Introduction to System Dynamics Modeling
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Dynamic of Negative Feedback
Lecture - 8.3
Negative Feedback Loop: Analytical equation

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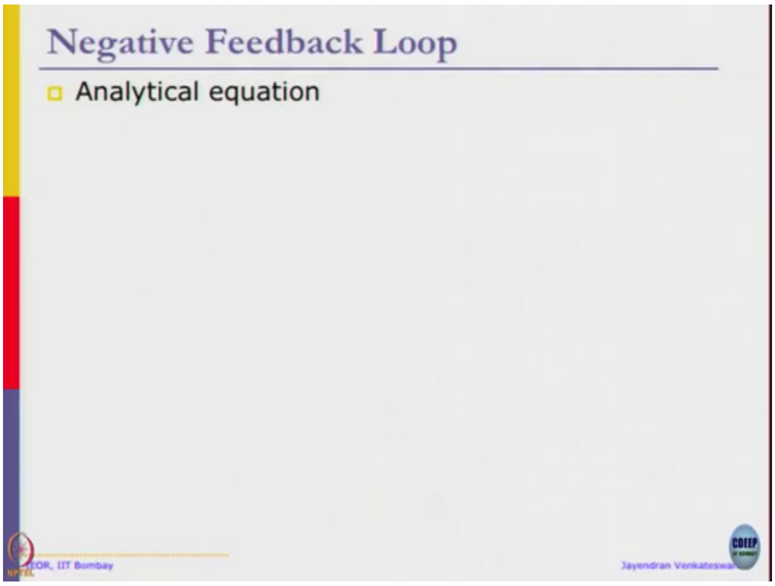

Let us look at for specifics on a vertical expressions of things.

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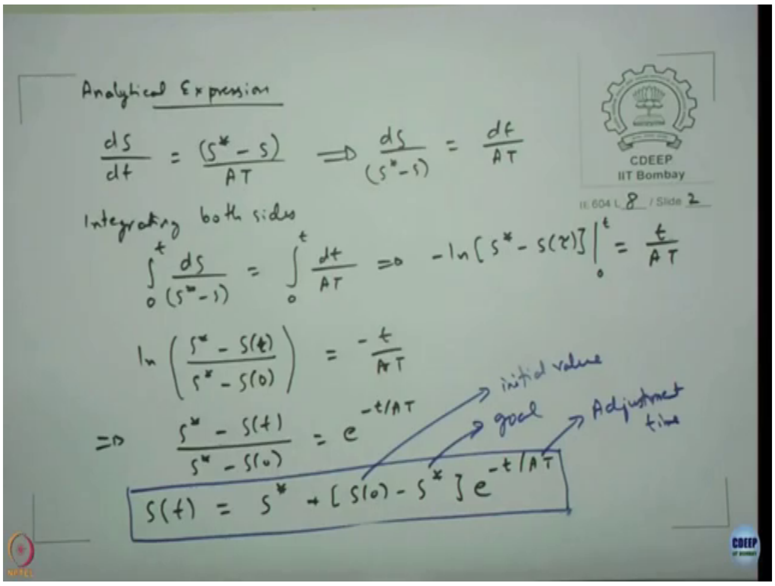

Analytical expression just like we did it for exponential growth, let us take a quick look at what we have here. The underlying equation is d S by dt is S star minus S divided by the adjustment time that is what we already have. Let us follow the same scheme that we did d S by S star minus S is equal to dt by AT.

Integrating both sides 0 to t d S by S star minus S 0 to t dt by AT is the single variable which is minus logarithm of S star minus S of anyways make it tau 0 to t is equal to that was t by AT. See here I have logarithm of S star minus S of time 0 divided by S star one second sorry t S star minus S of time 0. Probably S star comes of it surface where t S star minus S of time 0. Well minus t by AT which will be equivalent to S star minus S of time t divided by S star minus S of time 0 e power minus t by at which means my S f time t is equal to S star plus S naught minus S star e power minus t by AT.

Again these are linear systems. It is on non-linearity, hence we can nicely solve it like this stock cut time t is decide goal of this system S star plus initial start minus the goal e power minus t by AT. So, e power minus x should be the expression because then look at the curve that is happening. So, it is e power minus x. So, e power minus t by AT; so, given the desired given the goal as well as initial stock we can compute what happens at time t. For other values we need to know at we need to know S star and we need to know S naught.

So, S star is the goal S naught is the initial value at is your adjustment time. Of course, t is the total time limit right. So, one of the thing we saw was when adjustment time at unit passes, then what happens.

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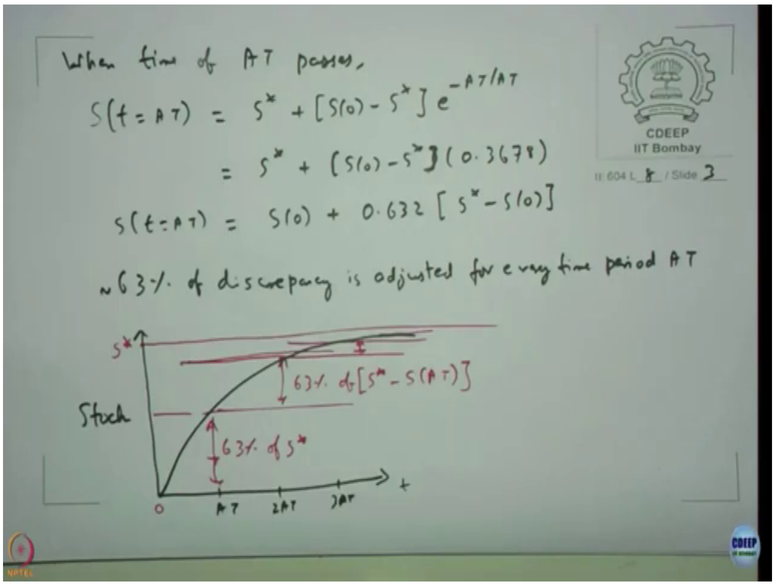

When time of At passes S t equal to at the S star plus S naught minus S star e power minus AT by at which is e power minus 1 which gives S star plus S naught minus S star into 0.3678 e power minus 1. So, S of time t equal to at it is S naught plus 0.632 times where S star minus S naught as time unit of at passes we adjust 63 percent of the discrepancy using the stimulations in time step was one the resolution was not great.

So, we took it about 62 to 65 percent, but if we reduce the time step we can see that the 63 percent of the discrepancy is about 63 percent of discrepancy is adjusted for every time period AT. Of course 63 percent of discrepancy is adjusted for every time period AT, right.

So, what we mean is if we have a graph like this. This is yours stock; this is your time. So, if you call it at 2 AT 3 AT and we get it like that stock exchanging from cross time 0 at 2 t 2 AT 3 AT times etcetera. So, up to this it will be 63 percent of S star. Here initial value of stock is 0, right. So, where ever it is reaching that is your S star it is reaching at infinite time of that.

Initial value is 0. So, 63 percent discrepancy is full filled when at time period passes. So, at this point at time 2 AT 63 percent of S star minus S at time AT is satisfied. So, every time you need 63 percent or remaining discrepancy gets satisfied, and so on until at 3 AT again it is 60 again some discrepancy get satisfied etcetera and reaches your goal asymptotically.

Yeah it is come very well , but what was the purpose the idea is 60 point discrepancy. I justify at every time period at to the remaining discrepancy.

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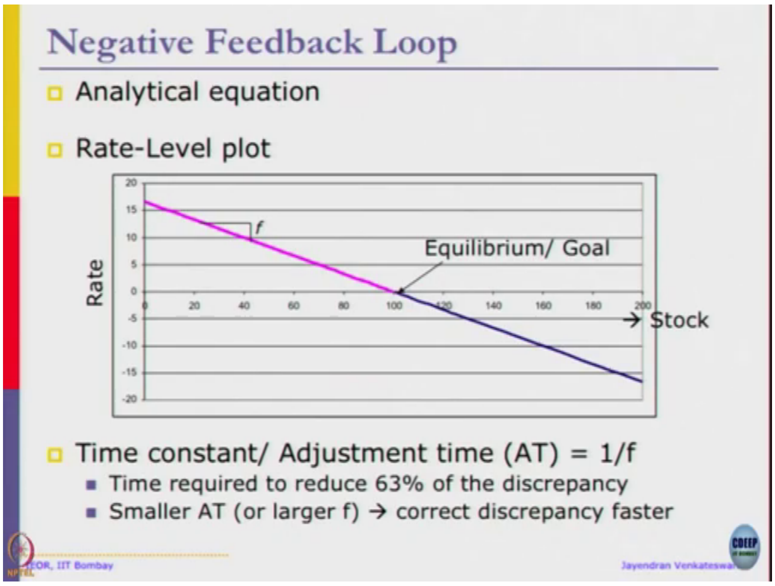

Now, we will ask as we saw a rate of level plot. We had the level on the x axis and rate on the y axis and we saw that it was at any positive slope g, then we will have the exponential growth. Since this is a goal seeking system, so we have a negative slope right. So, when you ask any negative slope system has the example that shows here, here the equilibrium point is at 100 because 100 was the goal of the system. The goal defines equilibrium point.

So, any push on either direction will bring the system back to that same equilibrium point that is how the system is going to go. And any again depends on a slope, it will it will affect the timer takes to reach the equilibrium and that is f nothing, but 1 over AT. Yeah, time constant adjustment time is 1 over f time required to reduce 63 percent of the discrepancy smaller at correct discrepancy is faster larger f faster.

We will stop here. In next class, we will look at a System Compensation and there is other end how do you control when there is uncontrolled variables in a system also.