Managerial Economics
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Lecture - 5
Basic Tools of Economic Analysis and Optimization Techniques (Contd…)

Welcome to the fifth session of managerial economics; we were discussing the first module of managerial economics; that is, introduction to managerial economics. And in this topic, if you remember in the last session, we discussed about the relationship between economic variable, we termed them in term of linear, non-linear. Then we discussed about that, how these variables are related and what is the method to capture the relationship? So, one was, what we discussed in the last class is slope; slope is basically, it captures the change in between the dependent variable and the independent variable. But when the change in the independent and the dependent variable is very small; in that case, the general method to calculate slope or just finding the relationship through the slope sometimes does not serve the purpose. And that is the reason we introduce the concept of differentiation. And differentiation is a method, differentiation is a approach through which we can calculate the or capture the change in the dependent variable, due to change in the independent variable.

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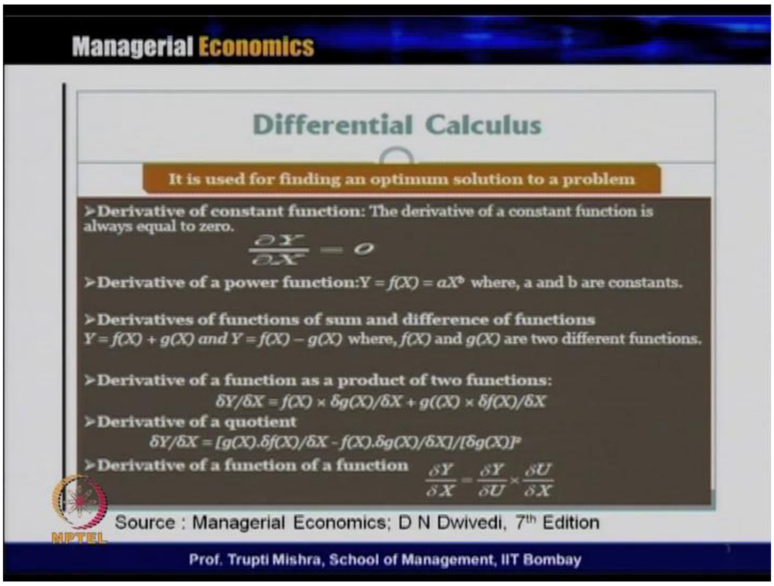

So, in the last class, we have taken just a general functional form, and identified the differential calculus method. In this class, what we will do? We will take some different kind of function like constant function, power function, derivative of function of sums and differences of function. Derivative of function is a product of two function, derivative of a quotient, and derivative of a function of a function, and also we will take a function, where there are multi variables are there. So, we will just check one by one how the differentiation is generally used, or how that through differentiation, how we calculate the relationship between the two different variable; independent and dependent variable, in the different kind of functional form.

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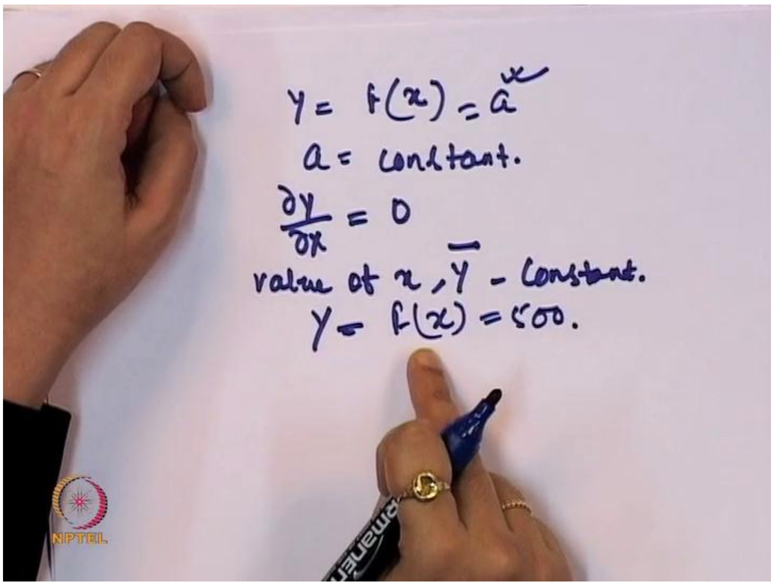

So, we will start with a constant function, and where if we look at functional form is, y is equal to it is a function of x. So, y is a function of x; and which is equal to a. Now what is a over here? a is the constant. Now, we will take a first order derivative of this functional form. In this case, what is the first order derivative? We have to take the derivative with respect to x, and which will come as 0. Why it will come as 0, because it is a function of x and that is in the form of a constant. And when you are taking the first order derivative with respect to a constant, the derivative the value of the derivative will be equal to 0. So, whatever the value of x, so whatever the value of x, y remain constant here. There is no change in the y, because y is represented in term of a function, and function is a constant over here. So, may be if you modify this weight and if you put a value, suppose you take a functional form; y is a function of x and which is equal to 500.

This is the value of the intercept; this is not the value of the slope here. So, in this case, whatever may be the value of x, there is no change in the y, y remains constant, because this is a derivative of a constant function.

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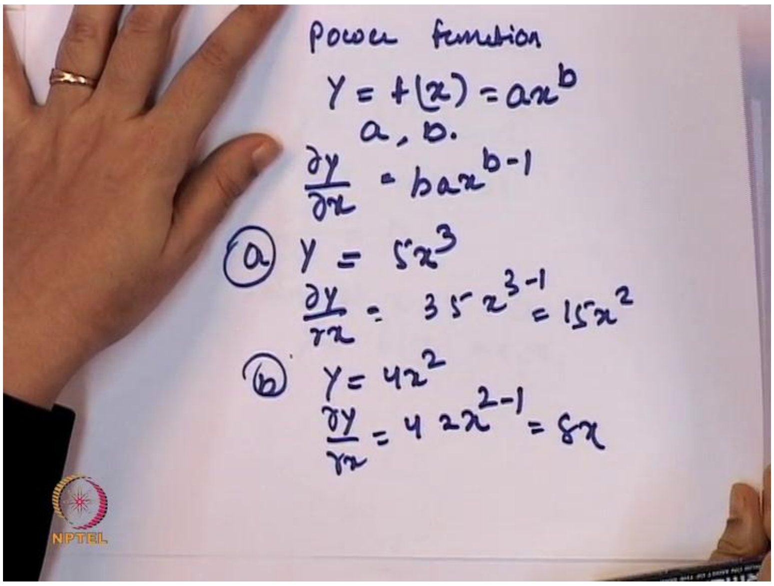

Now, we will discuss the second kind the derivative of a power function. So, here the functional form has a power on it. Suppose here again y is the dependent variable, x is the independent variable. It is a y is a function of x, and which may be a x to the power b. Now here there are two constant; one is a and another is b. So, y is a function of x, which is a x to the power b, and in this functional form we are getting two points; one is a and second one is b. Now if you take a derivative of this, how this will become, what is the outcome over here, del y with respect to del x; that will be b a x b minus 1. Now, let us give a number to this functional form. Suppose take a functional form where y is equal to 5 x cube. So, x to the power cube. So, in this case, y is a function of x and the x has also a power on it.

So, now what is the derivative or how we can check the relationship, del y to the power del y with respect to del x, so that will come as the 3 5 x 3 minus 1. So, this is 15 x square. Similarly, you take 1 more functional form, where y is equal to 4 x square. Now how we will find the differentiation over here. So, del y by del x is equal to 4, then 2 x, 2 minus 1, so this is 8 x. Similarly, suppose we take one more functional form, which has also a power on it.

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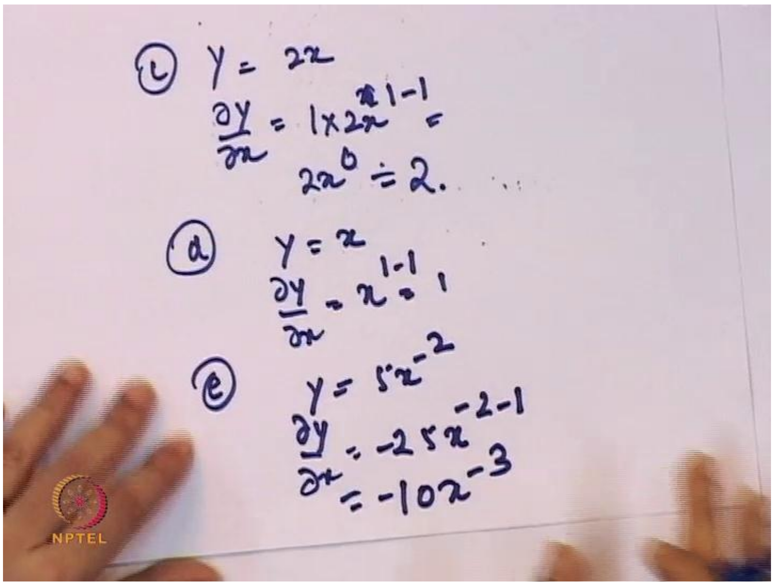

So, if you look at, suppose we take y is equal to 2 x, x to the power 1 over here. Now if you take del y by del x, then this is, this comes to 1 multiplied by 2 x then 1 minus 1, so 2 x to the power 0 which is equal to 2. Suppose, take one more functional form in this category of derivatives; like this is a derivative of a power function. So, let us take y is equal to x. Now what is del y by del x, del y by del x is equal to x 1 minus 1 which is equal to 1. Similarly, if you take one more functional form, which has a negative power y is equal to 5 x minus 2. So, this is del y by del x minus 2 5 x minus 2 minus 1, and this comes to minus 10 x minus 3. So, this is how we solve when we get a power function, and what is the significance of this power function? The functional form has a power, and it can take any value that is from less than 0 or that may be the greater than 0. Now, we will discuss about the third category of the derivative; that is derivative of a function of sum differences of functions. Sometimes if you will find it is not only a single variable, there is also a summation added to it.

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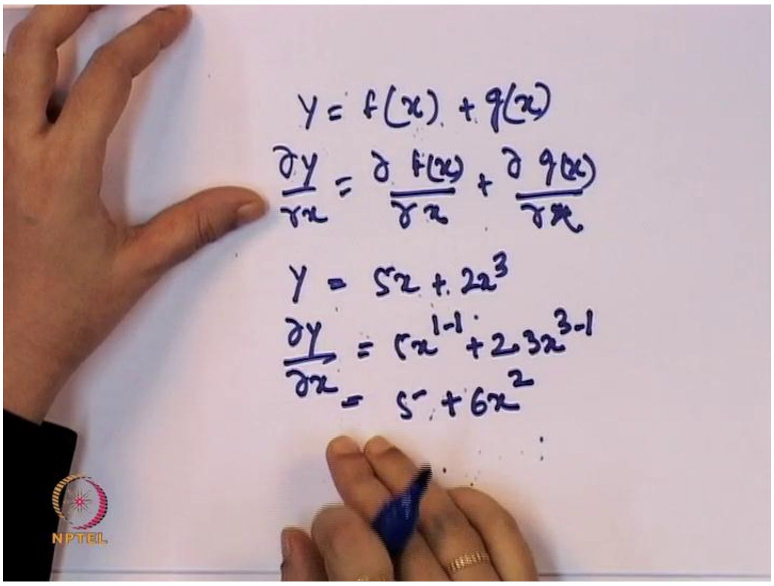

So, if you take a example where y is a function of x and also it is a g is a function of x. So, y is dependent on this x function of x and also function of this x. So, in this case how we take the derivative, in this case the derivative is del y by del x is equal to del f x with respect to del x, plus del g x with respect to del x. Similarly, now if you give a numerical term to this, suppose we take a functional form y is equal to 5 x plus 2 x cube. So, now taking the first order derivative del y by del x is equal to 5 x 1 minus 1, plus 2 multiplied by 3 x 3 minus 1. So, this comes to 5 plus 6 x square. Now what happens if it is not a case of addition, if there is a difference, it is not a sum rather it is a differences of function. Now, we will take another functional form, in order to show that when there is a subtraction, or when the y is dependent on x and the functional form has a subtractions of, the functional form has a differences between two variables, how to get the derivatives of this.

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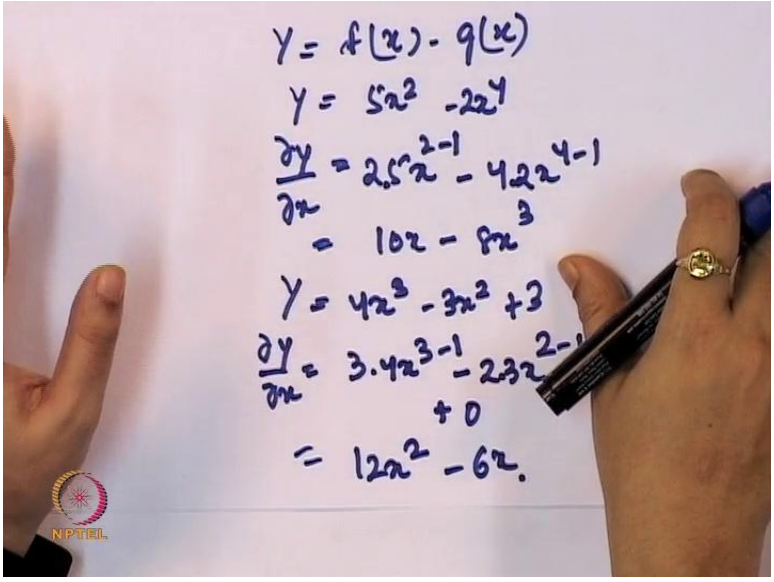

So, let us take here that y is a function of f x minus g x. Let us give a numerical value to this, so y is equal to 5 x square minus 2 x 4. Taking the first order derivative del y by del x is equal to 2, 5 x 2 minus 1, minus 4 2 x 4 minus 1. So, this comes to 10 x minus 8 x cube. So, it is like, if the y is dependent on a function which has the addition or which has the subtraction, in that case we have to take the partial derivative with respect to both the variable. So, in this case like it is f x and in the g x. Now, suppose apart from this two variables, let us add a constant also in the functional form. Let us take a functional form, where y is equal to 4 x cube minus 3 x square plus 3. So, now how we will get the derivatives over here, so del y by del x is equal to 3 4 x 3 minus 1 minus 2 3 x 2 minus 1 plus 0, because the first order derivative of a constant would be always equal to 0.

So, this comes to 12 x square minus 6 x. So, in case of a derivative, where the function is of sums or the differences, we get the functional form in term of the negative value; like there is a differences between two variable or there is a positive value, there is a summation between this two variables. Now, let us check, what is the next kind of a function? So, this is a derivative of a function as a product of two functions. So, in the last case we took care of summation, we took care of the subtraction. Now we will say that, when the derivative of a function, where the function is as a product of two function.

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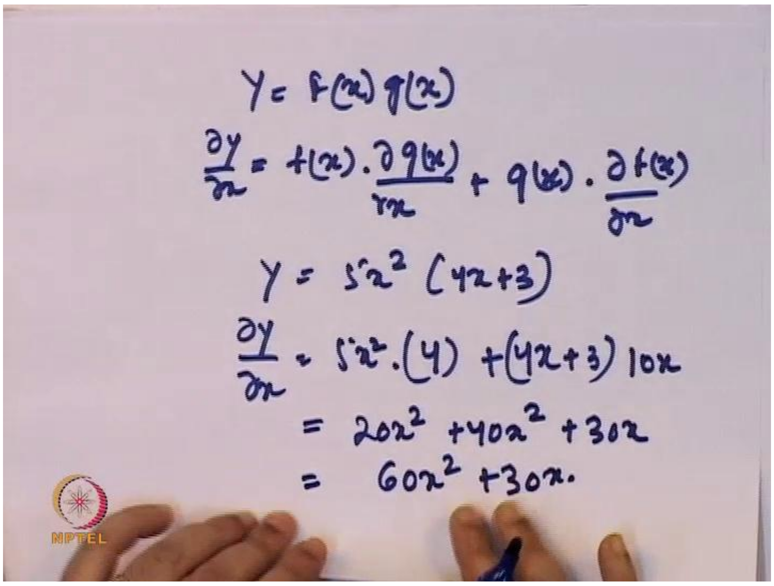

So, in this case how the functional form will be, the functional form will be Y is function of x and g x. So, the functional form is, product of two function; that is f x and g x. Now, how to take the derivative over here? Here, we take the derivative keeping others as constant, and in the second part we take the derivative of the second part keeping the first part as the constant. So, in this case, this is f x d g x, d x plus g x d f x, d x. So, now let us take a numerical function to get more clarity. Suppose we say that y is equal to 5 x square 4 x plus 3. So, here y is dependent on two function; one is 5 x square second one is 4 x plus 3. So, this is a case where the function is a function of 2 other function. So, in this case how to solve, how to find out the derivatives. So, in this case 5 x square plus multiplied by 4; that is the derivative of del 4 x plus 3, plus 4 x plus 3 as constant and taking the derivative of 5 x square.

So, if you're taking the derivative of 5 x square, how much it will come, it will come as 10 x. In the first case we have to keep 5 x square as the constant, and we have to take the derivative of 4 x plus 3, so which will come as 4. So, the first case f x is constant, the derivative is with respect to g x. Second the g x is constant, the derivative is with respect to f x. So, in this case, if you again simplify this, then this comes to 20 x square, plus 40 x square, plus 30 x, so which come to 60 x square plus 30 x. Now let us take another functional form and where the function is again as a function of two other functions. So, it is a product of two other function serve as a functional form for the independent variable, dependent variable where the value is defined by the independent variable.

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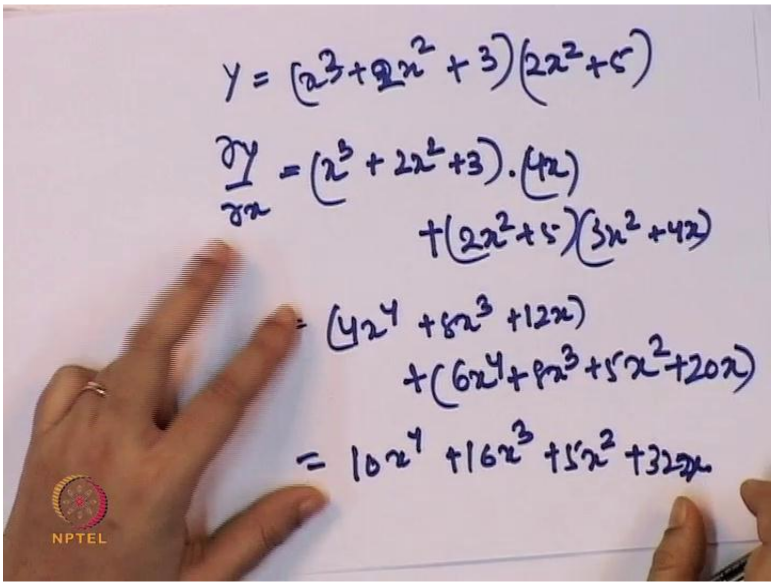

Now, suppose take a numerical example, where y is a function of x square plus 2 x square, x cube plus 2 x square plus 3. So, this is one function, and the other function is 2 x square plus 5. So, there are two functions over here. So, taking the first order derivative del y by del x is equal to. So, in this case the first one will be constant now; x cube plus 2 x square plus 3, multiplied by the derivative of the second function. So, the derivative of the second function is 4 x. Similarly, now the second term 2 x square plus 5 will be constant, and we have to take the derivative of the first function; that is 3 x square plus 4 x. So, simplifying this we get 4 x 4, plus 8 x cube, plus 12 x for the first one.

For second one, it is 6 x 4 plus 8 x cube, plus 5 x square, plus 20 x; that comes to 10 x 4 plus 16 x cube, plus 5 x square plus 32 x. So, when the functional form is a product of two function, in that case basically we take the derivatives by keeping the first factor, first function as constant and taking the derivative of the second function. And in the second case we keep the second function as the constant and we take the derivative of the first function, when the functional form is product of two functions. Next we will check how we generally solve or how generally find the derivative of a quotient.

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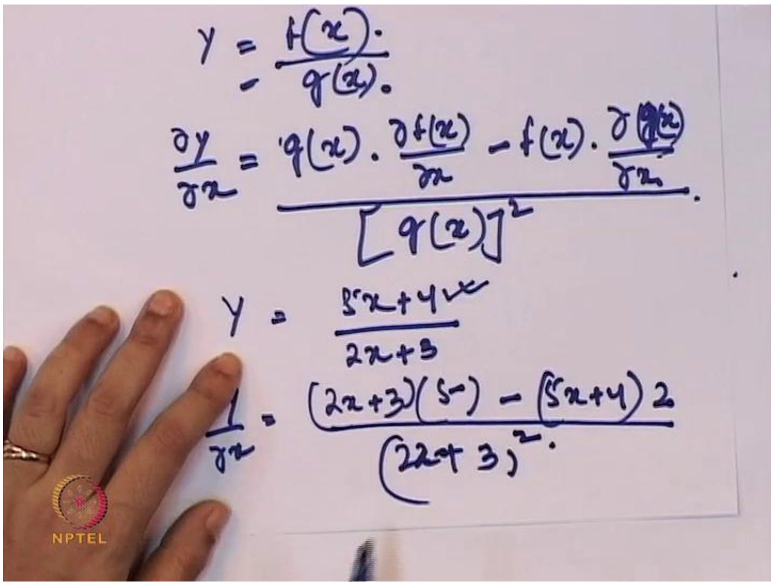

So, here the functional form is y is a function of x, so let us write a functional form. This is a general functional form, but when you take that, this is derivative of a quotient, here the functional form involve a quotient of two function; that is f x and g x. In this case how to get the derivative, what is the formula to calculate the derivative? So, del y by del x is equal to g x multiplied by derivative of f x with respect to del x. In the first case g x is constant the derivative of the f x, and second case f x is constant derivative of derivative of g x, divided by g x to the power whole square. So, when the derivative involves a quotient, or the function of a quotient, two function of a quotient, in this case the derivative is. Derivative generally we calculate derivative by following this formula; that is g x multiplied by d, derivative of f x with respect to x, second term minus second term remain constant, f x remain constant, we take the derivative of g x with respect to d x, as a whole this divided by g x to the power if it is the whole square of this. Now, let us take a functional form to understand this, in a numerical term. So, y is function of 5 x plus 4 and 2 x plus 3. So, in this case, how we will get the derivative del y by del x is equal to 2 x plus 3. Then we take the derivative of this; that is 5 x plus 4 that comes to 5, minus we have to keep f x as constant. So, 5 x plus 4 is the constant, we have to take the derivative of 2 x plus 3, so that comes to 2, as a whole 2 x plus 3 whole square of this.

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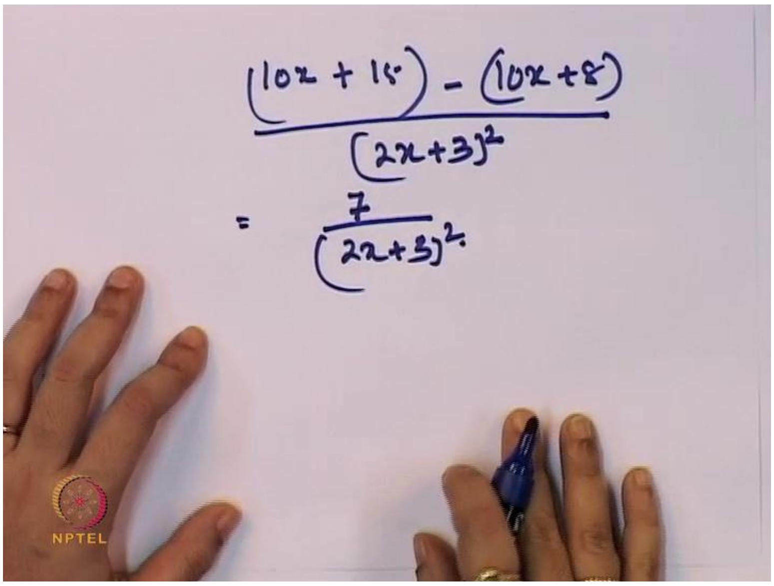

So, if you simplify this, this comes to 10 x plus 15 minus 10 x plus 8 divided by 2 x plus 3 square. So, this is equal to 7 by 2 x plus 3 square. So, in case of a functional form, when there is a it is a quotient of two function, in this case we generally follow a formula, which give one function constant by taking the other function, derivative of the other function, and finally it is divided by the functional form of the second, the whole square of the second function is divided by the, whatever the derivative and the constant of the other two function. So, now next we will see the last category; that is when the function is a function of function, in that case how to find out the derivative.

So, we discuss about a power function, we discuss about a constant function, we discuss about a function which has sum and differentiation differences, we discuss about a function which has the product, and we discuss about a function which has in the quotient form. So, now we will discuss of a functional form which is function of a function.

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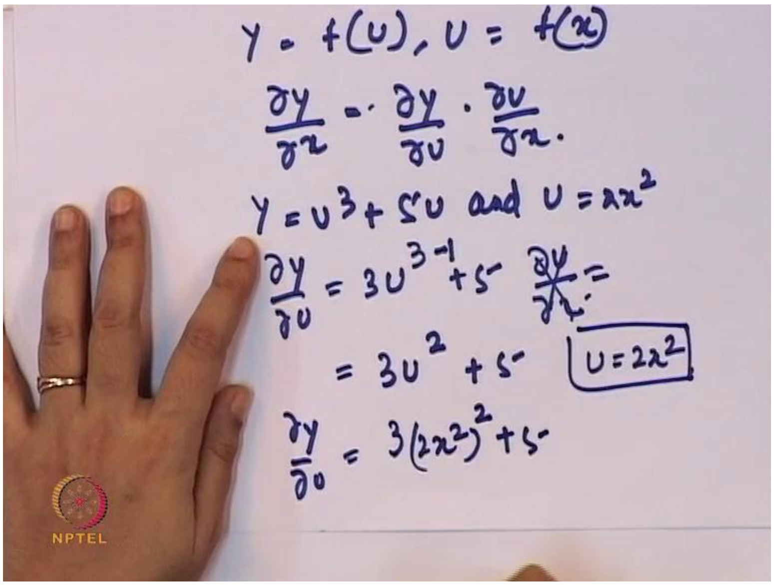

So, now takes a functional form, where y is a function of u and u is a function of x. So, in this case how to find out the derivative; del y by del x is equal to, del y by del u and del u by del x. So, here the derivative has also two term; one is del y by del u, and second one is del u by del x. So, let us take a again a numerical functional form to understand this more. So, y is equal to u cube plus 5 u ,and u is equal to 2 x square. So, del y by del u is equal to 3 u 3 minus 1 plus 5 and del u by del x is equal to. So, before finding this del u by del x, lets simplify it more this del y by del u, so this comes to 3 u square plus 5. Now, what is del y by del u. So, u is equal to 2 x square as we know, this is the functional form u is equal to 2 x square. So, if you put this value over here, so then 3 2 x square, to the power again square plus 5, so which comes to 3 2 x square, again 2 x square plus 5. Simplifying it further, so 3 multiplied by 2 x square again multiplied by 2 x square.

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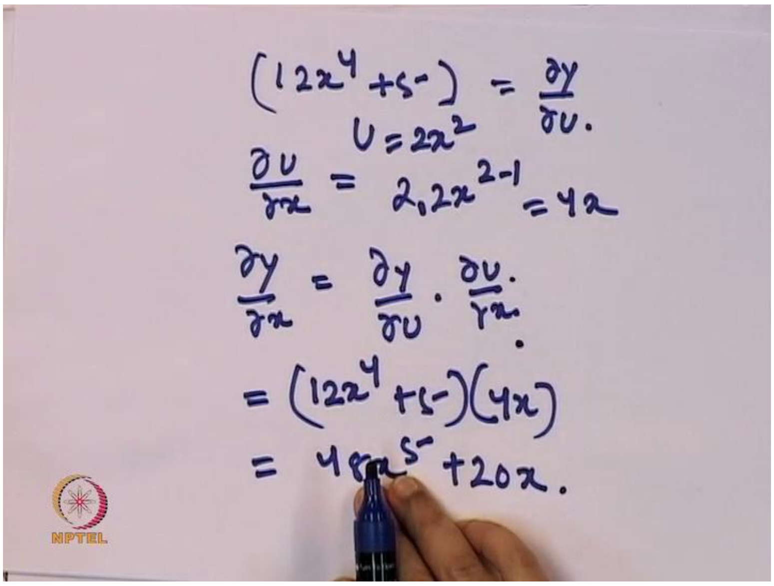

So, if you simplify this again, so this comes to 12 x 4 plus 5. So, this is what, this is our del y by del u, this is the first part of it. Now, coming to the second part, second part is derivative of u with respect to x, why the derivative is u derivative of u with respect to x, because the u is in a functional form, and u here is the dependent variable, the value of the u depend on the value of the x. So, taking the derivative of u with respect to x, we get 2 2 x 2 minus 1 which is equal to 4 x, because u is equal to 2 x square. So, del u by del x is 2 multiplied by 2 x 2 minus 1 which comes to 4 x. So, now taking this del y by del x, which is again a function of del y by del u, and del u by del x. So, that comes to 12 x to the power 4, multiplied by 5 multiplied by 4 x. So, this comes to 48 x 5 plus 20 x. So, what is happening over here. Here the functional form associated with the dependent variable, it has two functions, it is a function of function. So, the variable is directly not getting, directly not getting generated from the functional form rather, because the functional form is again a function of the other functional form.

So, in that case generally we first find out the value of y, which is with respect to u, then we find out the value of u with respect to x. And then finally, we find the derivative of y with respect to x, which is a product of derivative of y with respect to u, and derivative of u with respect to derivative of x. So, taking the same example it comes to 12 x to the power 4 plus 5 multiplied by 4 x, which come to 48 x to the power of 5 and 20 x. So, in this case, we discuss about a constant function, we discuss about a power function, we discuss about a function with sum and differences, we discuss about a function which has the product, we discuss about a function which has the quotient value, we discuss about a function where the function is again a function of some other variable. Now, let us analyze one more thing, where the functional form has several independent variables. So, here we generally take a functional form where y is a function of x.

Now we will introduce a case, where the value of y is not only getting valued in the form of the value of x rather than, there are several other independent variable those were deciding the value of y. We'll take a value, we take a functional form with the multivariable and then we will see how to find out, how to use the differential calculation, or how to find out a derivative the first order derivative in order to understand the relationship between the independent variable and the dependent variable. So, in this case how the functional form will look like. So, let us take a case of a demand function, because in case of demand function the primary variable it effects demand is price, but if you look at there are several other in variables, those who effect the demand for the product. So, we will take a case of a typical demand function, where we assume that demand is not only influenced by the price, there are certain other variables also influence the demand. And we will see how we find out the relationship by taking the derivative with respect to different variable.

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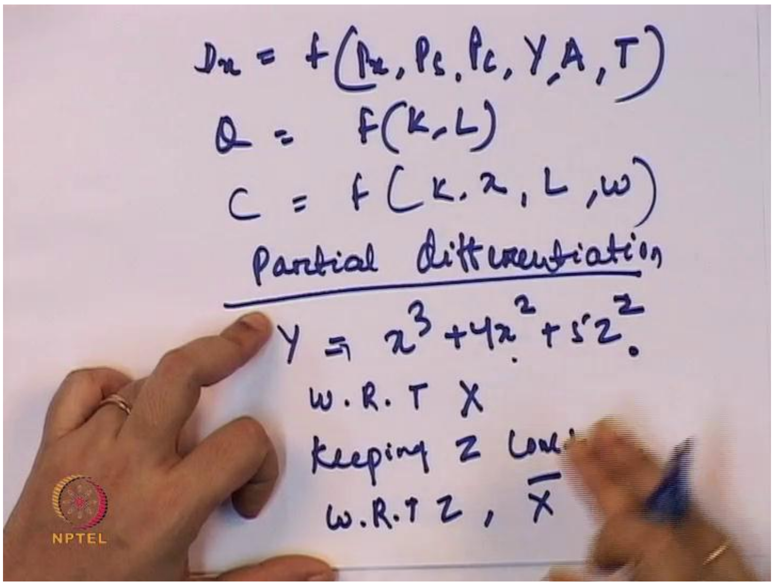

So, here if you're taking lets write a demand function; that is demand for x, it is dependent on price of x, dependent on price of the substitute goods, dependent on the price of the compliment goods, dependent on the income, dependent on the advertisement, and dependent on the taste of the consumer. So, here p x is the price of product, p s is the price of substitute good, p c is the price of compliment good, y is the income a is the advertising expenditure, and T is the taste of the consumer. So, similarly there are some other function what gets used in economics typically, where the dependent variable is dependent variables are dependent on many independent variable not only one independent variable. So, this is one example is demand function, similarly there is also example of production function. So, if you take a production function, it is always a function of capital and labor.

So, at least typically in the long run, the function is not only capital or the function is not only labor, rather its function of both capital and labor. So, if in this case if you consider Q is the output, which is dependent on the capital and the labor. In this case, the value of output dependent on two independent variable; that is value of capital and value of labor. Similarly, we can take a cost function, where it is a function of capital, where it is a function of the rent what we pay for the capital, it is a function of the whatever the price we pay to the laborer, it is a function on the whatever the wages and the salary we are paying to the. So, in this case particularly, when the functional form has many independent variable. In this case generally we use the partial differentiation partial differentiation, in order to understand the relationship between the two variable; that is x and y.

Now, let us take a functional form. So, here y is a function of x cube plus 4 x square plus 5 z square. So, we have two variables here, those who are influencing y, y is the dependent variable and x and z they are the independent variable. So, now how we will find the derivative or how we will establish the relationship between y and x, and x and z. So, the first thing what we will do, we will find out the derivative with respect to x keeping the z constant. And in the second case we will find out the derivative with respect to z keeping the x as the constant. We need to do a partial differentiation with each variable, in order to understand their relationship with the dependent variable, and we cannot take the derivative of both the variable simultaneously, rather we will keep one as the constant and the derivative of the other in order to understand the relationship.

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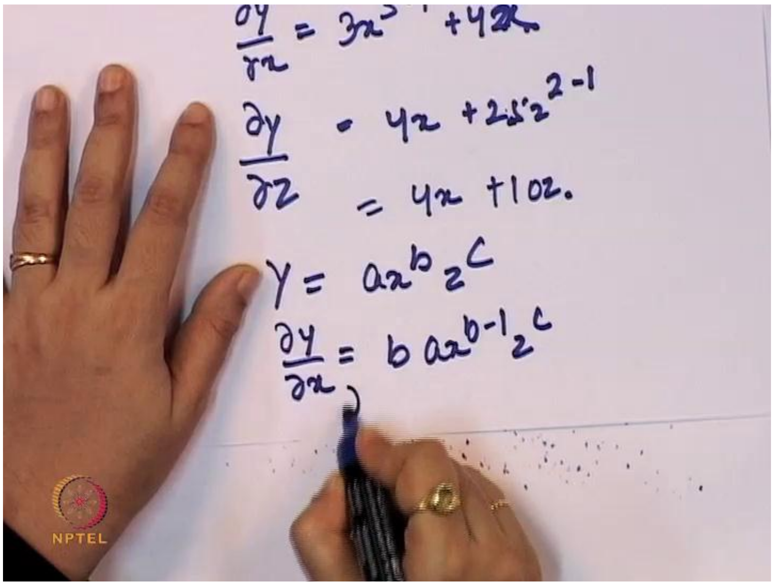

Now, taking the same functional form, where y is a function of, y is a function of x cube plus 4 x square plus 5 z square. We will first find out del y with respect to del x, so this will come as 3 x, 3 minus 1 plus 4 z. Similarly, how we will find the second one; that is del y and with respect to del z, which is 4 x plus 2 5 z 2 minus 1. So, this is 4 x, because z is constant. So, in this case we get 4 x plus 2 x, 2 multiplied by 5 z; that is 2 minus 1, so that comes to 4 x plus 10 z. Similarly, if you take a functional form, suppose y is equal to a x to the power b, and z to the power c. So, in this case, this is a case of your power function, where the y is dependent on x and z, so in this case how we find the derivative. Derivative of y with respect to x keeping the value of z constant; that is b a x b minus 1 z c. And second is del y and del z keeping the value of x remain constant. So, that comes to a x b c z c minus 1. So, when you have a functional form which has many variables, and many variables particularly in the independent variable category, and where the dependent variable is dependent on many independent variable.

In that case in order to find out the derivatives, first we need to keep one variable constant, taking the derivative of y with respect to the other variable, and second case we need to keep the other variable as constant, and taking the derivative of y with respect to the present variable. So, with this, we almost completed that what kind of derivative or how the derivative is used in case of different functional form, whether the functional form has the constant, it has the power function or maybe it is a addition subtraction product quotient, or if it is a function of function. Now, next we will move into the optimization technique. Now what is optimization technique, if you know till now we have talked about. The relationship between economic variables, and we have understood that how to find out the relationship between two variables; that is through the slope or through the calculus method. Next we will come to the optimization technique, and optimization technique is what. We optimize in such a manner that, we are achieving the desire result or the desire relationship between the economic variable.

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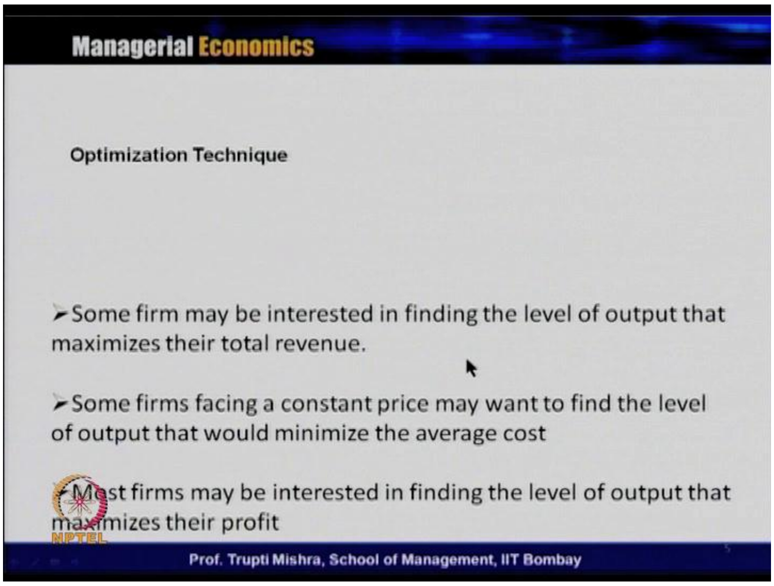

So, basically it is a technique of managerial decision making, maximizing or minimizing function, generally this optimization technique is used either for maximizing or for minimizing the function, and it is a technique of finding the value of independent variable which maximizes or minimizes the value of the dependent variable. So, basically we need to maximize the value of independent variable or minimize the value of dependent variable in order to, understand that how particularly those two variables are related. So, generally what are the cases where these optimization techniques are being used; like sometimes some firm may be interested in finding the level of output that maximizes their total revenue. So, it is basically finding the maximum level of output, which maximize their total revenue or the level of output maximize their total revenue. Some firms facing a constant price may want to find the level of output that would minimize the average cost.

So, may be a case, where the firm facing a constant price, or may they are finding a level of output which will minimize their average cost. And if you look most of the firm they are always interested to find out, what should be the level of output which maximizes their profit. So, if you look at this is the basic objective, basic aim of any firm, in order to understand the level of output which maximizes their profit. So, we will see how this optimization technique or what are the thumb rules, or what are the different approach or different methods to use this optimization technique in order to, for solving this managerial decision problem. So, we will take a function here, basically what we maximize; either we maximize the total revenue, or we minimize the cost, because the basic objective is to maximize the profit, and maximization of profit can take place; either by maximization of the total revenue or the minimization of the total cost, because profit is one, it is just the difference between the total revenue and total cost.

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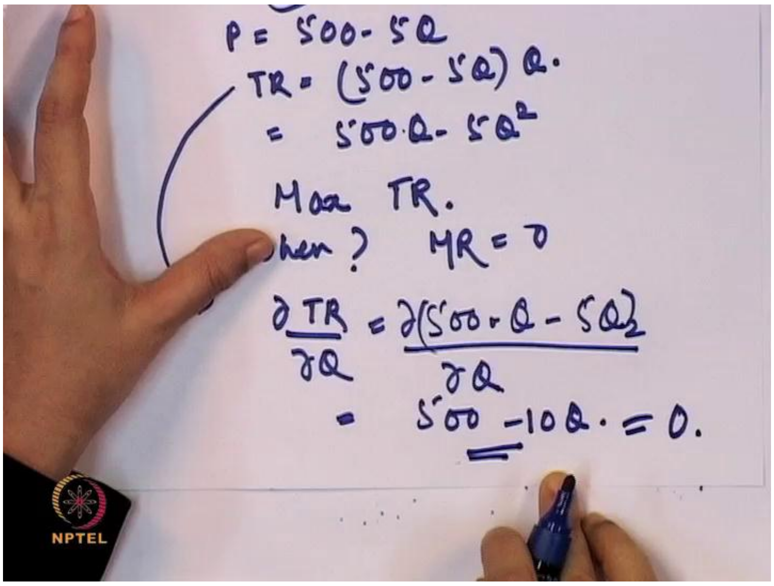

Now, what is total revenue, total revenue if you know, then total revenue this is p Q, p is the price and Q is the quantity demanded. Suppose we take that p is equal to 500 minus 5 Q. Now what is total revenue, total revenue is 500 minus 5 Q multiplied by Q, so that comes to 500 Q minus 5 Q square. So, total revenue is p Q, if the value of p is 500 and total revenue is 500, 5 Q multiplied by Q, that comes to 500 Q 5 Q square. Now, what is the role of optimization technique here or what is the role of, or how we can use this optimization technique over here, in order to maximize this total revenue. So, here the optimization problem is, maximization of total revenue; total revenue is p Q. Now, when this total revenue is maximum, total revenue is maximum, when marginal revenue is equal to 0.

So, the optimization problem here is to maximize the total revenue, and when total revenue is maximum, total revenue is maximum when marginal revenue is equal to 0. Now, let us find out marginal revenue. So, from this total revenue function, if you take the first order derivative, then we get the marginal revenue function. So, first order derivative the total revenue function with respect to Q; that will give us the marginal revenue function. So, if you take this, then this is 500 Q minus 5 Q square. We have to take the del of this with respect to q, so that comes to 500 minus 10 q. Now, what is the thumb rule, the thumb rule is when marginal revenue is equal to 0, total revenue is maximum. So, 500 minus 10 Q has to be 0, if the total revenue has to be maximum. Now, let us see what is the value of Q, when we set marginal revenue is equal to 0, and why we are setting marginal revenue is equal to 0, because the basic economic principle says that, if the total revenue is maximum, then marginal revenue is equal to 0.

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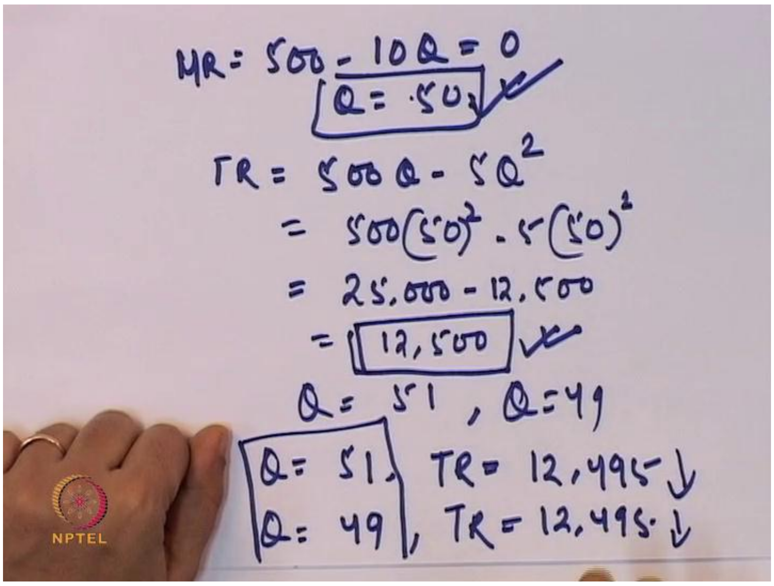

So, what is our marginal revenue, marginal revenue is 500 minus 10 Q, which has to be equal to 0, so this is our marginal revenue. Now, if you solve this then it comes to Q is equal to 50. So, what is this Q, when the level of output is equal to 50 units, the total revenue is maximized. There is maximization of total revenue when Q is equal to 50. Now we will find out what are the values of the total revenue. So, our total revenue is

500 Q minus 5 Q square. So, if you're putting the value of Q as 50; that is 550 square by 5 again 50 to the square. So, this comes to 25000 minus 12500, so it comes to 12500. So, the value of the total revenue is 12500. This is the maximum total revenue for the firm. And to achieve this, the Q has to be at least 50 units in order to maximize the total revenue.

Now, how we can check this, that this is the maximum amount of the total revenue, when the value of Q is equal to 50, we know the total revenue is 12500, but how to check that this 12500, is the maximum total revenue for the firm, which is facing a demand function; like p is equal to, if you remember this is 500 minus 5 Q, how to check this. We will take two different value of Q in order to check this. We will take Q is equal to 51 and we will take Q is equal to 49. So, if you take Q is equal to 51 and putting the value in the T R; total revenue equation, that is 500 Q minus 5 Q square, we get a value of total revenue which is 12495. Suppose we assume that Q is the level of output, if it is not 50, if you produce below this also still we can maximize the total revenue. So, let us assume Q is equal to 49, putting the value of Q as 49 in the total revenue function; we get a value that is 12495. So, we have 2 values; one if 51 and second one is 49. So, one is on a higher side when the level of output increases, whether it has any change in the total revenue, and second when the level of output decreases whether it has any change in the total revenue, and here we found that whether the Q increasing or whether the Q decreasing, the total revenue is decreasing.

If you look at total revenue is decreasing, because this is if what are the total revenue when Q is equal to 50. So, it can be concluded that 50 is that level of output, where the total revenue is maximum any level of output, either more than 50 or less than 50 is showing a decreasing total revenue. So, we can conclude that 50, when the level of output is 50, the total revenue is always maximum; particularly when the demand function is this, and when the total revenue function is this. Now, we will take in the case of the cost minimization, because the first case is revenue maximization, through revenue maximization the firm can increase the profit, and the second one when we can minimize the cost, again the difference between the revenue and cost is more and that leads to increase in the profit, which is in line with the basic objective of a firm, that is maximization of the profit.

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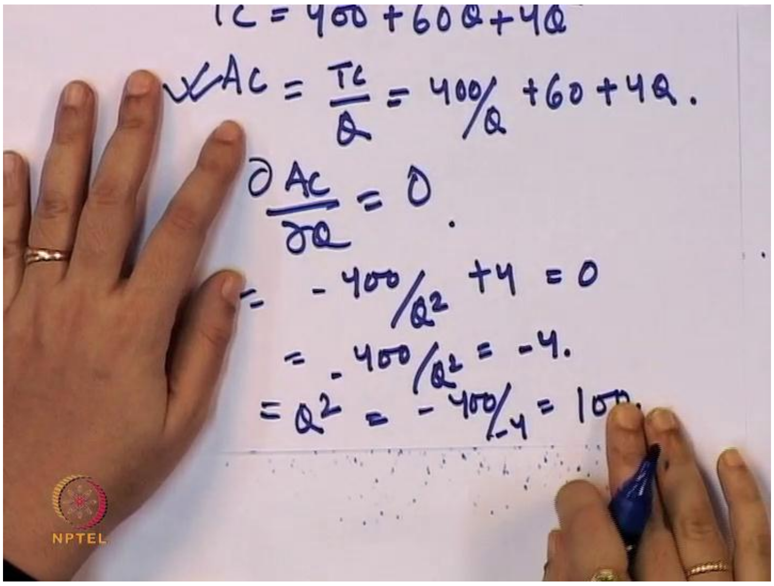

So, let us take a case of the cost minimization, in which situation generally there is minimization of cost, particularly when the firm is planning to setup a new production unit. They want to know, what is the minimum average cost through which they can setup a new production unit? When they are planning to expand their skill of production, they are looking for the minimum average cost through which they can expand the scale of production, or planning to raise the price of the product how it is effect the demand. So, these are the case, where the technique of optimizing output is require, by minimizing the average cost. So, here what is the optimization problem, the optimization problem is the cost minimization. Let us take a total cost function, what is total cost function here. Suppose this is 400 plus 60 Q plus 4 Q square. The cost minimization is not with respect to the total cost, rather with respect to the average cost. How to find average cost from here, total cost divided by Q will give us the average cost.

So, what is average cost, this is 400 divided by Q, plus 60 plus 4 Q, this is our average cost. So, we need to minimize the cost, in order to find out the difference to be more between the total revenue or the total cost. So, the first case what we are doing, we are trying to maximize the total revenue in order to maximize the profit. Now what we will try to do, we will try to minimize the cost, so that the difference between the revenue and cost is higher which leads to a higher profit. So, in this case the minimization is not related to total cost, rather the minimization is related to the average cost. Now what is average cost over here, average cost is the total cost divided by the unit of output; that is T C divided by Q, which is 400 by Q plus 60 plus 4 q.

Now, what is the rule of minimization, the rule of minimization is, the derivative must be equal to 0, if you remember in the previous case in order to maximize the T R, the rule was that marginal revenue has to be equal to 0. So, in this case minimization case we always take a thumb rule for this, that this first order derivative with respect to the average cost it has to be equal to 0. So, we need to find out, the derivative of average cost with respect to Q, and that must be equal to 0. Now we will find out what is the derivative of average cost with respect to q. So, that comes to minus 400 by Q square plus 4 which is equal to 0. So, this is again 400 Q square, which is equal to minus 4; that leads to Q square, if you simplify again, then Q square is equal to minus 400 by minus 4, which is equal to 100. So, if Q square is equal to 100, so we need to find out the level of output here.

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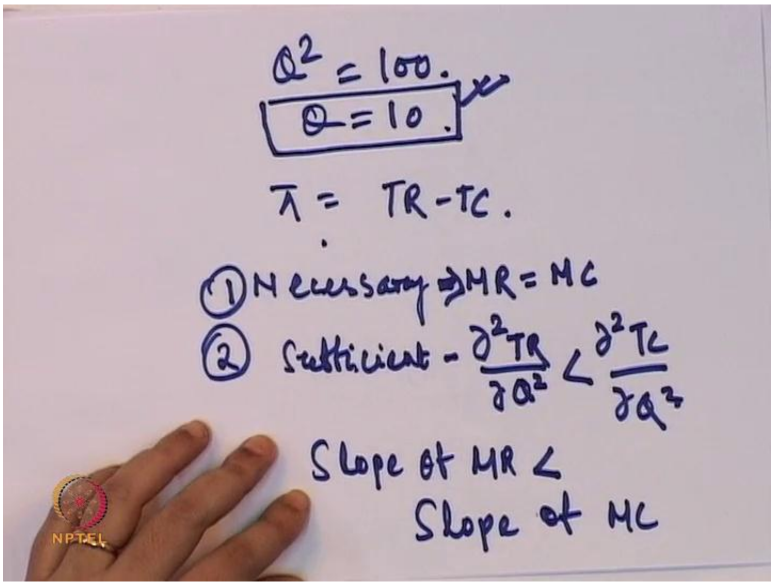

So, if Q square is equal to 100, then Q is equal to 10. So, when the unit of output or when the level of output is 10, this is the optimum level of output where the cost is minimum. So, at this level of output, the firm minimizes the cost, and if there guiding principle is on the basis of minimization of cost, the firm should follow a level of output that is equal to 10 units in order to minimize the cost. So, what we checked over here, optimization technique is used, either to maximize the revenue or to minimize the cost. So, we took a optimization problem which maximize the total revenue and there the thumb rule was to, maximizing the total revenue when marginal revenue is equal to 0.

And we took the second one second optimization technique which was the minimization of the cost, and here the optimization of problem is to minimize the average cost of production in order to maximize the profit. And here the thumb rule to minimize the cost, was to minimize the level of output, and for that is first order derivative of average cost with respect to Q has to be 0. Following that we got the level of output, and we say that this is the level of output, what the firm should follow in order to minimize the cost. If you look at all the business firm, they have a common objective. The common objective for all business firm is to maximize the profit. So, if you look at indirectly in the last two cases, last two optimization problem also we are trying to do. So, we are trying to in one case, we are trying to maximize the revenue, so that profit can be more, because their difference between the total revenue and total cost would be more, and the second case we minimize the cost.

So, that again the difference between the total revenue and total cost can be more which will maximize the profit. Now, we will take a problem where we will maximize the profit, rather than maximizing the total revenue or minimizing the cost. Let us see how we can do this by taking a profit function. And the basic need for this is if you look at the goal or the objective of the firm is to always to maximize the profit. So, now take a profit function and what is profit function; that is pi is equal to total revenue minus total cost. There are two conditions to maximize the profit; one is the necessary or the first order condition, which says that marginal revenue should be equal to the marginal cost. This is the first condition for the profit maximization, and second condition for the profit maximization, is the sufficient condition or the second order condition which says that; the second order derivative that is del square T R and del Q square should be less than del square T C and del Q square.

So, essentially it means, the slope of the marginal revenue function has to be less than the slope of the marginal cost function. So, for profit maximization, there are two conditions; one is necessary and the first order condition; that is marginal revenue equal to marginal cost. Second one is the sufficient condition or the second order condition where it says that; the second order derivative of the total revenue function should be less than the second order derivative of the total cost function, or on in other word the slope of the marginal revenue should be less than the slope of the marginal cost. Solet's take a profit function in order to understand, that how the profit is maximized, and how the first order and second order condition gets fulfilled when the profit gets maximized.

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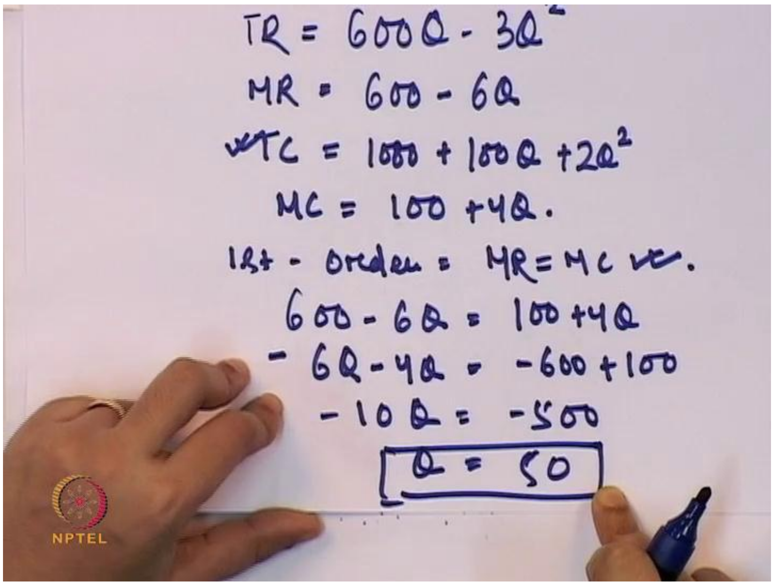

So, we will take a function that is total revenue, which is equal to 600 Q minus 3 Q square. So, what is marginal revenue, marginal revenue is first order derivative of this. So, this comes to 600 minus 6 Q. Then we will take a total cost function, total cost function is 1000 plus 100 Q plus 2 Q square, what is marginal cost function. The first order derivative of the total cost function. So, that comes to 100 plus 4 Q. Now, what is the first order or the necessary condition? The marginal revenue should be equal to marginal cost. This is the first order condition or necessity condition what is our marginal revenue that is 600 minus 6 Q is equal to, what is our marginal cost, 100 plus 4 q. So, if you simplify this is 6 Q minus 4 Q is minus 600 plus 100. So, minus 10 Q is equal to minus 500 and Q is equal to 50. So, the outcome of the first order condition is, we found out the level of output; that is Q is equal to 50. Now what is the second order condition, second order condition is that. The second order derivative of the total revenue function has to be less than the second order derivative of the total cost function.

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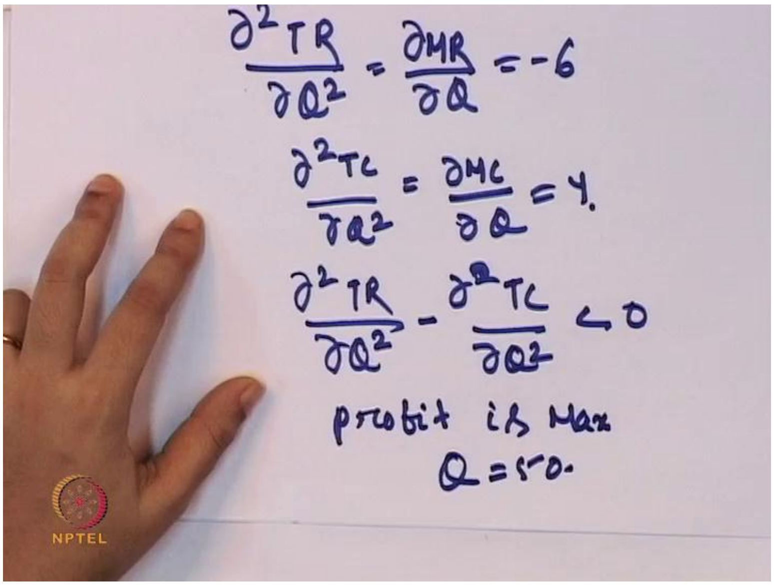

Now, let us see whether, particularly in this functional form whether we are fulfilling the second order condition or not. So, second order condition is del square T R, del Q square equal to del M R with respect to q. So, this is minus 6 del square T C del Q square. So, this is del M C with respect to del Q which is equal to 4 So, if you look at, this is less than this, and if the sum of both of this is also less than 0. So, this is del square T R del Q square minus del square T C, del Q square there is also less than 0. So, we know that, the second order condition gets fulfilled. So, we know that, profit is maximum when the necessary conditions get fulfilled; that is Q is equal to 50. Again we can do a random checking the way we did it for the other optimization problem; that you take any level of output which is more than 50 or less than 50. In order to understand, whether this is the level of output, which actually maximize the profit or not.

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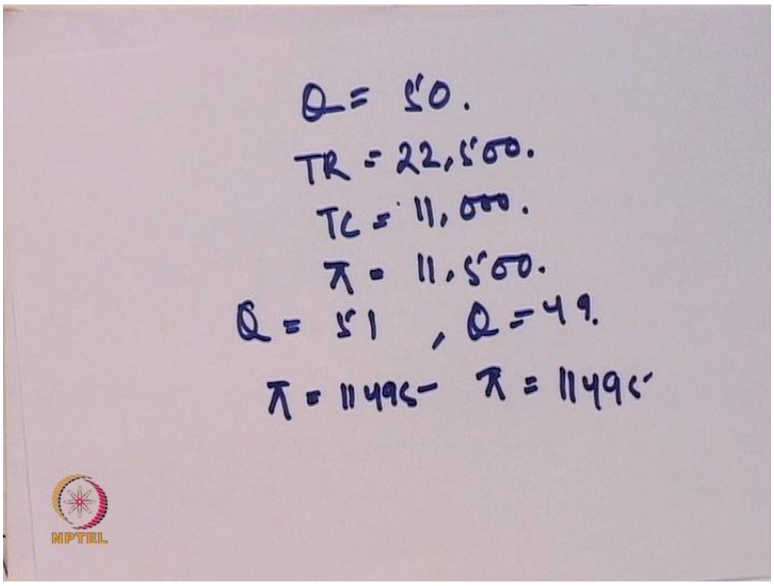

So, taking Q is equal to 50, we get the total revenue which is equal to 22,500 putting the value of Q, we get the total cost which is equal to 11,000. So, in this case, the profit is 11500. Suppose, you take a value Q is equal to 51 and Q is equal to 49. In the first case, the profit is equal to 11,495 and second case the profit is equal to again 11,495. So, we can conclude here that since the first order condition gets fulfilled, the profit is maximum when Q is equal to 50, because when we increase the level of output from 50 to 51, the profit is less; and when we decrease the level of output from 50 to 49 still the profit is less. So, we can say that, Q is that level of output which maximizes the profit. So, till now we are taking the optimization problem, and we are maximizing the revenue or profit or minimizing the cost without the constraints. So, next class, we will introduce the constraints, and then we will see how to use this optimization technique, in order to solve for the profit maximization or for the cost minimization.