Introduction to System Dynamics Modeling
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Diffusion model:
Fitting Data Estimating Parameters
Lecture - 12.1
Dynamic of Simple Structures:
SFD of New Products

So, in today's class we will be taking a look at Diffusion Models and for the specific example on how we can model from existing scenarios, how do you estimate parameters and beta diffusion models.

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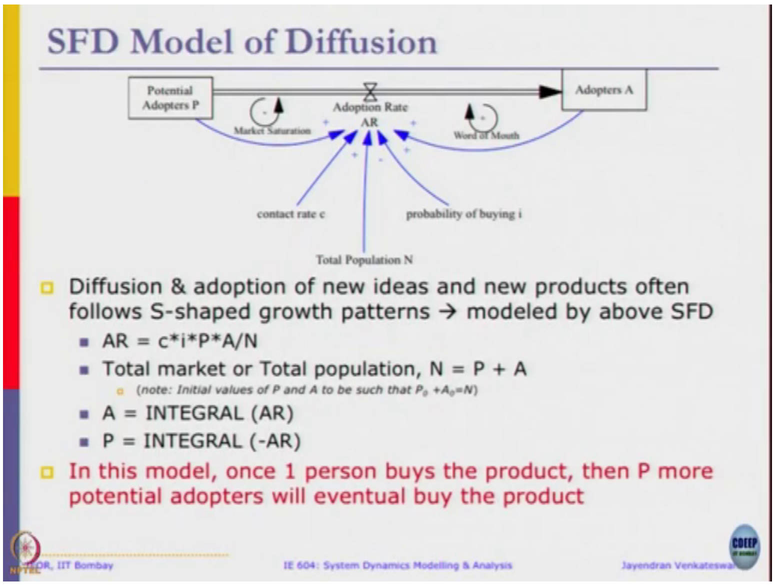

We have seen a Stock Fluid Representation of the Diffusion Model where we defined the divide the total population into two potential adopters of the product and the adopters of the product. And, an adoption rate or the sales rate will move people from potential adopters to adopters divided by the factors of contact rate as well as probability of buying.

So, the equations are shown. So, you can see AR is c into i into P into A by N total MA. The total population is conserved N is equal to P plus A. So, in this model the key thing that you understand is as soon as one person buys a product, eventually everybody will buy the product. So, the model will run until the potential adopters become 0 and adopters is equal to N. There is nothing stopping them or preventing them from stopping half-way. So, here as soon as one person buys the product, then P more potential adaptors will eventually buy the product. Now let us see how we can use it for more real examples.

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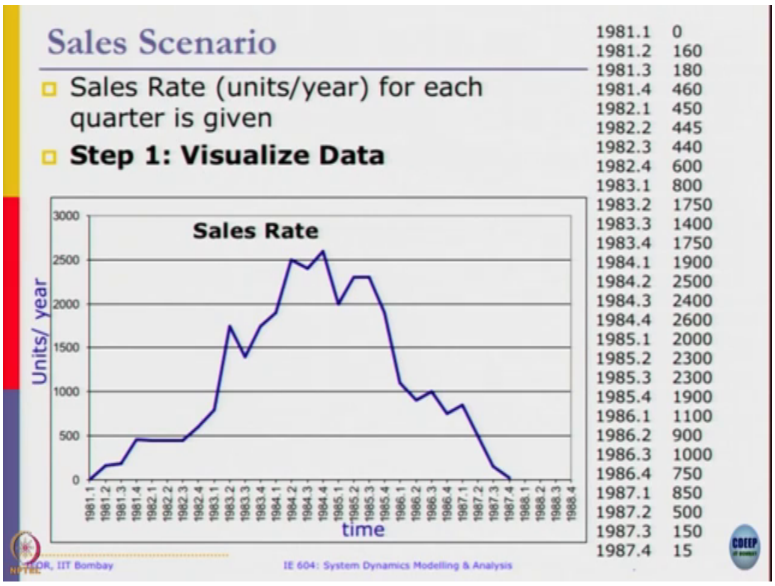

Suppose the sales rate that is the units per year for each quarter is given in the following table or the sales record is from 1981 to 1987, what are 1 to 4 per year on the units per year is given and when we plot it, we get a sales rate graphs as shown here that is the units per. Yeah the step 1 is once you get the data, we visualize what the point.

So, data here shows a kind of a bell shaped pattern. It increases peaks at around 1984 third quarter and then it rapidly falls down and kind of set 0 at 1987 fourth quarter. So, that is the data that has been provided to us. Now, you would like to see how we can kind of simulate this kind of behavior using the diffusion models that we have learnt.

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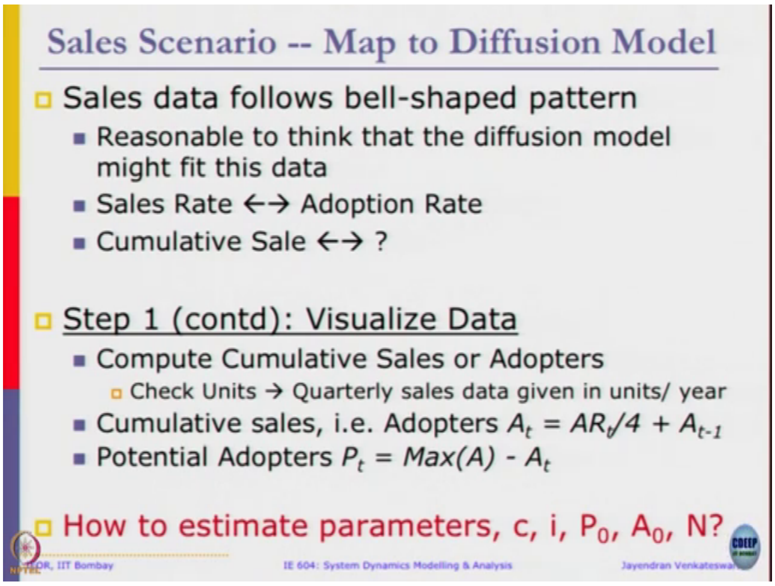

The first is to map it. Let us just make the observations sales data follows a bell shaped pattern. Here it is reasonable to think that the diffusion model might fill fit this data. So, what we have this sales rate it is nothing, but the adoption rate that is obvious. Then what will be the cumulative sales, we are going to map it to the diffusion model. The diffusion model we have seen the terms adoption rate. So, in our example that is a sales rate. So, what would be the cumulative sales in a diffusion model?

Owners are the adopters. The model wages define adopters. So, that will be a cumulative sales. So, if it is indeed a diffusion model, then these adopters was a cumulative sale need to represent a shared pattern. Now that the sales rate is at cumulative sales rate, we know that sales rate as it increases and then the sale rate is decreasing. Underling graph has to be S-shaped correct. It is intuitive, but we can go ahead and plot it and see also what happens.

That is what the first step is. We are going to do is you have the sales data that is only data we have based on that. Let us try to figure out what is the cumulative sales data, plot it and see what is the curve, then we will figure out how to estimate the parameter c i initial value of potential adopters, initial value of adopters and the total population N, ok. So, let us do that.

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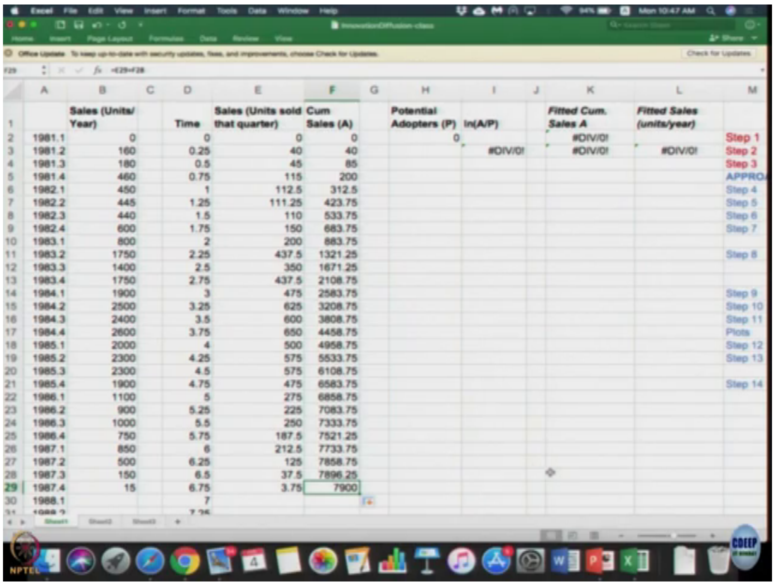

	Sales (Units/ Year)	Time	Sales (Units sold that quarter)	Cum Sales (A)	Potential Adopters (P)	ln(A/P)	Fitted Cum. Sales A	Fitted Sales (units/year)	
1981.1	0	0	0	0	0		#DIV/0!		Step 1
1981.2	160	0.25	40	40		#DIV/0!	#DIV/0!	#DIV/0!	Step 2
1981.3	180	0.5	45	85					Step 3
1981.4	460	0.75	115	200					APPRO
1982.1	450	1	112.5	312.5					Step 4
1982.2	445	1.25	111.25	423.75					Step 5
1982.3	440	1.5	110	533.75					Step 6
1982.4	600	1.75	150	683.75					Step 7
1983.1	800	2	200	883.75					
1983.2	1750	2.25	437.5	1321.25					Step 8
1983.3	1400	2.5	350	1671.25					
1983.4	1750	2.75	437.5	2108.75					
1984.1	1900	3	475	2583.75					Step 9
1984.2	2500	3.25	625	3208.75					Step 10
1984.3	2400	3.5	600	3808.75					Step 11
1984.4	2600	3.75	650	4458.75					Plots
1985.1	2000	4	500	4958.75					Step 12
1985.2	2300	4.25	575	5533.75					Step 13
1985.3	2300	4.5	575	6108.75					
1985.4	1900	4.75	475	6583.75					Step 14
1986.1	1100	5	275	6858.75					
1986.2	900	5.25	225	7083.75					
1986.3	1000	5.5	250	7333.75					
1986.4	750	5.75	187.5	7521.25					
1987.1	850	6	212.5	7733.75					
1987.2	500	6.25	125	7858.75					
1987.3	150	6.5	37.5	7896.25					
1987.4	15	6.75	3.75	7900					
1988.1		7							

So, I urge you to you download the excel file and open it the data whatever we saw is given. So, column B is given to you and based on column B if you scroll down, the graph is also plotted which is nothing, but the sales rate.

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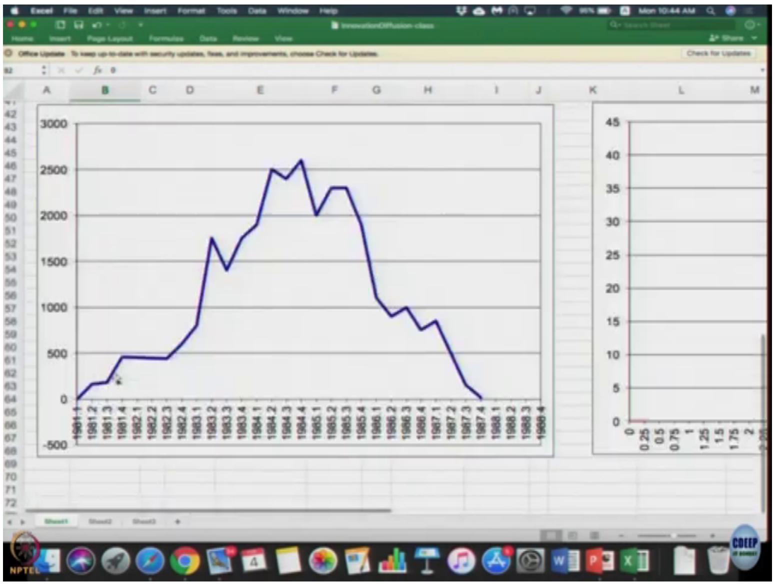

We have plotted those two values right here ok. Nothing else to do just observe what is there in the field, ok. So, first thing we are doing is the actual sales data gives it which year and which quarter and things like that whatever computation simulation purposes, we can reset the scale to 0, ok. So, the time starts at 0 and then since it is every quarter we can just make it 0.25 0.5 0.75 and so on and that is what the reset time column is put, ok.

Though we have data until only to 1987, 0.4 it is just extending it for a few more data points which is fine. So, all we have done here is reset the time starting at 0, ok. Now this and this is a slightly tricky example. The units of sales rate is units per year that is at every quarter I am selling what is the quantity per year ok, but we want what we want is what is the actual quantity sold in that quarter. So, to compute that first we divide sales that is units sold in that quarter as sorry; so, B 2 by 4.

Please note it the equation you see will be incorrect. You just change it to B2 divided by 4. See what happened is it kind of it is like you are accounting for that in a simulation time step or the level of accuracy every quarter they are reporting what is the sales rate per year, what do you want is actual quantity that is sold in that quarter. So, it since each year has four quarters, we are dividing the sales by 4. So, let us just divide it. So, it is B2 by 4.

We just drag the column all the way to the end. Now got it? So, sales is nothing, but unit sold that quarter is the total units sold divided by 4 that is a quantity sold in that quarter. The cumulative sales the first value is 0, the second value is that is I mean formula is given it is nothing, but current cumulative sales plus whatever has been sold in that quarter. So, the second row we can drag it to the end. You will get cumulative sales column is 7900. All of you got these values? So, all the steps that I am telling you right now is also written on the right side of the sheet.

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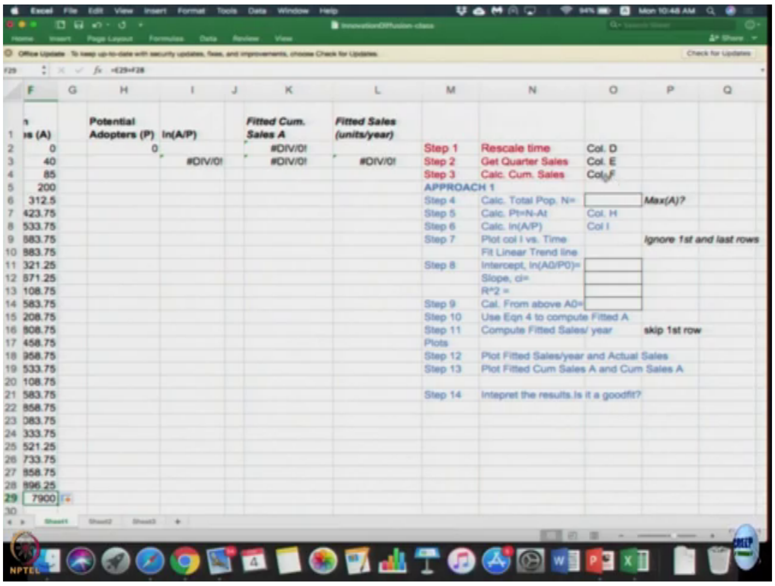

Step 1 Rescale time, step 2 get the quarter wise sales column E, step 3 calculate cumulative sales column F, ok. So, now they stop there. Up to column F is sufficient.

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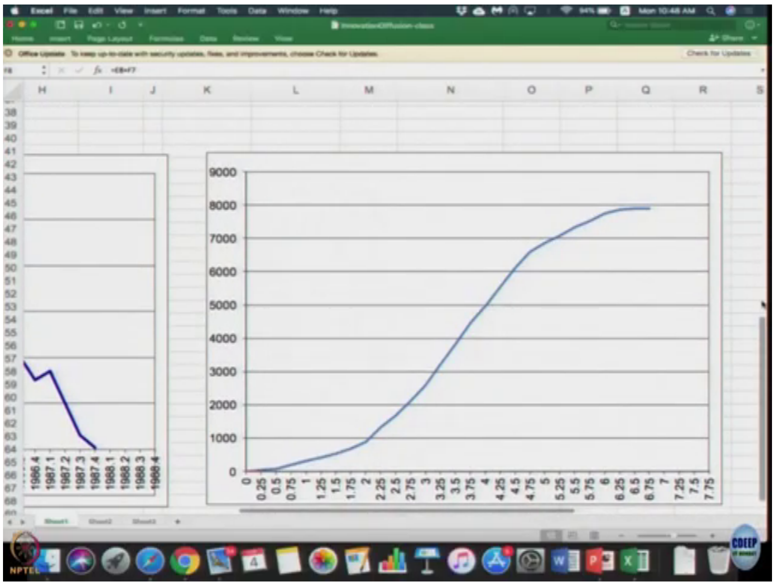

So, if we scroll down after doing column F, you will see a right side graph. I mean the formulas have been prefilled for use as soon as put your numbers, the graph will show this figure. So, this right side graph shows the cumulative sales data or the total number adopt the adopters over time which is a classical S-shaped. So, it looks like we can fit a diffusion model to see how well we can fit it. Please scroll down to that, so that you understand what you are doing. There are so many spreadsheets.

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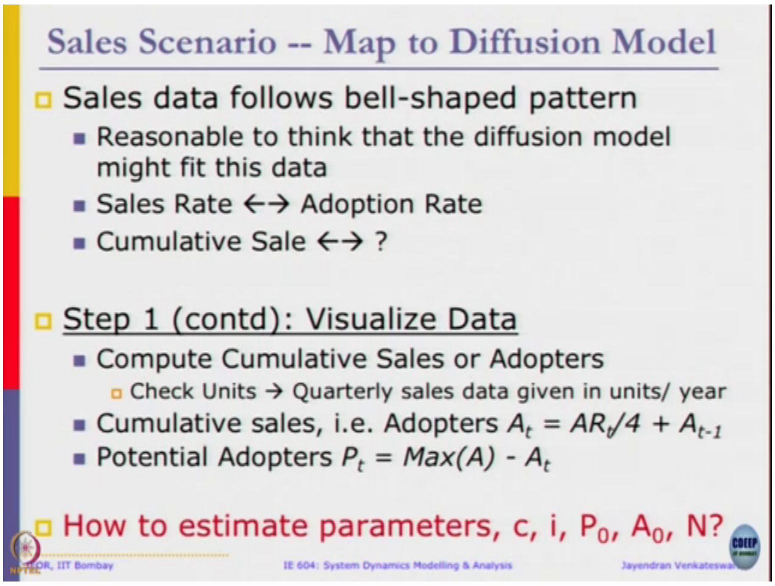

Now, we need to pretty much estimate these parameters c i P naught A naught and N from the data that is given. To estimate these parameters we need to look at a bit of Maths. Let us go ahead and do that.

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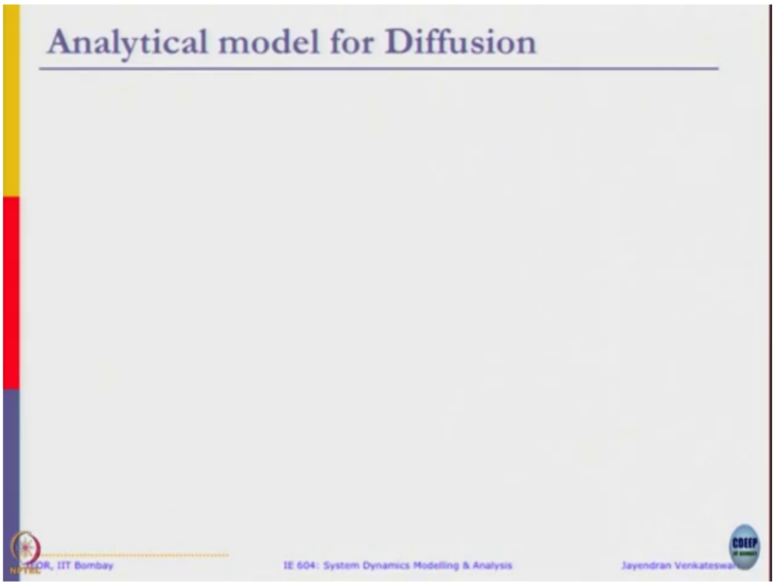

So, please go to the equations.

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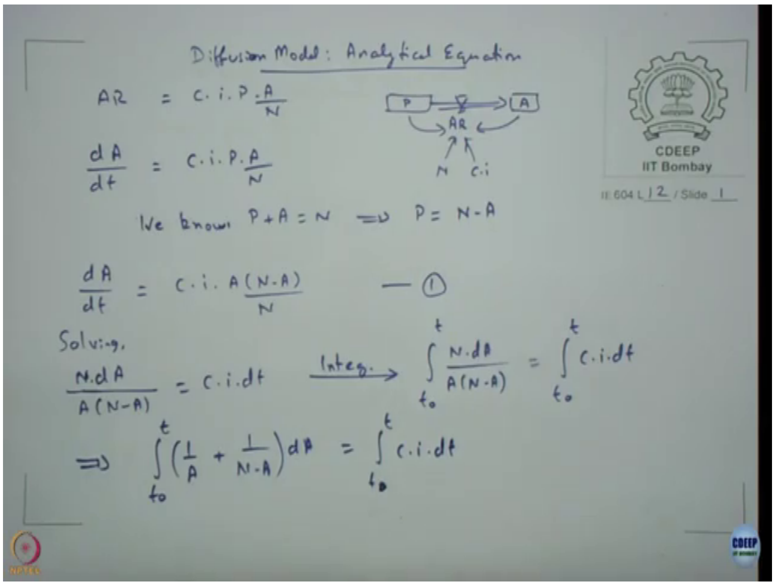

The diffusion model analytical equation it is going to be pretty lengthy derivative. So, you can get started. We have seen that the adoption rate AR is equal to c into i into P into A by N. So, this is what we had seen from the equation. So, whatever is the sales rate or the adoption rate, this is the same rate at which the actual adopters change. So this is nothing, but the change in the number of adaptors A. So, you want a smaller equation. What I am plotting here is this model. For simplicity sake I am just going to do it c into i this, ok. So, this is the model and underlying equation is shown; so, the change in a same as c into i into P into A by N.

We know that P plus A is equal to N or P is equal to N minus A. So, we can rewrite our d A by dt c into i into A into N minus A by N. So, remember what we need to solve for when you had a exponential growth or a asymptotic growth we try to solve the equations and if we try to figure out what is analytical solution for it, we are going to do a very similar thing right now. So, you would like to solve this. So, this let us just denote it as equation 1.

Solving let us say d A by N into d A by A into N minus A is equal to c into i into d t. Integrating it on both sides let us put it like t naught into t I am going to get that like dA by A in to N minus A equal to t naught t c into i into d t which that gives me, ok.

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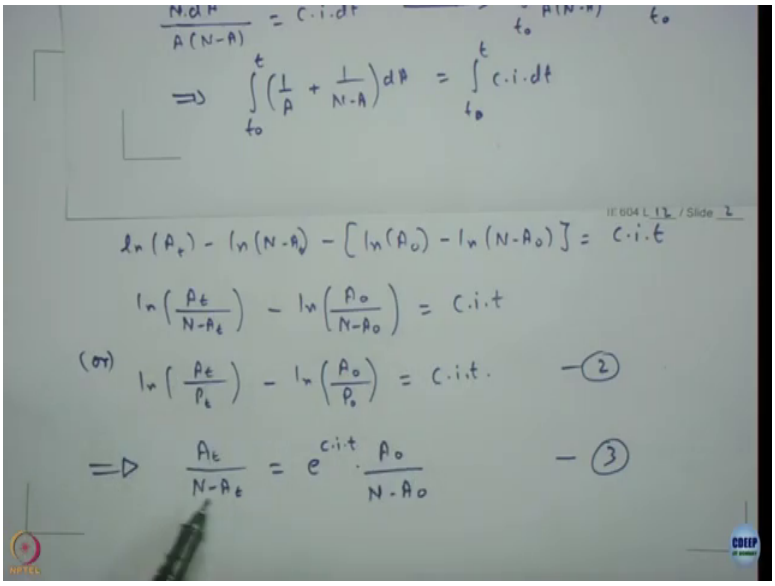

It is continuing the last equation I am just integrating it. So, it gives us log of A t minus log of N minus A t minus log of A naught minus log of N minus A naught equal to c into i into t. So, t naught is a time 0. So, the last term disappears.

So, this can be re-written as log of A t by N minus A t minus log of A naught the N minus A naught equal to the c into i into t or log of A t by P t minus log of A naught by P naught equal to c into i into t. Let us call it as equation 2, right. Then I can play with this main equation as per my convenience. This can be further solved or to give us another expression as A t by N minus A t is equal to E power c i t A naught by N minus A naught. Let us call it equation 3.

We do not need to go through this to get it here. This is from here we can get this expression. So, we can get it from here we can now compute the expression for A t. So, what you are trying to do is trying to come and solve for A t, it is just intermediate step. We can view it in this form or we can expand further N minus A t is nothing, but P t i. Just want to reinforce it that is why this has an equivalent either I represented as P t or N minus A t I can remove logarithm by raising to the power e which is what I did in equation number 3.

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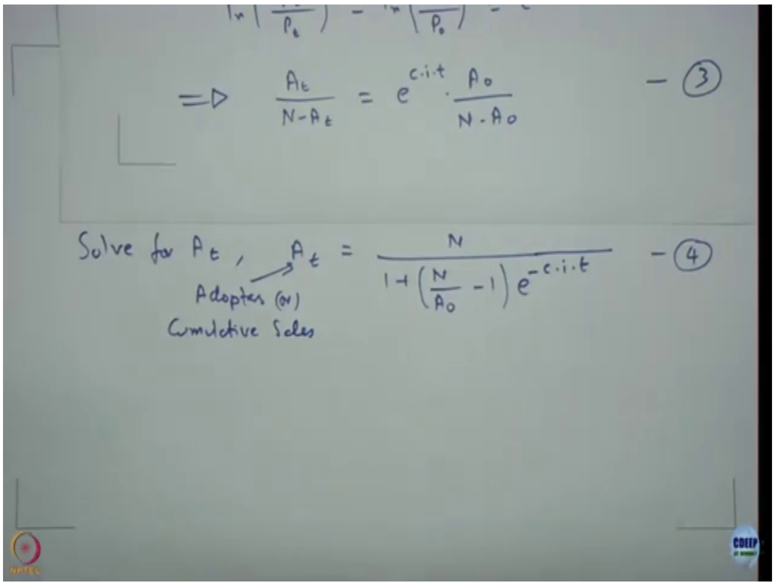

I can actually solve for from this equation you can directly solve for A t to get A t is equal to N divided by 1 plus N by A naught minus 1 into e power minus c i t. Let us call it equation number 4. So from 3, I have A t and N minus A t. You can take N minus A t now to the other side multiply then

Student: Sorry.

Trade to the other side, take a common factor of A t then, again go to back, divide it and to solve you will get this expression A t is equal to N by 1 plus N by A naught minus 1 into e power minus c into t by into c into i into t because t is the time. So, in this equation if you see the initial value of adapters is required. That is initial value of adapter 0. There is no diffusion model happening. So, there has to be initial value of adapters.

As soon as initial value of adapters it is going to be there, this equation come into effect c is known i is known N is known A naught is known. So, all the parameters are known. The only one thing that changes here is time. So, as we progress with time we can compute value of adoption rate or to the not adaptation rate adopters or in our example it is the cumulative sales equations you already come to so much, we can actually ask some very basic questions on you know.

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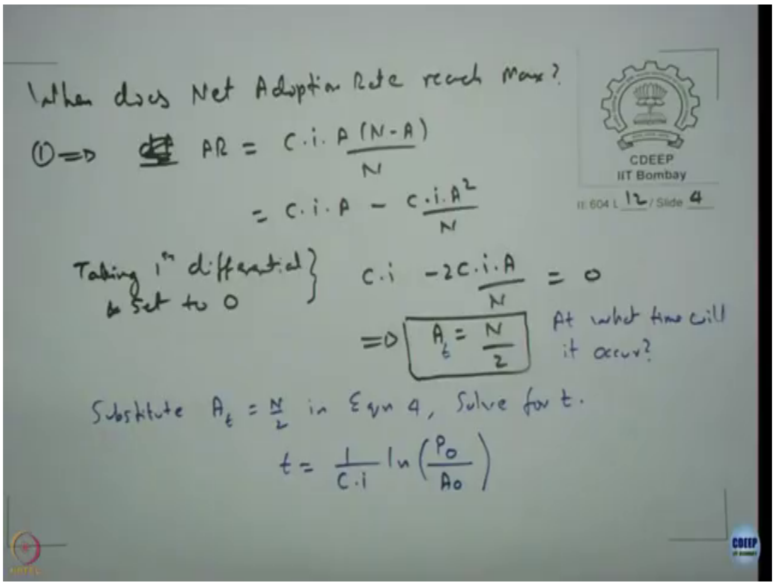

When net adaption rate it is maximum, we are taken it as from one adoption rate is already given as c into i into A into N minus A by N which is A minus c into i into A squared by N taking first differential and set to 0. We can get c i minus 2 into c into i into A by N equal to 0 which gives us A is equal to N by 2. So, when A is equal to N by 2, it is going to reach the maximum. You can take you can take second differential. If you do a second differential, we will find it is a negative. So, that means it is a maximum point so, but when does this occur at what time will this occur at what time will this occur how will I find it?

Student: Sorry.

At what time will it occur?

To figure this out.

Student: Substitute it.

Substitute A t is equal to N by 2 in your equation. What is it in your equation 4 and solve for t. If you substitute A t is equal to N by 2 in equation 4 and solve for it, this is the equation 4. These are equation 4. So, I substitute A t is equal to N by 2, solve it for t and you will get t is equal to 1 by c into i to the logarithm of P naught by A naught. So, this is the time at which it is going to hit the maximum.