Introduction to System Dynamics Modeling
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Dynamic of Simple Structures S-Shaped Growth
Lecture - 10.1
Dynamic of Simple Structures: S-Shaped Growth limited by Capacity

So, today we will be looking at expanding on the simple structures we saw, we have been seen both positive and negative feedback systems. To start with let us consider this scenario as shown in the diagram. So, we have population which is effected only by the birth rate or net birth rate that has sequency is a positive feedback system.

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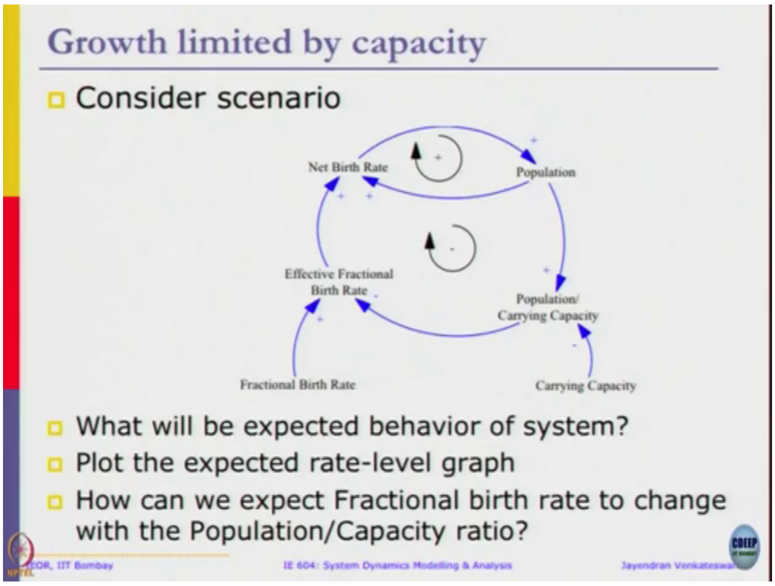

So, we are going to introduce a new variable called as a carrying capacity which is going to define how much the population can grow. The thing can grow forever. There has to be some limiting constraints some limiting resource which finally, flows downs the growth. So, in this simple example let us assume it is called as carrying capacity, ok.

Now, as you can see carrying capacity is external to the system. Now, in the first order system that we saw the simple positive feedback system we have seen that in population affects net birth rate, net birth rate effects population and that is net birth rate is defined by something called as fractional birth rate. Now, by introducing this carrying capacity what you are saying is we will as a population grows towards the carrying capacity, it is going to affect the effective fractional birth rate which in turn will affect the actual birth rate.

So, we have positive feedback system here followed with the, and a negative feedback system, right here, so as the population comes closer to the carrying capacity we would like the effective birth rate to fall down. And population is far for less than the carrying capacity, then there is no real restriction in the birth rate can be very high, as it as population approaches carrying capacity aspect the birth rate to fall or when population is exceeds the carrying capacity we can even expect the birth rate to be negative.

So, this is a very simple, quick scenario. Before we start simulating these things let us try to see what will the expected behavior of system be, we will try to plot the expected rate level graph. Level is another name for stock. And how we can aspect the fractional birth rate to change with population divided by the capacity ratio. So, let us look at those 3 things right now.

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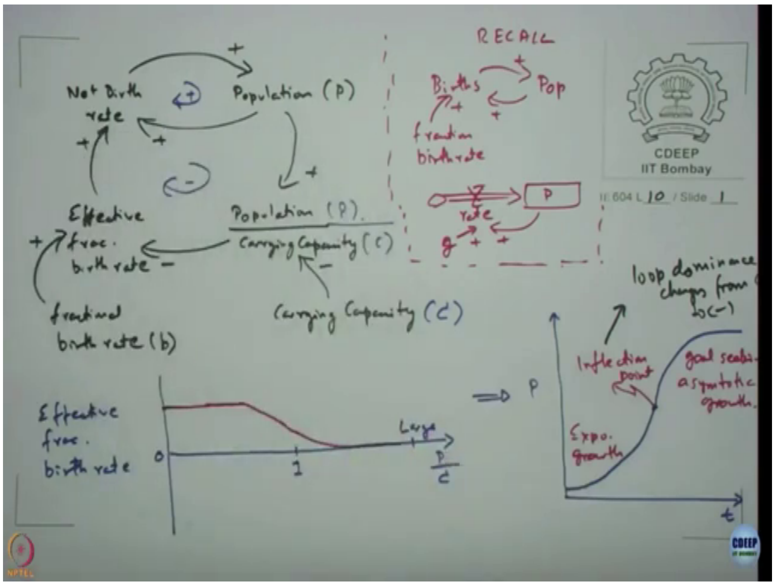

For convenience I have reproduce this same cause a loop diagram here. So, here we have a positive feedback system then here is the negative feedback system. Let us let us take this scenario, the questions from let us say bottom up kind of approach. Let us mark on the y axis, so effective fractional birth rate and on x axis here let us define it as P divided by C. So, let us denote carrying capacity as C, so your population is nothing, but the ratio is nothing, but P by C, ok. So, on x axis we have the ratio P by C, on y axis we want to plot what is going to be the effective birth rate.

To recall here; so, we can play certain context, we have population, we have births, we have. And this is a simple positive feedback system that we had seen and we had modeled it like a stock and flow like this and called a grade and here we put a growth factor g, right. This is what we had done when we studied positive feedback systems, where we defined as a stock and then we told it is going to increase the rate is equal to g into t and it became a positive feedback system.

So, this is a model that has been expanded on the left side where the fractional birth rate now g does not directly affect g, affect the rate which this right here. Here you seen the birth rate you are talking about put a small b to the g equivalent. We are saying that that is will change based on the carrying capacity, so that is the additional loop that has come into play right here.

So now, how would we like this fractional birth rate to change; so, for the convenience we can think you know as this P n, P by C ratio changes what will happen to this value of g what is value of d. What we expected to happen? So, for convenience let us just have some 3 values here. Let us just put 1, and let us put some large number (Refer Time: 06:19) a C, ok. Now, if population is very low, initially carrying capacity is very large then there is freely no restriction on the birth rate, right, we can allow the birth rate to be as high as it can be and so let us assume that birth rate starts here somewhere.

And for some time we can even expect the birth rate will be kind of, is not at all effected by the carrying capacity, ok. Even, suppose we are in excess of 10,000 suppose this is the carrying capacity and your current population is just 10, we can expect the birth rate to be constant, but again the current population is say 100, see 10,000 is quite far off per population is 500 or whatever number. So, up to some extent we can expect that the effective birth rate it can remain constant, nothing really effects it to change it is character, right. So, this can (Refer Time: 07:31) up to some point.

Another way to think about it is how much we are going to consume. For example, whatever we can take let us say the food you eat, suppose there is let us say you are hungry and there is say a 10 pizzas available, right. So, as per the normal rate, so whatever you can your hunger is satisfied when there are when you eat say one full pizza, your hunger is satisfied. So, there 10 pizzas till you eat only hunger is satisfied, right. But even if there is 5 pizza still you eat what hunger is still satisfied. So, when there is excess capacity your consumption continues to remain unaffected, right up to a point.

So, as the population comes closer to the carrying capacity or then I will start to ration the amount of intake; so, we can expect that this slope you know can slowly reduce over time. And as we approach 1, we continue to reduce and as P n C starts to be very very large at some point the growth get halted, right.

The population is way high as I told there is, to give an example I as you told as we started with there is a 100 pizza and 10 guys each of one it is fine then there is 50 pizzas 10 guys no problem or 50 guys 100 pizzas still no problem, then suddenly it is 100 guys 100 pizzas still may not be any problem. Imagine there are 10,000 guys and 100 pizzas, probably you can just get a small piece of it.

1 million guys and there is 10, 100 pizzas probably nobody will eat, you all be busy fighting, you may not get chance to eat, so that means, your consumption effectively has dropped to 0 as P by C flows large, right. Here pizza is your carrying capacity and population is the number of people who want to have a slice of the pizza or slice of the resource. Here we are denoting simply as a carrying capacity. So, this is a effective birth rate that we can expect.

Given this fractional rate value for g, if g is constant what will be the behavior? Even g is constant we can expect a exponential growth, right. Now, here in this graph it shows if this g is falling down, then what is the behavior you can expect? As g goes down you can expect the asymptotic convergence to the goal or a goal seeking behavior. So, this now if we model the population P over time then we can expect kind of a behavior like this, where initially there is a exponential growth and here it became a let us say a goal seeking or asymptotic growth, and this point we call it as the inflection point.

The point where the loop dominance changes; at inflection point, what happens there at inflection point? Loop dominance changes from positive loop to negative goal sequence. So, up to the inflection point this loop is active, after the inflection point I mean this loop is dominant rather, after inflection point this loop can be expected to be dominant given the ratio.

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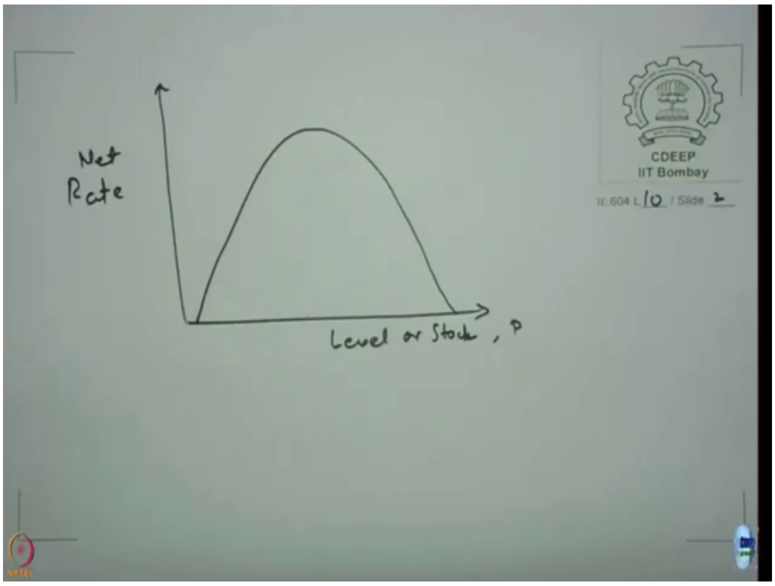

It is the eventually saturate then this kind of system. So, let us expand into one more type of graph because the rate level graph how will this graph be. How can we expect this graph to be? So, initially remember I need to get this growth behavior. So, if you recall in a pure positive feedback system when there is then population or the stock value those exponential growth then rate level graph, your rate level graph had a line like this.

And we saw whatever the positive slope it will show this exponential growth behavior and if it is goal seeking then we told that the line has to be like this, right. So, if I am going to get a to get like this then I need to get a triangular or a hump shaped graph like this, that you get something like this, this is your rate or the net rate that yeah just drawn. Here, this this net rate is what is plotted here.

So, just to confirm, so look at this graph we had shown 3 different graphs or 3 different x and y axis. The first one we told was a effective birth rate or how we can expect this to change with respect to this ratio, this hypothesized on that. And, based on that it will, here it is constant, so we can expect exponential growth where we did value of population over time graph and has this continues to fall and reaches 0 then it has to the addition to the stock will incrementally reduce. So, this is successfully reduced until it reaches the maximum value.

The third graph we plotted over net rate versus level. As I told level or stock in our case it is population P and this is net birth rate that will be kind of hump shape. Now, let us try to come up with the very simple model which can simulate this behavior for us; to do that I am going to make an assumption. See, this curve here as you can see is non-linear, right. Now, trying to put an equation for it whereas, P by C changes in value I need to come up with the curve like this.

This kind of little more difficult, and I need to come up with conditions or I need to draw it has a graph and then say if the value of P by C is able to 0 to 0.5 take this value, if it is between this take this value etcetera that is one way or if you know the equation for this curve then I write how this birth rate can change as a function of P by C.