Introduction to System Dynamics Modeling
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Dynamics of Simple Structures Positive Feedback
Lecture – 7.2
Doubling Time in Positive Feedback system

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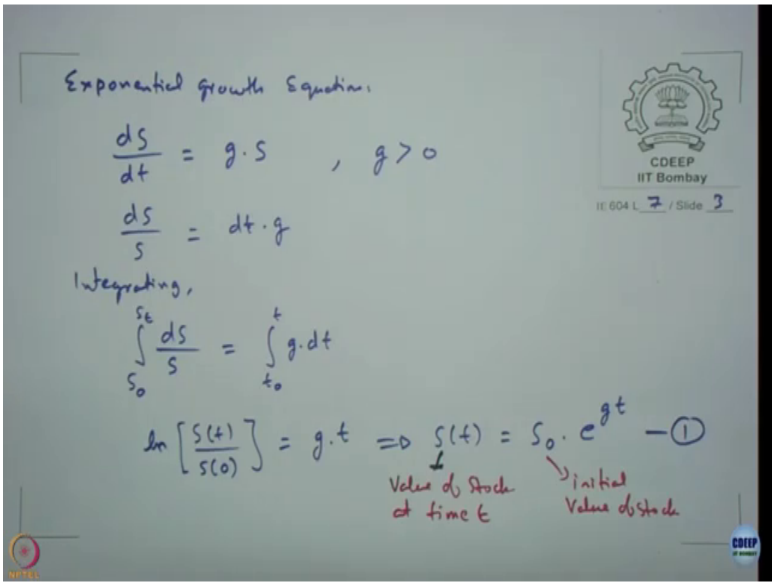

The Positive feedback systems are you can define, it as the exponential growth equation. Let us try to compute that. So, the initial system we started which is d S by d t is g into S that is what we start with. So, let us just d into g integrate both sides (Refer Time: 00:52) steps to the S naught time 0 into S of time t t naught to t g into d t which is integrate integral of 1 by S is logarithm of S. So, S of time 0, S of time 0 equal to g into g into t. It means that I can rise it to the power e and its all right and then I can get S of time t is S of time S naught into e power g t right.

Call then just raised e to the power exponential and then move the denominator to the right side. So, I get a stock at time t is nothing, but initial value of stock into e power g t and as you can see these are positive values g is greater than 0. So, g is greater than 0, then it is going to exhibit exponential growth just by defined by this term here and of course, S naught is a constant value I am just multiplying by the constant and given the initial value of stock. And if I know g and for whatever value of t of course, I can directly compute what is going to be a stock value at time t.

So, this is the value of stock at time t. This is the initial value of stock. Of course, we want to get any different point in time; we just take that as the initial value and then take the time difference. So, its you know let us take a one bad thing about putting these things, it means that there are many more equations coming that is why we want ok.

Now, let us see what can I (Refer Time: 03:23) that let me introduce this one you can remember this S of time t is S naught in to e power g t. It is not that we always use this fractional growth rate of g, we also defined something called as a time constant.

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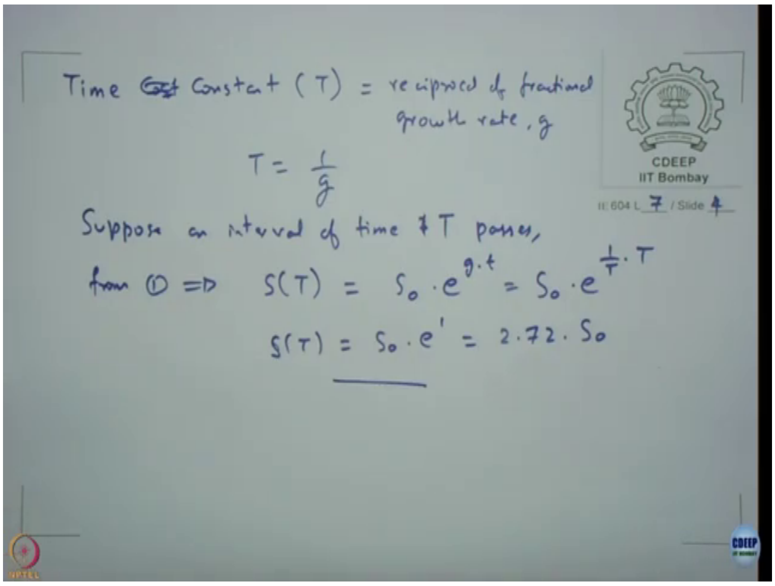

Let us define it as capital T, now which is nothing, but the reciprocal of the fractional growth rate fractional growth rate g whereas, g is just defined as one over g. Of course, since the time units of the units of g is one over time. So, capital Ts units is of course, time where it is useful is it is not that wherever we are looking at the interest rates sometime it could be the delay. So, instead of multiplying g by with the stock value, you can take stock divided by time value that is means the same thing.

So, the equations stock at time t is equal to S naught into e power g t instead of g, I can always use 1 by t 1 by capital T. So, suppose an interval of time T capital T passes from 1, you stop at capital t time it is proper time 0 into e power g t it is S naught into e power g itself is 1 by capital T capital T time units passes. So, a small t is also capital T so; that means, S of time T equals S naught into e power one which is about 2.72 into S naught.

So, after passage of one time constant, this stock value will be approximately 2.72 times to the initial value of stock right and if say two time unit 2 T u 2 T time passes if one t time passes, 2.72; if 2T time units pass, then stock value will be 2.72 into 2.72 times that will be the stock at time 2 T. So, this is defined as the time constant. Now, let us come to our one question that we ask what is the doubling time. Let us see whether we can compute that.

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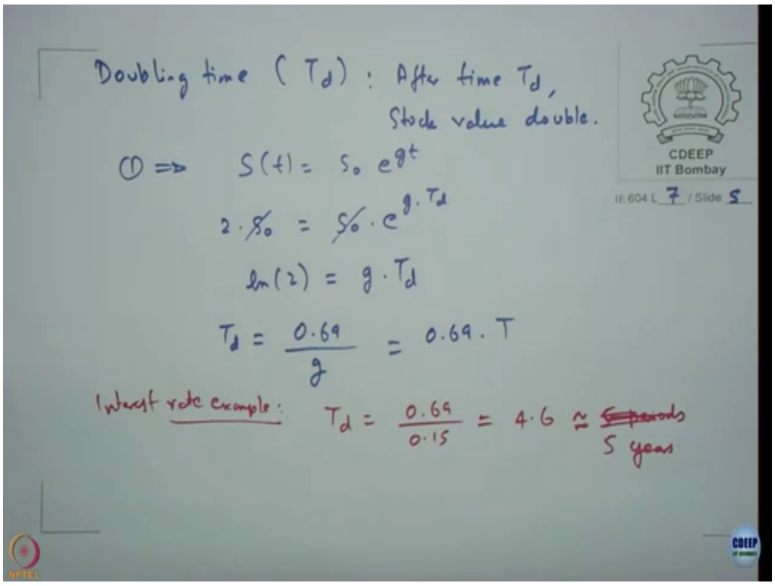

So, let us look at doubling time let us say T d doubling time it is the time after which after time T d stock value doubles, we need to compute this T d; let us see. So, one states that S of time t is S naught into e power g t. So, I know the stock value doubles. So, let us say 2 into S naught is equal to S naught into e power g times T d already have it.

I am cancelling S naught and taking a logarithm on both sides g into T d. It means that T d is equal to 0.69 divided by g or 0.69 into time constant capital T. So, this 0.69 will keep appearing because linear systems and we are looking at even goal seeking etcetera how the what computation or thumb rule purposes people sometime refer to that as 70 percent rule that after with every d T time units 70 percent are is nothing, but point 7 times your time constant or nothing, but roughly 3.69 divided by g.

So, far in our interest rate example, T d is 0.69 divided by 0.15 g is 0.15ah which is about 4.6. Hence our time resolution was time interval of integration was 1. So, only in the fifth period, we are able to observe that it has doubled the value. If we had used a smaller time stamp, then you would have observed more closer to 4.5, 4.6 ok.

So, this is so, every 4.6 years a 5 periods, I will take it as 5 years. So, every 4.6 years, the value of stock is going to keep doubling. So, and that can be computed just by giving the g values. So, to see now what is the fractional growth rate, then all we have to do is 0.7 by this is for approximate value of 0.69 by 0.154 0.6 in this case years is the doubling time yeah. So, there is nothing, but (Refer Time: 09:41) with exponential system rather than just calculating what is going to doubling time and identifying it. So, now, let us go back to a slides.

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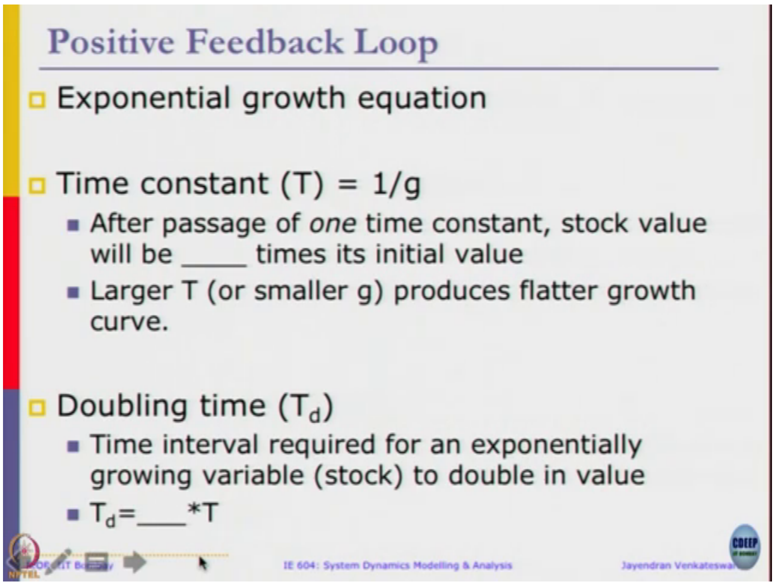

So, you have seen this, you have seen the time constant the passage of one time constants stock value will be 2.72 times the initial value larger T or smaller g produces flatter growth curve. So, now that we know all these equations, it is very easy to compute that is not the intent as we are looking at a more a modeling course, we need to understand what will happen when the based on the values of g. If your large value of g, then the growth has to be going to be cheaper.

If it is a smaller value of g growth is going to be more what is it flatter that it is be going to be much more time before you actually perceive the exponential growths. Doubling time; time interval required for us for an exponential growing variable to double in its value T d is a point six nine times your time constant.

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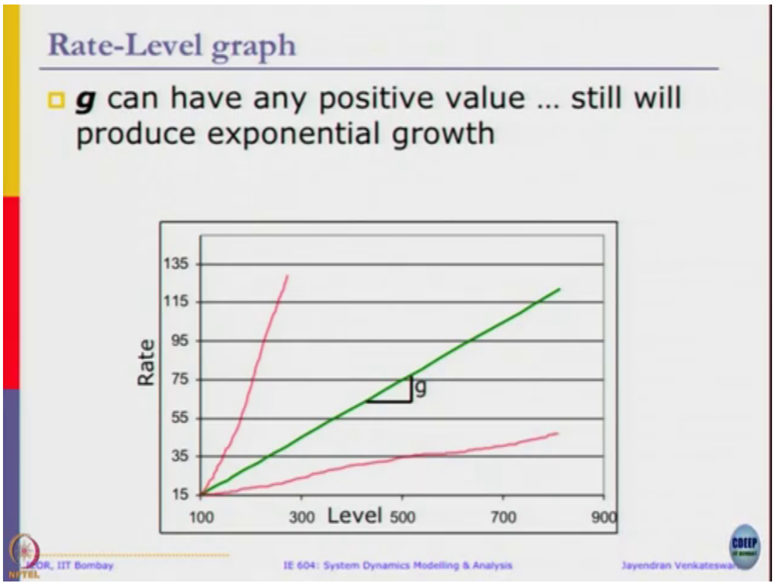

Now, if we start plotting with level and the rate that is we plot the rate for every value of the level that we just saw for the example; for same example, you will find linear line with the slope of g. It can be expected because a it is a linear equation right. So, for any positive value of g, we will still produce exponential growth.

So, even if this curve even if this line so, even if this line is like this, I am going to still produce the exponential growth or if the line is going to be like this. Of course, I am going to produce the exponential growth, we call this linear systems coming from this diagram right here where you can see that this line itself is linear that is why the entire system becomes the linear system right there. Any questions so far not at fully awake?

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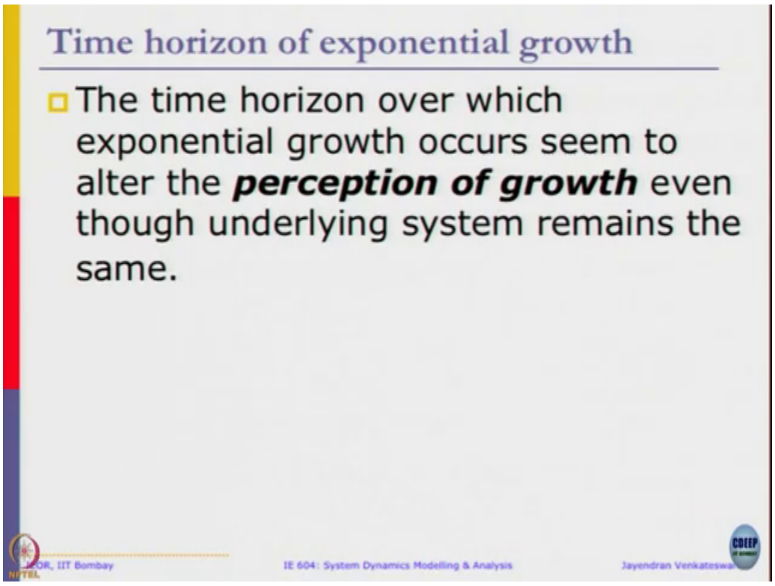

Now, let us go back and spend some time on looking at some graphs. The time horizon over which exponential growth occurs alters the perception of growth even though the underlying system remains the same. So, it depends on how much data that we want to look at from the past or the time series data we are looking at alters the perception. So, let us see what we mean by that and let us relate it with our variables of T d or the time constant t, let us see

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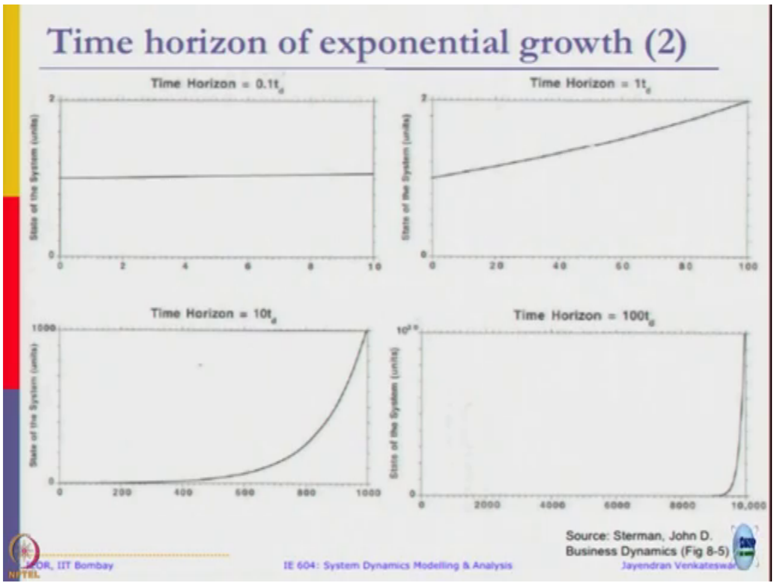

. Suppose the time horizons taken happens to be just point one times your doubling time, then it looks like there is a absolutely no growth. It is very easy to just dismissing; no this time is fine. It is just flat there is no growth in the system. It looks very flat. This time horizon is just point one times your doubling time.

The time horizon is one into doubling time if it; if it is 20 horizon or something then you may kind of start to pursue some exponential growth, but for practical purpose it looks like linear growth it just grows from whatever in this example as 1 to 2 in the time constants where it looks like a linear growth was. So, you do not start to pursue exponential growth at all even if you have observed system only until its doubling time right. Doubling time is depends on 0.69 by g. So, if g is going to be very small, then I am going to get a much longer doubling time.

Suppose this is 10 times your T d, then you can start to pursue some exponential growth which is shown here, but even then observe what is happening in the x axis; x axis is also changes with that time horizon 0 to 10, then 0 to 100 time units. Now, 0 to 1000 time units when we start to pursue it, but the difficulty comes in is when you look at data from say 0 to 400.

It is almost invisible to us and it seems to say that nothing is affecting the system there and suddenly things are active only after time 400 which is not the case underling system is the same. But if you again take too much data, then you do not see anything; you just see that something is just spiking all of a sudden that is an exponential growth.

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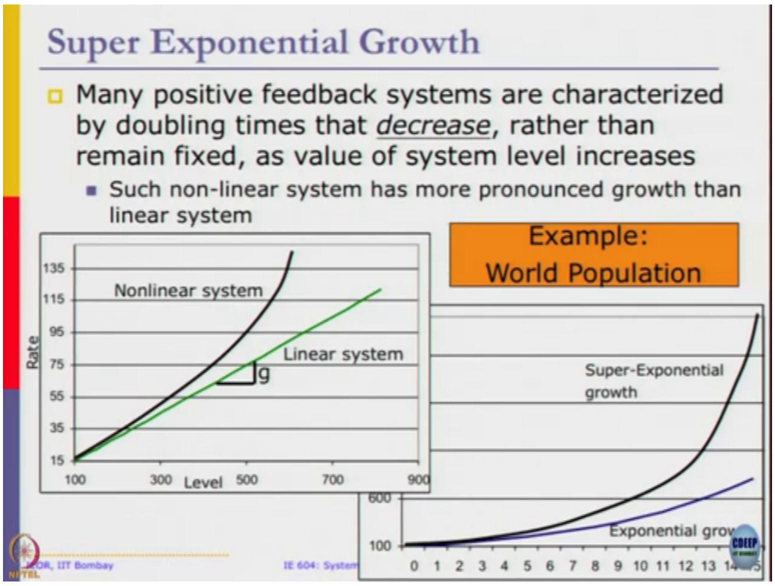

What is super exponential growth? Many positive feedback system is characterized by doubling time that decrease rather than remain fixed as a value of system level increases. If the doubling time is constant; that means, you are having an exponential growth. After every 5 time unit passes, your level value the stock value became 100, then 200, then 400, then 800; after every 5 years the doubling time is constant.

But system such as unfortunately population systems, the doubling time is not constant the doubling time keeps reducing whereas, the time it take 5 years to reach the double the value. Next time, it may take just 4 years to double in its value and following time, it may take just three and half years to double in its value.

So, as doubling reduces; that means, the value of g is increasing that is what it mean. So, if you plot as the stock value increases, the value of g changes. So, if you want to plot it, then world population; you will get a start looking at a non-linear systems because value of g is also now getting affected by some other factors. It is no more a external variable as we assume as long as g is constant does not change, then you get a linear system as well as a exponential growth.

So, system is linear growth is exponential. But if g itself starts to increase as we go along as the stock level increases, then you start to get an what we call as a non-linear system because the level and rate relation now became non-linear and the growth is going to be more pronounced.

Look at world population example. If you assume exponential growth, then probably you will get the blue line. Super exponential growth, you are going to get this black line that you see here suddenly exponential growth does not seem. So, bad you may you may be happy to have exponential growth, then the exponential growth is that then super exponential growth is ah. So, we want to move the exponential growth, then come to a (Refer Time: 16:20) goal seeking kind of systems.