Introduction to System Dynamics Modeling
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Dynamics of Simple Structures: S - Shaped Growth (Contd.)
Lecture - 11.1
Dynamic of Simple Structures: Examples of systems exhibiting S-shaped growth

So, we can needle look at Structures causing S-Shaped Growth today.

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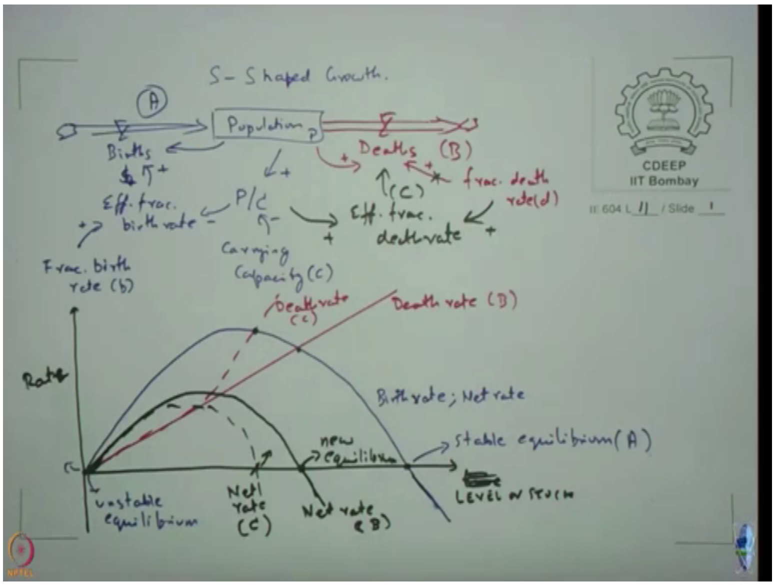

Basic structure of population effected by births as well as carrying capacity and we define the ratio P by C ,this is P n this is your c, then we have defined a fractional birth rate b and we define the effective fractional birth rate, I am sorry as the model.

So, we have taken up this basic model, where we were looking at how population growth gets of constrained at a later stage by the carrying capacity initially, when it is not constrained you got exponential growth when the, and as the population approaches carrying capacity we can see an asymptotic growth. So, that is the dynamics we saw; so, let us try to draw it right here. Now let us say this is your no ok, let us the rate level chart let us make it level our stock on the x-axis and rates on the y-axis.

So, for this system as it was the population increases the rate rather here in this case it is only the birth rate continues to increase and after the inflection point the birth rate falls down and it just an equilibrium somewhere later. So, the expected graph here we can see will be like this point here is your let me just write it here unstable equilibrium and this point here becomes your stable equilibrium, and this line here that we have drawn will be both the say this is the births birth rate as well as the net rate. This gives the birth rate net rate in the same graph we just have only one particular curve right here.

Then we had gone and drawn the deaths as another variable within the system. So, we had defined deaths and initially we defined deaths as just a fractional death rate d, we kept it as an external factor here. So, what we did for this particular scenario so, let us call it a say scenario that just make this part as a scenario A.

So, this is the curve I get for scenario A that is stable equilibrium for scenario A and when I include the scenario B let us see what happens. So, this is scenario B, since scenario B I have defined a death rate, which increase in proportion to the level of stock right, it just a d multiplied by your total population. So, it is going to be a linear curve. So, this is your death and death rate as in scenario B.

As a result you are going to have a net rate it is could be it is nothing but the birth rate minus the death rate. So, birth rate continues to get affected independent of this. So, let us assume that the same curve there will be some small changes in the population, but for practical purpose we will assume it to be or rather analytical purpose we will continue assume this is the birth rate and this is the death rate.

So, we will compute a new net rate for the scenario B which could be something like this, which will intersect this curve somewhere here. So, this is your net rate for scenario B and this is your now new equilibrium point; this is a new equilibrium right here. As you can see by adding a constant or proportionate outflow so, equilibrium moved from this point to this point here that mean system is going to saturate at a much lower value of stock. So, it may not reach the carrying capacity.

Once again change the model, we again introduce something called as an effective fractional death rate and then we computed that. So, this part written here is only for scenario C. So, scenario C this link is not there. So, as we make death also change with respect to the carrying capacity as the population approach carrying capacity death rate is going to increase further or increase non-linearly. So, this can be captured graphically as the level increases my death rate also changes.

So, let us assume that death rate kind of say change like this, let us assume this is death rate for scenario C. So, as per this my effective net my net rate for scenario C is further defined by this birth rate blue color and this dotted line as a death rate. So, the equilibrium point further shifts downwards and this becomes a new equilibrium point right here.

So, as more and more constrains keep happening. So, we can learn couple of things here as more and as a constrains start acting on the actual state of the system and as it starts affecting the flows, the point of equilibrium will be lower. As more constrain strip coming initially your constrain is in only on the births sorry stop here, even if I had a constant exogenous outflow a proportionate exogenous outflow I found with the equilibrium shifted downwards because more inflows and outflows are acting on, the more outflows are acting on the stock in this case.

Suppose both birth rate and death rate is getting affected by the carrying capacity then the new equilibrium point is much lower. See, equilibrium point refers to the value of the stock.

So, if you assume this is the carrying capacity you are able to reach it, that means, here we cannot reach the carrying capacity can just we can visualize it very easily that I am going to saturate at a point much lower than the carrying capacity of the system, because of the non-linear dynamics on the birth rate as well as the death rate that you see right here.

See, if death rate and the birth then they are independent of each other right then it makes sense to actually do the comparison among them, but the equilibrium point will be the point of intersection between death rate and birth rate that is the net. So, here this point; so, this is the equilibrium point compared to this, this is the equilibrium point again which this is happening it is a point at which net rate; this is the point at which birth rate equal death rate that is inflow is equal to an outflow. So, that becomes your equilibrium point, right.

Anything here, that means, birth rate is lower death rate is higher, 0 that means, level of stock has to go down so, that means, it will go up here which is defined by this net rate value. For this curve when you do not have this part of the model there is only birth rate and population, then there is nothing to stop at going all the way down to 0, it is becomes a new equilibrium point, right ok. So, this is what we did last class a summary of it.

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This pattern is exhibited various scenarios as population turns up many animals and plants, learning curves, diffusion of news, riots, epidemics, rumors are all exhibits this S-shaped pattern if you look at the aggregate level. Initially, just take the third point on diffusion of news and the rumors.

So, if think about it initially when the new spreads it actually we use the term it spreads like wild fire that means, it is actually going in exponential growth in the new. But, after sometime as entire population gets to know the information or gets to know the news or rumor or whatever it is then there is no new person hearing it. So, then it will saturate and hit the capacity. So, there the diffusion again becomes S-shaped.

When growth of new products and various the socio economic activities can also be attributed to this exponential growth initially everybody is full of enthusiasm and then we all exhibit the exponential growth in various aspects. Then, as the time goes on new information we find it more and more difficult to absorb or news to difficult to spread and so on.

And even the new products as and when the market get saturated. So, that is what you mean initially the growth is very nice lot of people are buying it, but then we use the term then the market become saturated or (Refer Time: 12:54) it is achieving a kind of hitting the carrying capacity or number of people who would like to buy the product is already reached no new products are sold. So, that is the time when we want to introduce another new product.

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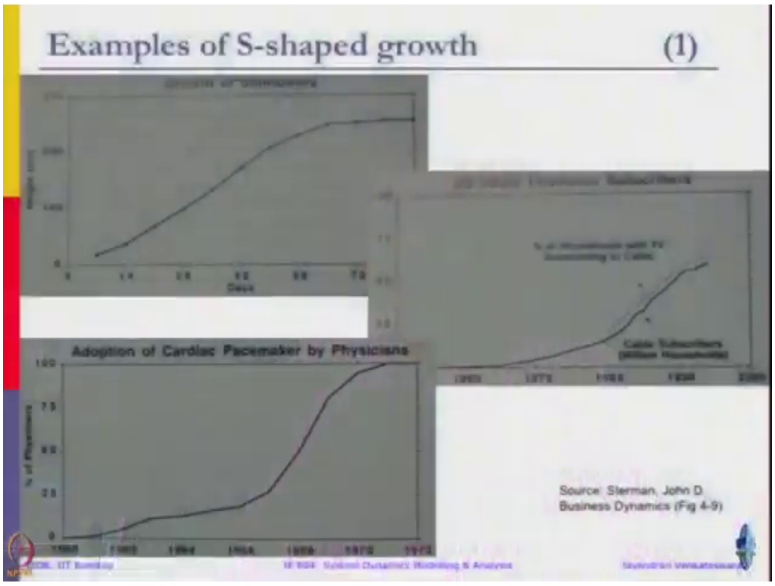

These are some example curves exhibiting S-shaped here is a plot of growth of sunflowers their height and days. So, if you plot your own again what kind is there your own growth per we will probably get S-shaped curved all your growth are saturated you are not you may grow wider, but not taller. So, that is kind of saturated right here. In cable TV growth again it is nice S-shaped pattern here and saturate as everybody starts your own cable TV or (Refer Time: 13:51) satellite dish.

So, only new customers are going to come or the people are to going own add in second TV's or going to buy a new TV's and so on. Adoption of cardiac pacemakers of physicians again exhibits a nice S-shaped pattern.

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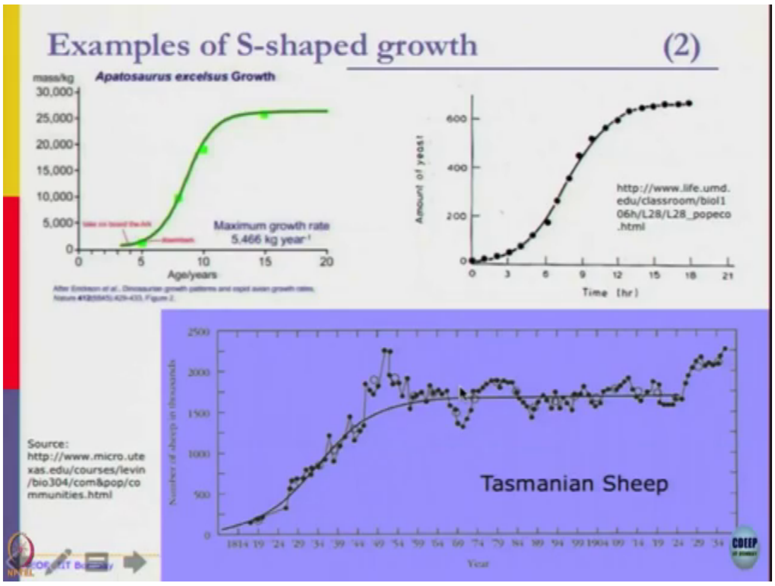

This is some various different papers that I found these curves this is the amount of yeast and the time it takes for it to grow in S-shaped growth. This is the growth of Tasmanian sheep which has been shown in various other literatures where again classical S-shaped pattern is shown after where it has reached a kind of a steady state and revolving around there fluctuating around which mean.

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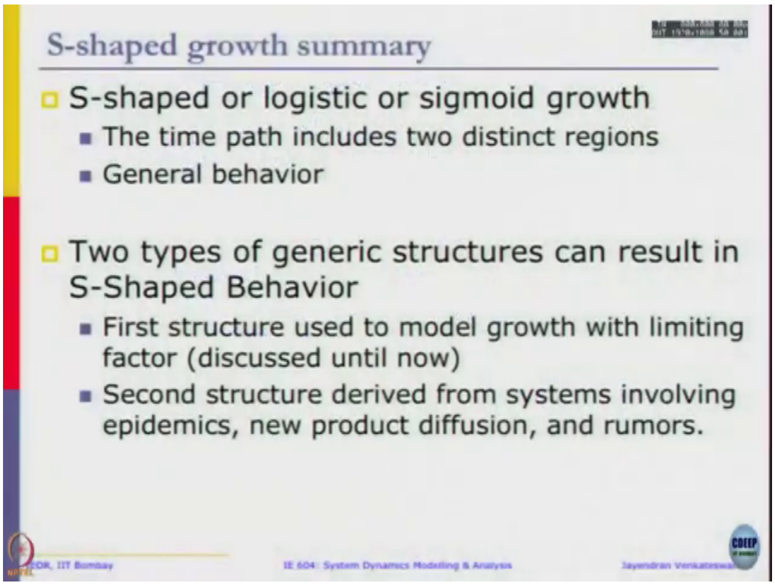

So, this growth is also called as logistic growth or a sigmoid growth. So, these other terms that you may come across in literature they all mean this S-shaped pattern. So, time path includes to distinct behavior as we saw, but general behavior is initially you have a exponential growth until inflection point and beyond the inflection point we have a saturation or asymptotic growth yeah because, so the general behavior.

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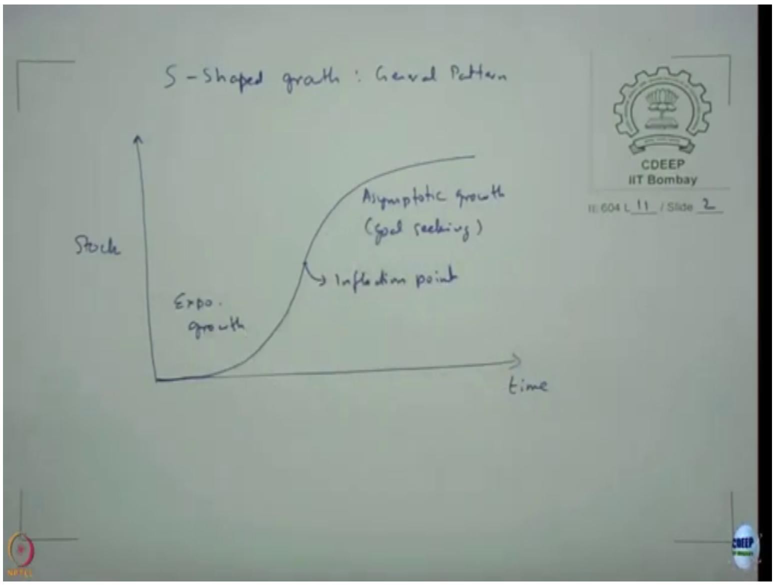

The general pattern this is your time this is your stock this is S-shaped pattern that you are looking at. So, initially we have exponential growth and later we will have asymptotic growth for goal seeking. So, this point here is referred to as inflection point. So, inflection point is a point at which after which the negative feedback loop starts to dominate the positive feedback loop. So, this is exact behavior.

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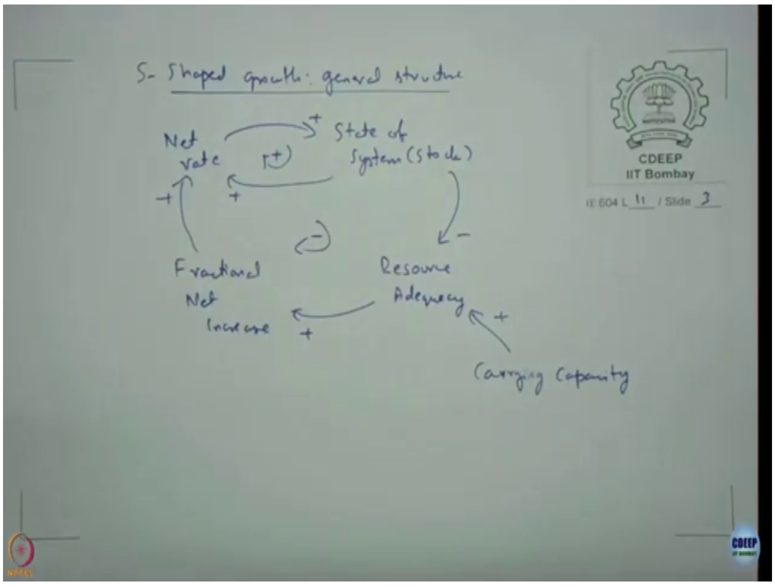

The general structure for that, the general structure of S-shaped growth is we have state of system or stock then we have the net rate, this is your typical positive feedback system you can model a resource adequacy in the final carrying capacity. So, the final fractional net increase this is your positive feedback loop and this becomes your negative feedback loop.

So, this is the general structure which is going to cause S-shaped growth. So, we are going to pretty much be having on positive feedback as well as the negative feedback which is constrained by your carrying capacity or which affects the resource adequacy. Kindly note there is slightly different from the birth model that we saw then birth model we had a negative link here because we divided by carrying capacity and we have positive link here and we made a negative link here, does not matter when so, loop is a negative feedback loop that is what we want as long as that is satisfied you are going to get a S-shaped growth.

Where the net rate as the state of system getting affected positive feedback and negative feedback. At point of inflection the negative feedback will start to dominate after the point of inflection, until then the positive feedback loop dominate causing exponential growth. So, when we rewrite or we write about it is always initially the system state is driven when exponential growth.

when the system when the resources are adequate. As the resource adequacy approaches the carrying capacity, the net rate flows down causing an asymptotic growth and finally, growth see this.

Having shown this generic structure actually this is the first structure that we have see, but it is sure it is can modeling growth with a limiting factor whatever we have discuss till now is it based on this limiting factor or carrying capacity. We can have a second structure derived from systems involving epidemics new product diffusion rumors where these two different loops may not be very apparent to you, but it still cause a S-shaped growth.