Introduction to System Dynamics Modeling
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Dynamics of Negative Feedback Structures
Lecture – 9.1
Dynamics of Negative Feedback System: Extension of Model

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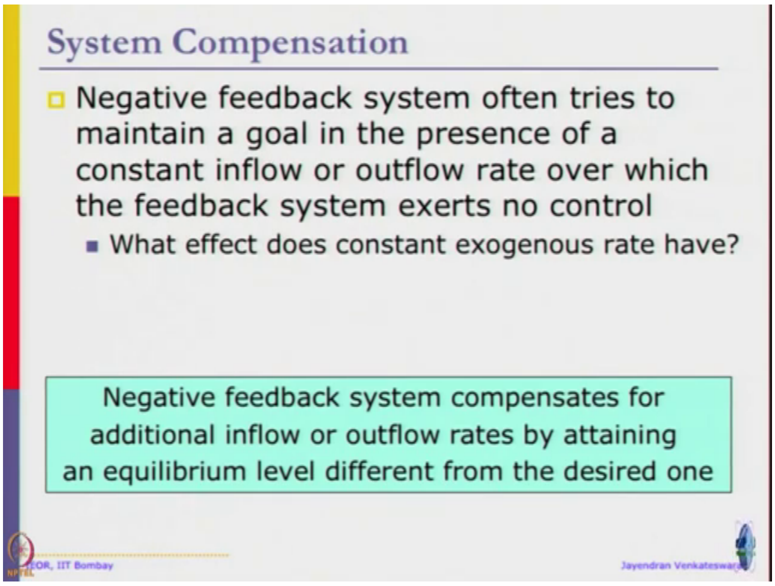

Let us introduce this notion of System Compensation. So, negative feedback system often tries to maintain a goal in presence of some other exogenous factor either constant in flow or a constant outflow rate or is a feedback system exists no control. So, in the presence of such an external or exogenous variable how does the system behave that is what we try to understand today, what affect will this constant exogenous rate have?

So, negative feedback systems will compensates for the additional inflow or outflow attaining an equilibrium which will be different from the desired one. So, though we; though we will have a goal, the system because of the constant exogenous variable will actually come to an equilibrium which will be different from the goal as you see in the see in the example. So, before we go into the an analytical version of it, let us take our laptops and expand the labor model that we did yesterday in the face of a constant exogenous outflow.

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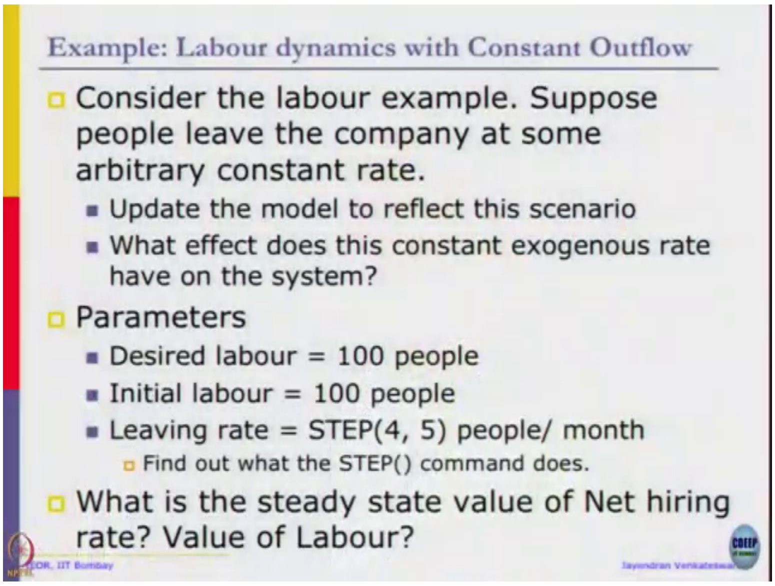

So, let us look at what we want to do in this model consider same labor example. Suppose; so, in the labor example there was a desired labor that a company desired and we had the current level and the gap was suggested based on some adjustment time, I think we the base cases assumes 6 months of adjustment time. So, we had build that.

So, in that consider labor example suppose people leaves a company at some arbitrary constant rate or which you have no control people may leave the company at any point they chose. Now, we want to update the model reflect this scenario and what effect does this constant exogenous rate have on this system.

We can start with parameter desired labor 100, initial labor 100 and the leaving rate as step we will come to that what it means. So, let us go to Vensim and try to model this. So, first one is this one, suppose people leave the company at some arbitrary constant rate, we need to update the model to reflect this scenario; so, people are leaving that has to be modeled.

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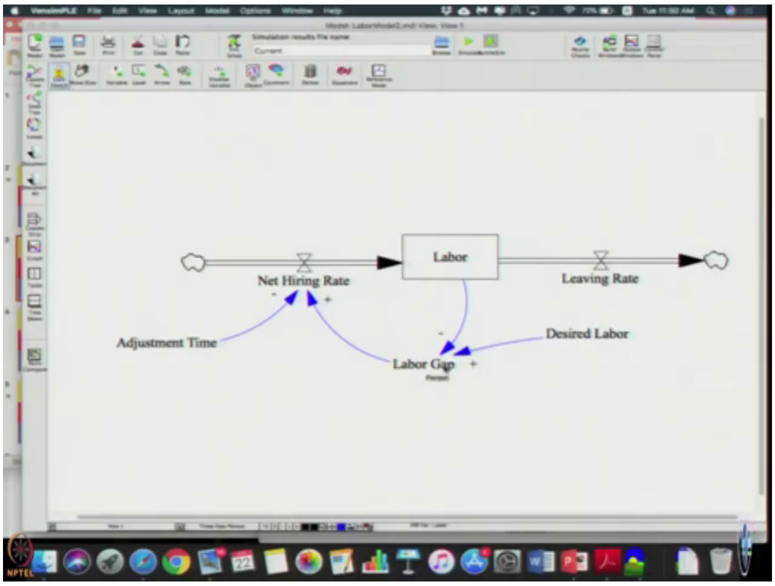

So, let us go to Vensim. So, this is the modeled we used yesterday, right.

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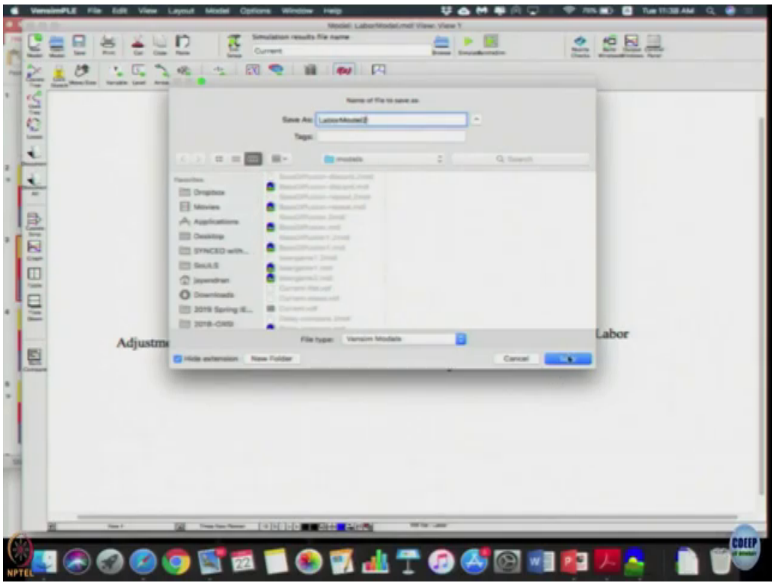

I am just going to save as 2, so, that new version. So, let us prefer a now change the model. So, let us just quickly click equations button f of x, click that.

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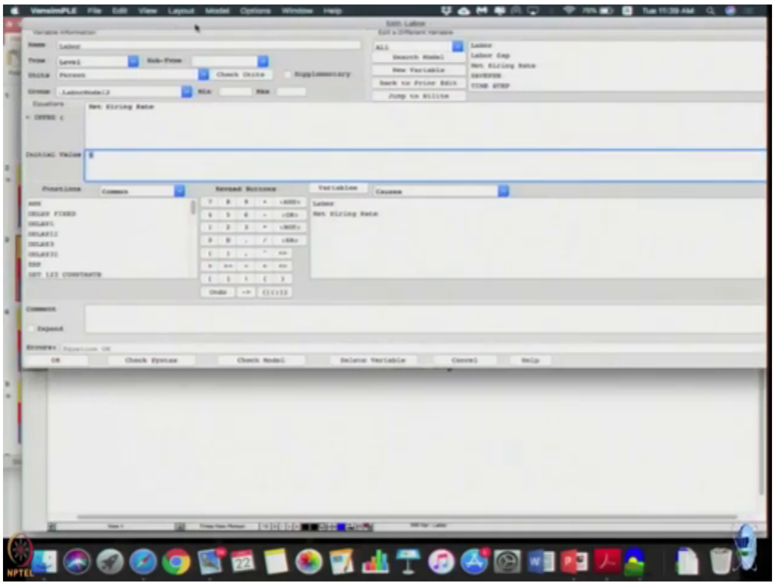

And just check if adjustment time is 6 months, net hiring rate is labor gap divided by adjustment time units is percent per month, labor gap is decide labor minus labor unit is percent, desired labor is 100 desired labor 100 percent. Now, let us take labor that is a stock, I just opened the stock labor initial value it will say 0. So, let us just change that to 100.

So, now, the goal has well as actual stock is 100. So, we do not expect any new people joining the company. Now, click rate we need to model people leaving the company. So, let us click rate click inside the labor; so, I just include a variable called leaving rate. So, there is since it is exogenous, we do not need to show anything coming into this it is already given as exogenous. Let us go to the equations as soon as I click equations both these things will become kind of on the black background.

So, let us click labor first and you will see that integration equation as changed, net hiring rate minus leaving rate is automatically get that. So, which is what we want, we want the stock to change hiring rate minus leaving rate. So, all you do is ok, we are done nothing to change there; come to leaving rate, let us just put a value 0 and what should be the units for this, persons per month it is a rate which always be per time unit, we use the same time unit as a inputs, ok. So, initial value of labor is 100, desired labor is also 100. So, then we can expect net hiring rate should be 0 and leaving rate also as 0. So, we actually simulate the model.

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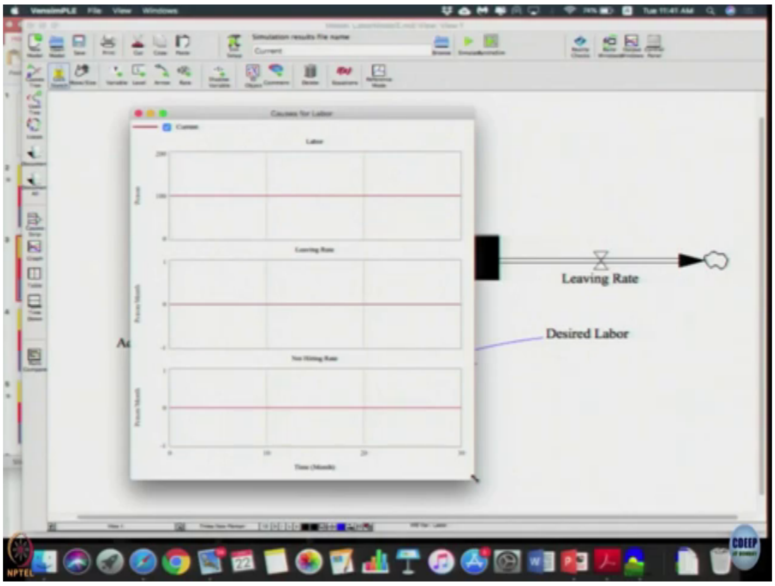

The labor does not change is always at 100, leaving rate is 0, net hiring rate is 0, this is what we mean by dynamic equilibrium. So, model starting at dynamic equilibrium nothing is changed now. So, we just included this one extra variable within the model.

Now, let us go to our model say that here it says leaving rate is step of 4 comma 5 people per month per or person per month I guess. Find out what step command us let us do that, before I change the equation; see there will be many such commands it is a pretty decent help.

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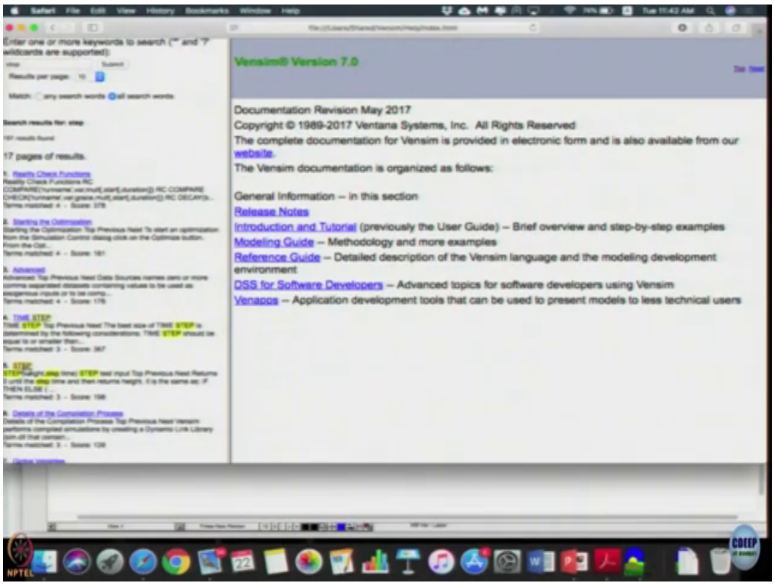

So, you just click help, click Vensim manual it gets installed by default you do not need internet for it, it is in local drive. You should open up a screen like this search, step, submit if a various things in my unit comes as option 5. In my unit comes as option 5, you click step; the step command is a you can restep function.

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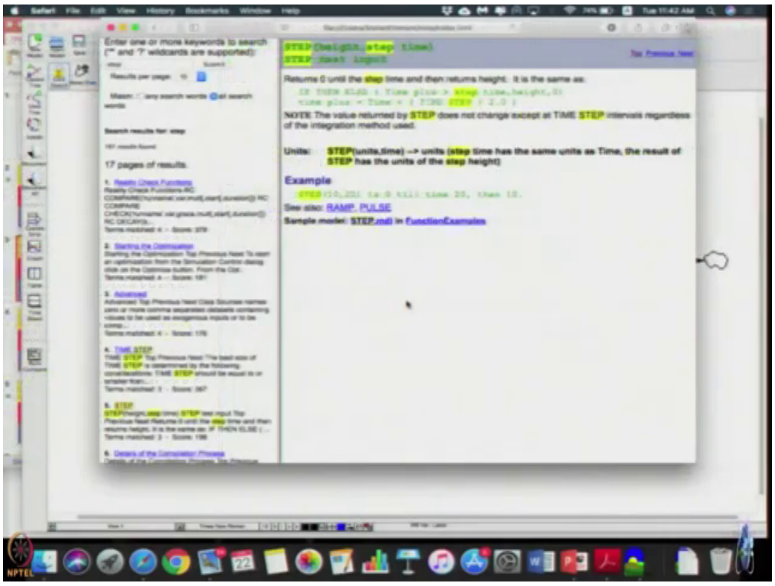

So the, it takes a value of height at the step time until it is 0. So, if the current time is greater than the step time, then it takes the value of height else it continues to remain 0. So, this is what the step function does.

So, there are various function that is available like you know another cause is most of you have either in the 4th year of under grade or already master students, you must have studied another course on how to give the various inputs in a systems.

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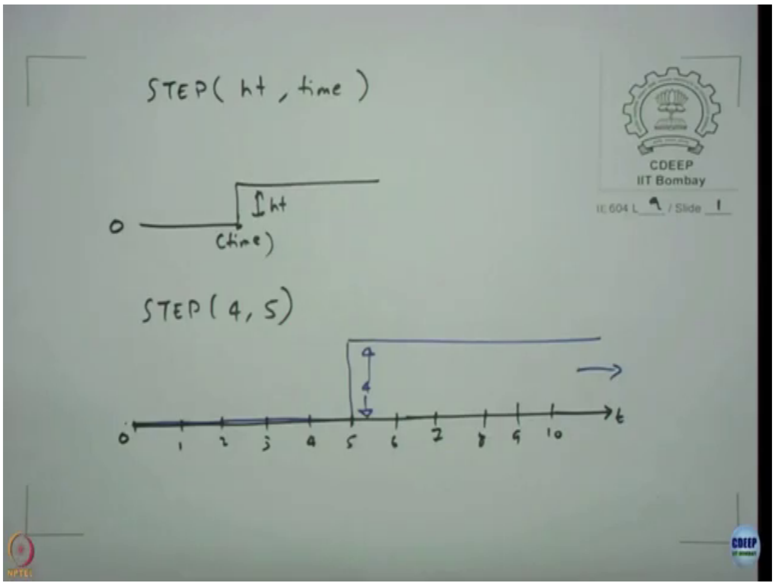

So, we can various types you can have pulse input or a ramp input or a step functions or steps. So, there are specific functions for each; so, this is just simple step function. So, we want step function of height comma time. So, pretty much what we want to model it as this input; so, this is your height and this is the time. So, value is 0 until then before the time and after that it takes the value of height.

So, in our example suppose step is say 4 comma 5; that means, a time we can have the access 1, 2, 3, 4, 5, 6. So, step of 4 comma 5 means at time 5, I am going to get a start getting step input of height 4; so, this is 4 units ok, it is going to go on. It can take a positive value or a negative value of the height, you can put a minus 4 also it will come pulse in the turn out direction. So, you can use multiple step functions to construct whatever a functions you want ah. There are other functions like step called RAM, pulse, pulse strain etcetera which we can use to model variety of inputs in the system as required.

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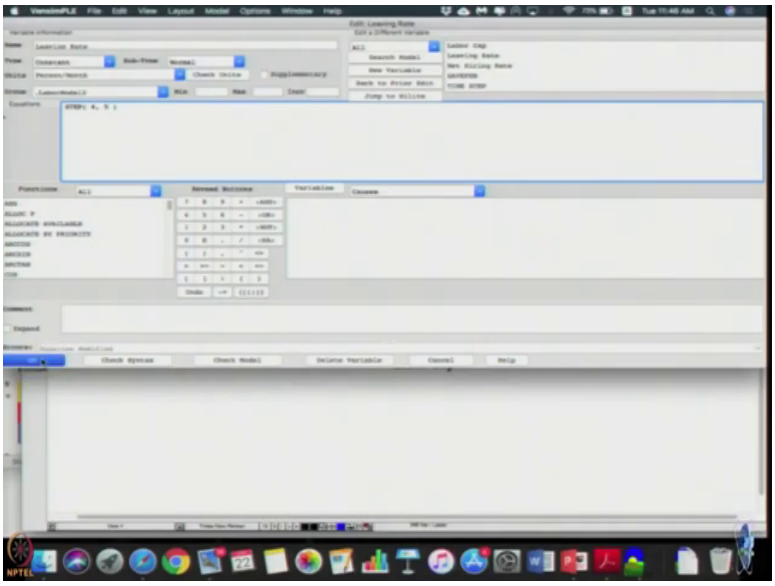

Now, let us go to the leaving rate and in the equation we need to type step. So, list of all the functions that is available is a that Vensim supports is listed here, you can see a drop down called as function and common. So, these are the common functions that is listed here, but if you want all the functions you just select all, then every single function that Vensim supports is put here. So, you can look for step.

So, once you click step the basic key also comes there. So, the step of height comma start time, yes time; so, we can pretty much guess it. So, height is what we want is 4, on start time we want is 5, that is it; I just model a step function into the system. Now, let us save it and let us run the model and see what happens. Remember desired labor is 100, current labor is 100 as soon as people leave let us see what happens.

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Let us just click play, click run, let us then touch click labor, click your causes strip; so, that I can get all the graphs at 1 shot, got it ok. So, let us see, the leaving rate became a step function because the time resolution we can get you know time step is one. So, after that you get a increase in next time unit, it looks like a ram, but is actually step function to reduce the time unit, we can make a more clear step function.

So, this is a leaving rate. So, as soon as that happened the same goal seeking behavior is observed where; so, to compensates of people leaving there is 4 people are leaving. So, for system to reach again in equilibrium; system leaves equilibrium when inflows is equal to your out flow right, it will be steady state again is inflow is outflow.

So, system is going to keep hiring people until the net hiring rate is equal to 4 when it balances at the net; net leaving rate, right. So; that means, how many people I am going to hire and that will have a nice goal seeking approach that we see here. Now, let us see what happens to an actual labor, we can see that the labor does not remain at the desired level of 100, instead it now finds the new equilibrium at a point about 70, near 75 to 78, I guess is there it is attaining it is new equilibrium. So, it is right here which is different from the desired goal that we achieves it.

So, in presence of a constant exogenous variable for example, this outflow system will come to a equilibrium, but it is different than what their desired state will be. So, that is how system is going to behave.

Student: (Refer Time: 12:28).

But, a system will have equilibrium at that point that hiring will be equal to this, the system when it reaches steady state hiring will equal to your people who are leaving. So, the system will reach steady state only at that point. So, at that point system will becomes balanced.

Student: (Refer Time: 12:46).

Correct, but you are not considering any extra information in the system, right. Every time only you look at the current labor and based on that you are making a decision to hire, you do not you do not you do not account for this information in making it decision at all within the model, right. So, you looked at every time you look at the labor and say 4 people are sort. So, I am going to hire some 4 people and I keep doing that then I find that I am achieving an equilibrium which is lower than that, pretty I can show you analytically in a minute.

So, let us go back to the graph. Here the labor gap is at the very time when you are looking at desired labor and current level of labor and taking their difference only and based on adjustment time trying to do the hires, ok. And, system will already reach equilibrium when as soon as inflows equal to your outflows system reaches equilibrium. So, net hiring keeps increasing until it reaches 4 when it becomes equal to your net leaving rate.

So, I am not taking the leaving rate information in my labor gap. So, system will achieve a different equilibrium than your desired labor when your desired is 100 based on the system will actually achieve equilibrium at a much lower level, because of this constant exogenous flow; it is going back to the causes script. So, this path here until time 5 when everything is constant there is no change; so, that phase is called as dynamic equilibrium. After that say until time 28 or so, system is what we call as in transient phase and after that it reaches steady state, ok.

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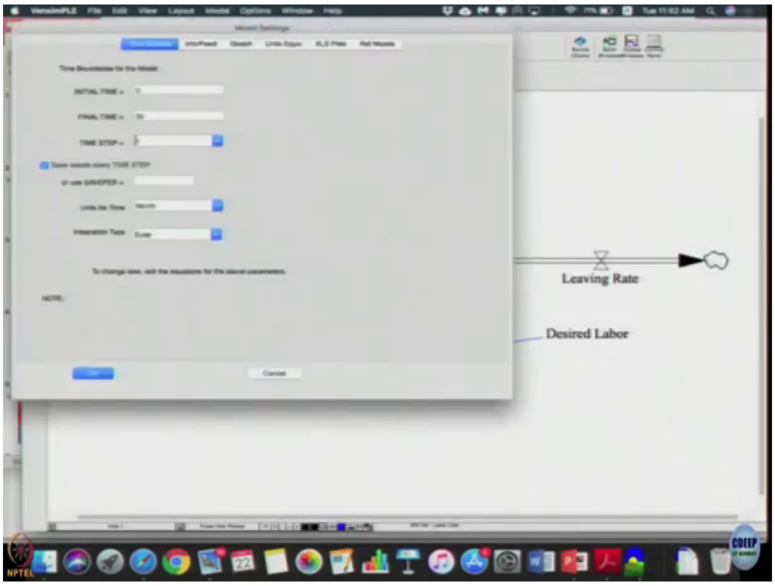

So, most of for analysis so, you just to expand on there, let me just increase this model setting and is going to model setting increase in the final time to 50, simulating it again. So, we can view it nicely yeah, ops I want to same yeah.

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So, about say from time 5 to time say approximately 25 or 30 system is in transient phase and after 30 system is in steady state. So, most of our analysis especially analytical computations we end up focusing only on the steady state systems when does it will be steady state or what is the value of the steady state and system, and then operate during the steady state domain, but using simulation we need to spend sufficient time and analyzing the transient phase also. So, how the transient shape is happening because that is what is causing dynamics in the system, because in reality there is hardly any system operating in steady state all the systems are in transient state externally. So, we need to focus on transient phase also.

So, we have dynamic equilibrium from 0 to 5 then we have transient phase and then we have steady state. At steady state we can observe that the leaving rate equal to net hiring rate or inflows equals to your outflow rate then system is steady because the stock will not change, right. Stock change will happen only these inflows and outflows are different if they are equal change in stock is 0, this stock level saturate is there.

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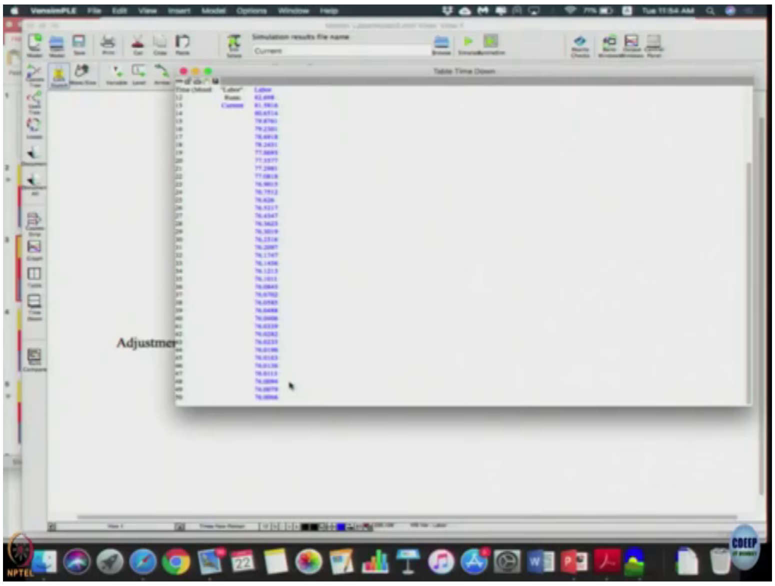

And the new equilibrium value we wanted to see that is I just opened the tables to see exact value, the new equilibrium value of seems to be about 76. So, instead of 100, it has saturated at 76 as a new equilibrium value or the labor it does have labor saturated at 76. So, let us see why that happens, ok.

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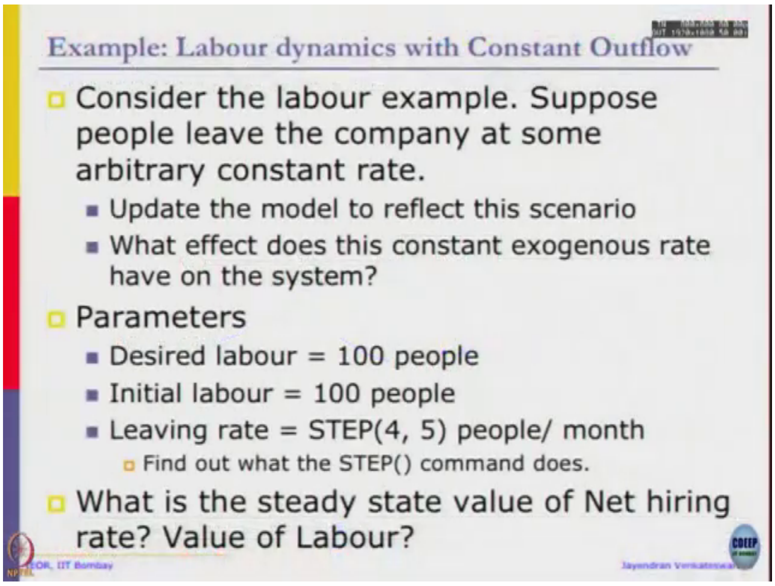

So, steady state value of net hiring rate became 4 persons per month and value of labor saturated at 76 percents. So, let us look at this behavior of steady state system or simple linear system analytically; you better understanding, but in this we going to flip the direction.

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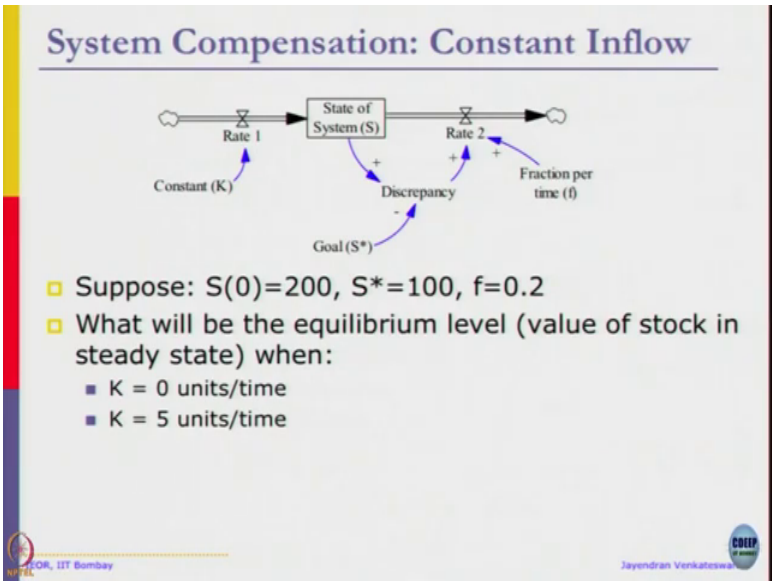

Suppose, there is a constant inflow and I have an negative feedback loop in the outflow. So, in case of a constant inflow system is correcting discrepancies from the goal in the state of system to as per fraction time as f it corrects it. So, for this system let us try to compute it then you can use the same analogy to do the math's for the example that we just saw, ok.

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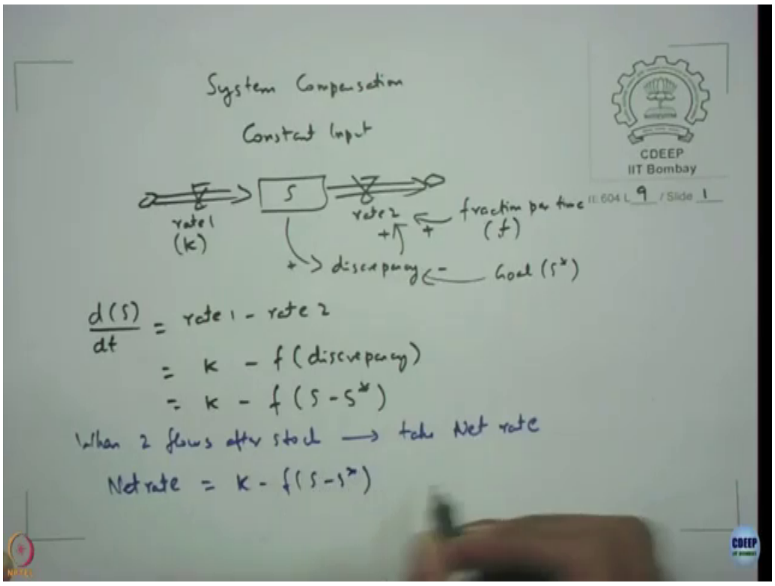

So, let us assume the same system where as 1 inflow and then outflow. So, outflow is a negative feedback system, inflow is a exogenous; so, where the constant K. So, just to write the equations we get d S by d t is equal to rate 1 minus rate 2, rate 1 is K, rate 2 we are going to adjust the discrepancies. So, let us call it f into discrepancy, f is a fraction per time; let say f, I am getting f from here. So, rate 2 is fraction per time into discrepancy, K minus f into S minus S star, ok. So, this is the system that I am simulating, ok. When 2 flows affect a stock when needed graphical integration, what did we do?

Student: (Refer Time: 19:57).

We took the net rate, right. So, when 2 flows affect a stock we just take the net rate and if stock does not need to change then net rate has to be, 0 right. So, at steady state the net rate is going to be 0. So, that we can write 2 flows affects stock it take net rate, your net rate net rate is same as this right, whatever you see here.

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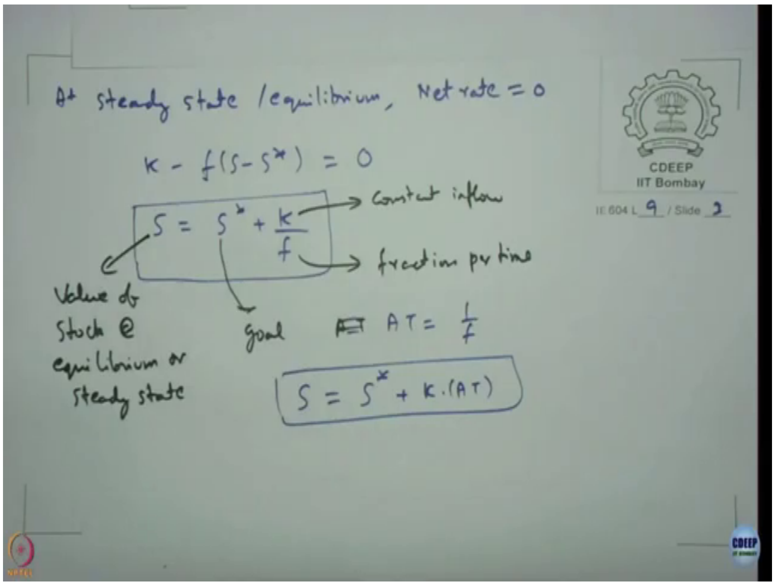

At steady state or equilibrium, net rate will be equal to 0 does imbalancing inflows is equal to your outflows; that means, K minus f into S minus S star is going to balance itself out, it is going to be equal to 0; that means, I want to write it in terms of S, what is the new value of stock at which I am going to balance this out. So, if I rewrite I can get S is equal to S star plus K by f, this is the new value in which system is going to achieve it is equilibrium.

So, S; so, this is the value of stock at equilibrium or steady state this is your goal, K is your constant inflow, f is the fraction per time in which I am adjusting the discrepancy. Then yesterday, we saw another expression adjustment time is equal to 1 by f. So, that case I can rewrite my expression as S star plus K into adjustment time, it is the same thing.

So, we can try it out for the example that we did, there you might you find that S is a S star minus of exogenous. So, the new level will be less than your desired goal by that in proportion to your inflow size. So, in this particular example. So, suppose the initial value is 200 and S star is 100 and f is 0.2; what will be equilibrium value that is value of stock in steady state when K is equal to 5 units per time, when K is 0 what will be equilibrium steady state value of stock will be 100.

Yes, it will reach S star when K is equal to 5 125 yeah, it will be saturated 125 itself that is at a value higher than the goal in case so, there is a constant input. So, input is coming with you are trying to get rid of it, but it gets keeps coming all the time. So, that accumulates over the time which results in not saturating it at 105, but stops at 125. In proportion to the rate at which I am able to adjust the discrepancy because average time to adjust the discrepancy is 1 over f which is 5 and 5 times K, it becomes 25. So, the new steady state value becomes 100 plus 25 so, it is 125.