Introduction to System Dynamics Modeling
Prof. Jayendran Venkateswaran
Department of Industrial Engineering and Operations Research
Indian Institute of Technology, Bombay

Dynamic of Negative Feedback Structure
Lecture – 9.2
Zero Value Goal system, Positive and Negative Loop Systems

But not all systems we need to explicitly do the goals.

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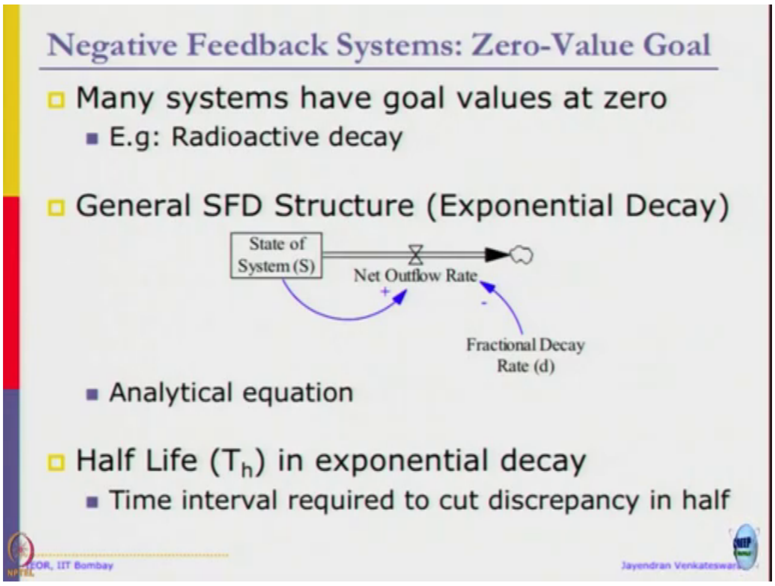

There are some systems where the goal is zero value, many systems have zero value goals; example is here let us say you can assume radioactive decay. There is also exponential system to exponential decay. The general structure is that, we do not really explicitly specify the goal; because the goal is anyway is zero. So, you do not need to explicitly define it, so the system becomes much more compact. So, here state of system is net outflow rate is simply S times the fractional decay rate d. So, here change in stock or net yeah outflow rate is just stock times t. So, we do not need to subtract with the zero.

So, let us see what happens and typically in these systems we are interested in half-life; for all in these goal the example also radioactive decay we are interested in half-life, it is nothing but the time interval required at the discrepancy in half. Let us quickly check that out.

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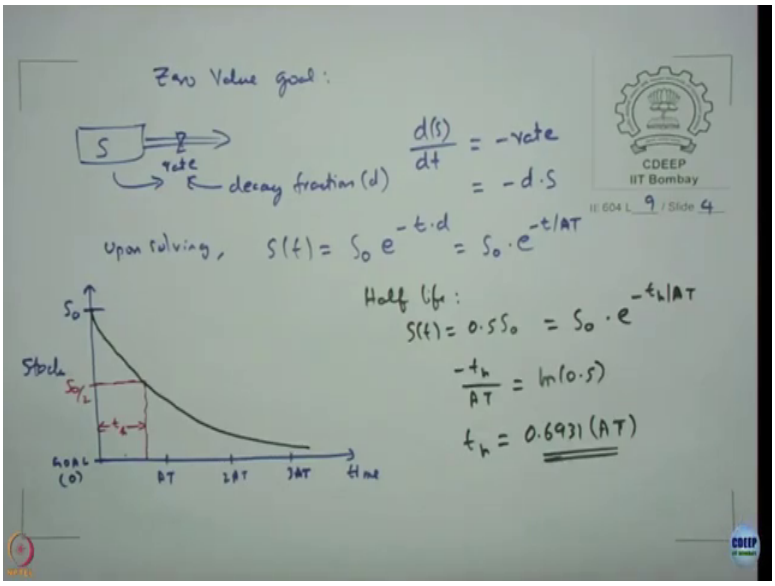

Zero value goal; what we want to do is, we have stock S and they have rate and a fractional adjustment of decay rate d. So, let us decay fraction d. So, the equation for this is equal to minus rate correct which is minus d into s; because I am just reducing it. You can solve it similar to the way we solved for exponential growth or the previous scenarios by moving S to the other side and integrating.

So, upon solving we can get a nice expression for a s at time t; which is nothing, but the initial value e power minus t into d or S naught e power minus t by same adjustment time. See A t again stands for adjustment time, which is just 1 hour the fraction d this is an A.

This curve has, so we need to get a expression e power minus x, so that we can get the curve similar to this, right. So, for all these systems this is your S naught and this is your goal right, goal is 0. So, this is your S naught and this is your goal; and let us assume we have a adjustment time, twice adjustment time 3 times adjustment time and so on. So, this is your time axis A T, 1 A T, 3 A T etcetera; that we have to scale that length curve like this.

So, this is a behavior expected because e power minus s function and we are plotting that. So, one thing we are interested in this is, we want to know when is the half-life; so we want to know at half point where I will reach this. So, let us denote this as to half life t subscript h; the time at which the stock value becomes 50 percent. We can quite straight forward to compute it half life; half life is when S of t is 50 percent of your initial value. So, from the above equation we get this is equal to e power minus t h by A T.

So, once you solve it, we will get it as minus t h by adjustment time is 0.5, so; that means, your adjustment time is nothing, but 0.6931 into adjustment time where as in the half life is equal to about 0.6 times your adjustment time. This will be very similar to your exponential doubling time. In fact, it will be exactly the same.

So, the exponential doubling time and your half life here, we get the same multiplier of 0.69 times your adjustment time. So, there is a required to cut a discrepancy half. So, here you are going to double or half you just multiply with 0.69 or 0.7, you can just imagine it like a 70 percent rule. So, after 70 percent of time has passed, the stock value is cuts into half or stock value doubles in case of exponential growth systems.

Still now we have looked at positive feedback systems, then we looked at negative feedback system; then we looked at negative feedback system with constant exogenous rate either inflow or an outflow and then we had a zero value goal a special case, but again negative feedback system.

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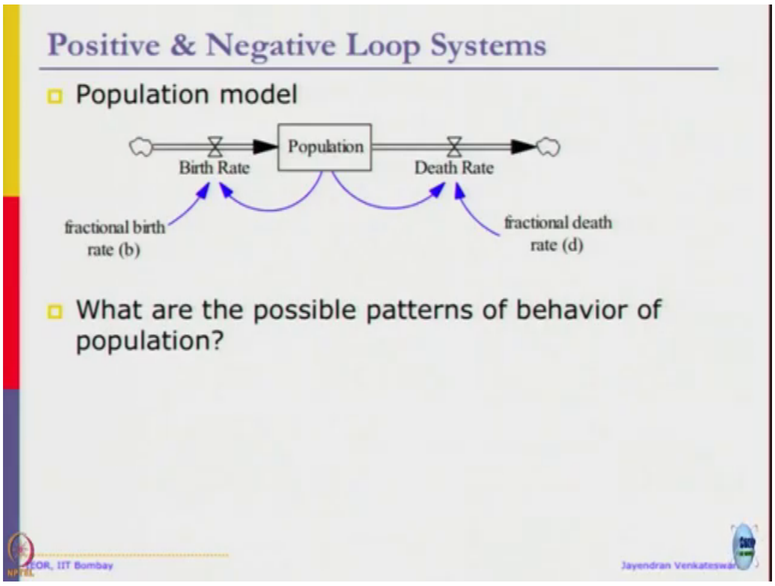

Now, we will be interested to take it to the next step of natural question is what happens when both positive and negative feedback occurs in the same system. So, if you only look at one part of it, there is if just focus your attention on population birth rate that we expected to be exponential growth system.

On the other side if you focus your attention on only population and death rate, we expect it to be a negative feedback system or a zero value goal system where system is the population is eventually going to die off. If there is no additional birth rate in system at some point like the system is going to die off. So, if the death rate here we are going to have an exponential decay because zero value goal or with birth rate here we are going to have a positive feedback system which result in exponential growth, correct.

So, now our birth rate is gone by fractional birth rate and death rate is gone by fractional death rate d. So, what are the possible patterns of behavior? Is there anything else, then both these values both these state values how do we expect the system to behave? Individually we know, that both are there together, both b and d are there then how well the system behave.

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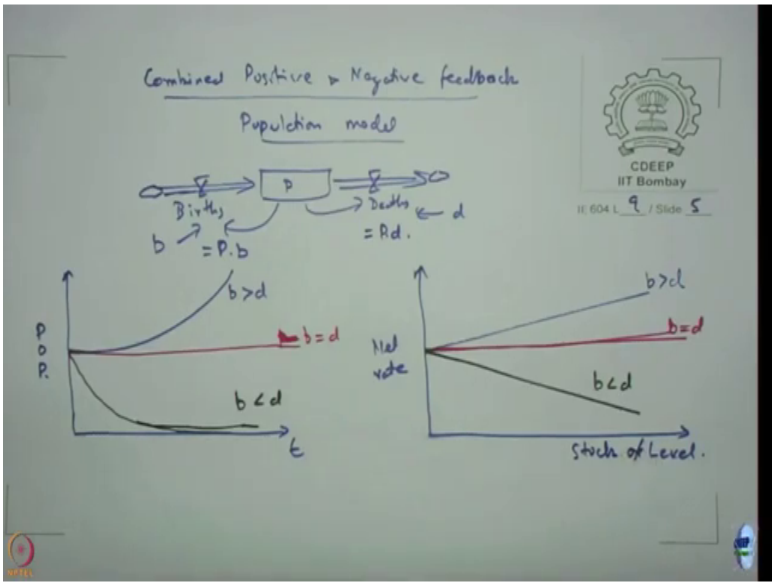

Combined positive one negative feedbacks; you are looking at population model, deaths with the fraction d, births with the fraction b, ok; this is the system we are looking at. Here only three possible ways, the system cannot oscillate, this system cannot have produced in S shaped growth; because this nothing causing any other dynamics within the system. So, we just have two constants b and d. So, we are going to get only three modes of behavior for your population.

If in steady state when births is exactly equals deaths, when population will remain constant, when births will be equal to deaths; see here deaths the equation for deaths is nothing, but deaths is equal to P into d, births is equal to P into b, the equation of births is just P into b, equation of deaths is P into d. So in this system, if P is equal to b I am sorry d b is equal to d; then system is going to not oscillate nothing system will always be in steady state as this.

Now, if you have more births then deaths what will happen? It will be in an exponentially increasing system. So, this when b is greater than d, b is less than d; less births than deaths then what will happen? It will decay to the zero value goal right, it would not be a mirror image of this, instead you will get a behavior like this, this is one b is less than d, right.

So, this is your goal seeking behavior, zero value goal; this is the shape we drew. That is what we will get when deaths are more than births, the population is going to keep declining at a rate zero; if births are greater than deaths is going to have exponential growth, births are equal to deaths then you are going to get a constant rate.

So, it comes out because of this, the net rate as suppose to the level if you plot it; the net rate b equal to d case is constant sorry this is constant line, b equal to d it is constant. When b is greater than d, it is again a I am going to get a b is greater than d or b is less than d.

So, if the slope is positive I am going to get exponential growth; slope is negative, then I have I have to get exponential kind of decay; and this constant I mean it is equal, then I am going to get a linear growth. These are only three possible modes of behavior within the system, right.

And again the system continuously to be linear and at any point we just figure out which is dominating and appropriately you can compute the net rate. And if suppose I know the value of b and value of d, I take their difference and depending on that I can figure out whether the system is going to increase or decrease and I can compute what is going to be for example, as doubling time or the half time for half life time yeah, level is a stock P, this is a stock or level.

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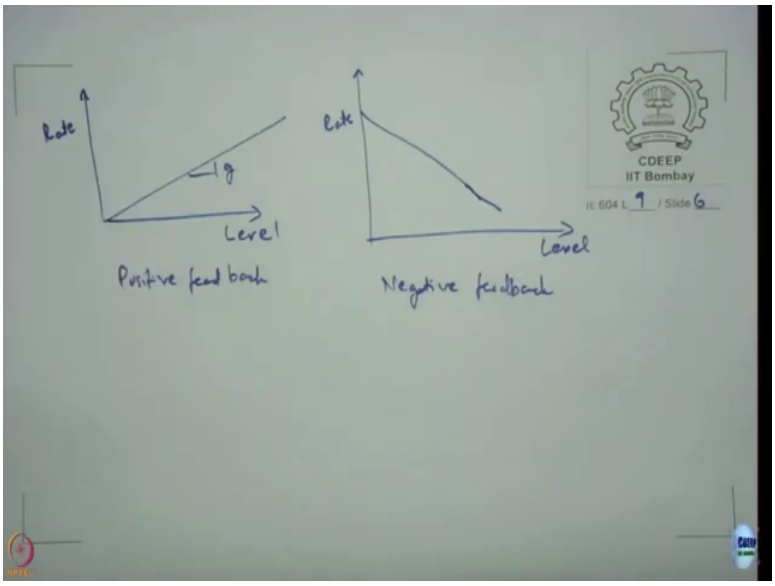

Then we saw this like let me just know this; for positive feedback systems we had made a rate level chart, I had shown you the slide. So, there is a rate level chart was like this and you told that this slope here you find your growth rate g. For negative feedback systems, we also had a rate level chart; where we drew it something like this, this direction it went on negative side also if you remember. So, you just drawing the same graph expect that we already taking the difference and drawn it here; the same graph that we drew from the same point of as g, yeah and that is why the system continuous to be linear, yeah.