Introduction to System Dynamics Modeling
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Dynamic of Simple Structures S-Shaped Growth
Lecture - 10.4
Dynamic of Simple structures: Extension of model to include death rate

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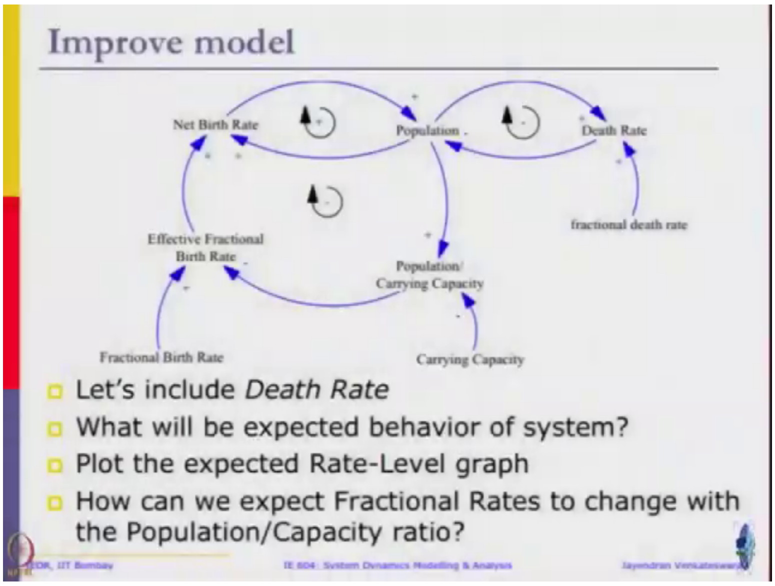

Now, let us improve the model let us add one more thing. Suppose there is a constant birth within the system not constant what I meant is, there is a birth rate to the system, but it is not influenced by the carrying capacity. Birth rate is affected by the fractional death rate and death rate is not influenced by the carrying capacity at all.

What will be expected behavioral system? Plot expected rate level graph? And how we can we expect fractional rates to change with the population slash capacity ratio? So, let us keep this graph in mind keep this diagram. So, again the left half is exactly the same, only thing now is population is also decreasing at a proportional rate as per the fractional death rate in addition to the net birth rate.

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Improve model include including the deaths rate, this is nothing but the fractional death rate d into the population P. So, let us so that so that is changing in proportion what can we expect our behavioural system to be will continue to be s shape?

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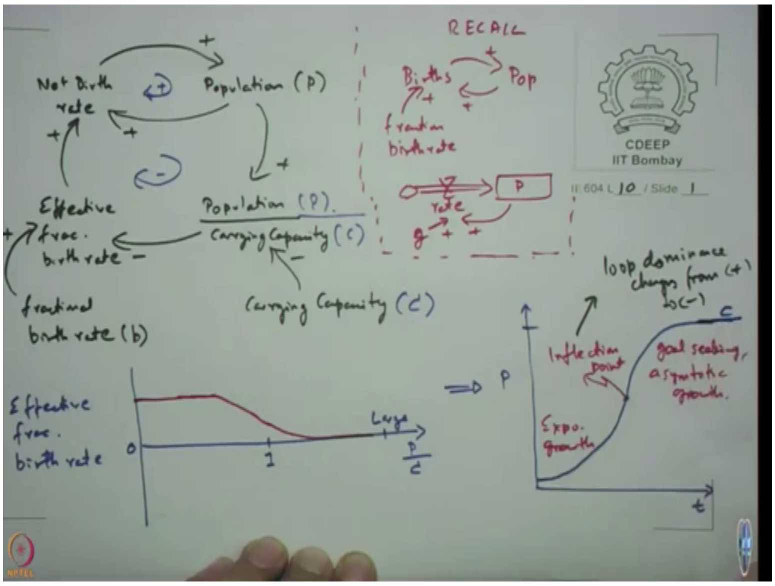

My first case we reached the carrying capacity c this point to carry it became equal to the carrying capacity basically recall. So, we are looking at one graph like this one like this and one for the rate level.

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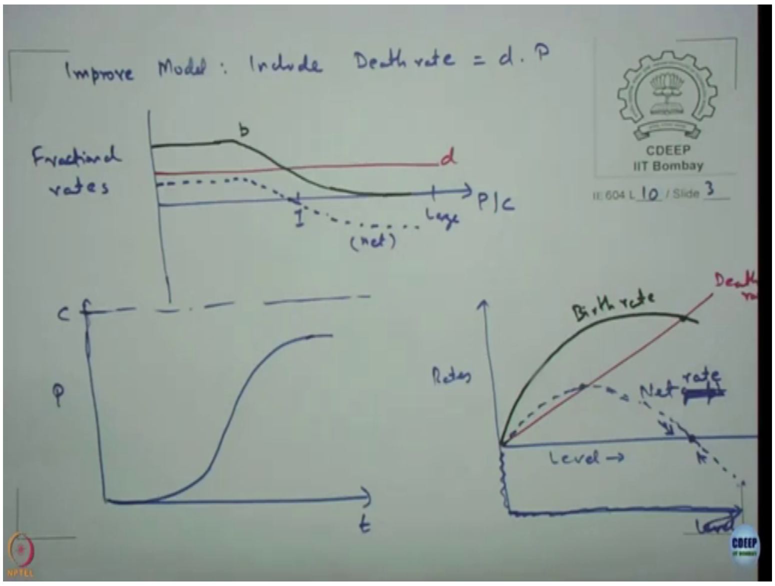

Instead of drawing that fractional rates then we have a (Refer Time: 02:39) there and (Refer Time: 02:41) over time; then we have rates over level graphs. So, the population will continue to exhibit a safe growth but because there is a see when you studied system composition what happened when there is a constant outflow what happened?

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We stopped below the below or above the goal right. When there is constant exogenous factor then we are not able to reach the goal. We either saturate lower or higher depending on direction whether it is exogenous inflow or outflow. Similarly, when we have a proportional outflow; outflow is proportional to the total stock, but the proportionality is constant. Hence we will continue to exhibit a shift, but we will be topping below the goal we will not be able to achieve the same carrying capacity.

So, here probably the carrying capacity is here, much higher then where we are we might be able to stop we will see why that is in a minute. So, let me just for convenience (Refer Time: 04:07) this as a level there is a level cut. So, as the level is large what happens to the death rate? As the level increases death rate keeps increasing right its a constant proportion right say this is your death rate.

But birth rate was exhibiting a hump shaped behaviour right. So, let us model birth rate birth rate, was exhibiting a exponential I mean not hump shaped graph previously. So, we will continue to have the same graph, but as I draw the hump shape it will intersect the death rate at some point correct.

So that means, at that point it has to be 0; because when system attains the equilibrium birth date has to be equal to the death rate. Earlier when the acquired equilibrium when the death birth rate net birth rate became 0, but here it will attain the equilibrium and birth rate is equal to the death rate right that happens at this point.

So, we can actually plot that is birth rate minus death rate as a kind of a net rate graph. That this point now becomes the stable equilibrium point, this is the unstable equilibrium point that is point 0 and your point of inflection is now not net that graph it is called net rate sorry.

And this point now is the peak of the net rate where it achieves that represents the point at which there will be the point of inflection. That need not be same as the point at its birth rate peaks net rate where it can peak at a different point then peak of birth rate ok. This is expected behaviour we can continue to visualize it here fractional rate this is P by c this is large this is I.

So, here deaths is constant d that nothing changes in a similar fashion if we are going to draw the curve. Probably you are having a looking at a birth rate which is constant. And then we want that the birth rate to kind of reduce and then come to this point. So, if you are assumed that as the b then the net rate is minus this probably net rate is somewhere here and at this point it starts to reduce and intersect somewhere this becomes your net rate net that is not (Refer Time: 07:39) net fraction. Now, let us incorporate this kind of system in a simulation model and see what how does that look like.

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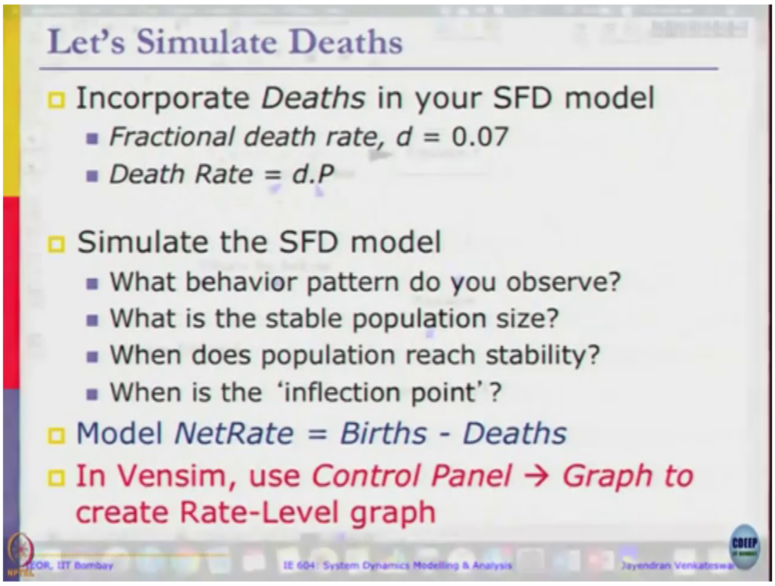

So, let us simulate the death incorporate deaths in your SFD model fractional death rate d is 0.07 death rate is d into P. Once we do that we will stimulate the SFD model and look at what patterns of behaviour we observe what is the stable population size? When does population reach stability? And when is the inflection point? We will look at all that. And the model from you can also include a variable called net rate and then we looked at in Vensim we will also update this rate level graph.

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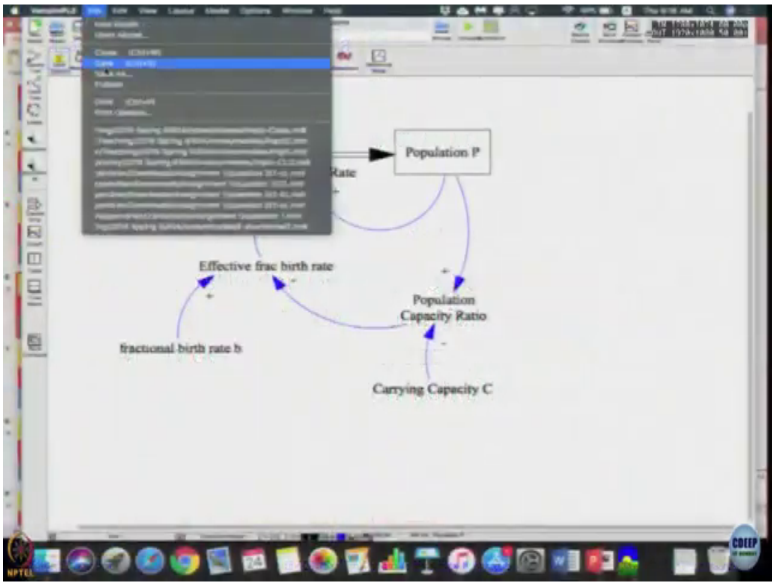

So, I am going to go back go to Vensim now and is going to file save as it is dangerous for that.

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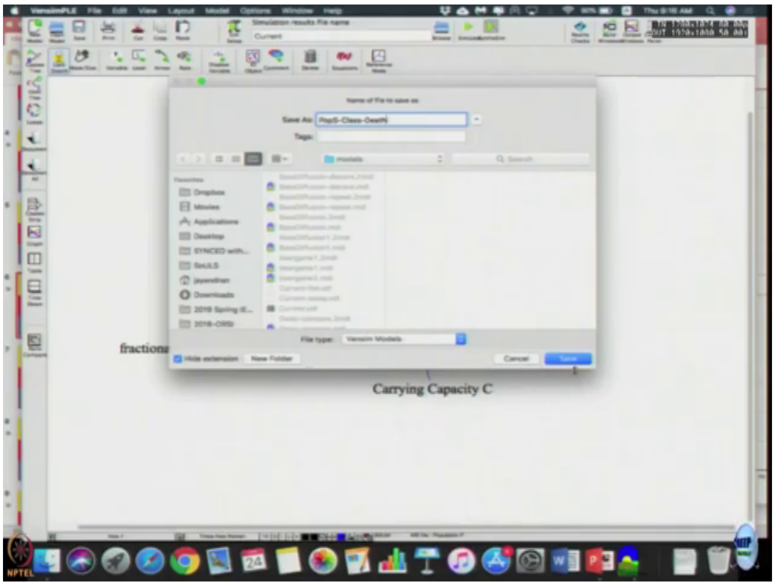

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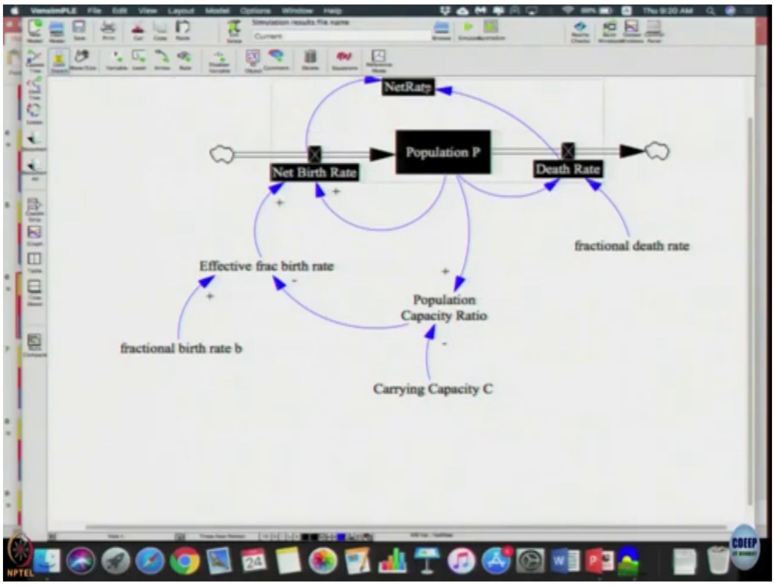

I need to include the deaths, arrayed a flow death rate. And have another variable called as fractional death rate arrow connecting population to death rate fractional death rate to death rate.

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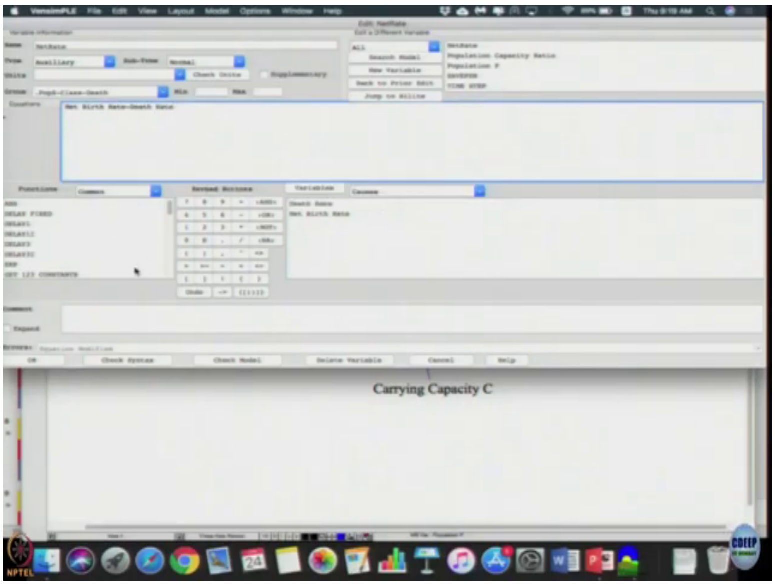

Click equation fractional death rate you wanted 0.07 one over month is a. The fractional death rate is 0.07 death is (Refer Time: 09:33) per month is nothing but population times fractional death rate. The death rate is product of these two fractional death rate is 0.07 a constant and population P. So, once you click it you will find that death rate is also included here. So, leave it and initial value we had 2000 just change it to 2 change inside a population back to 2 let us click ok.

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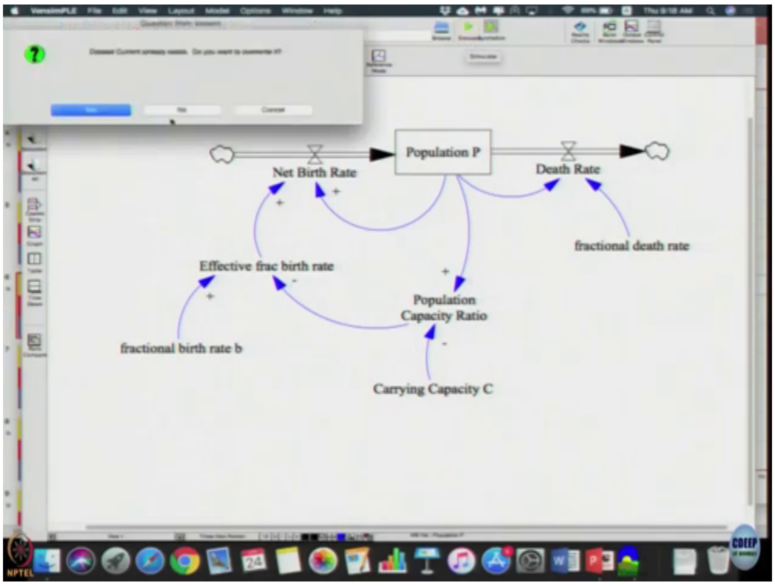

Let us click the play button click population click the causes strip.

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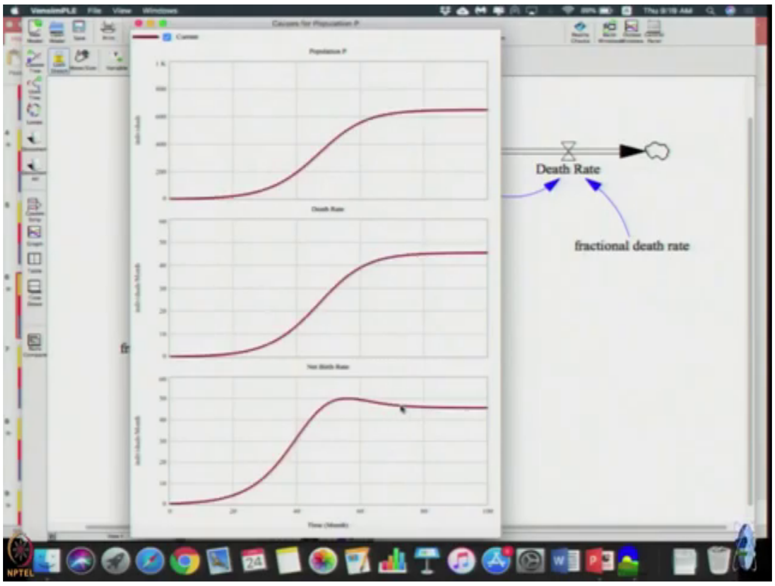

And you get interesting graph the population of stock continues to exhibit a s shaped pattern you are seeing here. So, the s shaped growth is always with reference to the stock value ok. Death rate just to in proportion to the population; so, it also has to exhibit the same shape this is a constant proportion constant times death rate.

But if you look at net birth rate in the previous time when you observed it increased to a peak and then fell down and get 0. But here now net birth rate went up and as it is coming down it is intersected with the death rate and then both became equal at around 40 46 and then both are now constant value.

To see the actual net rate falling to 0; we need to actually model the net rate right. So, to model the net rate we can just introduce a new variable called as net rate connect your birth rate and death rate to this net rate, click equation, click net rate, click birth minus deaths birth minus death. So, its the death rate so I can simulate.

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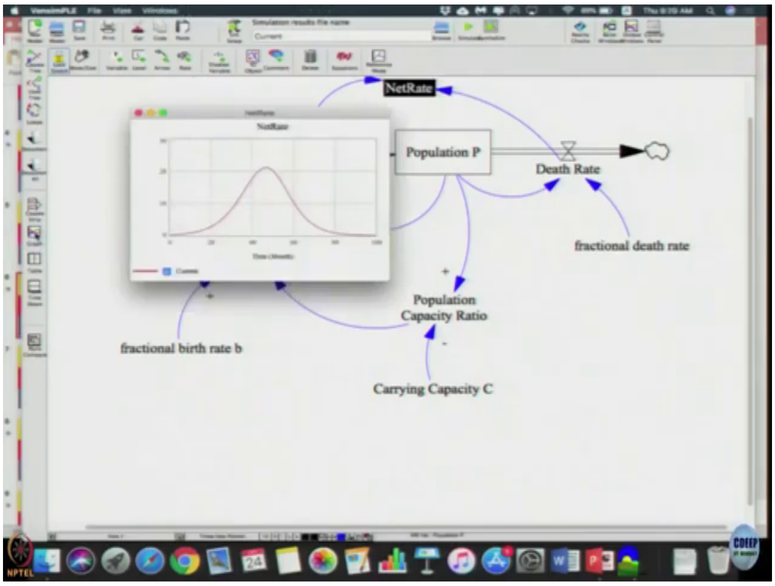

Now, if you plot net rate, you will find that the net rate will fall to 0; that you see here and the value of net rate.

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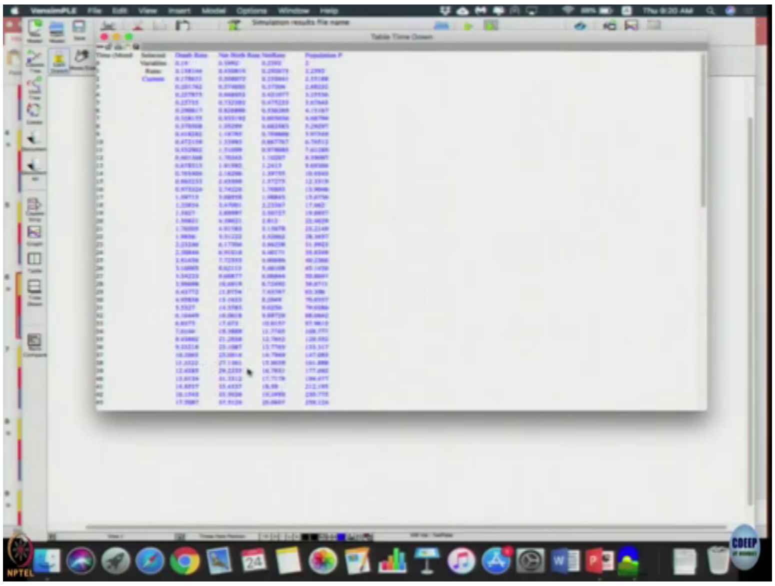

So, to visualize all the values select all four click the tables you got a table of values its rolling. The net rate here peaks at about around times of peak where is net rate here around the same value of 49.99 its around time 56 quite close to when this population value appears near 500. So, the death rate and net birth rate are still different but as that.

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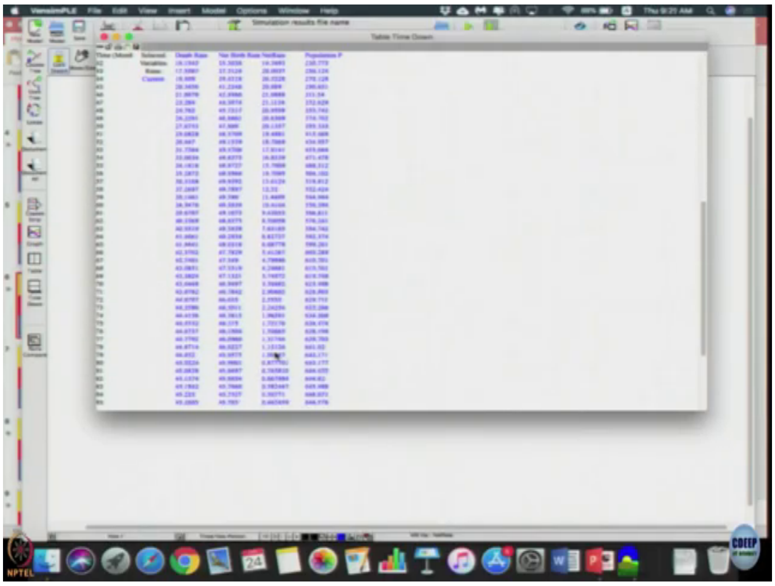

But this is the net rate net rate peaks.

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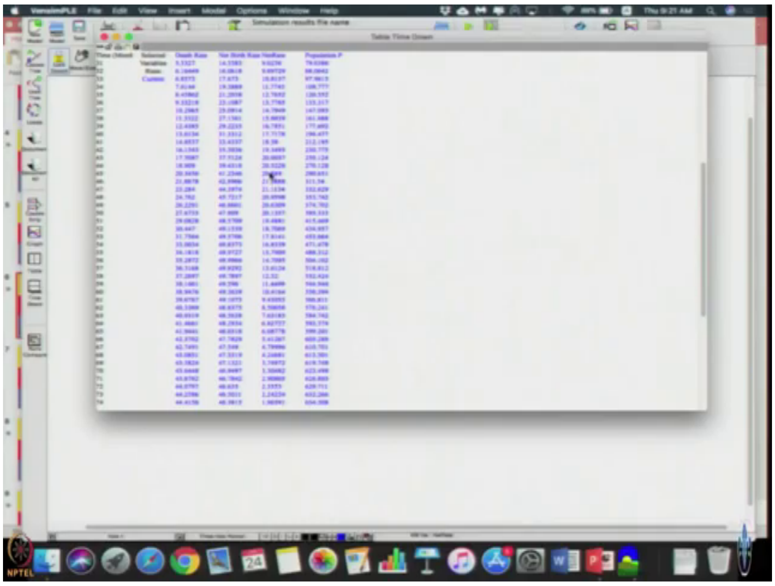

The net rate peaks at actually 20 at time 47 do you observe? The net rate is at third column for me is that 21.1134 value very small to read, but you can feed from in your computer its around 47. Just observe that the net birth rate actually peaks much later at time 52 or 53 rather 56 is when birth rate peaks, but net rate had peaked much earlier at around time 47 itself, but it can be different. And then death rate and birth rate both will converge to a in this case around 45.5.

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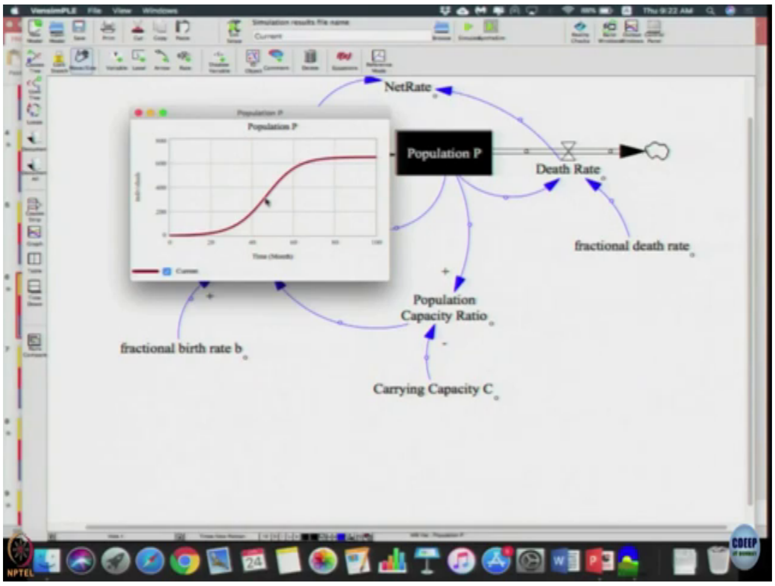

If the population peaks at around 650 or something when it peaks is about around time 80 the inflection point is somewhere around 44, 45 it coincides with the peak 47. It coincides with peak of the net rate not the birth rate or the death rate at the peak of the net rate this is the inflection point.

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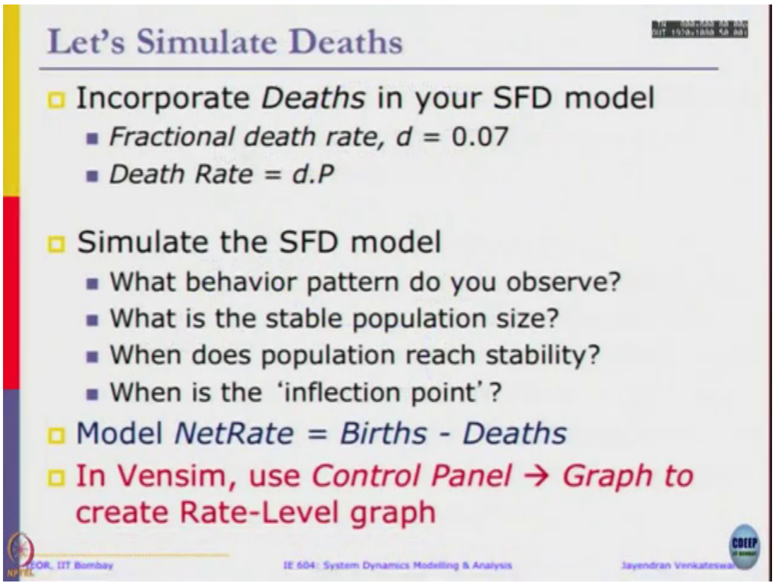

You can try this to include that net rates two rates are also in the graph. And see whether we are getting the graph very similar to what we have plotted straight line and a curve intersecting and everything can be visualized that is the I find on this. Suppose the other limiting factor such as starvation affects death rate non-linearly birth rate be affected.

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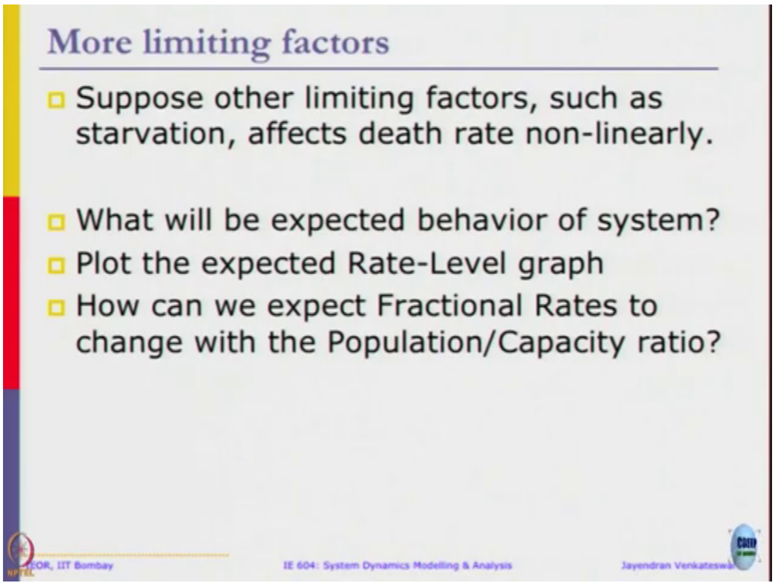

Let us assume it also affects death rate what will be expected behavioural system? Plot the expected rate level graph? How can we expect fractional rate to change with population capacity ratio so for that? So, suppose death rate is also changing non-linearly what will happen that is the question.

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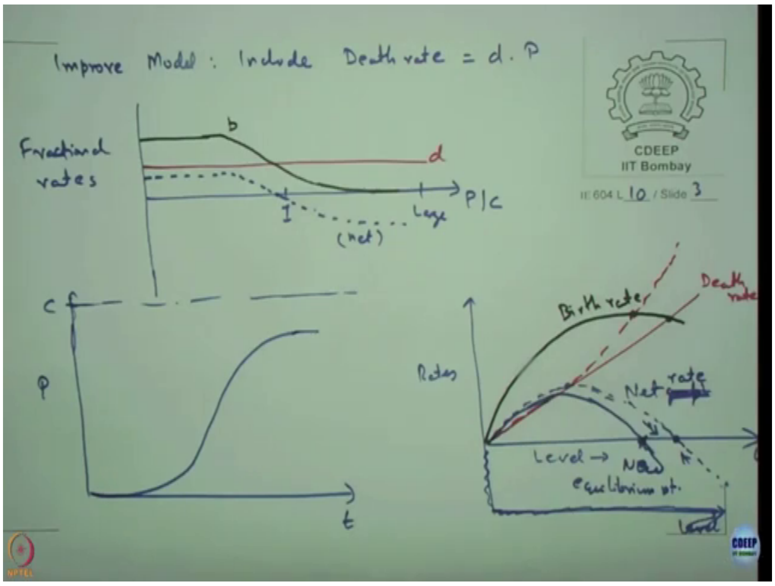

Let us understand it here. So, if death rate also changes non-linearly, just let us look at this graph; you need to change this non-linearly this shape is going to remain the same perhaps it might the point of saturation maybe lower right. So, what does it mean by changing death rate non-linearly; that means, death rate is going to increase. As I come closer to as the level becomes larger and larger as level goes closer to carrying capacity my death rate is going to increase right. Suppose the death rate increases like this I mean death rate is changing non-linearly now ok.

So, now birth rate we already the same graph now you can see the point of intersection is here and my new net rate graph comes like this. So, this becomes a new net rate graph. So, what you are observing is as death also changes non-linearly my equilibrium point shifts left to it or equilibrium point becomes lower or the level at which I expect to saturate becomes lower.

So, if I did not have the death rate at all as a constraint the first case the birth rate increase then it fell down and gets leveled that is the maximum. I can reach if I have death rate linearly increasing then my birth rate graph is going to intersect at much early point; that means, equilibrium point is shifts lower. And if death is also increasing non-linearly then the equilibrium point will be even lower this will be the new equilibrium point it will be somewhere here.

So, you can expect the similar behaviour and equilibrium point is lower you can also model this. We will continue it in the next class.