Introduction to System Dynamics Modeling
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Dynamics of Stocks and Flows (Contd) & Patterns of Behavior
Lecture – 6.2
Graphical Integration

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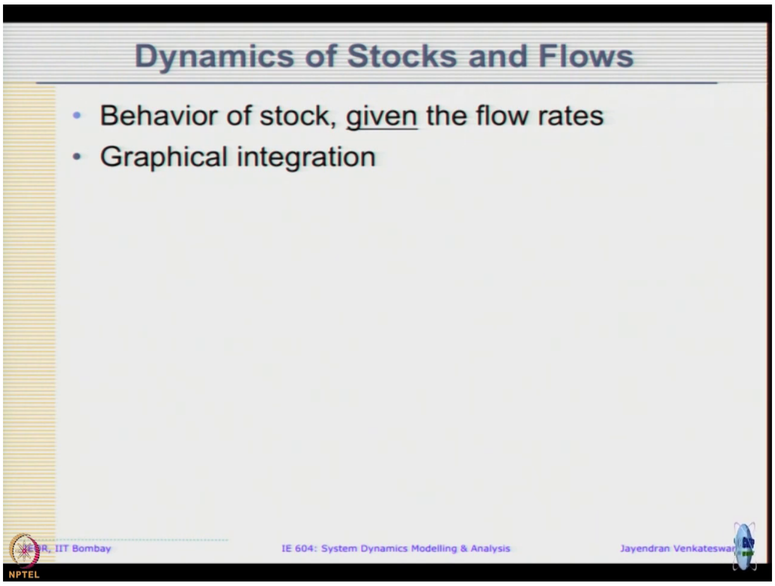

Now, let us get some intuition on the behavior of the stocks, as I told now we have kind of learning how to define stocks and flows. So, let us try to see what will be the behavior over time, the entire thing is called system dynamics; the dynamics may be evaluation over time. So, let us see when this values are the input stock if it the flows when it changes, then what kind of behavior we can expect in the stocks. And we can do that graphically we do not really need any math to it, so that we can build a intuition.

So, this given the, what do you want to say compute the behavior of this stock given the flow rates. We know how the flows are happening or do you know what is happening over the stock that is called as now we are just integrating it, so we can do it graphically. So, before that we will try to understand, what is it that we want to do.

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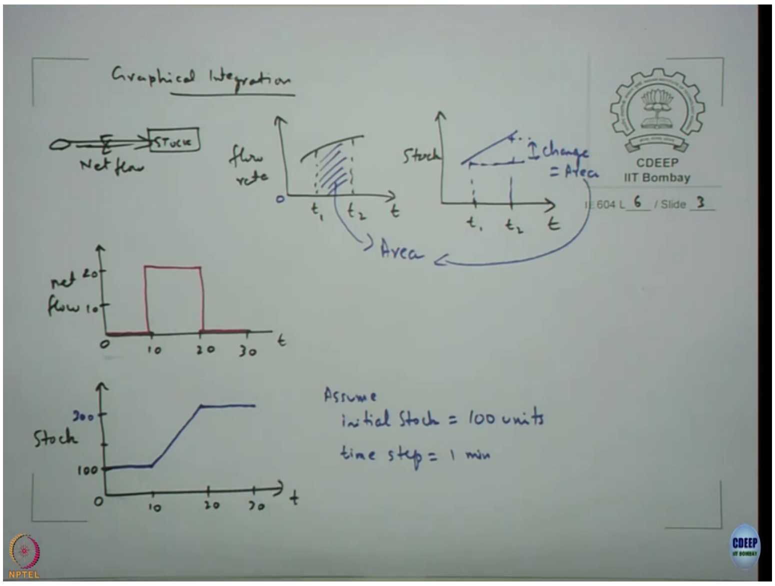

Let us consider a simple example like this. I have a stock and I have a net flow into the stock. So, what we see is x axis is always time unless otherwise stated. Suppose the flow rate is changing like this over time, we want to see how the stock changes.

Flow rate suppose changed some say time t 1 to t 2; the same time t 1 to t 2 we want to figure out how the stock changes. Flow rate is positive, let us assume it is positive, let us take this as

0. So, we expect the stock is going to increase, correct. How much will be a quantum of increase?

Let us not worry about the shape, let us just take it like suppose it changed like this. So, what should be quantum of increase? The change should be the area under this curve that ok. Let me draw a more simpler example, let us take net flow time t, 0, 10, 20, time unit is 30; y axis says 0, 10 and 20. Let us assume the flow is 0, then there is a rectangular pulse from time 10 to time 20 and then again it falls back to 0.

So, there is a rectangular pulse is given as a net flow into the stock. So, let us assume this is a behavior of the flow over time. Now, I want to plot the behavior of my stock over time. Let us assume initial value of stock is 100; let us assume initial value of stock is 100. So, until time 10 what will be the value of stock? It is going to remain the same 100, there is no change in flows. So, for this diagram the net flow is 0; that means, the value of stock remains at 100.

So, here we are going to assume what kind of; assume initial stock is 100 units, ok. Central time 10 it is 100; then what happens on time 10 to 20, it will increase? How will it increase? So, if you give rectangular plus what happens; is going to increase? It is a ram function, it is going to increase linearly, right. Until the value of what, what will be the maximum value of the stock? So, it is going to go up to 300.

And after that what happens at time 20 onwards, 20 to 30; again this falls to 0, but the stock remains unchanged, stock does not fall to 0, stock continues at 300. You can assume time step of say 1 minutes say, assume time is in minutes, so it is quite easy. So, every minute I am adding 20 units in to the system, right; from here time 10, 11, 12, 13, 14 time 11 I have added 20 time unit 20 units, at time 12 I have added another 20 units.

So, from here from time 10 to time 11 it goes from 100 to 120 then go to 140, 160, 180 to 200 to 220, 240, 260, 280, 300, right. And so, every time step, I am getting a equal amount which is just being put here is added here. So this, now coming back to the figure here; so what happens is as the flow changes over time, from say time t 1 to t 2 this area will be equal to this change. The change in the stock value will be equal to the area under the curve for the flow rates, ok. This is kind of the change in stock is equal to area for the flow rate between the same time intervals say t 1 to t 2.

So, this is why we actually say that this stock has memory, the stock stay remains at the value of 300. So; that means, it has a memory of all the past events that has occurred, it has accumulated it and now it stores it and the value of 300. So, that is what we refer to as memory. And there is inertia, unless the flow changes nothing influences stock; stock continues to remain the same, it does not leave it anytime or it does not increase it automatically, it continues to remain that. So, that is what it indicated by the word inertia.

In yesterdays lecture I talked about stocks are influenced by inertia and memory, so this is what I meant, ok. So, let us look at more interesting example. Let us just quickly escalate it; for the graphical integration let us look at a more some example yeah.

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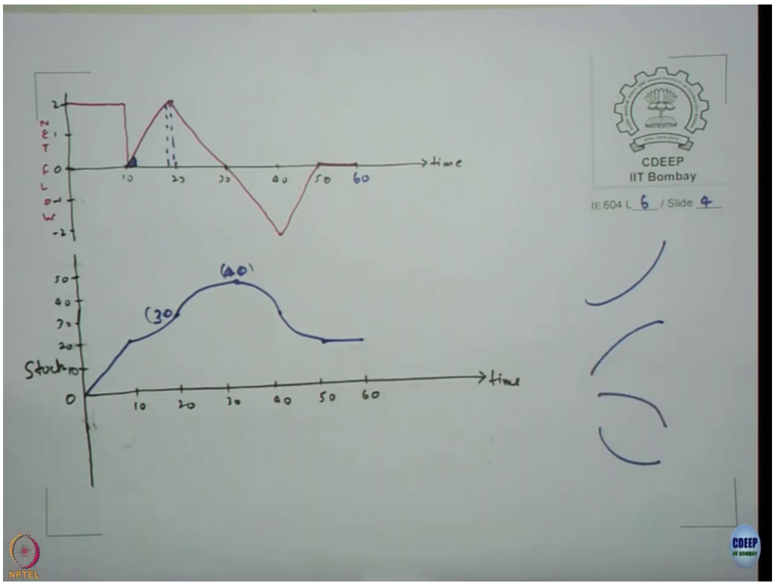

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So, same example earlier, assume this is a net flow now; you have all rectangular pulse, the net flow is now ram function, it increases and it decreases linearly to time 30 at to 0 and then it goes to minus 2; then again comes back to 0 and then remains 0 from time 50 to time 60, ok. So, y axis goes from 0, 1 and 2; 0, minus 1, minus 2. So, that is the y axis and this is the input function given.

So, how will the output stock look like. I am just going to mark the time units here 10, 20, 30, 40, 50, 60, ok. Assume the initial value of stock is 0; if it is not specified assume it 0, ok. We will assume a initial value of stock is 0, then what will you what will happen to stock from time 0 to 10? So, the area under this is 20. So, it has to go to 20.

So, since they already had 20 let me; since I am not sure how the dynamics are going to go later, let me just make intervals of 10, 20, 30, 40, 50, let us see what happens. So, we expected to linearly increase to 20, that is the value of stock very same as the previous example, right ok. Now, what happens, from time 10 to 20 what will happen to the value of stock?

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What will be value of stock at time 20?

Student: 30.

Ha 30. So, what you did is you calculate the area under this curve, it is half into base into height that is 10 units. So, you are already at time at, not time at time 10 you already had value 20; so 20 plus 10 your 30. So, you are going to reach.

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30, but how are you going to reach it?

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Which side? It is going to be like this or is it going to be like this.

Student: First one.

First one. So, simple reason the rate is increasing, from time 10 to 11; from time 10 to 11 only small unit is added to your stock; but from time say 19 to 20 large quantities added to the stock, right. So, the amount added here, the growth should be higher than the amount here. So, that an intuitively asked us to draw a curve like this, anyway this is 30. So, it is not that accurate with this figure, but it is 30. Then what happens from 20 to 30?

Student: Increase.

It will increase or decrease? Stock will increase. The flow shows a decreasing rate, but still every time unit some positive quantities added to the flow. So, as long as there is a positive quantity look at it, it says net flow; net flow means inflows minus outflows. So, if it is positive means, it is being added to the stock.

So, large quantities added first and then at every time units smaller and smaller units are added until it hits the total of 40, right; because area under the curve continues to remain minus same as 10. So, now, the curve goes like this. So, this is 30, this is 40; y axis value is 30 and 40, sorry.

Then what happens from 30 to 40? Now, net flow is negative; that means, I am removing from the stock. Again what is happening here, I am removing less quantities and then I am removing large quantities; so the shape will be at two only options either this or this, this one.

Student: (Refer Slide Time: 13:55)

Comes like this and then; again though the flow net flow is actually increasing, but it is still all negative values. Again I remove large quantities initially and then bring it back to these quantities. So, this values will be again back to 20 and then since there is no change in flows, it continues to remain straight at 20.

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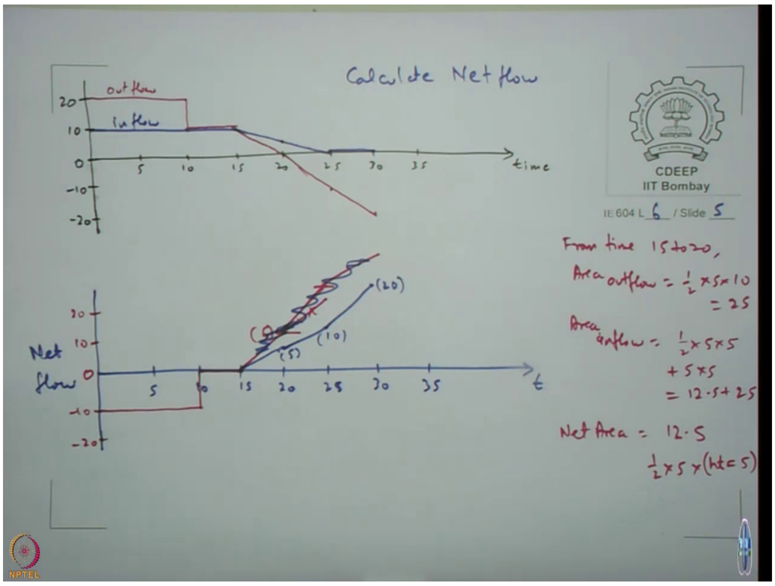

Calculate net flow. Given a net flow we can integrate it nicely; but when we have both inflows and outflows given separately like this figure, here the inflow starts at 10, it is constant to 10 until time 15. And then from time 15 to 25 it linearly decreases and in 25 it hits 0 and remains constant at 0 till time 30.

Outflow is starts at 20, then drops to 10 at time 10, then from time 15 onwards it decreases at a constant rate until it hits minus 20 at time 30. Once we have the net flow we can always calculate the this one stock; so we will learn how to calculate the net rate.

Again x axis is time let us do 5, 10, 15, 20, 30, 35; but we want this net flow. So, from until time 10 anyway both inflow or outflow are constant. So, what should be a value of net flow; what is the value of net flow? At every time instant you are taking the difference, right. So, you have to work out the same thing. So, inflow is 10, but outflow is 20. So, a net flow becomes minus 10, net flow is minus 10.

So, this is your net flow to minus 10. It just inflow minus outflow 10 minus 20 is 10, I am just taking subtraction. At this point both inflow and outflow are the same so; that means, net flow is 0 and I am not neither adding nor subtracting. So, this has to jump back to 0 up to time 15 is 0.

Now, it becomes tricky, I am having a linear decrease in both outflows as well as inflows. So, what should be the value, how does it increase? Now, you start worrying about areas, earlier it was the same thing. So, you just go, say one time step nah or rather upto time 20; at time 20 outflow is produced by so much. And inflow, what is the area under inflow and area under the outflow, if we do the subtraction?

Area under the outflow from here to here is 25, half into 5 into 10 it is 25. Area under inflow; inflow graph if you see; you have a rectangle here and then a triangle here, ok. So, from time 15 to 20; area under outflow is half into 5 into 10, 25 units; area under inflow is half into 5 into 5 plus 5 into 5, so you get 12.5 plus 25.

So, inflow minus outflow adds about 12.5 units in to the system; because inflow has higher than outflow, by solving how we got it. So, this is the area under the curve of inflow and this area under the curve for outflow. So time, at this time point in time I am having it. So, now 12.5 units are added.

So, net flow then becomes, stops at 12.5 here; am I doing a math correct. And the same slope comes here, so I am going to get a line something like this here. And here there is no inflow, but outflow is negative; so negative outflow means you are adding to the system.

Whenever there is a negative outflow means you are just adding to the system further, then you are removing from the system, right. Here you are adding and here outflow is also positive; so you are adding here and removing here. But when an outflow became negative that means, you are adding at two places; you are adding here plus adding here. So, actually this slope becomes higher and here I am not adding any more, I am adding only here, so I will get a flow like that.

So, it kind of has a crazy pattern right there ah; but here we got the concept. So, what you are trying to do here is calculate the net flow first and then using the net flow compute it; because of net flow is constant, then I know it is a linear increase in stock. This net flow is increasing, then I notice some sort of parabolic function; whether it is and I get different shapes in the stock, so once I get this I can get the other one. So, that is one there is one more math involved that is area covered under.

Student: Half.

Under this curve, so that equals this. So, then what you essentially have is it increases up to 0.5. So, pretty much your net flow will go up to 0.5, so this has to be at 0.5, got it. So, inflow and outflow net area is 12.5, correct. So, 12.5 means the time unit is fixed; half into base when time unit is fixed. So, I have to multiply it by the height which will be then equal to 5 right; height equal to 5, so that means the time 5 is one it goes linearly.

So, then you calculate the other one. So, here the only thing is the slope will increase, because this also getting added and this also gets added; because I am kind of. And here then slope will be lesser than in the previous level ok, it will have linear function. So, this point is at 5. So, this also has to come at 5. So, it is fine it is a linear function it will work out the same thing so.

Student: (Refer Slide Time: 21:51).

Total change is 25 ok. Let me get rid of this part, so let us draw a line here. So, let us keep it 5. So, it goes up to 5 here and then it is going to. So, here slope should not change it here, it should go to 10; then it will change to 20, is it what we are getting.

See if this it was a linear example, so it was easy. Suppose this net flow itself had an hyperbolic things, then we need to kind of double check it; but for this case it will just work if we just take the differences and start plotting it, it should work yeah, it should work.

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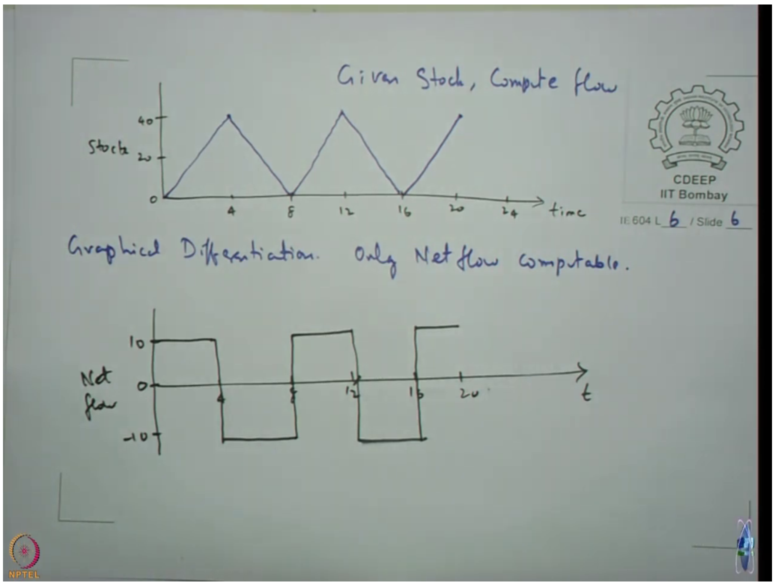

Now, let us do one more example. This example is kind of trivial, given your stock compute flow. So, this is called as graphical differentiation. Given a stock we cannot compute inflows and outflows separately; we can only compute net flows, only net flow computable. We already know it for this trivial case.

What will be the values here, how will it change? Time units goes from 0, 4, 8, 12, 16, 20 etcetera. So, 0 to 4 eventual increase has been 40 linearly. If it is linear increase in stock, then it must be a rectangular constant value; for this it has been increasing, so it must be, so it must be 10 up to time 4, 8, 12, 16, 20. So, up to 4 then it must be constant at 10, from time 4 to time 8 it reduces and until I hits 0; that means, this flow has to be negative.

Let us take minus 10, then flow has to be constant at minus 10; then again there is an increase, so it must be constant here upto 12, then again there is a decrease, it is this 12 then again there is an increase and so on. Or what you have is a kind of just a linear input which changes constant, but you can see the dynamic that is happening.

Just by the input is constant, just like the net flow is constant is always at 10 and then again it comes to falls to minus 10 and again increases plus 10. And even with that simple inputs we can find that a stock keeps fluctuating in this fashion. Or the other way, if the stock fluctuates in linear fashion, that all it tells us is the net flow is actually constant.

So, this is an important; the idea here is not this graphical integration in different how many ways can easily do it. It is you are not here without trying to figure out what how to do differentiation integration and visualization. The idea is here is, when we start looking at actual pictures and graphs; many times we will be finding many such you know graphs which keeps increasing and decreasing over time.

The main thing I understand is, it need not be because even if say the entire thing is changing, even if things are constant. And even if fluctuating at only two different endpoints, still it can give a kind of a saw tooth kind of a behavior or a kind of will kind of a sinusoidal behavior can be seen, just by keeping the value is constant. So, those kind of intuitions is what we need to until build.