Introduction to System Dynamics Modeling
Prof. Jayendran Venkateswaran
Department of Industrial Engineering and Operations Research
Indian Institute of Technology, Bombay

Bass Diffusion Model
Lecture – 13.2
Bass Diffusion Model (Contd.)

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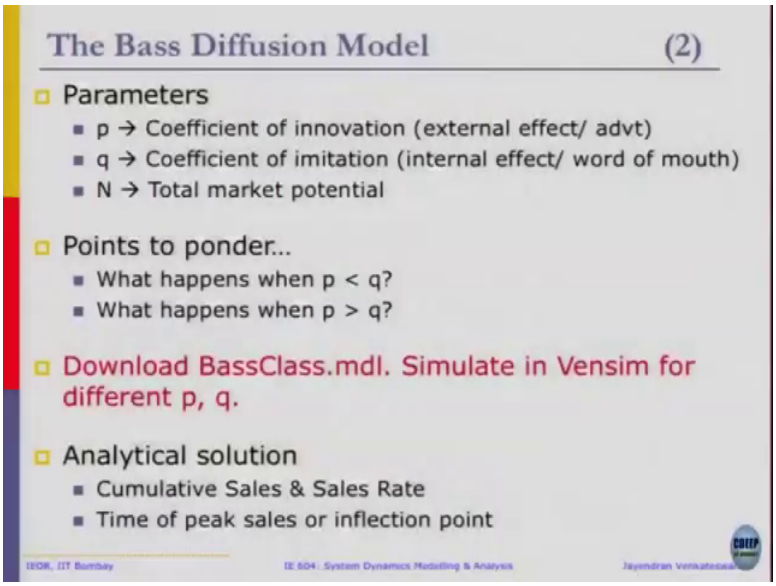

Before we go on do fitting the model for a more realistic examples, let us look at this analytical solution for the cumulative sales. As far as sales rate nothing but total adaptors and the adoption rate and time of peak sales or the inflection point. So, time of peak sales will coincide the inflection point, so there is a different terminologies for same thing, just make a note.

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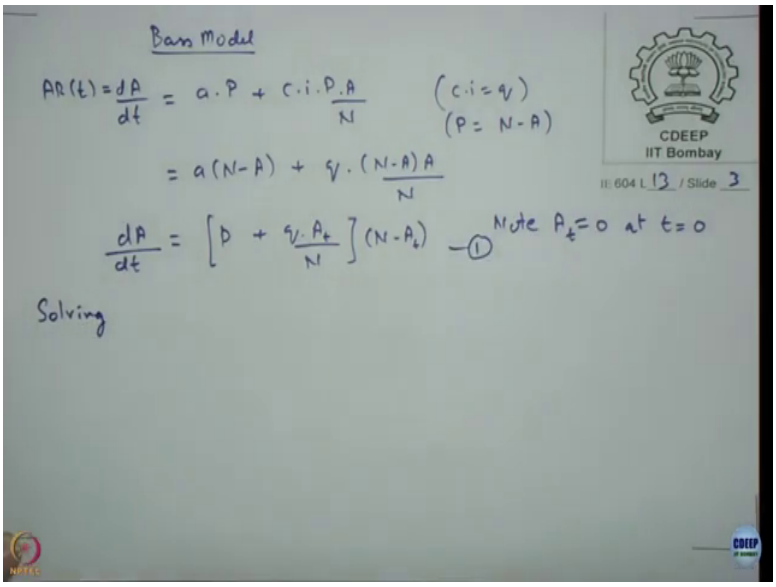

So, in bass model written that AR is nothing but dA by dt that is change in the adaptors. This a into P plus c into i into P into A by N. Since, we set, we will set c into i is equal to q and we already know that P is nothing but N minus A. We can rewrite this as a into N minus A plus q into N minus A into A by N. It is can be further rewritten as p plus q into A by N into N minus A.

Again note, A is equal to 0 at t equal to 0 rather A t is equal to 0 there should be subscript t for all A, all A in P has a subscript t in it. We started off with similar d A by d t equations in the previous case and then we had a lengthy derivation. I will take the liberty to skip that part and directly jump to solving 1 yeah.

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A sorry, it should be small p.

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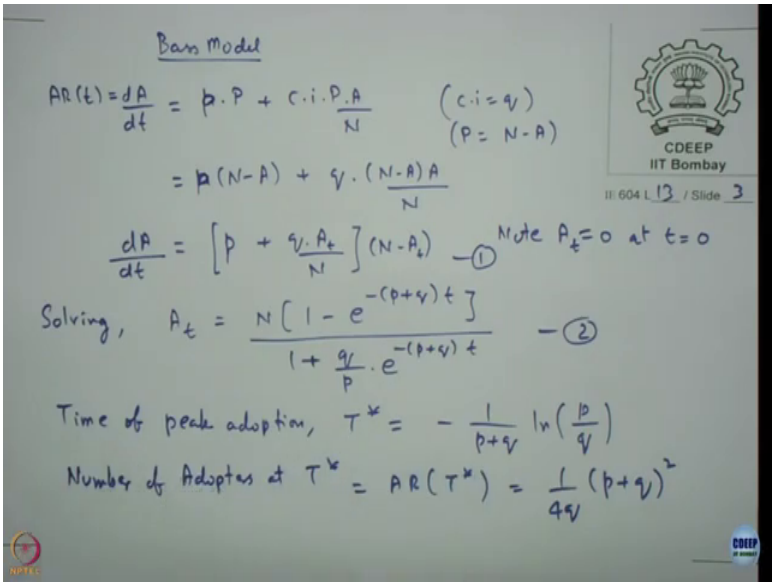

I am sorry, small p, small p

sorry, solving 1 we can get adopters at A time t as N into 1 minus e power minus of p plus q into t divided by 1 plus q by p d power minus p plus q into t, we call it equation 2, based on this we can get our time of peak adoption in the similar fashion or your T star, what you solve here is minus 1 by p plus q lan of p by q.

So, this T star for the cases when p is greater than q becomes negative right so; that means, T is a negative. So, that is why you are having a p get a T is equal to 0 and the number of adaptors at T star that is A R at T star is equal to 1 by 4 q into p plus q the whole square. This can be derived in similar fashion as we saw last class, it is just little a time consuming, but not that difficult you can solve that, because it is just one variable, you just have to integrate it, called bass model.

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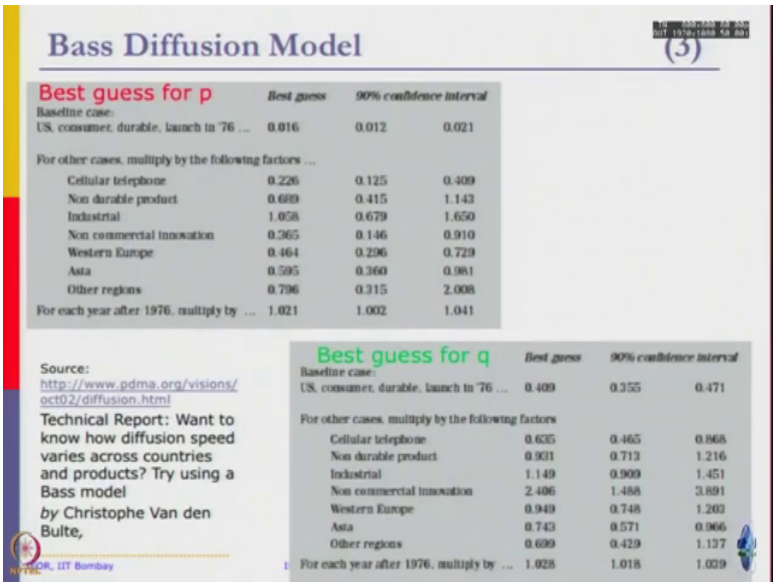

So, people have been using the bass diffusion model for quite some time, the various studies that has been reported to look at the diffusion speed across various countries and products resemble that paper the link is given. So, they have given some best guesses of p. So, various type of products the ps and qs are going to differ based on whether it is a FMCG good, fast moving consumer goods or it is a durable goods, etcetera.

For example a baseline case of US consumer durable is as in 76 was given as the best guess and based on that people have given various estimates for p's, similarly, various estimates for q's. As you can see even in the best guess, the best guess for p is 0.02 and best guess for q is 0.47 where word of more just clearly dominating here as compared to over advertising effects. Similarly, things can be compared across, it is a non durable goods for example, let us see. A things industrial goods, which are quite comparable is 1.149, it is 1.05 things just some

more comparable in case some of the items as supposed to the other ones where it is quite far apart.

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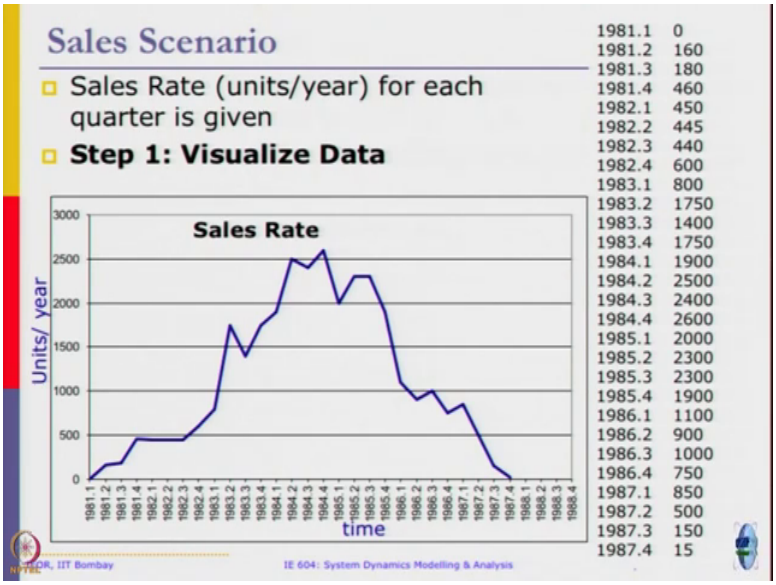

Now, we with that let us move on and figure out, how we are going to do for the same example we will take up what we did last class. So, the suppose we are given the sales rate again, we have a nice hump shaped pattern of the sales rate or the adoption rate. Now, how will you fit a parameters for a estimator parameters for the bass model which in this case is happens to be p q and N. When it was only q and N that is c into i and N, we use the use couple of ways that we solved it. Now, we have another factor called as p. So, let us see how we can do that.

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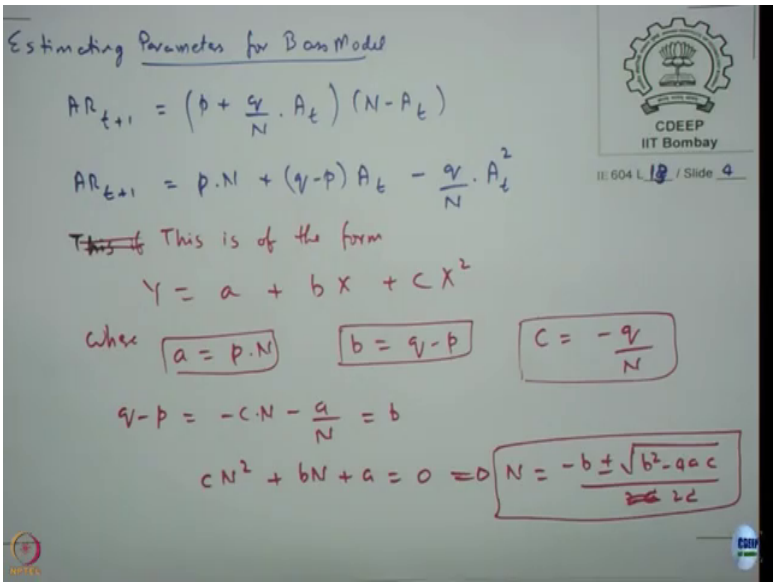

Let us take the equation as it is. Let us take what is happening to our say AR, say let us just put AR at time t plus 1 is p plus q by N A t into N minus A t which can be further expanded as A R at time t plus 1 is p into N plus q minus p into A t minus q by n into A t square. This particular equation is of the form, this is of the form Y equal to a plus b x plus c x square right, where a is nothing but p into N b is nothing but q minus p and c is nothing, but minus q by N. So, now, what you have is a nice polynomial already here, which we have the output variable Y in terms of input x which x and x square I am just getting a polynomial to that.

So, now I need to estimate my a, b and c based on this and use it to compute my values of p N and q., to the one step further we can write this for example, q minus p is nothing, but what is q? Let us take q from here which will nothing, but minus c into N minus a by N.

So, q is equal to minus c into N is coming from equation three here and p is equal to a by N which is coming from here, q minus p is equal b which comes from here, this if you solve it you can get c into N squared plus b N plus a is equal to 0 which just implies that N is nothing, but the roots of these, is equal to minus b plus or minus root of. So, total population is nothing root of these equation and once you have N you can use it to compute your q n p.

You are not required, but just you can do it, we changed it sorry. So, now, we can actually use this to build a regression model, this is what you guys are going to do. So, you can just make note of the equation a b and q. again, once you get a, b and c you can always solve these equations directly or you can compute N and then you can solve it.

So, now let us go back to our slides.

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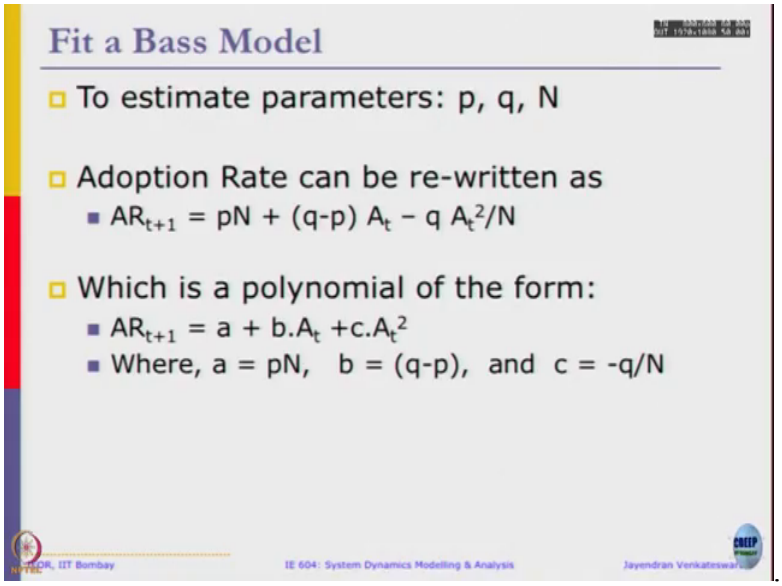

So, not a fit a bass model to estimate these parameters p, q and N, we need to estimate and we wrote it, we rewrote the equations adoption rate and we got this polynomial of this particular form.

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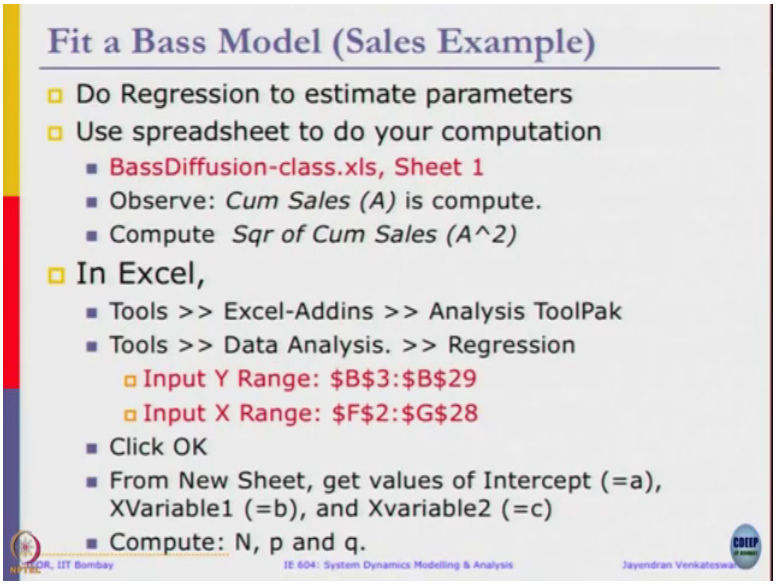

So, we need fit a polynomial of the same form. So, we are going to use regression to estimate these parameters. In fact, you guys are going to do it. So, use spreadsheet do your computation BassDiffusion class dot xls is presented there, compute cumulative sales is already computed in the sheet is available model please, download it. So, cumulative sale A is already computed.

You compute this square of cumulative sales and then in excel at the data analysis ToolPak and then you click tools, data analysis, regression and give these exact ranges that is given here. So, it is in the same spreadsheet that is what you will be giving and click and then get

you're a, b and c and using the equation that we just give you why do not you solve for N p and q? The same data I have already precomputed the values for A, because you already know how to do that you just have to include A square.

And use the regression let us follow the instructions, because you both the columns, see you got, because your regression includes A column as well as A squared column. So, you need to have both the columns selected. So, the parameters can be fit. In excel, we pre compute second parameters and give it.

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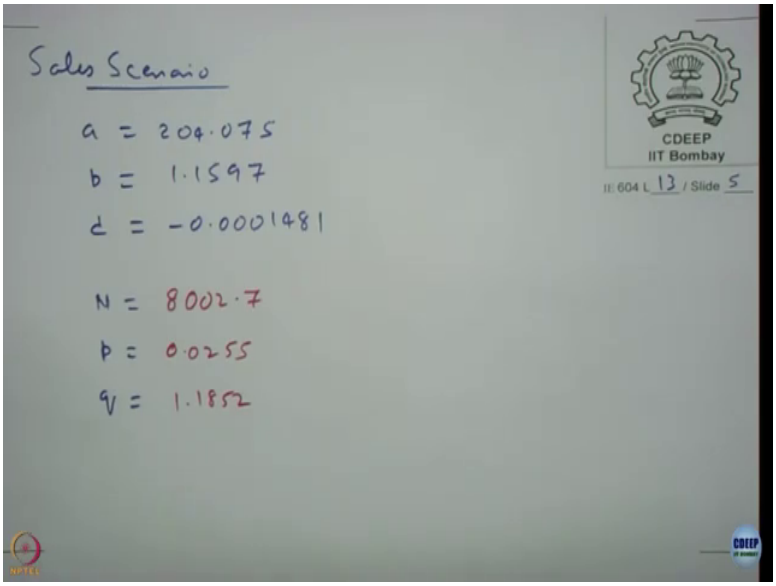

Even if we did not get ,a b and c at least using a b and c tell me values of N, p and q, is the values of a b and c, what do be N p and q, because this simulate it this what we want. Anyone value of N? 8002, p N q, we already have N, you can directly compute p and q, p is nothing but a by N.

We already have a divided by N should give you p; p 0.0255 something and q is 1.1852. So, now, these values we can put it in your bass diffusion model in your Vensim file and simulate it to see whether you are getting bell shaped curve for adoption rate. You can substitute these values directly in your Vensim file right away and simulate the model. What are the Vensim model file you have for bass diffusion substitute the values of this N p and q since, p is less than q we are going to get a bell shaped pattern for the adoption rate. Please check it.

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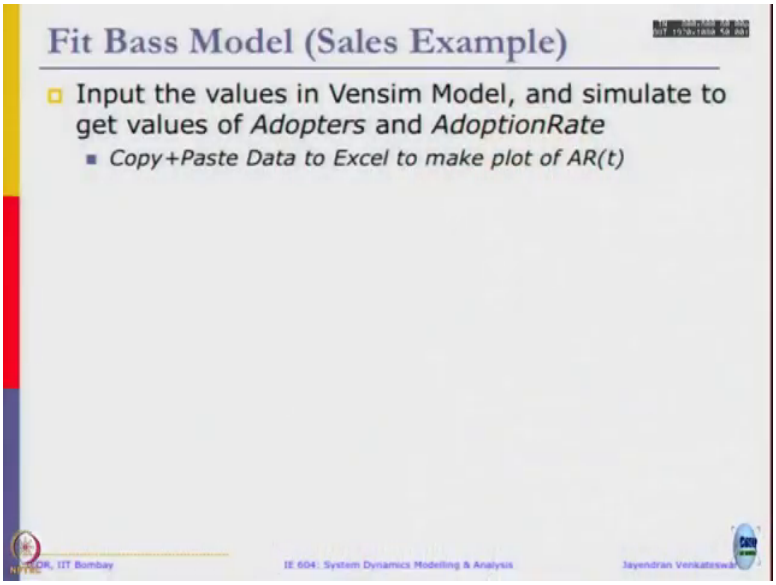

Input the values in Vensim of N p and q, simulate it to get the values of adoption adaptors and adoption rate once you get it. I have already shown how to calculate the values for the how to visualize, how to see the values in a table form. Once you open a table there is a featured copied, you can copy and paste the data value into your excel, then you can compare the actual data with simulated data, if you do that you will get this curve.

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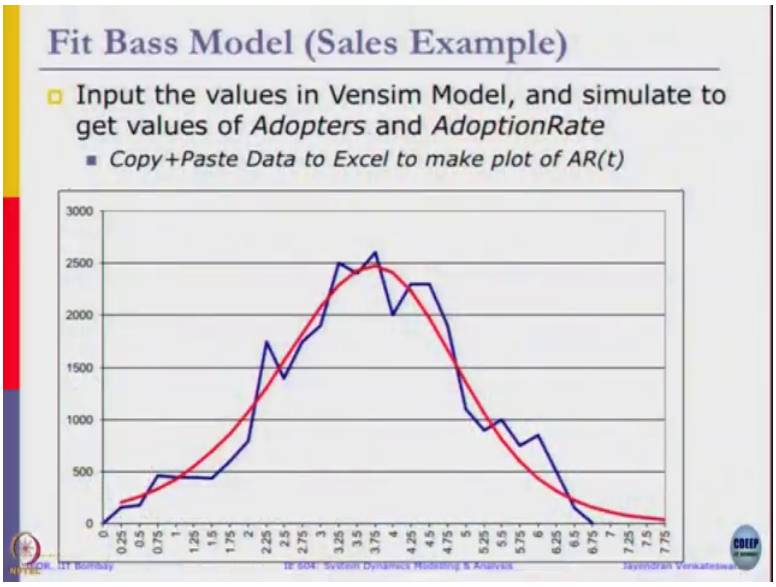

So, the red curve is what I got through Vensim which is you, which is exact same thing you will get it in your Vensim model also there is so, randomness you will get the exact same thing and the blue line is a real data that we actually have pretty reasonable fit using the bass model.

So, to get this graph you have to copy paste the data from Vensim, you need to plug the data in Vensim, simulate it, copy the data, paste it back in excel and then do this. There are ways in which we can get the input data itself the, not input the original sales data the blue line that you see here into Vensim, but we will I will show that later. Now, to summarize this full sales example.

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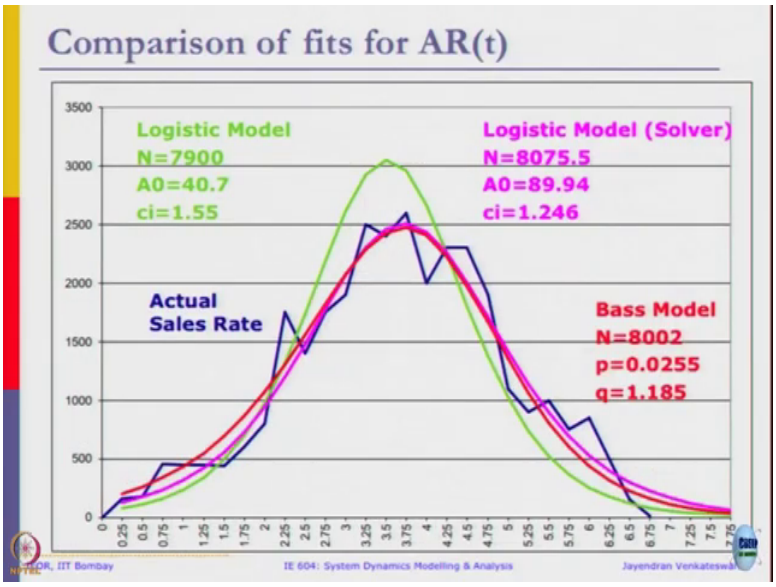

Let us look at a comparison of all the fits that we have been got for the adoption rate first fit. So, the blue line represents the actual sales data remember that the (Refer Time: 15:47) line and the green line was a logistic model that we first fit. Remember the logarithms that we took and fit a linear regression line that is represented in this green line which had a N value of 7900, A naught as 40 and c i as 1.55 the logistic model and then we did using solver, where we directly use expression for 80 and use a solver determined the values of N, A naught and c i and this is the best estimate, we have to minimize the means the square error.

So, these are the values we got N is a 8075, A naught is a 89, c i is a 1.24 on the bass model, which is showed in red. The logistics solver is in pink, this is a red in red color this is latest data that we just got with N is 8002 and p is 0.02 and q is 1.18. So, if you see this c i N p are equivalent. So, you can see here the c i p are reducing from 1.55 to 1.24 to 1.18 at the same

time. So, you can observe here, here N is very low and A naught is also quite low compared to the second case.

So, the it, I have to take a corresponding q value. So, it can try to fit a curve as close as possible. In this case A naught is much higher nearly, 90 while the c i or the q is 1.24, but here we overcame the start up problem as well with the very small value for effective advertisement and the p value again is kind of comparable to the logistic model solver scenario.

So, for the same another thing we have to observe is for the same data set, we had used slightly different models and we got different results of course, we can just focus on logistic model solver and the bass model here, the curves are very similar to each other and both are correct, it is not like one is better over the other, it depends on how we want to start if there is indeed advertisement happening, then we might want to prepare bass model.

If there is no advertisement happening we actually you know, see rate the market by giving people freebies or initial setup people were directly approached and they immediately bought this product, after which word of mouth predominate took place then you might want to represent it as the as a simple logistic model itself.

This one green line is to illustrate that even with all these models, if you do not pay attention and be careful you will get some plot and then we will say ok, this is also s shaped. Let me use it, but we can definitely get a better fit, if you use solvers or we use exact analytical solutions to compute the values.

Please remember that and observe how good a fit is and how we can improve it further based on the data given. So, this bass model can be extended similar to other diffusion model, one is to include discard and replacement purchase.

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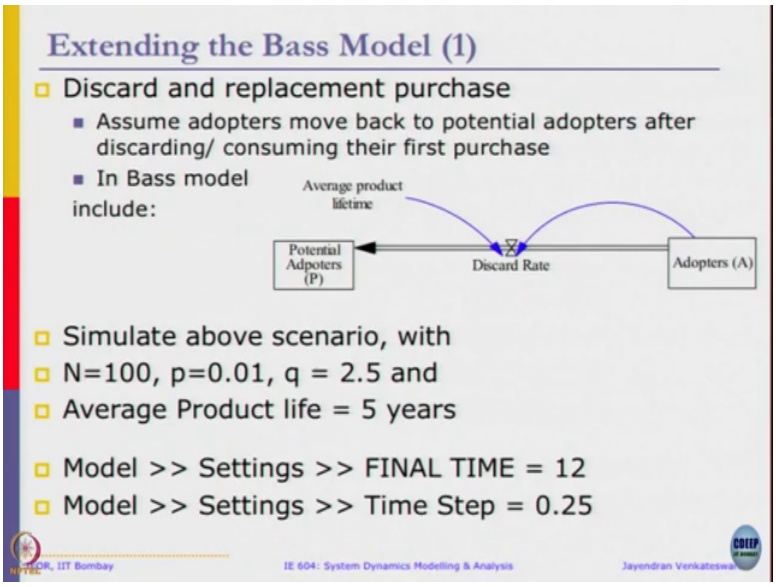

Assume adopters move back to potential adopters after discarding and consuming their first purchase, they buy it and then they leave it, they have abandoned it. We can include it by adding additional rate which comes from adopters to the potential adaptors which is shown as a discard rate.

So, in this case the equation with discard rate will be adopters divided by product lifetime. You can simulate this above scenario, when average when discard rate is not considered then all the potential adopters that is N is 100 everybody will buy the product, when the discard rate intuition says that some fraction will keep discarding the product every time unit.

So, the steady state that we will achieve will be lower than 100, what the exact values we can try to, we can simulate it right. Now, you already have the bass model you have to open it and

input these values N is 100 p is point 01 q is 2.5 average product life time is 5, because you have to include this rate in your model and define the equations for that.

What is the value it is saturating? The rest of the model is same as your bass model say in the bass model you have to add this. So, do not end up just creating two stocks and one flow, there is another flow from potential adopter say adoption rate, vodafone advertising affect, everything is a same all you are doing is adding one more flow within an existing model ok. I did not show that part, equation for discard rate is adaptors divided by left hand. Other extensions are also possible.

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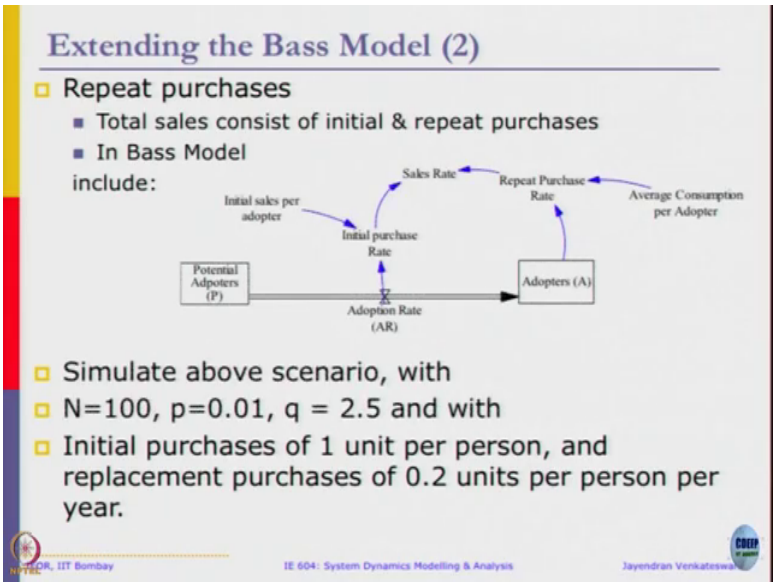

For example, if you want to model repeat purchases, there is no discard, but suppose we have an adoption rate, but we want to model repeat purchases. There is they already purchase a product and they are already adopted, but now I am just going to keep you know purchasing

again and again. So, here we can distinguish between sales rate and adoption rate. So, adoption is you become kind of a what can I say, you adopt a brand and sales rate is how much units are being purchased every time period. So, that can be simply modeled as for example, adoption rate we can create a separate variable call sales rate and initial purchase rate will be initial sales per adaptor.

So, this suppose assume initial sales is 1 say initial rate purchase rate will be adoption rate into sales rate sales per adaptor. So, sales rate will be equal to adoption rate only from this side.

Now, for the repeat purchases, I can simply add a variable called as repeat purchase rate and we can come up with an average consumption per adaptor. Suppose, you are going to repeat it how often or what quantity they are repeat return and I can add it to the sales rate.

So, me effective sales rate is nothing, but the sum of repeat purchases as well as initial purchases. So, adopters are people who have actually owning the brand and I am going to do it again and again. There are already into the product now, how often they are going to keep buying it again and again. Again, we can, there is no discard rate in this remember.

So, you can modify the bass model we can, you now set N is a 100, p is 0.01, q is 2.5 initial purchase is 1 per unit per person 1 unit per person and replacement just 20 percent units per person per year. We can see what happens to the actual sales rate is sales rate is at 100 will be more than 100 eventually, it has to be little more than 100, because everybody would adopt the product, but then how does that change is something you can try at your home ok.

Thank you.