here. And dumping happens only once a week which is verified by the bottom graph which shows there is a dumping happening every week.

Now is actual fun part with SD where we have to do sensitivity analysis just making this is ok, but then to get insight. We can find out say for example, what happens if the natural absorption time instead of 2 days become 4 days because of various reasons; it has like capacity to observe as reduced and now the natural absorption time is 4 days.

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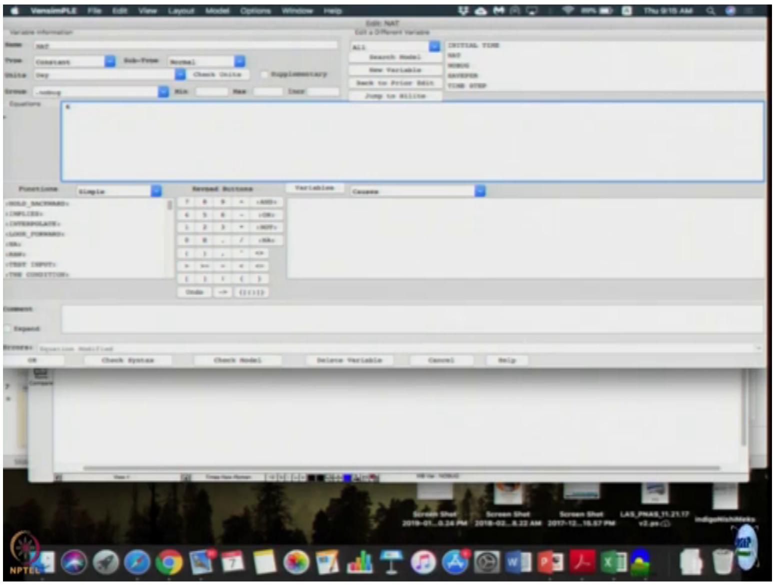

So, let me just make it 2 to 4; in that case what do we expect to happen? Can we have the same behavior? Behavior has to be the same right once it dumps is start absorbing correct, but then let us see what happens. Let us simulate NOBUG; I just made the absorption time 4 from 2 days, the dumping is the same absorption also happens. But now you see steadily the peak from 420 has moved closer to 500 right and 480 is a peak and it does not go now below say 90, there is always some NOBUG in there in the river constantly.

So, as absorption time increases say logically speaking over the absorption time becomes average become say 7 days itself; then by 7 days we can expect roughly little more than half of the NOBUG would have been absorbed. So, you start accumulating. So, the accumulation effects is captured in this stock right here. So, if we did not have the stock we may assume that as soon as a dump eventually it will get absorbed, but it is not going to get absorbed until I stop the inflows again.

And then allow it for some time to slowly absorb and decay completely; that is interesting one average absorption is 4 days or 12 days of course, it is going to keep increasing instead of we have tried it 4 days, you can try with 12 days. The next is interesting instead of dumping 420 every 7 days; what if we do 60 every day? We expect same behavior a different behavior? The quantity is the same they got permission with government to dump 420 gallons a week what difference does it make if I do it every day or do it once a week. Let us see what happens.

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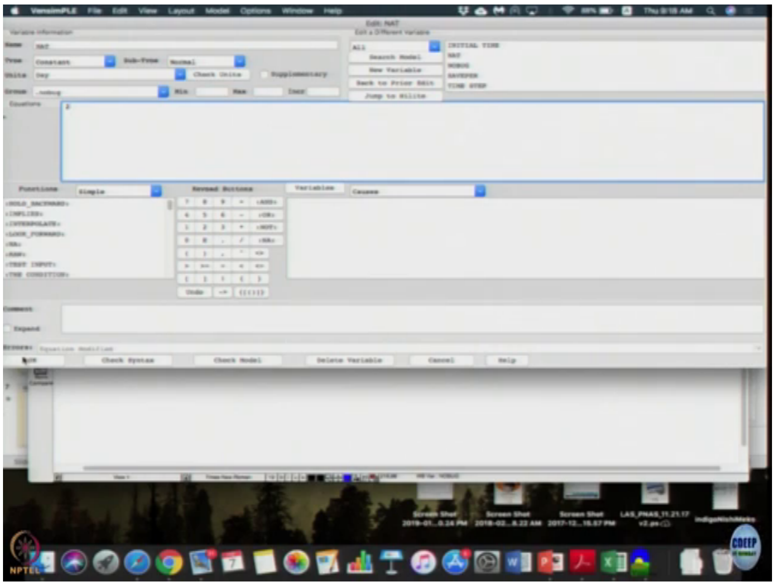

So, I am going to go back and take the natural absorption time back to 2; in two case it came up and then went down very consistently nice graph I got.

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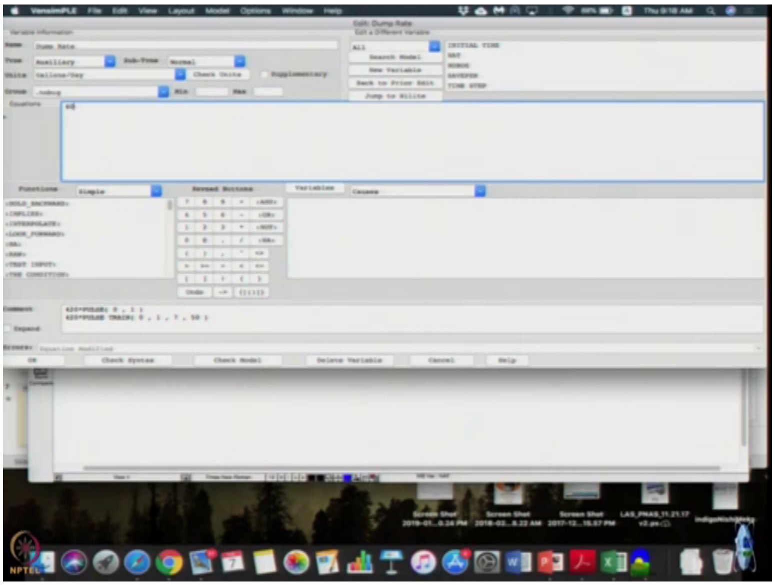

Now, I go to the equation of dump rate and you are doing 60 gallons a week every day timestep is day. So, I just write 68 means every day it will be 60. So, let me simulate see the absorption time is for whatever quantity; whatever quantity I put takes an average of 2 days to deplete correct.

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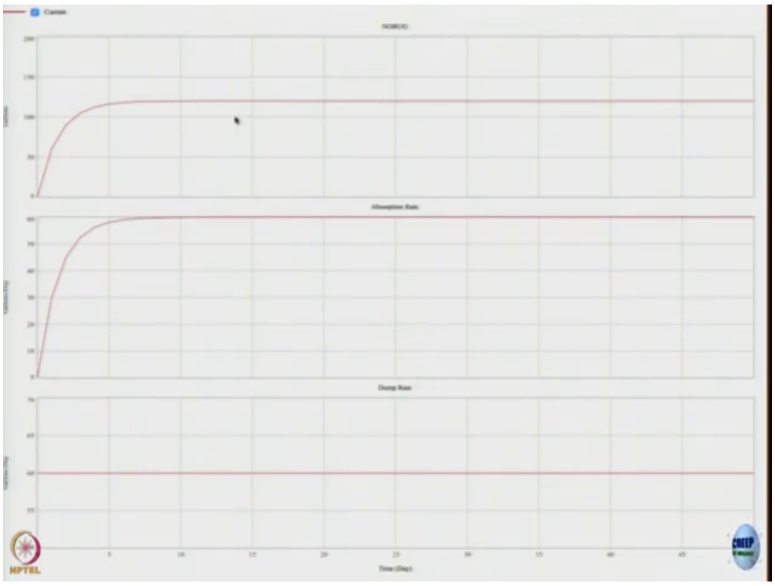

So, now if I dump every day; it does not give you enough time for the river to absorb everything. So, eventually the total amount of say NOBUG in the river actually increases to what is it about 120 looks like right; that is a interesting thing. But if you think about it logically what we did is we dumped it once a week we gave it gave time for the river to recover. In this case, a dumping 60 every day; so by the time it starts processing 60 the next 60 is coming in every day it takes time river cannot process so much, but still.

So, there is a some amount of about 120 gallons that is end up in the river even though nothing else changed right. This is the simple act of the same quantity, but different times actually has a nice kind of. So, its better when there is a time given forever to recover rather than when it is very less though the quantity is the same.

So, these kind of insights you can get only if you know play with it, simulate it, try it with different values and this is the kind of policy which is just a simple model can use to explain different policies; if a people want to you know the same example do you allow them dump once a week or dump every day; the quantity is the same, we can find that actually analytically there is difference in behavior and it does not tend of not a very complicated model to explain.

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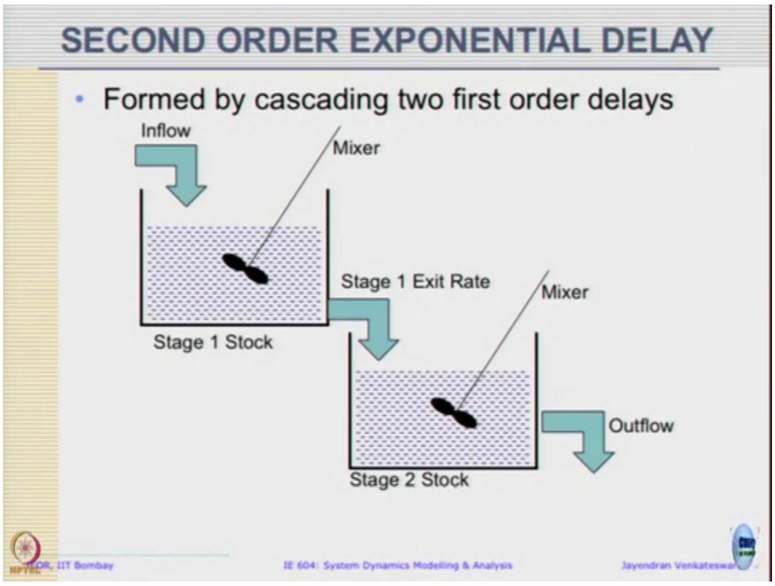

Second order delay can imagine as cascading two first order delays; that is it that is called a 2nd order delay. So, the tank example looks very similar. So, a stock flow also we may able to guess based on this.

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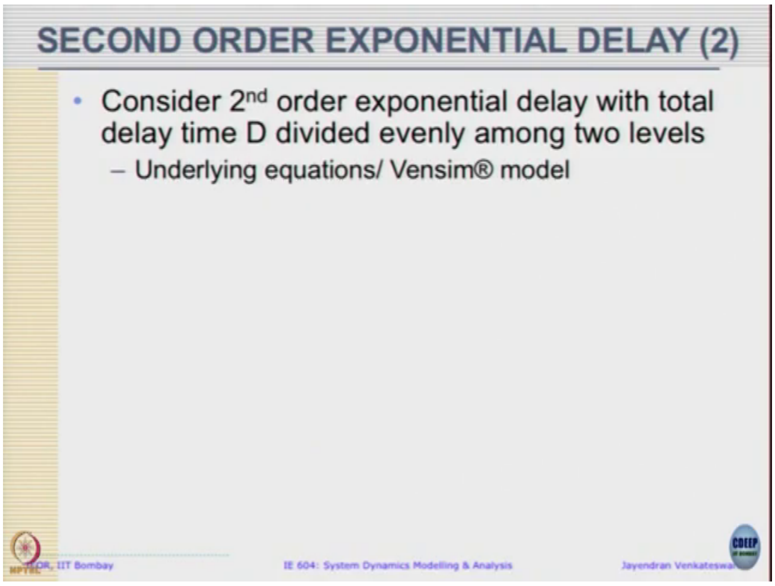

So, what we are going to do is; when you do a 2nd order delay we will assume the total delay time is divided evenly among both the levels.

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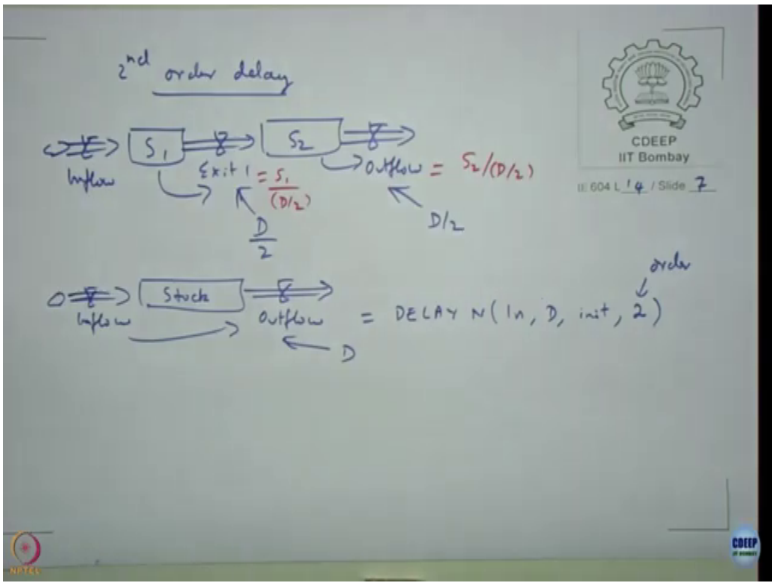

So, if we want to model the 2nd order delay; we just take up inflow stock 1, it is called exit 1, stop 2; this is your final outflow. This is affected by D by 2 and this is affected by D by 2; if you want a model it vensim directly as function; will do it as in flow stock outflow D.

Here we will write the equations for outflow as delay N as your in D initial value if any; then we will write the order as 2. So, this is your order here the equations for exit is nothing, but S 1 by D by 2. So, the equation for exit is S 1 divided by D by 2. So, exit is S 1 by D by 2, outflow is S 2 by D by 2 ok.

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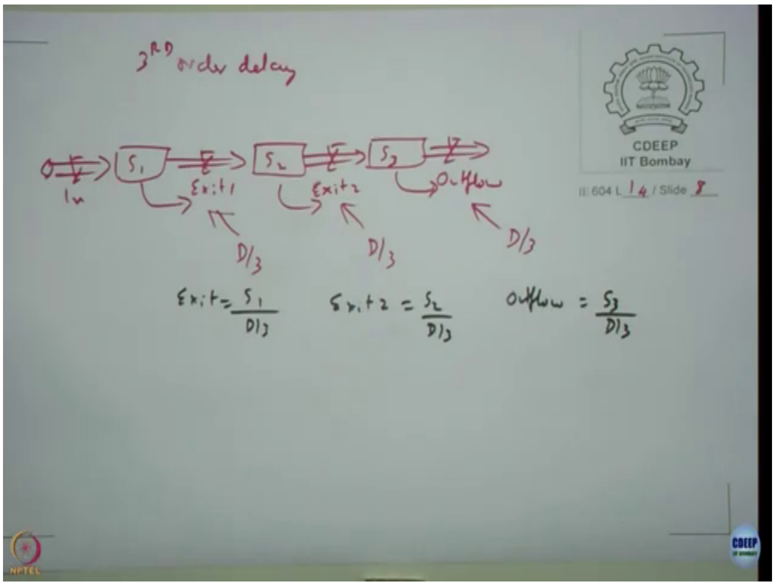

So, this is the 2nd order delay; do a 3rd order delay that is nothing, but your in flow stock 1, let us call it exit 1; then I will stock 2, then I will stock 3 out flow. So, when I say first order delay there is 1 stocks, 2nd order delay 2 stocks, 3rd order delay I am looking at 3 stocks unsymmetrical is conserved.

So, all the stocks and flows are getting interconnected and now delay time is evenly divided among three; among three of it. So, the equation for exit 1 is S 1 by D by 3, exit 2 is S 2 by D by 3, outflow is S 3 by D by 3; so evenly divided. So, the average delay time continues to be by D ah; here as soon as outflow happens D by 3 amount starts flowing out of exit 1; as soon as inflow happens. And after D by 3 one third of the time, this stock will start to peak and then of two thirds of the time this stock will start to peak.

So, this so that is a peak of the outflow when this one peaks is when this peak outflow will occur. To model it and vensim again we can use the similar delay and function with the 3rd order delay. Since 3rd order delay, I just showed you delays divided evenly among three levels.

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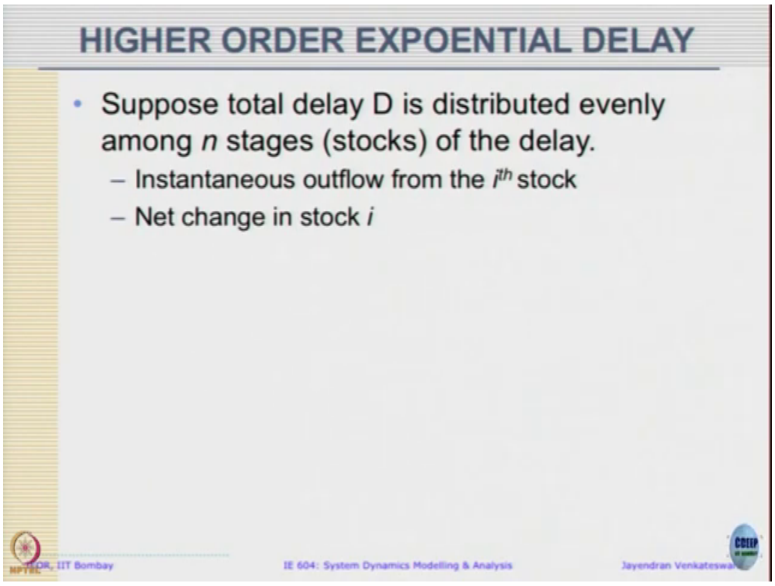

Suppose, delay these divided distributed evenly among n stages then instantaneous outflow of stock i will affect the subsequent stock; the delays evenly divided by n stocks. So, for each level I have D by n; so we have 5th order or 6th order delay.