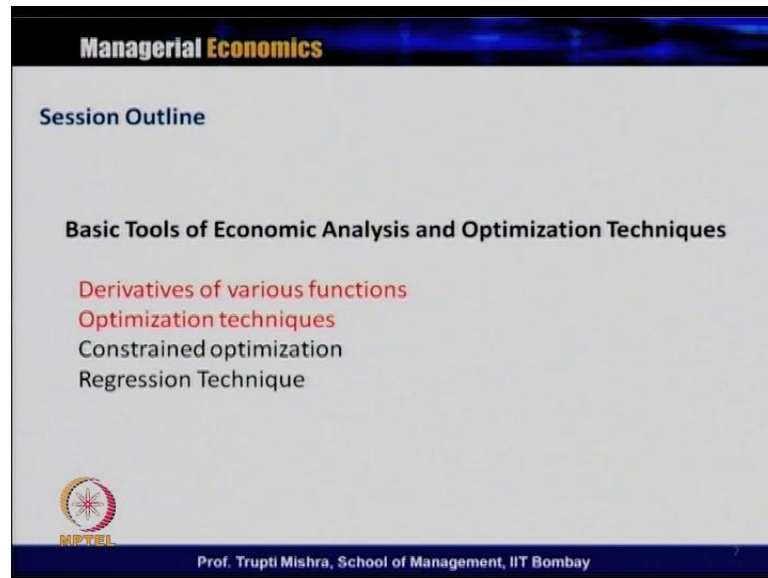


Managerial Economics
Prof. Trupti Mishra
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Lecture -6
Basic Tools of Economic Analysis and
Optimization Techniques (Contd...)

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So, we will continue our discussion on relationship between different economic variables, given a quantification or through different methods graph or the mathematical equation. So if you remember in the last class, we have started discussion about the derivative of various function, how to solve the various function. Then we introduce the optimization technique, where we did two type of optimization; one is the maximization of revenue or maximization of profit, and second one is the minimization of the cost. So, whenever we are doing this optimization technique, either it is a maximization or it is a minimization problem, we did not consider the case of a constrained, and we just optimize the maximization of a profit function or we just optimize the minimization of a cost function.

So, today I will discuss the optimization technique with a constrained, either in the form of the income or in the form of the cost, when it comes to cost, and then it comes to the revenue, either its maximization or it is a minimization of the cost.

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Managerial Economics

Constrained Optimization

The techniques used for achieving a target under constrained situations or conditions is called constrained optimization

Substitution technique

Lagrangian multiplier method

Source : Managerial Economics; D N Dwivedi, 7th Edition

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So, in case of constrained optimization, this is a technique used for achieving a target under constrained situation or condition, is called constrained optimization. So, may be the motivation for optimization is remains same. It is achieving a target, either to maximize the profit or to minimize the cost, and but here the difference is that there is a constrained along with the objective function, and how to do this constrained optimization? We generally discuss two type of technology, we will talk about the substitution technique, and later on we will take the Lagrangian multiplier method.

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Managerial Economics

Substitution Technique

Applied to the Problem of Profit Maximization and Cost Minimization

For Profit Maximization

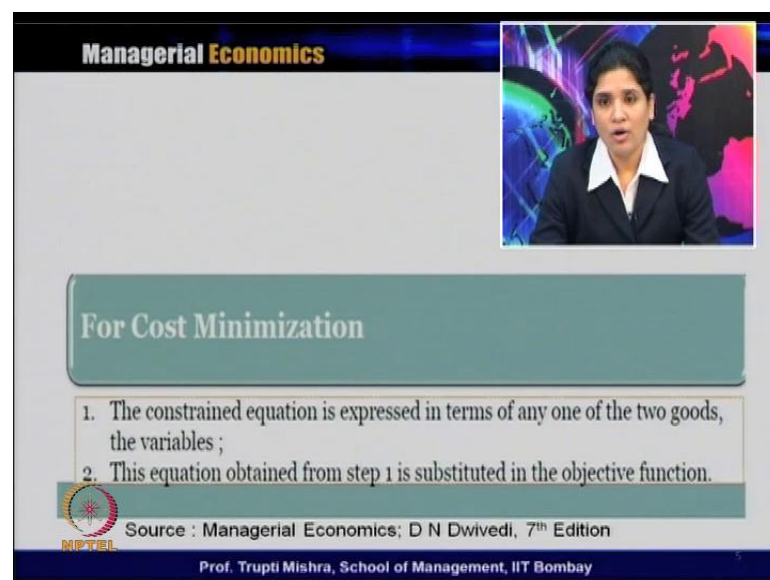
- One of the variable is expressed in the terms of other variable and solve the constraint equation for obtaining value of one variable.
- The value obtained is substituted in the objective function, which is maximized and solved for obtaining value of the other variable.

Source : Managerial Economics; D N Dwivedi, 7th Edition

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So, taking the substitution technique, it can be applied to the problem of profit maximization or it can be for the cost minimization. For a profit maximization, one of the variable expressed in term of the other variables, and solve the constraint equation for obtaining the value of one variable. Suppose, there are two variable x and y, so the best way for solving it through the substitution technique, is to represent one variable with the other variable, and then you solve for that variable, and finally you substitute the value of one variable, in term of what you have solved to the other variable. And here the value is obtained is substituted in the objective function, which is maximized, or solve for obtaining the value of the other variables. So, whatever the value is obtained by substituted, it will be again substituted back in the objective function, which is maximized and solve for obtaining the value for the other variable.

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For Cost Minimization

1. The constrained equation is expressed in terms of any one of the two goods, the variables ;
2. This equation obtained from step 1 is substituted in the objective function.

Source : Managerial Economics; D N Dwivedi, 7th Edition

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So, we will see how we use this substitution technique in case of a profit optimization problem, and in case of a cost minimization. And how this is different for cost minimization; may be the method again remains same. The constrained equation is expressed in term of any one of these two goods of the variables, and the equation is obtained from step one, is substituted in the objective function. So, whether it is a cost function, whether it is a profit function, the basic rule for this substitution technique, is that we expressed one variable in term of the other variables. We get the value of one variable, and finally again substitute back to the objective function. So, we will just take an example, how generally we do the constrained optimization, along with the

constrained with the objective function, whether the objective function is a profit maximization or whether the objective function is the cost minimization?

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$\pi = 100x - 2x^2 - xy + 180y - 4y^2$
 $x + y = 30$
Max π function
w.r.t $x + y = 30$.
1st step - x in term of y
OR
 y in term of x
 x OR y

So, we will take a case of profit maximization first; and in case of profit maximization, we will maximize the profit. So, here profit is equal to $100x$ minus $2x$ square minus x y plus $180y$ minus $4y$ square. This is the profit function, and the profit, here the optimization problem is the profit maximization. Since we are saying that this is the case of a constrained optimization, there is also a constraint attached to this, and the constraint is in the form of x plus y is equal to 30 . So, now what is the optimization problem? The optimization problem is, maximization of profit function, with respect to the constraint, that is x plus y which is equal to 30 . Now, how we will do this, the first step is, we will express x in term of y or we can express y in term of x . And after getting the value of x or y , again we will substitute this value of x and y in the profit function. So, now we will see, let us substitute the value of x and y , before it converting into another term, or may be the profit maximization problem.

(Refer Slide Time: 05:52)

The image shows a whiteboard with handwritten mathematical work. At the top, three equations are listed and grouped by a right-facing curly brace: $x + y = 30$, $x = 30 - y$, and $y = 30 - x$. Below this, the profit function π is derived. It starts with $\pi = 100(30 - y) + 2(30 - y)^2$. The next line shows the expansion: $+ (30 - y)y + 180y - 4y^2$. The final line shows the simplified expression: $= 1200 + 170y - 5y^2$. In the bottom left corner of the whiteboard, there is a small circular logo with a star and the text 'NPTEL' below it.

So, suppose x plus y is equal to 30. So, this can be written as x is equal to 30 minus y . So, in this case x , we are representing in term of y , or y can be 30 minus x . So, substituting the value of x and y in the profit equation what we can get; π is equal to 100, it has hundred x , so x we are representing in term of y . So, this is 30 minus y , plus 2 30 minus y square, because it was $2x$ square, minus 30 minus y y , because it was x y , plus 180 y minus 4 y square. So, if you look at the profit function now, all the terms in term of y , there is no x over here in the case of the profit function.

Now again, if you will simplify this, then this comes to 1200 plus 170 y minus 5 y square. So, what we did, the first step is this, where we represent x in term of y . Now substituting the value of x in the form of y in the profit equation, which gives the profit equation, which is equal to 1200 plus 170 y minus 5 y square. Now, to find out the value of y what we have to do. We have to take the derivative of π with respect to y , and which we need to set equal to 0. So, if you are taking this, then this comes as the first order derivative, because for any maximization minimization rule in order to get the value, always the first order derivative has to be equal to 0.

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$$\begin{aligned}\frac{\partial \pi}{\partial y} &= 0 \\ 1200 + 170y - 5y^2 &= 0 \\ 170 - 10y &= 0 \\ -10y &= -170 \\ y &= 17 \\ x &= 30 - y \\ &= 30 - 17 = 13 \\ y &= 17, x = 13 \\ \pi &= 2800.\end{aligned}$$

So, $\frac{\partial \pi}{\partial y}$ is equal to 0, which is like 1200 plus 170 y minus 5 y square, which is equal to 0. Now solving this, this will give you 170 minus 10 y which is equal to 0, or may be minus 10 y is equal to minus 170, and y is equal to 17. Now, what is our x, x is equal to 30 minus y. So, this is equal to 30 minus 17, which is equal to 13. So, we get a value y which is equal to 17. We get a value of x which is equal to 13. Now, putting the value of y and x in our profit equation, we get profit which is equal to 2800. So, here how we maximize the profit with respect to a cost constrained, and with respect to a value of x and y. The first step is always to represent one variable in term of the other variable. So, in this case what we did, we represented x in term of y.

And after represent the value of one variable in term of the other variables; then we put the value in the objective function. So, if you remember in the previous slide what I have showing that, we represent the profit function only in term of variable y. Then after getting the profit function, we took the first order derivative equal to 0, in order to get the value of y, and through that we got the value of y which is equal to 17, and from there we got the value of x, which is equal to 13. By putting with a value of x and y in the original profit function, we get a profit which is equal to 2800. So, by substitution technique following the two steps we got the profit, we got the value of x, and we got the value of y. Now, we will see through substitution technique, how we can do a cost minimization problem.

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Handwritten mathematical derivation on a whiteboard:

$$\begin{aligned} \checkmark \\ \text{Min. TC} &= 2x^2 - xy + 3y^2 \\ & \text{[36 units of } x, y\text{]} \\ \text{w.r.t. to } x+y &= 36 \\ x &= 36-y \\ \text{TC} &= 2(36-y)^2 - (36-y)y \\ & \quad + 3y^2 \\ &= 2592 - 180y + 6y^2 \\ \frac{\partial \text{TC}}{\partial y} &= 0 \end{aligned}$$

The whiteboard also features an NPTEL logo in the bottom left corner.

So, here what is the optimization problem, the optimization problem is to minimization of the total cost. Now, what is total cost, let us take total cost is equal to $2x^2$ minus xy plus $3y^2$. Now, the firm here what is the constrained. The firm has to get a 36 units of x and y as the combine order, now what is the optimum combination. Optimum combination is to, what is what should be the minimum cost to produce this 36 unit of x and y . So, in this case, what should be the constrained? The constrained is again, if you look at x plus y is equal to 36. So, optimization problem is to minimize the total cost with respect to, or may be subject to, x plus y is equal to 36.

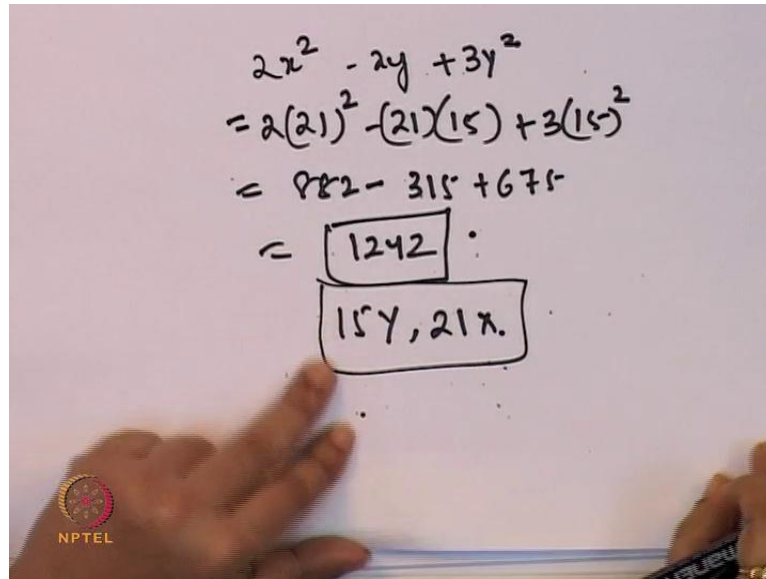
Now, following the substitution technique, what is the first step. The first stage we have to represent one variable in term of the other variable. So, x is equal to 36 minus y , because if you remember the first step substitution technique is always representing one variable in term of the other variable. So, here x is equal to 36 minus y . Now, putting the value of x in the cost equation $2x^2$. So, this is $2(36 - y)^2$, x and y . So, this is $(36 - y)y$, plus $3y^2$. So, this is $3y^2$. So, if you again simplify this, this comes to $2592 - 180y + 6y^2$. So, after putting the value of x in the cost function in term of y , we get a total cost function which is equal to $2592 - 180y + 6y^2$. Now, in order to get the value of y ; and in order to get the optimum combination or the optimum cost, what we have to do? We have to take the first order derivative of total cost function, with respect to y and we have to set it equal to 0, in order to get the value of y .

(Refer Slide Time: 12:55)

The image shows a whiteboard with handwritten mathematical work. At the top, the first-order derivative of a cost function is set to zero: $\frac{\partial 2592 - 180y + 6y^2}{\partial y} = 0$. Below this, the equation is simplified to $-180 + 12y = 0$, then $-12y = -180$. The solution for y is boxed as $y = 15$. The relationship between x and y is given as $x = 36 - y$, which is then solved to $21 = x$. The final optimal values are boxed as $15y, 21x$. An NPTEL logo is visible in the bottom left corner of the whiteboard image.

So, now we have to take a derivative, the first order derivative of 2592 minus 180 y plus 6 y square, and this has to be equal to 0. So, if you do this, then we get the value 180 plus 12 y; which is equal to 0, which you further simplify, then it is minus 12 y is equal to minus 180, and y is equal to 15. And if y is equal to 15; then x is equal to 36 minus y, which is equal to 21. So, y is equal to 15, x is equal to 21. Now this is the optimum combination, the firm should produce 15 unit of y and 21 unit of x, and this is the optimum combination for the firm. Now, what is the next best task for us? The next best task for us is to, whether producing this combination, the firm is incurring the minimum cost of production, or what should be the minimum cost to produce this combination. So, for that what we need to do, we need to put the value of y, we need to put the value of x in the cost equation, and we need to find out the minimum cost. So, what was our cost equation?

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$$\begin{aligned} & 2x^2 - 2xy + 3y^2 \\ &= 2(21)^2 - (21)(15) + 3(15)^2 \\ &= 882 - 315 + 675 \\ &= \boxed{1242} \\ & \boxed{15, 21} \end{aligned}$$

The cost equation is $2x^2 - 2xy + 3y^2$. So, putting the value of x is equal to 21, and y is equal to 15, this comes to 882 minus 315 plus 675, which is equal to 1242. So, this is the minimum cost what the firm incurs, in order to produce 15 unit of y and 21 unit of x , so what is the optimization problem here. The optimization problem here is to, minimize the cost with a constrained, that at any cost the firm has to produce 36 unit of both the goods; that is x and y . So, this is the optimum combination for the firm, and this is the minimum cost to produce the optimum combination of the firm. Next, we will see the second method to for this constrained optimization, and that is the Lagrangian multiplier method.

So, apart from substitution technique, the most popular or may be the most commonly used technique to do a constrained optimization is always a Lagrangian multiplier method. So, what is Lagrangian multiplier method, it is again one of kind of method to solve the constrained optimization, and it involves combining of both the objective function and the constrained equation, and solving by using the partial derivative methods. Basically, it takes the partial derivative with respect to both the variables, and then it gets the value of x and y , and by getting the value of x and y , it maximizes the profit or minimizes the cost. So, we will see how it works for the Lagrangian method.

(Refer Slide Time: 16:31)

The image shows a hand-drawn mathematical derivation on a whiteboard. At the top, it is titled " π Maximization". Below the title, the profit function is given as $\pi = 100x - 2x^2 - xy + 180y - 4y^2$. This is followed by the constraint equation $s.t. \ x + y = 30$. The constraint is then written as $x + y - 30 = 0$. A Lagrangian multiplier λ is introduced, and the constraint is written as $\lambda(x + y - 30)$. The Lagrangian function is then defined as $L\pi = 100x - 2x^2 - xy + 180y - 4y^2 + \lambda(x + y - 30)$. Finally, the variables to be optimized are listed as x, y, λ . A small NPTEL logo is visible in the bottom left corner of the whiteboard.

Let us take a profit maximization case. Suppose, the profit equation is, hundred x minus $2x$ square minus xy , plus $180y$ minus $4y$ square, again subject to x plus y is equal to 30 . The same profit equation what we took for the substitution technique, and the same constrained what we take for y using the substitution technique method. So, x plus y is 30 ; that is constrained, and profit is what we take for the substitution method. Now, how it is different from the other method. In case of other method we are substituting the value of x and x for y or y for x , here we will not do that; rather we will use a partial derivative method, to solve this profit maximization problem. In this case what we do, so x plus y is equal to 30 . So, we will find another variable here; that is x plus y minus 30 is equal to 0 , and the λ x plus y minus 30 . Now, we will reframe the objective function using the, adding a Lagrangian multiplier over here. And what is the Lagrangian multiplier here; that is λ x plus y minus 30 , this is the another term what we are getting here.

So, what is our new profit function? New profit function is $100x$ minus $2x$ square, minus xy plus $180y$, minus $4y$ square. This is our original profit function, along with that we add a Lagrangian multiplier; that is λ x plus y minus 30 . So, if you look at, now the constrained also we have added in the objective function. So, this is our Lagrangian function. Lagrangian function comes from the constrained and what we add in the objective function, in order to maximize the profit.

Now, this is the profit function now. Now we have to find out the value of unknown server here, what are the unknown server here; the unknown is x, the unknown in y, and the unknown is lambda. So, we need to solve for the value of x, we need to solve for the value of y, we need to solve for the value of lambda. Now, how we will do that; we will take a partial derivative of the profit function with respect to x, we will take a partial derivative with respect to profit with respect to y, and we will take a partial derivative with respect to lambda, which is the Lagrangian function or which one is the Lagrangian multiplier.

(Refer Slide Time: 19:46)

The image shows a whiteboard with handwritten mathematical equations. At the top, equation (1) is written as $\frac{\partial L\pi}{\partial x} = 100 - 4x - y + \lambda = 0$. Below it, equation (2) is $\frac{\partial L\pi}{\partial y} = -x + 180 - 8y + \lambda = 0$. Then, equation (3) is $\frac{\partial L\pi}{\partial \lambda} = x + y - 30 = 0$. Below these, a subtraction is shown: equation (1) minus equation (2) results in $-80 - 3x + 7y + 0 = 0$. An NPTEL logo is visible in the bottom left corner of the whiteboard image.

So, we will take the first one; that is, I may be first order derivative of this with respect to x. So, this we will get as 100 minus 4 x, minus y, plus lambda which is equal to 0, and let us call it the equation one. Similarly, for the second one, we have to take the derivative with respect to y. So, this comes as lambda, this as x plus 180, minus 8 y plus lambda which is equal to 0, and this is equation two. The third unknown is, with respect to lambda, so this is dell l pi with respect to lambda; that gives you x plus y minus 30, and this is equation three. Now if you make a summation and if you can make a two equation, then it comes to 100 minus 4 x minus y plus lambda, is equal to 0, and if you add the second 2 equation; this is 180 minus x minus 8 y plus lambda is equal to 0.

So, if you do a subtraction from 1 to 2 over here, then you would get minus 80, minus 3 x plus 7 y plus 0, which is equal to 0. So, if you look at what we did over here. We

basically in order to get the value of x and y, we got a got two equation, may be two joint equation in order to get the value of the unknown. So, in the first case, this is 100 minus 4 x minus y plus lambda, and second case it is 180 minus x minus 80 y plus lambda which is equal to 0. If you subtract the second one from the first one, then we get minus 80 minus 3 x plus 7 y plus 0, which is equal to 0. Now, if you look at the equation three; x plus y minus 30, this is our equation three, we will multiply 3 with this equation.

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$$\begin{array}{r} \lambda + y - 30 = 0 \\ 3x + 3y - 90 = 0 \\ -3x + 7y - 80 = 0 \\ \hline 10y - 170 = 0 \\ y = 17 \\ x + y = 30 \\ x = 13 \\ \lambda = -31 \end{array}$$

So, this is x plus y minus 30, which is equal to 0. So, if you multiply 3 in equation three, this comes to 3 x plus 3 y minus 90 equal to 0, and what was our previous equation when we did for this, this is minus 3 x plus 7 y and minus 80. And if you take this again, this comes to minus 3 x plus 7 y minus 80 which is equal to 0. From these two equations, if you sum it, then it comes to 10 y minus 170 which is equal to 0 and y is equal to 17. So, we got the first unknown value; that is y is equal to 17. Now we can get the value of x from here, because x plus y is equal to 30. So, from that we can get the value x, which is equal to 13. And now we can get a value from the. This is our first unknown, this is our second unknown; our third unknown is lambda. So, from the value of x and y, we can get the value of lambda, and lambda will come to minus 31. So, we know the value of all these three unknown, once you put in the value in the profit equation, we get the profit, and we can see whether the profit is maximum or not. Similarly, using this Lagrangian method; we can also solve a cost problem, where the optimization problem is to minimize the cost. So, let us look at to the cost minimization problem.

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The image shows a whiteboard with handwritten mathematical work. At the top, the cost function is given as $C = 100x^2 + 150y^2$. Below it, the constraint is written as "S. to $x + y = 500$ ". The Lagrangian function is then defined as $\lambda(500 - x - y) - L.C.$. The next line shows the Lagrangian cost function: $L.C. = 100x^2 + 150y^2 + \lambda(500 - x - y)$. A box is drawn around the variables x, y, λ . Below this, the partial derivatives are listed: $\frac{\partial L.C.}{\partial x}$, $\frac{\partial L.C.}{\partial y}$, and $\frac{\partial L.C.}{\partial \lambda}$. The final result is given as $x = 1.5y$, $y = 200$, and $x = 300$. An NPTEL logo is visible in the bottom left corner of the whiteboard image.

Now, the cost function over here is, $100x^2 + 150y^2$, and this is subject to $x + y = 500$. Now, Lagrangian method; what is the first step. The first step is to get the Lagrangian function from the constrained equation, then again form a cost function adding the Lagrangian multiplier or the Lagrangian function. So, in this case, the Lagrangian function is $\lambda(500 - x - y)$. So, this is our Lagrangian function. Taking this, what is our Lagrangian cost function; that is, $100x^2 + 150y^2 + \lambda(500 - x - y)$. So, if you look at, now we have again three unknowns; that is x , y and λ . In order to find that what we will do, we will follow the same, maybe formula what we did for the profit maximization. We will find out $\frac{\partial L.C.}{\partial x}$, we will find out $\frac{\partial L.C.}{\partial y}$, and we will find out the $\frac{\partial L.C.}{\partial \lambda}$. And after getting the equation, again we can get the value of x as $1.5y$.

So, that comes to y is equal to 200 and x is equal to 300 , and from there we can get the value of the cost, and we can get the value of the Lagrangian multiplier. So, what is the essential difference between the substitution technique and the Lagrangian technique? We used both the methods to solve the constrained optimization problem, and what is a constrained optimization problem. A constrained optimization problem is one, where we maximize the profit function or minimize the cost function, with a constraint that is in the form of the other variable. So, in case of substitution technique what we do. We substitute the, we represent the value of one variable in terms of the other variables, and

then we substitute that value in term of the other variables in the objective function, whether it is a profit function or whether it is a cost function. Then we solve for it, and when get one value we represent that in term of others and get the final value of the other variable also.


And in case of Lagrangian multiplier, Lagrangian multiplier method, we form a Lagrangian function on the basis of a constrained, at that objective function and solve the objective function through the partial derivative method, in order to know the value of unknowns, and what are the unknowns over here. Unknowns always in term of the variable, two variables those are in the objective function, here typically in the case of the x and y. So, we started our discussion for this typical topic. We started our discussion with the relationship between different variables, whether its linear, co linear may be non-linear or curvilinear.

Then we discussed different function attached to different kind of functions; linear, non-linear, and the curvilinear. Then we saw. Then I think we discussed the method of how to solve the different functions, and then we talked about the optimization problem, where we maximize the profit and minimize the cost. And in today's class, we have talked about the optimization with a constrained, using both the method; that is Lagrangian method and the substitution technique.

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Managerial Economics

- Nature of the managerial problem and use of functions
- Method of formulating a function
- Estimation of a linear function : A simple regression technique
- Multivariate regression
- Tests in multivariate regression estimates

 Source : Managerial Economics; D N Dwivedi, 7th Edition
Prof. Trupti Mishra, School of Management, IIT Bombay

So, now we will move to a new kind of technique; regression technique, that generally used, again to understand the relationship between the two variables, two economic variables. And what are the things, what we are going to discuss in case of a regression technique. We will talk about a nature of managerial problem, and here there again the use of function, how we use the regression technique. Then we see how to formulate a function, so that we can estimate with the regression technique. Then we do an estimation of a linear function using the regression technique, and also we will see the multivariate regression and few taste in the multivariate regression estimate.

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Managerial Economics

Regression

A statistical technique used to qualify the relationship between interrelated economic variables.

Used in physical and social studies, where problem of specifying the relationship between two or more variables is involved.

↑ Estimation of the coefficients of the independent variables.

↓ Measurement of the reliability of the estimated coefficients

Source : Managerial Economics; D N Dwivedi, 7th Edition

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So, before going to the regression techniques and method to solve, or method to get the value through it regression technique, now we will understand, what is a regression? So, regression is a statistical technique, used to qualify the relationship between the interrelated economic variable. So, we know that economic variables are interrelated, and there are number of methods may be through graph, through function, or through may be the mathematical relationship, we always explain the value of the economic variable. So, regression technique is a mathematical technique, what basically quantify the relationship between the two variables. So, in general sense, we always says that two variables they are positively related or the negatively related. So, regression technique is one step ahead of this, and it gives the exact magnitude of relationship between two variables, that how they are related, even if they are related positively or negatively, what is the extent of relationship, what is the magnitude of relationship?

So, regression technique, that is why regression is the statistical technique or the mathematical technique, used to quantify the relationship between the interrelated economic variable. It is generally used in physical and social studies, where the problem of specifying the relationship between two or more variable is involved. So, either it used in the physical, or it is used in the social studies. And particularly in this case, where the relationship is, whether may be the problem is to specifying the relationship, or specifying the magnitude of relationship between two or more variables. Now, here in the regression technique what we do. We do the estimation of coefficient of the independent variable, and also we do the measurement of the reliability of the estimated coefficient. So, using the regression technique, the first step is to estimate the coefficient of the independent variables, and the second one is the measurement of the reliability of the estimated coefficient.

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Managerial Economics

Nature of Managerial Problem : An Example

Suppose a Manager spending Money on advertisement to promote sale of his Firm's product.

Sales has been increasing but not continuously.

Manager's problem is to find an answer to:

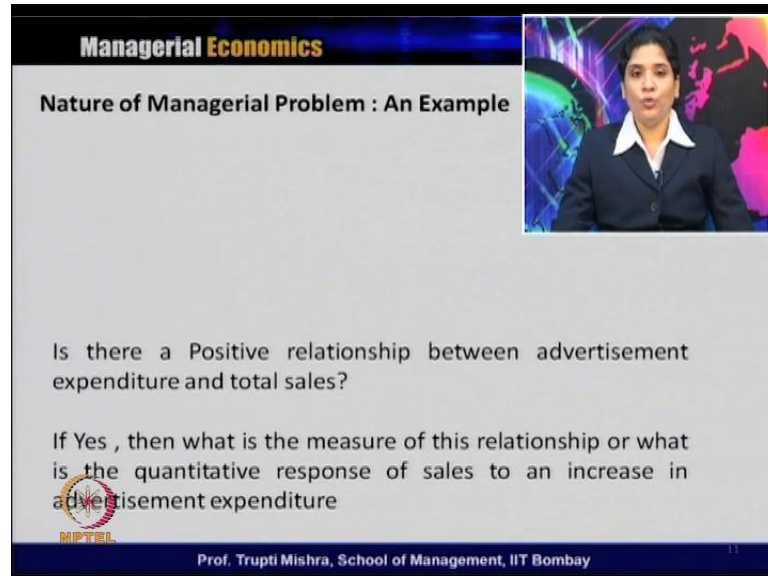
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So, before getting into the regression technique, let us understand, may be in what kind of managerial problem we need to use the regression technique. Or where is the case where we use the regression technique, in order to understand the relationship between the two variables. Let us take an example, suppose a manager spending money on advertisement to promote sales of his firm's product. So, manager is spending money on advertisement on promotional activity to promote the sale of his firm's product. Reacting to this, spending money on advertisement, sales has been increasing, but not continuously, sometimes it is increasing more, sometimes it is increasing less and some

time it is constant. Now, here what is the managerial decision problem or what is the nature of the managerial problem.

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Managerial Economics

Nature of Managerial Problem : An Example

Is there a Positive relationship between advertisement expenditure and total sales?

If Yes, then what is the measure of this relationship or what is the quantitative response of sales to an increase in advertisement expenditure

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The managerial problem here is to find an answer to, is there a positive relationship between the advertisement expenditure and the total sales, because the manager is spending a good amount of money on the advertising expenditure. The first question always comes whether it is affecting positively to the total sales. Second one if at all it is affecting positively to the total sales, then what is the measure of relationship, or what is the quantitative response to the sales, to an increase in the advertising expenditure. So, first one is, how they are related, and that always we have discussed, and always we can represent in the different form. Here the focus is the second one, what is the measure of the relationship or what is the quantitative response of sales, to an increase in the advertisement expenditure.

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Managerial Economics

Nature of Managerial Problem : An Example

Year	Ad Expenditure	Sales
1995	5	45
1996	8	50
1997	10	55
1998	12	58
1999	10	58
2000	15	72
2001	18	70
2002	20	85
2003	21	78
2004	25	85

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So, this how we can do, maybe we take the data from the, in a specific time period. Or maybe we take a time series data and there we can say, how the advertisement expenditure and the sales they are related. So, if you look at in the table; the first one gives a time line in term of the year, second one is the advertising expenditure, and third one is the sales. So, if you look at in different years, the advertising expenditure there is some amount of money it is going on increasing or sometimes it is. If it is look at, it is going on increasing from 5 to 8, 8 to 10, 10 to 12 and so on and so forth. And the sales also, it is going on increasing, but if you look at the increase in the sales is not may be on a increasing manner; sometimes it is increasing, sometimes it is increasing less, sometimes its increasing high, or sometimes it is increasing, maybe it is just constant. So, if you look at between the year 1998 and 1999, the sales is remain constant, even is the may be the expenditure has increase or the expenditure has decreased.

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Managerial Economics

Nature of Managerial Problem : An Example

A more clear and certain answer to these question can be found by plotting the sales data against the advertisement expenditure.

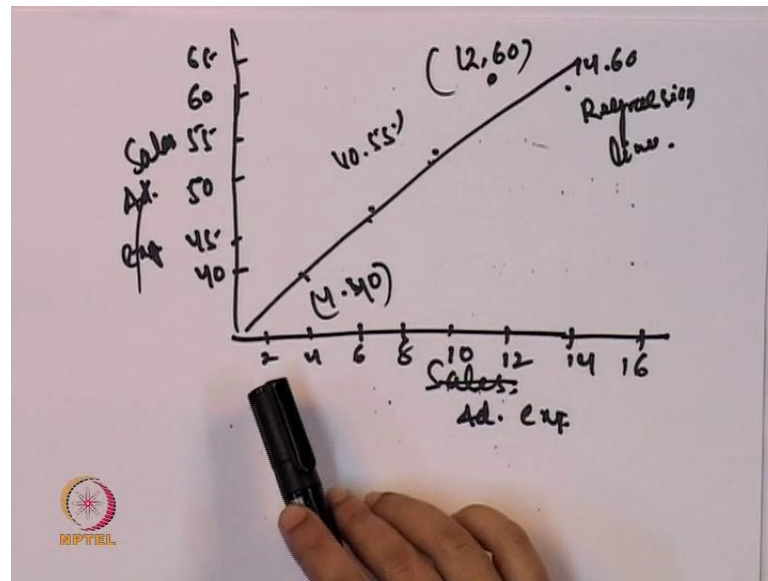
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14

So, in this case in order to understand this, may be how to plot it, or how to, may be presented this kind of relationship. So, may be a more clear and certain answer to this question can be, find out by plotting a sales against the advertisement expenditure. So, whatever we have shown in the previous graph, may be previous table, maybe we can represent that in a graph and we can say that, whether is a clarity that what kind of relationship, between the advertisement expenditure and the sales, whether it is positive whether it is negative; that is getting address the first question. And the second one is, how they are related; like whether what is the magnitude of relationship, whether the advertisement expenditure is increasing, and correspondent with that there is more increase in the sales, less increase in the sale, or what is the percentage change in the sales?

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So, if you plot this in a scatter diagram. So, maybe we take sales on the x axis, and advertisement expenditure on the y axis. If you can plot it, then its 2 4 6, maybe 8 10 12 14 16. Similarly, we can say 40 may be 45 50 55 60 65. So, now if you look at, maybe you take a specific year. Specific year may be the sale is 6, sales is 8, sales is 10, or sales is that and this is. There is a small change over here, maybe we are taking the add expenditure over here, and we are taking the sales over here. So, if you can plot it, then maybe this is a point, where the advertisement expenditure is 4 and the sales is 40. Similarly, maybe we are taking one point here; that is 8 and 50 point over here; that is 10 and 55.

So, if you look at, each combination gives us a, combination between the, or each point gives us a combination between the advertisement expenditure and the sales. So, similarly maybe this is a point, where it is 10 is 55 or we get a combination over here that is 14 and 16. So, if you draw a line over here, this is basically the regression line. So, regression line is used for what, maybe this regression line what we can say, or this is the line where, there are different combination of sales and advertisement expenditure. It is not clear that all the point will be on the same line, there are possibilities that, there may be one combination, where the advertisement expenditure is 12, and the sales is 60. So, in that case this is the combination; that is supposed 12 and 16. So, if you look at this combination is this point is not line on the regression line. So, if you put it in a scatter diagram.

So, this is what, this representation is generally called as a scatter diagram, and if you put this in a scatter diagram, it shows a relationship between two variable and again it is not sure, that when the advertisement expenditure moves from 2 to 4 or 4 to 6 or 8 to 10, what happens to the exactly the change in the sales from 40 to 45, 45 to 50 or 50 to 55. What is the exact nature of change, between the advertisement expenditure and the sales. So, scatter diagram, just we are doing a graphical representation of what we presented on the table, and it gives the different, may be combination, and each combination gives you a data on the advertisement expenditure and the sales. So, the primarily how the manager solve this problem, primarily manager solve this problem, by plotting it in a scatter diagram and they generally say that how they are related, but the question comes here, that whether through the scatter diagram, or whether through this table, or whether through this real data and real information, we get the exact nature or exact relationship between the two variable, and in this case typically the advertisement expenditure and the sales.

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Managerial Economics

Nature of Managerial Problem : An Example

For a Manager the requirement is to know the exact relationship between advertisement expenditure and sales for future planning.

Scatter Diagram does not answer this.

This question can be answered by Regression Techniques

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May be the answer is no, because it is not shows the exact relationship between the advertisement expenditure and the sales of future planning. Neither scatter diagram nor the real life examples, or not the real life data generally gives this answer. So, this question can be answered by the regression technique, and particularly in this case we use the regression technique, and regression technique gives the exact magnitude of relationship. The exact relationship between the two variables, those are in question,

maybe it is, in this case may be this is the advertisement expenditure and the sales. Now, for the regression technique, again there are two steps. So, we have already understood, what is a regression technique? So, regression technique is one, which gives quantification to the relationship between two economic variables.

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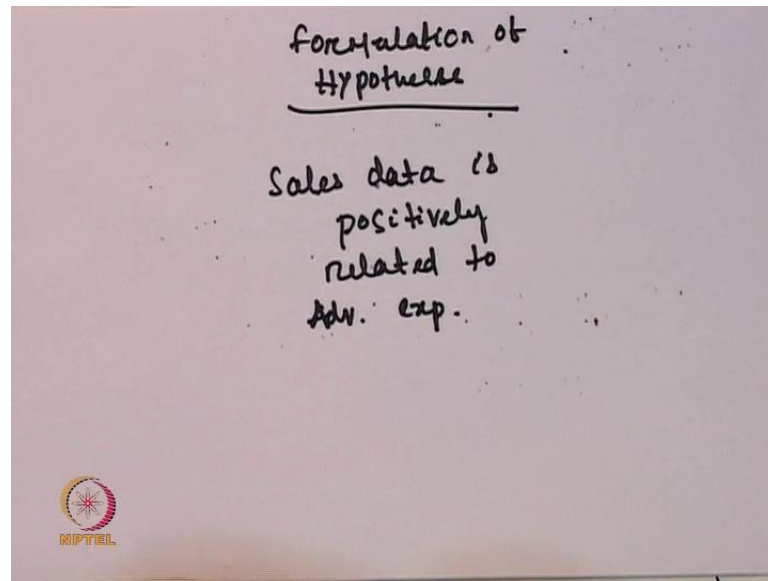
The slide is titled "Managerial Economics" and "Method of Formulating a Function". It outlines two steps in a flowchart:

- Formulation of a hypothesis**
 - It is done on the basis of the observed relationship between two or more facts or events of real life.
- Translating the hypothesis into a function**
 - Suppose a hypothesis, the sales growth is a function of ad-expenditure, this hypothesis can be translated into a mathematical function $Y = A + Bx$ where, $Y = \text{sales}$, $X = \text{ad-expenditure}$ and A and B are constants.

Source : Managerial Economics; D N Dwivedi, 7th Edition
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And how they do that, they follow two steps for that; one, initially they formulate a hypothesis, and it is done on the basis of the observed relationship between two or more facts or the events of real life. And second one, they translating the hypothesis into a function, and finally they evaluate, or finally they value the function in order to get the value of the data. So, there are two steps; one is formulation of the hypothesis, and second one is the transforming or the translating the hypothesis into a function. So, we will see the first one, how generally the hypothesis is being form, or how there is a formulation of the hypothesis.

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So, hypothesis is, if you look at, it is not may be a relationship between the two economic variable, rather it is a estimated relationship between two variable at, it is a probability that this two variables can react in a certain way. But the result is not known here, the result is unknown here. Suppose, we are saying here for example, advertising expenditure and sales growth, they are positively related. So, this is a hypothesis, this is statement on the basis of the observed data. We do not know that what is the outcome or what can be the result, whether it gives a positive relationship, or whether its gives a negative relationship. So, this hypothesis is one kind of herbal statement, from where generally we formulate a hypothesis, we formulate a function. We evaluate the function in order to note the exact relationship between these two variables. So, we will start with a formulation of hypothesis, generally how we formulate a hypothesis. So, here what means, take the hypothesis, we take the hypothesis sales data is positively related to advertisement expenditure. This is the hypothesis. For that what is the background observation we have to take, or what the background, may be the information we have to take. Now, what is hypothesis, as I was telling it is may be a formal statement about the relationship between two variables.

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Managerial Economics

Formulation of Hypotheses

Hypotheses is a postulate, an untested proposition regarding the relationship between any two or more variables of the real world phenomena.

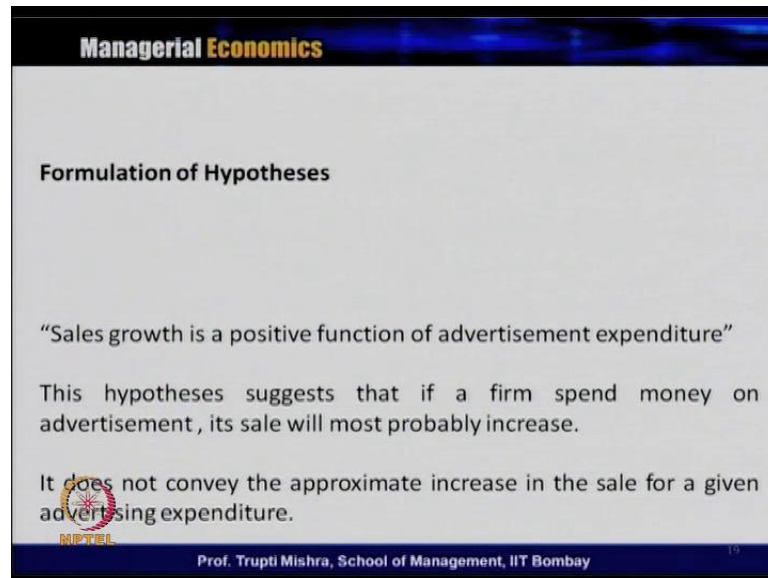
It shows only a probability of the event and serves as a guide for future action, cannot predict result of an action.

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So, it is a postulate, it is an untested proposition we have not yet tested, regarding the relationship between two or more variables in the real world phenomena. It shows only the probability of the event and serve as a guide for the future action, but cannot predict the result of an action. So, as we are discussing, the outcome is not known over here. It can be always take what be the probability of the event; like in this case if the advertisement expenditure increases, what will happen to the sales. So, the outcome of the event is unknown, only it can predict; that since the both of them they are positively related, when you increase the advertisement expenditure, the possibility is that the sales will also increase. So, this is your hypothesis is a postulate, untested proposition, regarding the relationship between two variable in a real life phenomenon.

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Managerial Economics

Formulation of Hypotheses

“Sales growth is a positive function of advertisement expenditure”

This hypotheses suggests that if a firm spend money on advertisement , its sale will most probably increase.

It does not convey the approximate increase in the sale for a given advertising expenditure.

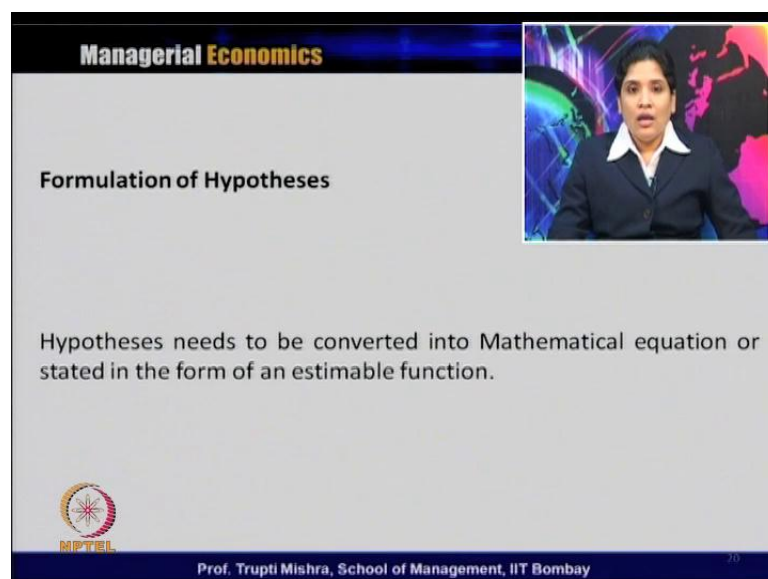
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19

Now, as we are just formulating the hypothesis, suppose you take the hypothesis, sales growth is a positive function of the advertisement expenditure. Now on the basis of that, the hypothesis suggests that, if a firm spends money on advertisement, its sales will most probably increase. Again it is a probability, there is no certainty that if the firm is spending money on the advertisement, the sales has bound to increase. There is a probability that the sales will increase, if the firm is spending money on the advertisement. It does not convey the approximate increase in the sales for a given advertisement expenditure.

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Managerial Economics

Formulation of Hypotheses

Hypotheses needs to be converted into Mathematical equation or stated in the form of an estimable function.

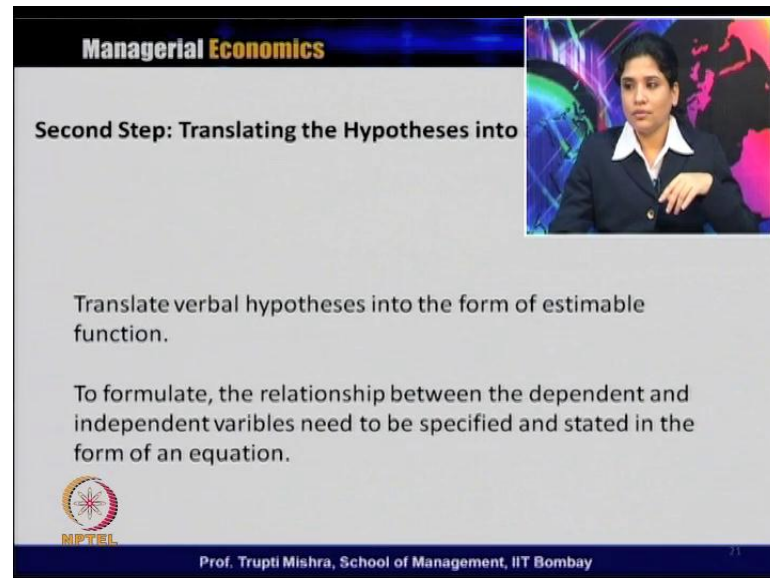
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20

So, hypothesis need to be converted into mathematical equation or stated in the form of an estimable function. Since it is a probability of event, we need to support it through the data, we need to support it through the function, in order to check whether the hypothesis is validated or not, or rather hypothesis goes or not, for that we need to convert it to a mathematical equation or convert it in the form of the estimable function.

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Second Step: Translating the Hypotheses into

Translate verbal hypotheses into the form of estimable function.

To formulate, the relationship between the dependent and independent variables need to be specified and stated in the form of an equation.

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So, there it comes to the second step, and what is the second step. The second step, when you translate the hypothesis into a function. So, it is a hypothesis, as I was mentioning it is a verbal statement on the basis of some observed relationship between two variables. So, in the second step we translate the hypothesis into a function, we translate the verbal hypothesis into the form of a estimable function. And to formulate the relationship, to formulate the verbal hypothesis into a estimable function, we need to identify, what kind of relationship is there, between the dependent and the independent variable. So, the relationship between the dependent variable and the independent variable need to be specified, and it is in the form of an equation. So, the form of equation can be linear, it can be non-linear, depending on the relationship.

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Managerial Economics

Second Step: Translating the Hypotheses into a Function

The form of equation can be linear or non linear depending on the relationship.

Hypotheses can be translated as:

$$Y = a + bX$$

Where Y = Sales, X = Advertising Expenditure, a and b are constant.

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22

So, hypothesis can be translated, as suppose in this case we take a equation, that is Y is equal to a plus b x, where Y is the sales x is the advertising expenditure, a and b are constant. So, Y is here dependent variable, x is independent variable. So, y is the sales x is the advertising expenditure a and b are the constant.

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Managerial Economics

Second Step: Translating the Hypotheses into a Function

The constant 'a' is the intercept, it gives the quantity of sales without advertisement, when X = 0.

Constant 'b' is the coefficient of Y in relation to X- Gives the measure to increase in sales due to a certain increase in advertisement expenditure.

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23

The constant a over here is the intercept, and it gives the quantity of sales without advertisement when x is equal to 0. So, there are two constant; one is constant a, another is constant b. Constant a is the intercept, it gives the quantity of sales without

advertisement when x is equal to 0, and constant b is the coefficient of y in relation to x , and it gives the measure to increase in sales, due to certain increase in the advertisement expenditure. So, if you look at b is directly related to the value of x . It gives the measure to increase in the sales, due to certain increase in the advertisement expenditure. So, b is the slope, a is the intercept, a gives the quantity of sales without advertisement, when x is equal to 0. And b is the coefficient of y in relation to x , and it gives the measure to increase the sales, due to certain increase in the advertisement expenditure. Now, task of analyst come here, to find out the value of a and b , because a is unknown over here, b is unknown over here, a gives us a value of sales, when there is no advertisement expenditure, and b gives us the value of the slope, which tells us that if advertisement expenditure increases by certain proportion, what is the exact proportion change in the sales.

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Managerial Economics

Second Step: Translating the Hypotheses into a Function

The task of analyst is to find the values of constant 'a' and 'b'

- Rudimentary Method
- Mathematical Method- Regression Technique

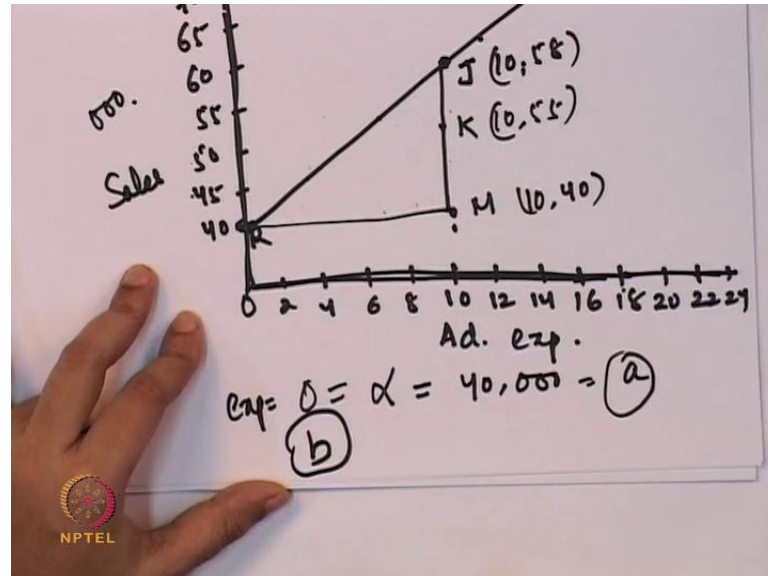
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So, the task of analyst is to find out, what is the value of a , and what is the value of b . There are two methods to find out the value of a and b ; one is the rudimentary method or the elementary method, and the second one is the mathematical method or the regression technique. So, we will see through using the elementary method or using the rudimentary method, how to find the value of a and b , and then again we will see the mathematical method or the regression technique, using that how to find the value of a and b . So, using in order to find the value of a and b , through the elementary method, we will use again the same data of the, what we are taking from the managerial problem, that is in the form

of what we present in the form of the scatter diagram, and then we will find the value of a and b over there.

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So, what we take in the y axis, and what we take in the x axis; x axis we take the advertisement expenditure, and y axis we takes the sales. So, this is 2 4 6 8 10 12 14 16 18 20 22 or 24. And here we take, may be 40 45 50 55 60 65 70 75 80 and 85 and 90. So, since we have, suppose the value of may be intercept it starts from here and we take a regression line. Suppose this is R and suppose this is L. So, if you take this may be R is 1 and L is the other combination, maybe there are different combination here of the advertisement expenditure and the sales. Suppose, we get a combination; that is, 10 and 58, so this goes to here, where your, suppose this point, this is point J, where it is 10 and 58. Or may be below somewhere that we get a combination K, which is again 10 and 55, or maybe we get a combination M, where it is, maybe it is somehow between 40. So, this is combination M, where we get 10 and 40.

Now, if you remember just now we discussed that the value of the intercept. So, if you look at, when the zero expenditure on advertisement expenditure, the total sales is 40. When the advertising expenditure is 10, maybe we have different combination where 10 is equal to, 10 is advertisement expenditure and 40 is the sales, K is the advertisement expenditure, 55 is the sale, and at the point J 10 is the advertisement expenditure and 58 is the sale. So, when it is equal to zero, we get the value of intercept; that is, alpha which

is equal to 40,000, if you are taking this in the zero unit. Now, what is the next task? Next task is to find out the slope of this, because this is the value of a . When expenditure is equal to 0, α is, value of intercept and this is the value of a . Now, second what we have to find out through the rudimentary method or through the elementary method, we need to find the value of b .