Introduction to System Dynamics Modeling
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Dynamics of Simple Structures
Positive Feedback
Lecture – 7.1
Dynamics of Positive Feedback systems

Today we are going to look at modeling of positive feedback systems in detail and their underlying behavior.

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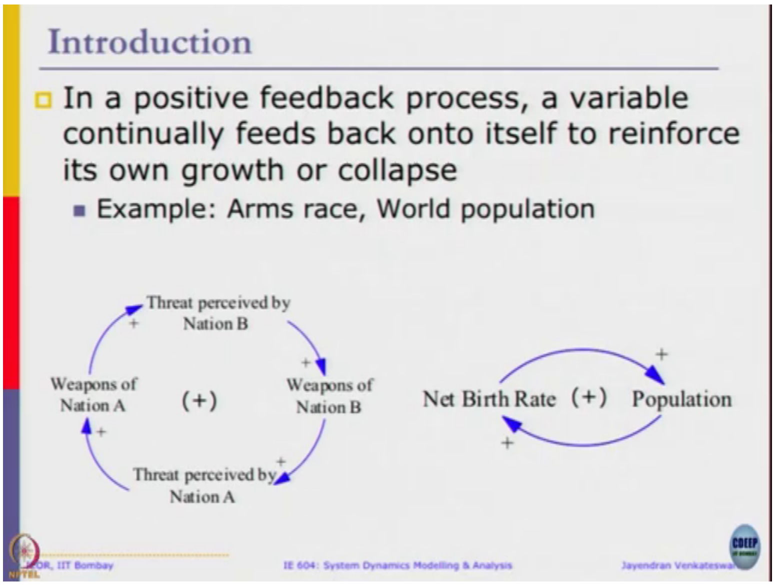

So, the positive feedback system or reinforcing type of systems a variable will continually feed back onto itself and will reinforce its own growth or collapse. Examples would be the arms race we can see the threat perceived by nation B will drive the weapons of nation B which then affects the threat perceived by nation A which then drives of the weapons of nation A to this entire loop to the positive feedback system.

Which continuously reinforce itself and which in popular literature is also known as the arms race. For more simpler example should be the population growth where if the net birthrate is positive then the we expect the population to grow here the population will not grow linearly because whenever there is more births it increases the population and after some time the that increased population will contribute to further increase through the higher birth rates right. So, this is going to continuously feed itself and will result in a exponential growth of the variable population.

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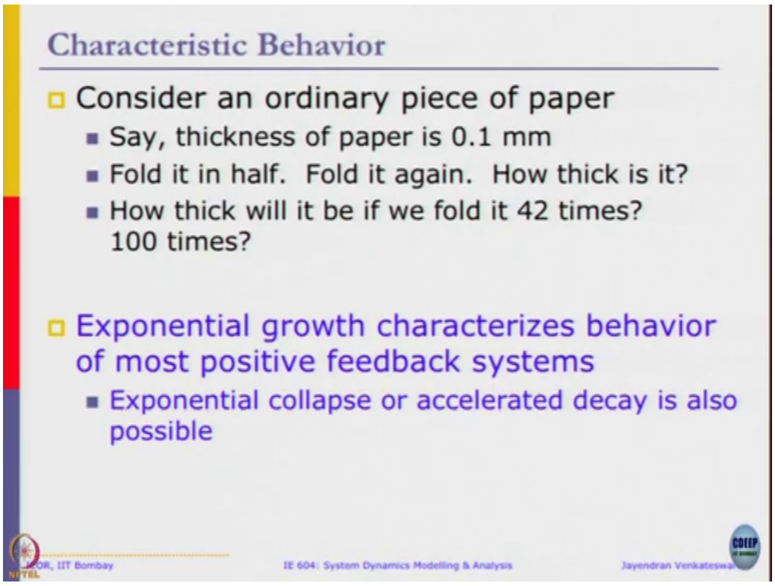

So, the characteristic behavior I think we saw it last time also its an exponential growth, but to get a more realistic feel of what is it that we are looking at because it is not perceivable in a short run we need to give it some time. So, consider a paper thickness 0.1 mm we fold it in half then we again fold it how thick will it be? It should be 0.4 mm its (Refer Time: 02:12) if we when we fold it in half it becomes 0.2 then we fold it again then it becomes 0.4.

So, for the every fold it doubles right there. How thick will it be? If we fold it 42 times or 100 times imagine the paper is big enough any ordinary piece of paper probably not more than 7 times you can actually fold it, but imagine if we can actually end up doing it just 42 times we are just 2 times 3 times it is not really infinite number of times they are folding it. It is countably finite number of 52 times to do the math's that is more than 4000 kilometers.

Kilometer we are just change the units from millimeters to kilometers 4000 which is more than distance from here to the moon. So, that is how big exponential can grow without if it is left unchecked. So, this exponential growth that we just saw characterizes most of the positive feedback systems the other one is characterized by accelerated decay or exponential decay. So, with time initially when it seems like you know systems is going fine, but after some time you can find that there is exponential growth that has occurred.

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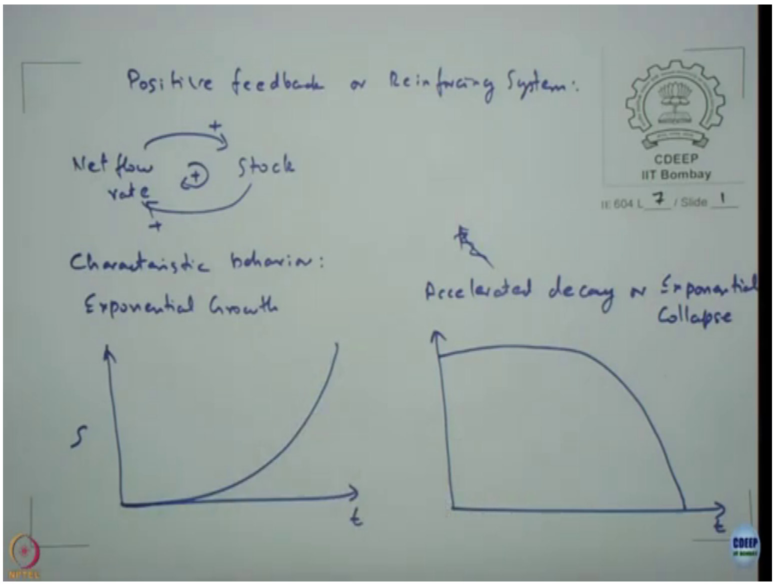

Say positive feedback or reinforcing systems. What you are looking at as net flow rate that is the (Refer Time: 04:08) stock. So, this is a positive feedback loop or a reinforcing loop. Either plus or r can denote it. Now, the characterizing characteristic behavior would be exponential growth. What was the variable of interest let us assume it is a stock.

So, this is your exponential growth other part is accelerated decay or exponential collapse x axis is always time. How will the shape of this be accelerated decay exponential collapse we have how many options do we want to it going to be the same or (Refer Time: 05:39) growth what other options we have. So, its going to be this way this is your accelerated collapse.

Since very less quantity seems to get down this does not increase its active flag this part in the diagram, but after some time it just after crossing point it just starts collapsing faster and faster before it hits 0. The example I illustrated was panic selling in stock markets or panic buying not panic buying panic selling. Nobody would buys in panic why people sell in panic.

Panic selling or withdrawing cash from the banks just message that the bank is running dry of cash is enough for people to start doing up initially if they get less and less amount then suddenly your face with riot scenarios. So, that we characterized as the exponential collapse in the system.

So, there is what characterizes these two is while system is here or even system is here it seems that things are all fine, but then if the same amount of time you wait suddenly the system becomes at unmanageable size up to here things are and then you just waited a little longer and system becomes unmanageable, but here it was fine till here then you waited a little longer and now system becomes unmanageable size which is here if you did not want.

So, that is a characteristic behavior of exponential system. So, that thing seems to be ok, but the growth is being unchecked using the system. Now, to model this as a stock flow diagram let us see how we are going to go about doing that.

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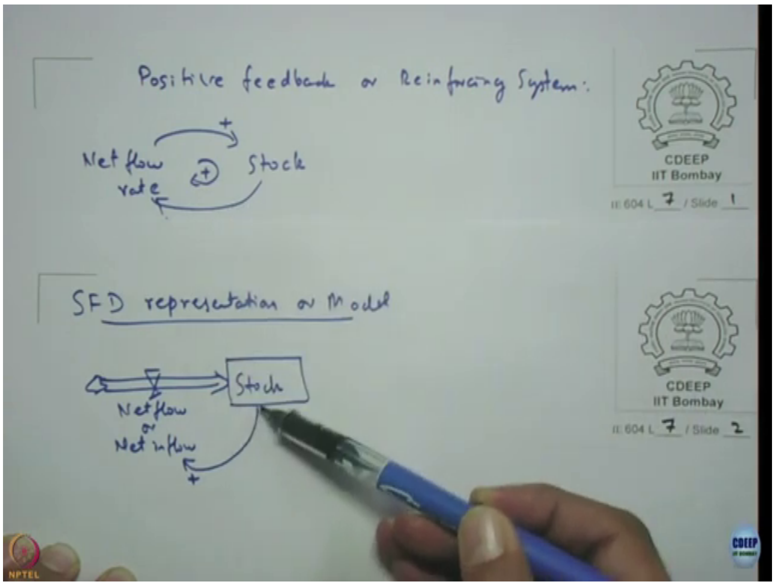

Let us see look at the SFD representation or model. There is a stock. So, its a rectangle and there is a net flow rate net flow or net inflow when we see that the stock again feeds back into a net flow rate. So, I have to represent that the positive sign. So, the top causal link is represented by this thick arrow with a valve. The bottom causal link is represented by an explicit causal link in a stock flow diagram.

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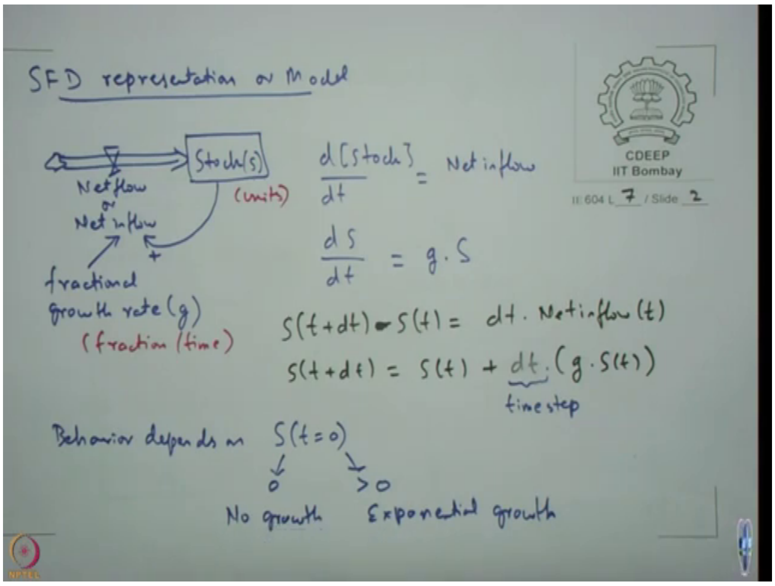

And to now capture the relation between the net inflow versus stock. Let us introduce a new variable called as fractional growth rate. Let us have the this was g let us defined the stock as just a variable s for simplicity. So, the underlying equations here so, in stock would be given as change in stock for d t is nothing, but net inflow.

There is d s by d t my net inflow let me just simply defined as g multiplied by stock s if at all g is the fractional growth rate. So, units can be fraction per time. The stock units are just simply units net inflow rate becomes this units per time. So, this is simple equation that is underlying this diagram that we just draw it here drawn here the stock is just d s by d t and fractional per net inflow rate is nothing, but g into s. So, at every point in time I am adding g into s to the stock values right.

So, if I want to simply simulate what I want to do is stock at time t plus d t is minus. So, stock at time t is nothing, but d t into net inflow at time t or stock at t plus time d t to stock at is a minus sign to stock at time t plus d t into g into s its again a time t, but dt is here d t is the time step. So, till using Euler's method they say time step of 1 then d t is simply 1. So, s at time t plus 1 as s at time t plus 1 into g into s of t and to simulate this model all we are going to do is keep solving this equation again and again right.

So, if you want to do it manually how will you go about doing it? First they say we have to initialize some value of stock. So, actually the behavior it depends on the initial value of stock right; behavior depends on initial value of stock at t equal to 0 right. Why it is behave, why it is it depend on that? The simple reason is stock value 0. What will be the behavior? It will be nothing it will be just the stock value will continue to be 0 forever because the initial value of stock is 0.

So, if it is 0 then we can expect a no growth. If it is any value greater than 0 then we can expect exponential growth. There are two possible behaviors are there if it is I mean assume g is greater than 0 as long as g is strictly positive we are going to get a exponential growth. First stock has to be nonzero for this to get start l system will not will exhibit no growth.

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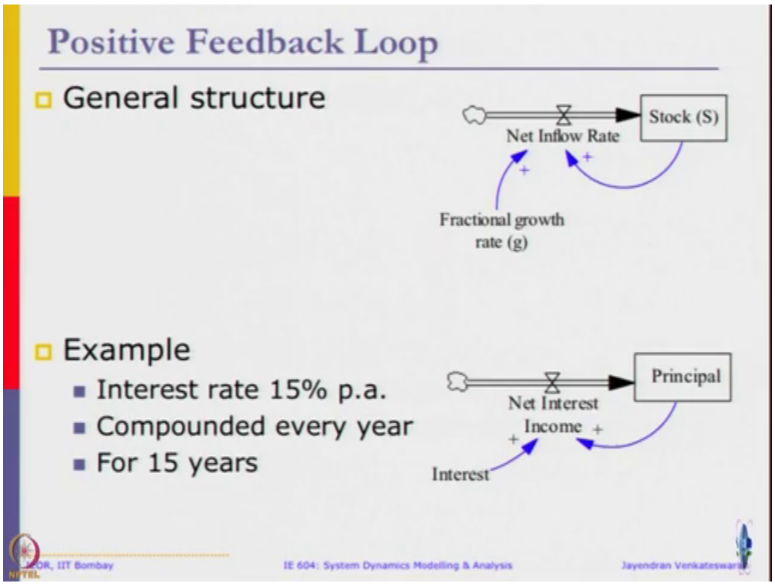

So, let us do the diagram we just saw the underlined equations. So, if you want to contextualize it to this very focused example assume you put money in a bank and it is going to accumulate interest compounded interest every year. For the next say 15 years to this simple diagram here represents how the interest accumulates in a bank and though you may feel that bank is not giving you adequate interest and a money does not seems to grow the speed. Which you may want it, but actually the growth is exhibits a exponential growth.

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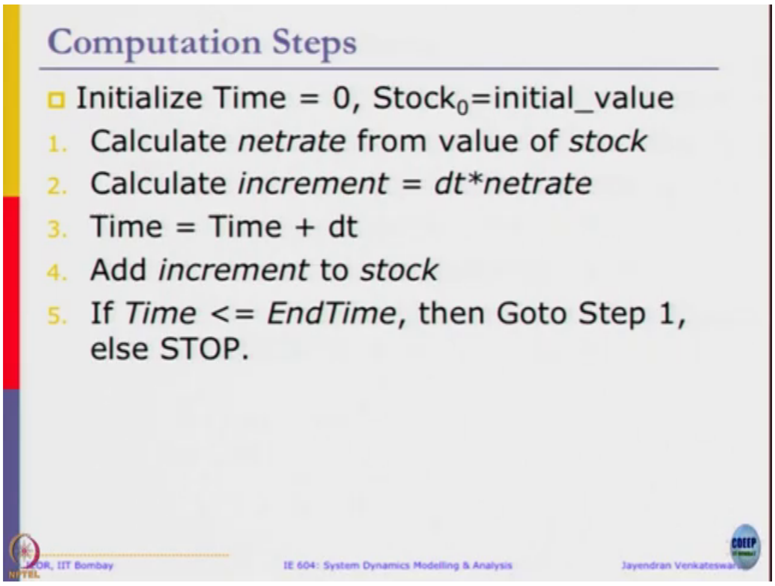

To simulate it so, let us we will let us do a quick hand simulation. It typically initialize the time at 0 and initial stock at the initial value then see first the net flow is calculated based on the current value of stock then we add the increment that is delta t into the net inflow increment time and then we add this whatever increment we calculated to the stock. So, this time is not recent time you keep loop it. This may look unnecessarily confusing, but what do you want to do is shown here.

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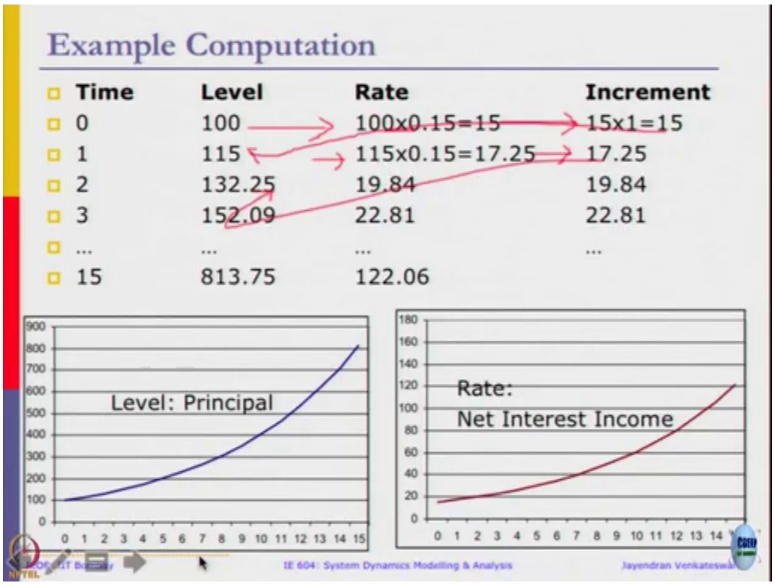

So, let us assume for that interest rate example the initial value of stock is 100 right. So, first at time 0 the level is 100 then at time 0. So, up to this is what is given time 0 initial value of stock per level is given at 100. So, we calculate the rate this 15 the value of g is taken as 15 the interest rate is taken as 15 percent. So, we take 0.15 into 100 which is 15 and let us assume a time step of 1 the d t is equal to 1. So, 15 and 1 is 15.

So, the next time period the increment time period by 1 then level will be 115,100 plus 15. Then we repeat the same step 115 into 0.15, 17.25. So, the next period we add 17.25 we add 17.25 to the stock, but from this I calculate this and I calculate this then I add this till here. Then from this I calculate this value 17.25 of course, calculate the increment and add the increment oops sorry to that is fine.

So, I can see when you start with 100 with every time unit passing I am adding an increment value equal to interest rate multiplied with that current value of stock. I mean we are assuming we are not taking the money out. So, let us see what happens when we plot it so, we started with 115 value is now 813.75. It is easier to visualize to see this graph.

So, this now it starts to exhibit a exponential growth. So, this is a principal value that is a stock value another net interest income. Which is your net inflow rate which will also again exhibit there is nothing net income is nothing, but a constant multiplier of the stock. So, stock exhibits a exponential growth it is just a fraction of that it will also exhibit exponential growth offset by the value of g which engages 0.15.

Let us see initially I started with 100 and then by the time it reached 200 was 1, 2, 3, 4 around time period 5 it reached 200 let us see let us assume how long 200 takes to double from 5 I start 1, 2, 3, 4 I mean around time period it reaches 400 that has double.

And from as you can see so, after 5 time period passed it doubled from 100, 200. After another equal interval of same 5 time period passed the current value of stock is 200 doubled to 400. Under the same equivalent amount of say another 5 time units passed the stock value doubles to 800 right. So, with a constant so, interesting thing about these exponential systems is there is a constant doubling time.

That is after constant time unit passes many stock value doubles in value which is the characteristic of such a exponential systems. So, in this particular example it happens to be 5; 5 time period approximately. But we can actually compute this.