**Managerial Economics**
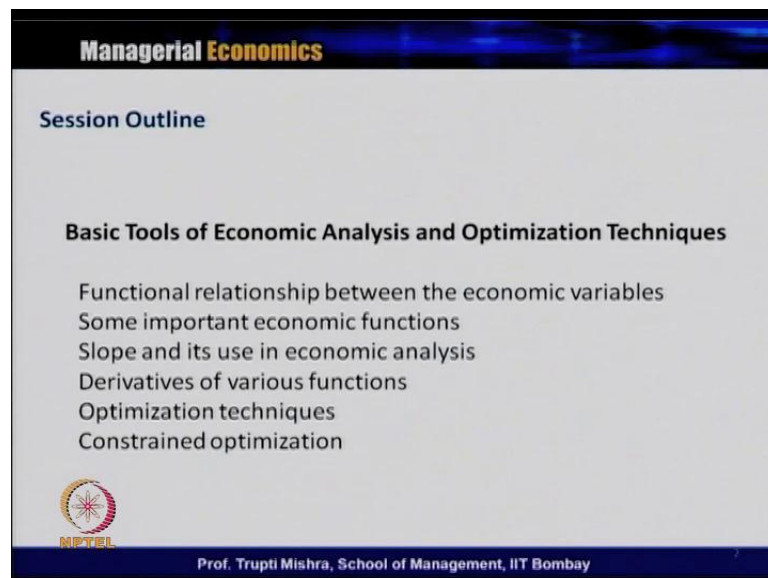**Prof. Trupti Mishra**
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**Lecture - 4**
**Basic Tools of Economic Analysis and**
**Optimization Techniques**

Welcome to the fourth session of managerial economics. Basically, we are on the first module of managerial economics, which talks about introduction and fundamentals to managerial economics.

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So, if you remember in the last class, we just discussed about the functional relationship between the economic variables, how they are related, and what are the different forms to represent them. Then we discussed some of the important economic functions like demand function, bi-variate demand function and multivariate demand function.

So, today's session we will focus on the different types of function that gets used typically in a demand function, how to measure a slope and what is its use in the economic analysis, different methods to analyze the slope and find out the slope or measurement of the slope. Then derivative of various functions and in the next session, we will basically take the optimization technique and constant optimization.
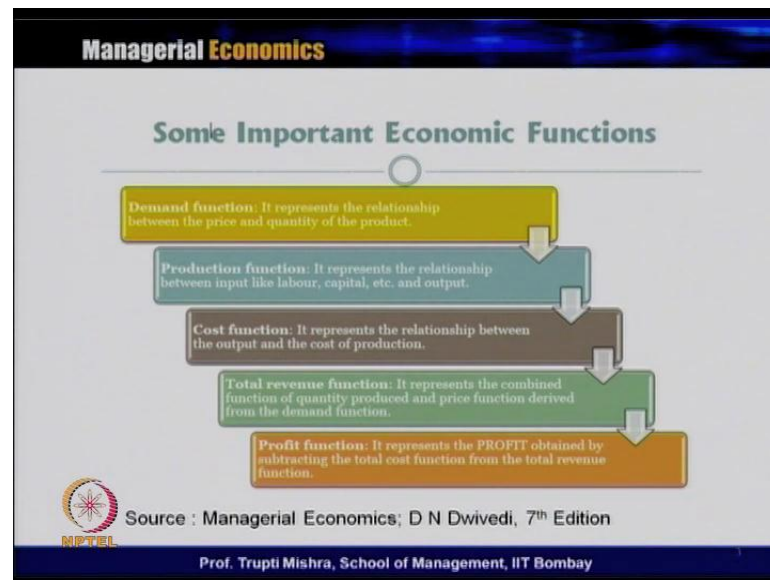
So, till now, all our discussions, if you look at it just focuses on the demand function. But apart from the demand function, there are certain other topics also where we generally use the relationship between two variables in a functional form, like production function which represents the relationship between the inputs like labour and capital with the output.

We talked about the cost function, where it is basically the relationship between the output and the cost of the production associated with that. When you talk about the total revenue function, it represents the combined function of quantity produced and price function is based on the demand function. Sometimes also we talk about a profit function. This is the profit basically, as you know it is the difference between the total revenue and total cost function. So, whenever there is a change in the total revenue and wherever there is a change in the total cost, it generally affects the profit. So, profit function is basically the relationship between the profit revenue and cost.
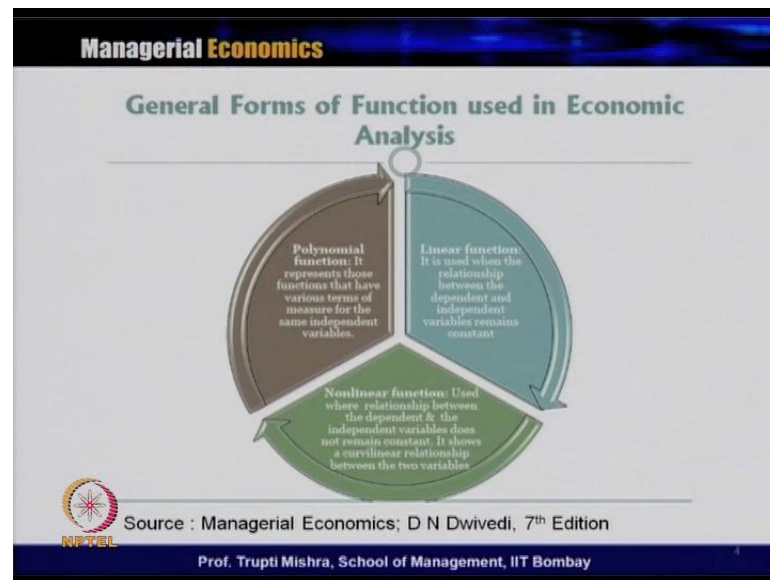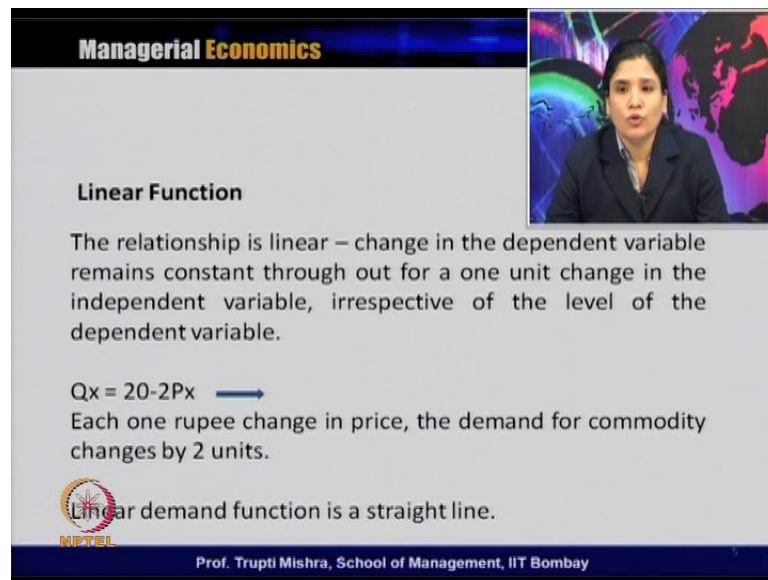
Then, we will discuss what are the general forms of function used in the economic analysis. So, one way we are clear that we use a functional form to understand the relationship between two types of variable. The variable, typically in this case, all the variables are economic variables. So, there are three types of function we use in analyzing the relationship between the variables. One is linear function, second one is the non-linear function and third is the polynomial function.

Linear function is used when the relationship between dependent and independent variable remains constant. Non-linear function is used where the relationship between the independent variable and dependent variable is not constant, but changes with the changes in the economic variable. Polynomial function represents those functions that have various terms of measure for the same independent variable.

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So, we will check all this functions in more detail by taking them individually. So, in a linear function, the relationship is linear. The change in the dependent variable remains constant throughout for one unit change in the independent variable, irrespective of the level of the dependent variable. Whatever the change in the independent variable, the change in the dependent variable remains constant in case of a linear function.

Suppose you are taking a demand function, which says that Q x is equal to 20 minus 2 P x. What does it signify? For each 1 rupee change in price, the demand for commodity changes by 2 units. Because, if you look at the second term of this functional form, there it is minus 2 P x. So, for 1 rupee change in the price, the demand for the commodity changes by 2 units.

When you represent graphically, the linear demand function is always a straight line because the change in the dependent variable remains constant for one unit change in the independent variable.
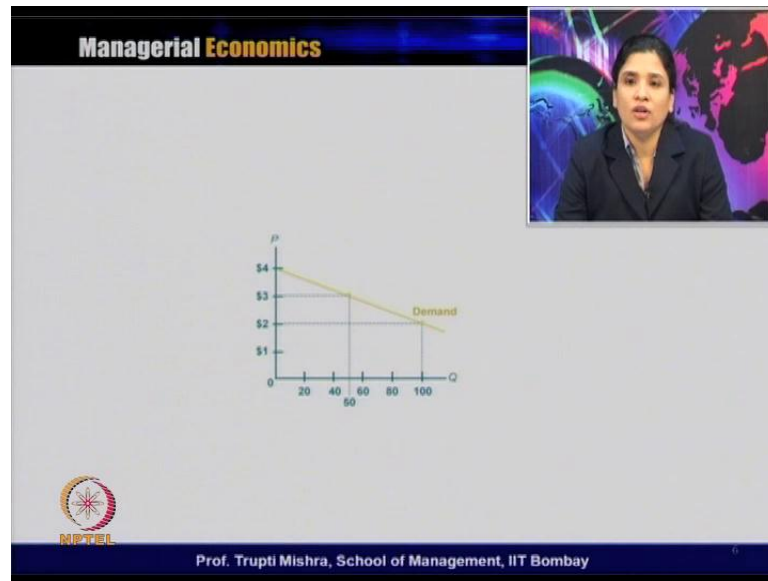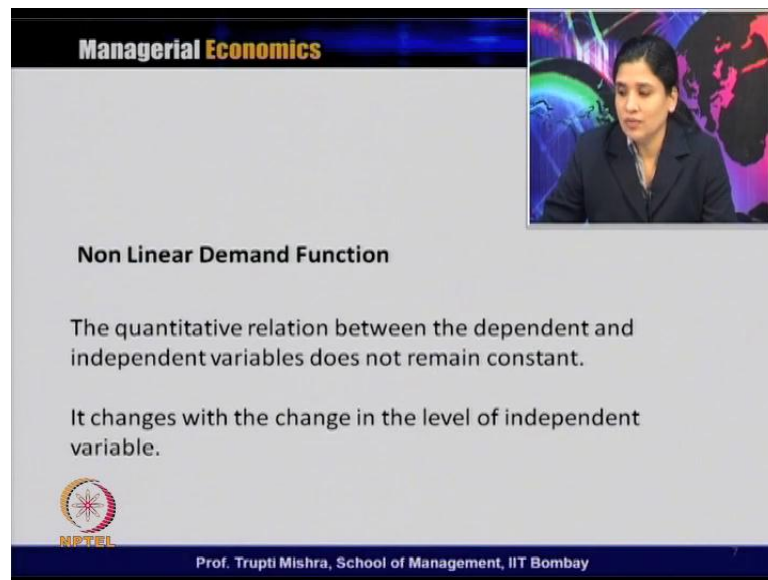
So, this is just a hypothetical way to understand the linear demand. In the vertical axis, we are taking the price and in the horizontal axis, we are taking quantity. So, if you look at it, when the price is changing, the quantity demanded is also changing. So, initially when the price is 2 dollar, the quantity demanded is 100 units. When the price increases from 2 dollar to 3 dollar, the quantity decreases by 100 units to 50 units.

So, if you look at the demand curve, at each point it gives a price and quantity combination. Here, the quantity demanded is the dependent variable. Whenever there is a change in the price, that leads to change in the quantity demanded also. If you look in the percentage wise also, when the price changes from 2 dollar to 3 dollar, there is 50 percent change in the price. When the quantity demanded decreases, it decreases from 100 to 50. Again, this is a 50 percent decrease in the quantity demanded. So, 50 percent increase in the price is leading to 50 percent decrease in the quantity demanded. At this point, the relationship between these two variables is linear. As it is constant, the price point changes from one to another.

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Then, we will discuss the non-linear demand function, where the relationship between the dependent and independent variable is not constant. It changes with the change in the level of independent variable. So, in the previous case, we are discussing that 50 percent change in the price will bring 50 percent change in the quantity demanded. However, in case of non-linear demand function, the unit of change may not constant with each change in price. When the price changes from 1 dollar to 2 dollar or 2 dollar to 3 dollar, that may not necessarily be the same kind of change in the level of the quantity demanded.
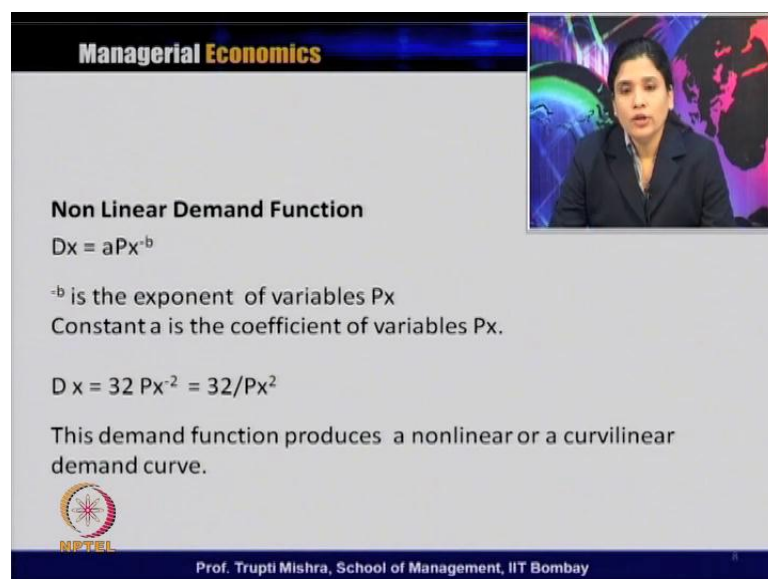
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So, if you are taking a non-linear demand function, that is d x which is a function of price x. So, here x is the product, P x is the price of x and d x is the quantity demanded of x. Taking the functional form in a non-linear, d x is a P x to the power minus P. Here, a and b they are the constants. Minus P is the exponent of variable P x and constant a is the coefficient of variable P x.

If you simplify it further, may be you are taking a number term over here. Suppose, d x is 32 P x to the power minus 2. Maybe, we can take just a reciprocal of this 32 minus P x square. So, in this case, the demand function produces a non-linear or curvilinear demand curve. It means it is not a straight line. The change in the independent variable is not constant throughout whenever there is change in the price.

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So, this is an example of a non-linear demand schedule that shows how it changes when there is a change in the price. So, when price is 1, quantity demanded is 32. When price is 2, quantity demanded is 8 and when it is 3, quantity demanded is 3.5. Similarly, for 4, 5 and 6, if you look at the trend, the quantity demanded is going on decreasing when the price is increasing.

But, here the point is not to establish a negative relationship or inverse relationship between the price of x and d x. The point what we are discussing here is that, with each change in the price point, the change in the quantity demanded does not remain constant. The change in the quantity demanded with respect to each price point becomes different.

This is a typical feature of a non-linear demand curve. When you plot this in a graph, we generally get a curvilinear relationship, which is in the form of a curve. We do not get a line. We do not get a straight line, which is generally the representation of a linear demand curve.

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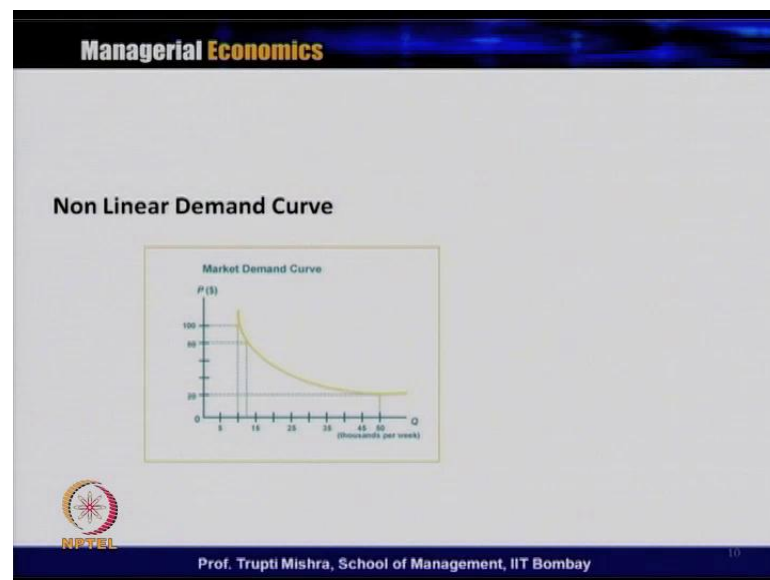


So, this is the graphical representation of a demand curve. If you look at the different points in the demand curve, the change in the quantity demanded does not remain the same. So, if you are taking the p here which is the price, it is represented in the vertical axis and q is the quantity, which is represented in the horizontal axis. When the price is changing from 100 to 80, the quantity demanded is increasing. Again, when it is decreasing from 80 to 20, the quantity demanded is again increasing.

But, if you look at the change in the price point from 100 to 80 and the corresponding change in the quantity demanded from may be 10 to 12, that does not remain constant. With the next change in the price point from 80 to 20, there is a significant amount of change in the quantity demanded, that is from 12 units to 50 units.

So, in case of a non-linear demand curve, even if the demand changes along with the change in the price point, there is always a difference in the amount of change at different price point.

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The third kind of function generally used in the economic analysis is polynomial function. What is polynomial function? The function that contains many terms of the same independent variable are called polynomial function. So, we consider a short term production function here, where output is a function of the labour and output is represented as Q and labour is represented as L. So, putting it in a functional form, Q is the function of L over here. The polynomial function takes different types of functional forms such as quadratic functions, cubic functions, and power functions.

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So, taking the example of the same short run production function, where Q is the output, L is the labour and a, b, c and d are constants associated with the different coefficients. It takes a quadratic function or it takes the form of a cubic function or it takes the function of the power function. So, when it is becomes a quadratic function, Q is equal to a plus b L minus c L square, where a, b, and c are constants. When you take a cubic function, then it is a plus b L plus c L square minus d L q, where again a, b, and c are the constants associated with the coefficient. When it takes as power function here, it is a L to the power b, where a and b are the constants and b is the coefficient associated with variable L.

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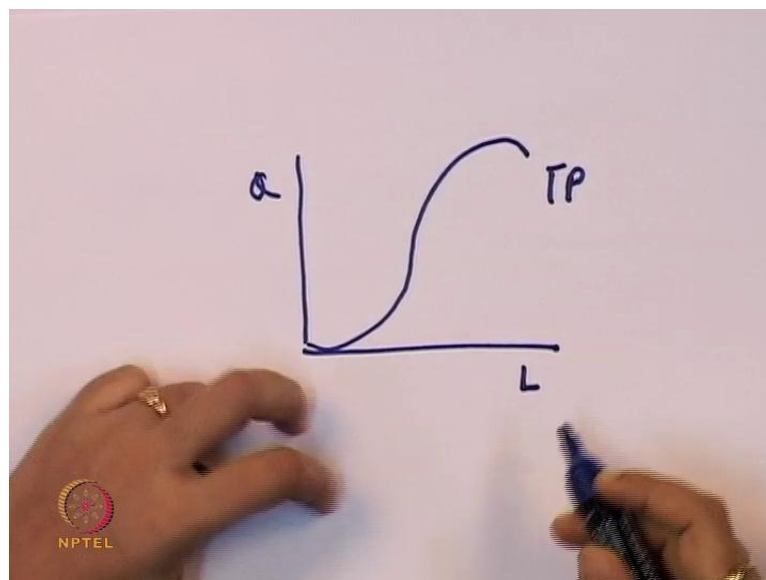


So, polynomial function may take a quadratic function or a cubic function or a power function. You can represent this polynomial function graphically, with all these three types of function, whether its quadratic cubic and function. So graphically, if you look at a cubic function, when the polynomial function takes a cubic function, suppose we take L over here, L is the labour and Q is the output. Now, the cubic function takes this type of shape. Now, what is this curve? This curve is the total product curve and total product is dependent on the output and the labour.

So, if you are taking Q over here and L over here, cubic function takes a form which may be not a straight line and not exactly a curve. It follows a different kind of change at each change in the L. So, how this Q and L are related here? L is the independent variable and

Q is the dependent variable. So, whenever there is a change in L, that will bring in change in Q. So, in this case of a cubic function, L changes when Q changes. But the change in the Q is not constant with each change in the labour.

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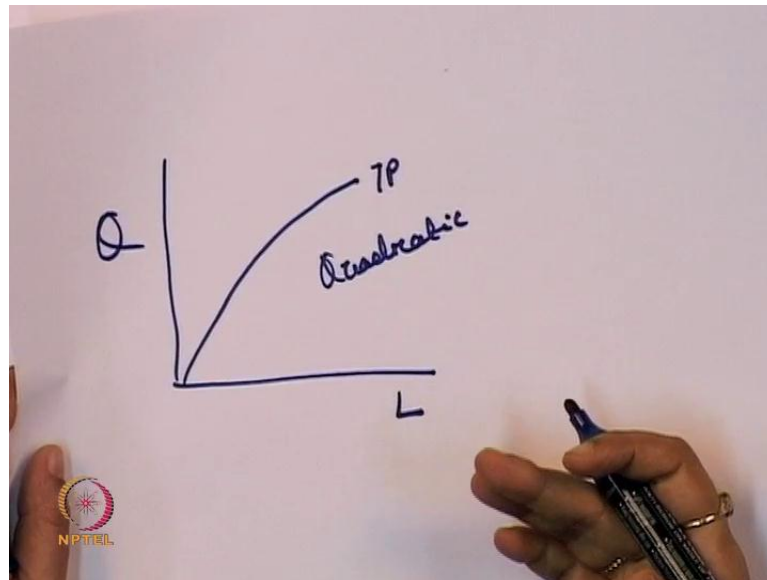


Now, take a case of a quadratic. So, with the same short run production function, we take L in the x axis and Q in the y axis. Now, it is a quadratic. So, you just follow it. There is no cyclical function over here and there is no much fluctuation here. Total product curve is this and this is a typical example of a quadratic function.

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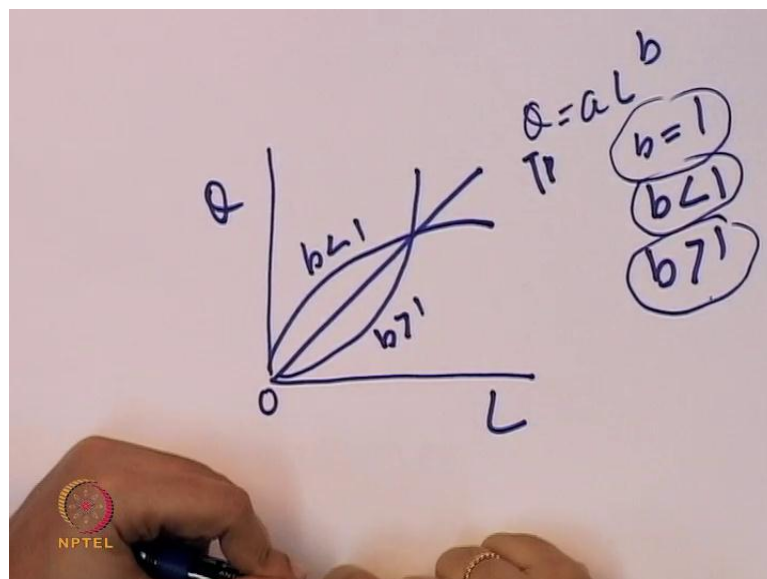
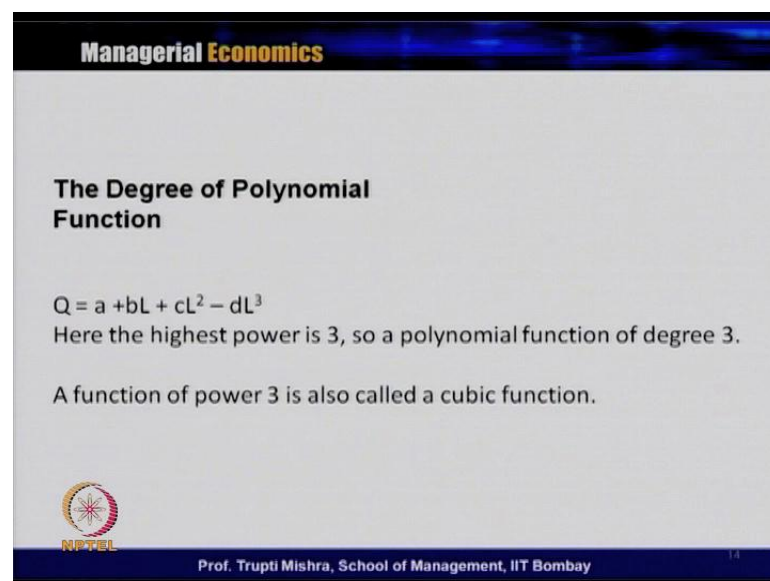Now, graphically we represent the power function of the polynomial function. So, the power can take any value. The coefficient associated with b or the coefficient b associated with L can take any form. So, if you remember the power function is Q, it is equal to a L to the power b. So, b can take a value which is equal to 1, less than 1 or greater than 1. So, in this case, if you represent graphically again by taking the same formulation, here it is labour and here it is output. When we get the value of b equal to 1, it is a straight line. The total product curve is a straight line. When b is less than 1, we get this kind of shape and when b is greater than 1, we get this type of shape.

So, if you take a power function in case of a polynomial function, the power associated with the coefficient b can take any value. May be, sometimes it is 1 or sometimes it is less than 1 or sometimes greater than 1. So, whether it is 1 or less than 1 or greater than 1, when you represent that graphically or when you represent that geometrically, this is the shape that we get for different kinds of function.

So, polynomial function takes the quadratic function. Polynomial function can also be represented through cubic function and polynomial function can also be represented through a power function. Each time the value of b changes, the graphical representation also changes dependent on the value of the coefficient.

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Now, how to find what is the degree of a polynomial function. So, degree of a polynomial function is, if we are taking a functional firm, which is q, it is equal to a plus

b L minus c L square. Here, the highest power is 2. So, this is a polynomial function of degree two. A polynomial function of power two is also called a quadratic function.

So, in order to identify what is the polynomial function, it is always the highest power associated in this functional form. So, in this case, the highest power is 2. So, the polynomial function of degree 2 we can say is having this functional form. So, polynomial function of power root is also called a quadratic function.

Let us take one more example in terms of a cubic function. So here, what is a functional form? The functional form is q is equal to a plus b L plus c L square minus d L cube. Here, the highest power associated with the coefficient is 3 and this is a polynomial function of degree 3. A function of power 3 is also called cubic function. So, in the previous example, in the functional form, the highest power is 2. So, that is why it was a quadratic function of degree 2 and in this typical functional form, the highest power is 3. That is the reason, it is called as a cubic function because the function of power is 3.

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Then, we will see what is the degree of polynomial function, when the polynomial function is in terms of a power function. So, here in this case, the functional form q is equal to a L to the power b. The range of power is between b greater than 1, b is equal to 1 and b less than 1.

So, in this case, except 0 it can take any power. So, it may be less than 1 in the negative form, equal to 1 or it may be greater than 1. So, in this case, b taking the value of 0 is not possible. It takes any of the value and this is the example of a power function under the polynomial function.

So, there are three types of functions. One is linear, second one is non-linear and third one is polynomial. In case of polynomial, in polynomial function again we represent in terms of quadratic function, in terms of a cubic function or in terms of a power function. Every time, the degree changes on the basis of the power associated with the functional form. The highest degree in case of a quadratic function is 2 and the highest degree in terms of a cubic function is 3. The highest degree in terms of power function is greater than 0 or it may take a negative value or a positive value.

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Now, how to solve a polynomial function? It can be solved either through the factoring methods or through the quadratic formula. What is the property of a quadratic or a cubic equation, when there is more than one solution? So, polynomial function can be solved by factoring method or by quadratic formula. It can be solved and the property of quadratic and cubic equation is that it has more than one solution.

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Now, solving this quadratic and cubic equation, we have two methods. One is factoring method and second one is the quadratic formula. Now, we will see what is factoring method and what is cubic equation or what is the quadratic formula to solve this cubic equation.

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So, we will take a function. That is, y is equal to x square plus x minus 12. So, in the factoring method, what is the first step? The first step is that we have to set y is equal to

0. So, taking this x square plus x minus 12 is equal to 0, now what is the second step? We have to factor the equation.

So, this x square plus x minus 12 can be also represented with x square plus 4 x minus 3 x minus 12, which is equal to 0. Now, simplifying again, this takes x plus 4 and x minus 3, which is equal to 0. If you simplify, we get two values of x over here. One is x is equal to minus 4 and the second is x equal to 3. If you look at minus 4 has no meaning in economic analysis.

So, basically we will go with the positive value, that is x is equal to 3 and we solve this functional form with a value of x which is equal to 3. So, even if we are getting two values, one is minus and second one is plus. Typically, since we are applying this in economic analysis, there is no significance when we get a negative value of any variable. So, that is the reason we are ignoring the first value of x, which is minus 4 and we are going with the second value of x, which is equal to 3. So, if you look at it, this solution is through factoring method. So, this is the solution of a polynomial function by using the factoring method. Now, we will check the second one through the quadratic formula.

Now, what happens in case of a quadratic formula? The quadratic equation is set equal to 0. That is the first step and the equation is again the factor for obtaining the two values of the variables x and y. Following the formula, that is minus b plus minus b square minus 4 a c divided by 2 a. So, let us see, how we can solve a polynomial function through the quadratic formula.

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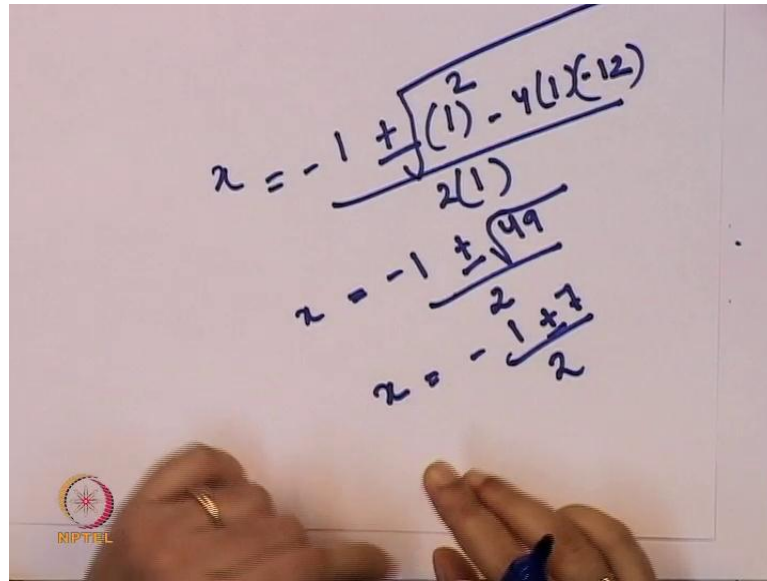


Now, what is the functional form over here? The functional form is y is equal to x square plus x minus 12, which is equal to 0. Because, what is our first step? The first step is to set the quadratic equation or whatever the functional form, we have to set that equal to 0. Now, what is the implication of this equation? Now, x is equal to, how to factor it again to get the value of x? Because, the first step is always to set it is equal to 0 and second, we factor out this equation in order to get the values of the variable x and y or if there is only one value, the value of the x.

So, if you are following, then this is minus b plus minus b square minus 4 a c by 2 a. Now, what is the implication of this equation? Now, a is equal to 1. This formula is taking this equation, that is a is equal to 1, b is equal to 1 and c is equal to minus 12. So, this is the formula to factor out the equation. This is the second step. The first step is to set this equal to 0, that is x square plus x minus 12 is equal to 0. From this equation, we get the value, and that is a is equal to 1, b is equal to 1 and c is equal to minus 1.

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Now, we will substitute this value of a, b and c, in case of the quadratic formula. So, it is x is equal to minus 1 plus minus 1 square 4 a c and 2 a. So, this is 2 and 1. So, x is equal to minus 1 plus minus 49 root divided by 2, which you simplify again, this is minus 1 plus 7 by 2.

So, in the previous case, once we have identified the value of a is equal to 1, b is equal to 1 and c is equal to minus 12, we will put this value into the quadratic formula and we are getting the value of x. Now, this is 1 plus minus, it means x is having two values here, which satisfies the quadratic equation. Because, this is minus 1 plus minus 7, which is divided by 2. So, we will get two values of x and it satisfies the quadratic equation.
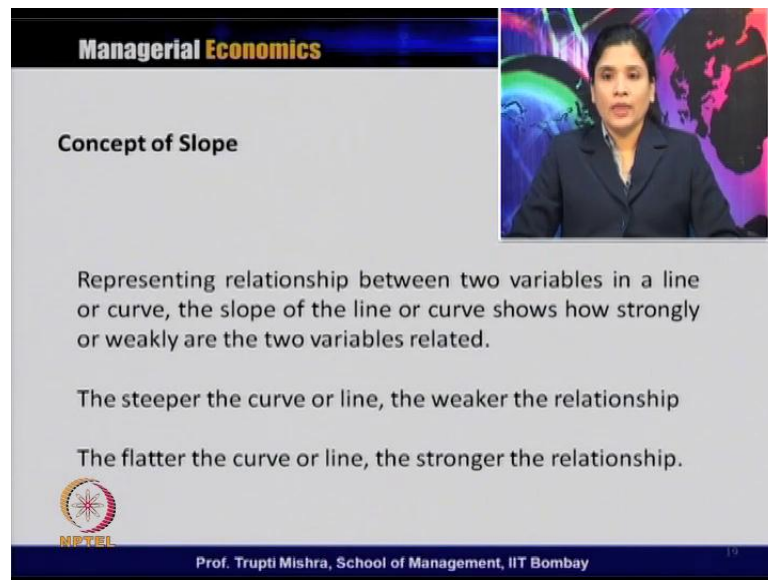
Now, if we take the first value, that is x is equal to minus 1 plus 7 by 2, we get x is equal to 3. If we take the second value, that is minus 1 minus 7 divided by 2, then we get x is equal to minus 4. So, we are getting one negative value and one positive value. Anyway, this is negative and still it is satisfying the quadratic equation. So, we have two values. One is positive and one is negative.

So, basically we ignore the value, which is with negative sign. We always go for the positive sign value because it makes some sense in the economic analysis when we go for the positive value. So, if you remember, in the previous solution that we did through the factoring method, we also got two values of x, that is 3 and is minus 4.

So, whether you solve the quadratic or cubic equation or whether you solve the polynomial function, either by taking factoring method or by taking the quadratic formula, you always get two values of x. On the basis of the value we get for x, we decide which value to take for further analysis and which value to ignore. So, polynomial function can be solved by using two methods. That is, one is the factoring method and second one is the quadratic formula. We get the same value for x taking any specific formula, either through the factoring method or through the quadratic formula.

Then, we will discuss the concept of the slope over here. Now, what is slope? If you remember your marginal analysis what we discussed before, may be few sessions back or may be the last session, the marginal change is always whatever change in the dependent variable due to one unit change in the independent variable.

So, slope is to measure the relationship between the marginal changes in two related variables. It can be also defined as the rate of change in the dependent variable as a result of change in the independent variable. So, through marginal analysis, we know how the dependent or the independent variable changes, when there is a change in one variable. Through the marginal analysis, we know that when there is a change in the independent variable, it leads to change in the dependent variable.

But, through slope, we can measure the exact nature of the relationship, that is whether they are positively related or whether they are negatively related. We can also quantify the change, that is what the percentage change is or what is the amount of change that has taken place in the dependent variable due to change in the independent variable.

So, geometrically if you look at what is a slope, it represents the relationship between two variables in a line, in case of a linear relationship and or as a curve in case of a non-linear relationship. So, the slope of the line or the curve shows how strongly or weakly two variables are related. So, in order to find out the slope, we represent graphically the relationship between the two variables. So, if the two variables are linear variables, or

linearly related, we get a line. If the two variables are nonlinearly related, we get a curve. Slope generally says, how strongly or how weakly these two variables are related to each other.

The steeper is the curve or the steeper is the line, the weaker is the relationship. Implication for this is that they are not strongly related or they are not related, if there is a steeper line or steeper curve. It means that there is no change or no more change or no significant change in the dependent variable, even if there is a change in the independent variable.

However, if the curve or the line is more flat or becomes more flat, it signifies that there is a stronger relationship between these two variables. Or we can say if there is a small change in the independent variable, it leads to a greater change in the dependent variable. So, when two variables are represented through a line or a curve, the slope measures the change between these two variables. That is, the amount of change, the nature of change or in the other words, we can say that they can quantify the relationship between these two variables. So, if it they are steeper, they are not much related and if they are flatter, then they are related to each other.

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Now, taking a typical example of a demand function over here or demand curve over here, the slope is the ratio of change in the dependent variable and change in the

independent variable. So, if you look at in case of a linear demand curve, we always get a straight line demand curve and in case of a non-linear demand curve, we get a curve.

So, with respect to demand curve, what is the slope? Slope is the ratio of change in the dependent variable D to the change in the independent variable. So, movement down the demand curve gives the decrease in the price and if it is upwards, there is an increase in the demand. The ratio of the change in the price and the change in the demand gives the slope of the demand curve. So, demand function is a function of price. So, price is independent over here and demand is dependent over here.

So, how to measure the slope over here? The change in the demand due to change in the price becomes the slope of the demand curve. Because, this slope measures the change in the demand curve due to change in the price. So, in this specific case, the slope is the change in the demand due to change in the price. So, we will see how generally we get a slope in case of a linear demand curve and in case of a non-linear demand curve.

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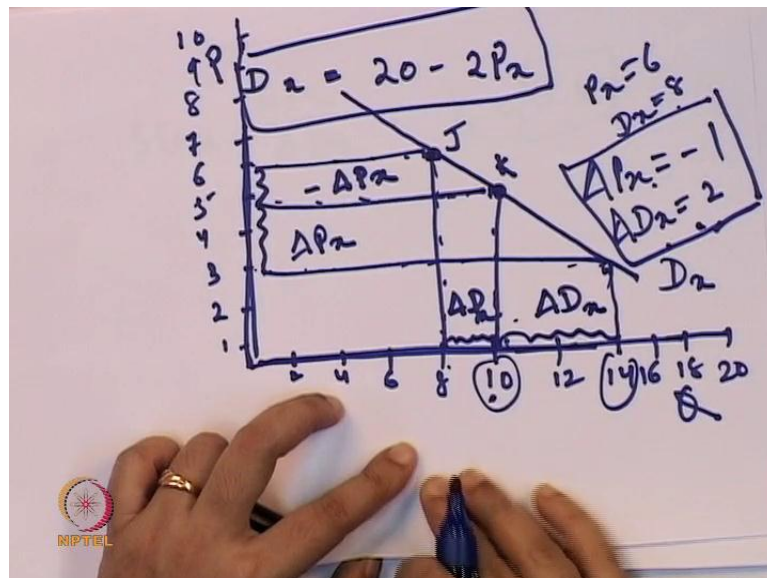


Suppose, we take the example of a linear demand function and that is, d x is equal to 20 minus 2 p x. This is a demand function. Now, how to find out the demand curve from this demand function. So, let us say this is 2, this is 4, this is 6, this is 8, and this is 10, 12, 14, 16, 18 and 20. Similarly here, we can say 1 2 3 4 5 6 7 8 9 and 10.

So, when the price is 6, suppose we say the quantity demanded is 8 and when the price is 5, the quantity demanded is 10. When the price is 3, the quantity demanded is 14. So, price is 6 and quantity demanded is 8. We get one point of the demand curve. Then price is 5 and quantity demanded is 10. We get the second point of the demand curve. Price is 3 and quantity demanded is 14. We get the third point of the demand curve. If you join these three points, we get the demand curve. So, maybe this is point j, and this is point k.

Now, what is the demand curve showing over here? If it is demand for x suppose, what is the demand curve showing over here? This is the change in the price of x and the consequent change in the quantity demanded of x. Price, we are considering here and quantity we are considering here. So, if you look at y axis is p and q is represented in x axis. So, this demand curve is essentially showing the relationship between the change in the quantity demanded due to change in the price. So, suppose the initial price, as we mentioned the initial price p x is equal to 6. So, the quantity demanded is 8 with respect to that.

Now, suppose p x decreases from 6 to 5 and quantity demanded increases from 8 to 10. So, this is what this is the change in the p x and this is the increase in the d x. So, this is minus because there is a decrease in the price and this is positive or this is plus because there is an increase in the quantity demanded.

So, when price of x decreases from 6 to 5, quantity demanded increases from 8 to 10. So, what is the value of del p x? del p x is equal to minus 1 because it changes from 6 to 5 or we can say, maybe it is from 5 to 6 and then it becomes 1. Then del d x is from 10 to 8 because this is the change in the x. So, del d x is 2. So, p x is the independent variable and d x is the dependent variable. Due to change in the p x, there is a change in the d x. So, given the value of del p x is equal to 1 and del d x is equal to 2, what is the slope of a straight line demand function?

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The slope of a straight line demand function is the ratio of the del p x by del d x. So, this is the slope of the demand function. So, del p x and del d x becomes the ratio and through this ratio, we can find out the slope of a straight line demand curve between the point j and k. So, if you see the previous curve, this is the point j and k. So, through this ratio, we can find out the slope between the two points j and k and this becomes 1 by 2, which is equal to 0.5.
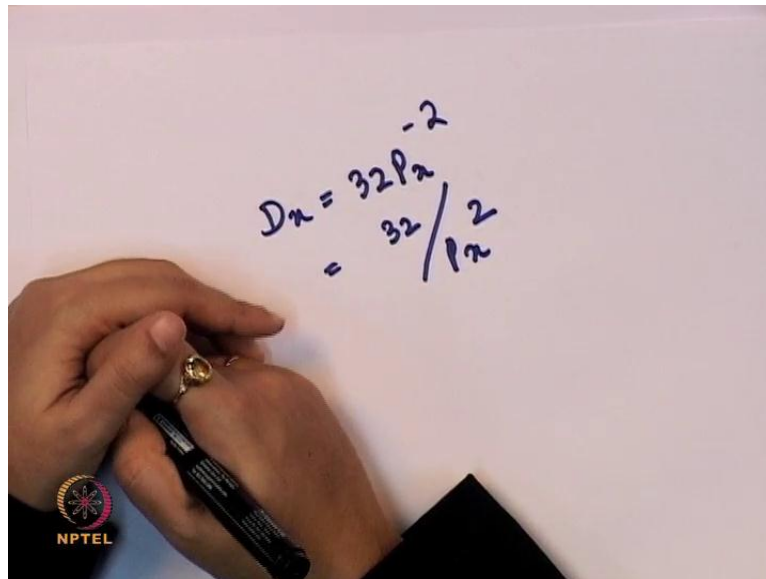
So, given the demand function d x is equal to 20 by 2 p x and p x is equal to 6 and d x is equal to 8 initially, there is a change in the p x from 6 to 5 and change in the quantity demanded from 8 to 10. That leads to the change in the price of x, that is del p x and change in the quantity demanded d x. So, which is 1 by 2 and the slope is 0.5. This slope, if you look at it, this is the case of a linear demand curve. The slope is constant throughout the demand curve.

Now, suppose we consider that if price of x decreases from 5 to 3. This is again the change in the price of x and the quantity demanded changes from 10 to 14. So, this is 10 and this is 14. This is the amount of change in the quantity demanded of x. So, if price decreases from 5 to 3 and quantity demanded increases from 10 to 14, we will find out what is del p x over here and what is del d x over here. So, del p x is 3 minus 5 and that is minus 2 and del d x is 14 minus 10 and that is 4. So, in this case, when you identify

what is the slope between these two points, then this is again the ratio of del p x by del d x and which is again 2 by 4 and we are getting a value, which is 0.5.

So, in case of a linear demand curve, you get a constant slope throughout all the points of the demand curve because the change in the dependent variable remains constant with respect to change in the independent variable.

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Next, we will see how we measure the slope of a non-linear demand function. Let us take a functional form, that is d x is equal to 32 p x minus 2 or we can say, this is 32 by p x square. Now, in case of a non-linear demand curve, the slope of the curve can be measured between any two points and then we can compare what is the slope between these two points.

What is the essential difference between a linear demand curve and a non-linear demand form? In case of linear demand curve, the change in the dependent variable remains constant throughout the entire demand analysis or entire analysis period. But in case of a curvilinear or in case of a non-linear demand curve, the dependent variable changes in a cyclic manner or in a different proportion at each point of the demand curve. That is the reason to necessarily measure the slope between the two different points and again compare whether the slope remains the same or slope is decreasing or slope is increasing or to identify what is the trend of the slope between different points of the demand curve.
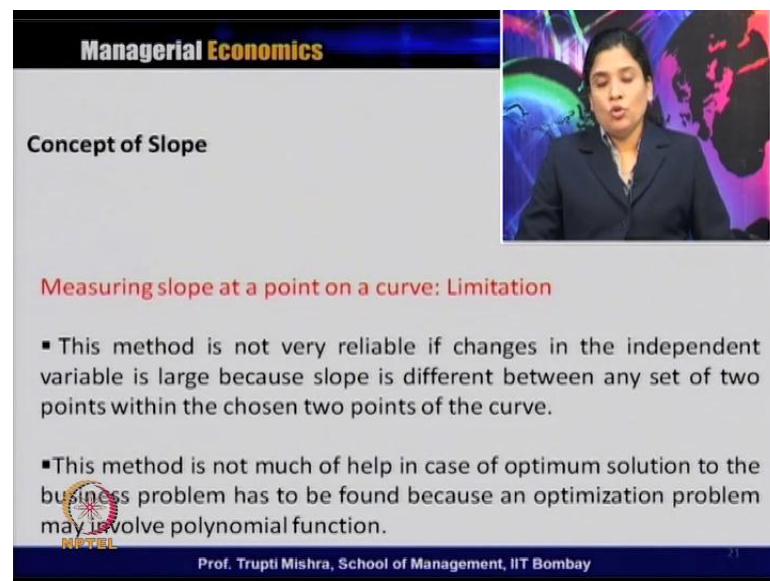
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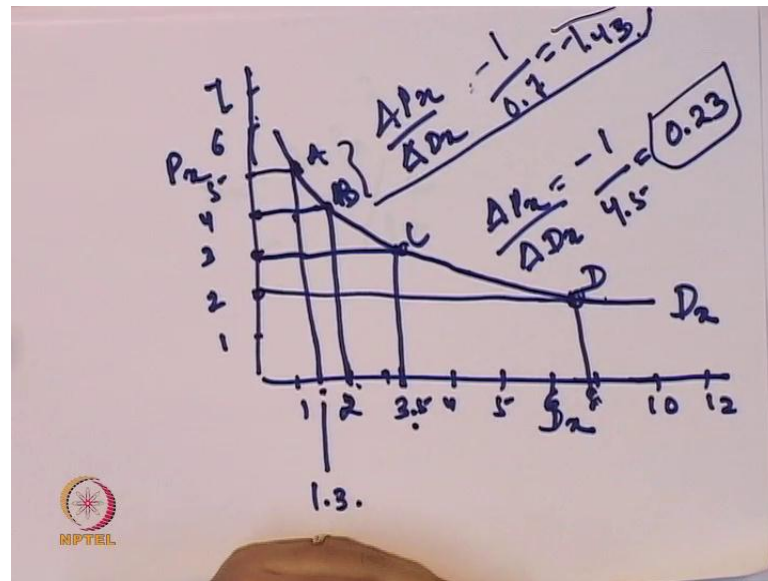


We take p x on the vertical axis and d x on the horizontal axis. So, here we get 1 3 or maybe, we can get 1 2 3 4 5 6 8 10 12 and so on. In case of p x, we can say this is 1 2 3 4 5 6 and 7. When price is 5, the quantity demanded is somewhere between 1 and 2. So, let us say this is 1.3. When price is 4, the quantity demanded is 2 and when price is 3, the quantity demanded is 3 or maybe, say it is somewhere 3.5 and when price is 2, then the quantity demanded is 8.

Basically, if you join these points, suppose this is point A, this is point B, this is point C and this is point D. If you join all these three points, all this four points rather, we get a non-linear demand curve. Now, how we will identify or how we will measure the slope between these two points? Now, what is the slope between point A and point B? Now, what is del p x over here? del p x is the difference between 4 and 5, that is the change in the price and that is 4 and 5. So, from point A to point B, what is the slope? That is, del p x by del d x. What is del p x? It is the difference between 4 and 5. So, that comes to minus 1. What is the difference in the quantity demanded? That is, the difference between 2 and 1.3. So, that comes to 0.7. Now, what is the slope over here? The slope over here is minus 1.43. That is, between point A and point B, the slope is 1.43. Now, what is the slope between point C and point D? So, what is the change in del p x? The change in del p x is between price 2 and price 3. So, this is minus 1. What is the change in the demand? The change in the demand is between 8 and 3.5. So, that leads to 4.5. So, this comes to 0.23.

So, if you look at it in a non-linear demand curve, the value of the slope changes or the value of slope is not constant at all points of the demand curve. So, when we measure or when we calculate the slope between point A and point B, we got a figure which is 1.43. When we calculated the slope between point C and point D, the value of the slope is 0.3. So, we can say that the slope of the non-linear demand curve is different between the different points.

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Now, when you measure the slope at a point of the curve, what may be the limitation or what maybe the constant over here. In case of a non-linear demand cure, what we do? We calculate the slope at two different points taking the change in the price and change in the quantity demanded and then we measure the value of the slope.

So, what are the limitations when you measure the slope at a point on a curve? This method may not be reliable because particularly in this case, when the change in the independent variable is large because slope is different from any set of two points within the chosen two points of the curve. This method is not much of help in case of a optimum solution to the business problem, that is to a firm, because an optimization problem may involve a polynomial function.

So, measuring slope, particularly for a linear and non-linear, it is possible when it comes to polynomial. It is basically difficult to use the same method to measure the slope and

that is the reason, the difference between two variables may be sometimes too large that is difficult to do analysis by measuring the slope in this way.

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That is the reason that there is a technique of the differential calculation has come into existence, in order to understand the margin or in order to measure the marginal change in the dependent variable, due to change in the independent variable. Particularly, when the change of change in the independent variable approaches 0 and the measure of such marginal change is generally known as the derivative.

The derivative of a dependent variable y is the limit of change on y when the change in the independent variable x approaches 0. So, because of the limitation to measure a slope at a point in a curve, the technique of differential calculation generally comes into picture.

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So, differential calculation is generally used to find an optimum solution to the problem. This is used in the derivative of a constant function, derivative of a power function, derivative of a function of the sum and difference of the function. Function is a product of two functions, and that is derivative of a quotient, and derivative of a function of a function.

So, we will check each function individually and how we use differential calculus over there. But before that, we will see how we can find out the differential calculus or how we can represent the differential calculus graphically. So, we are considering y is the dependent over here and x is the independent over here.
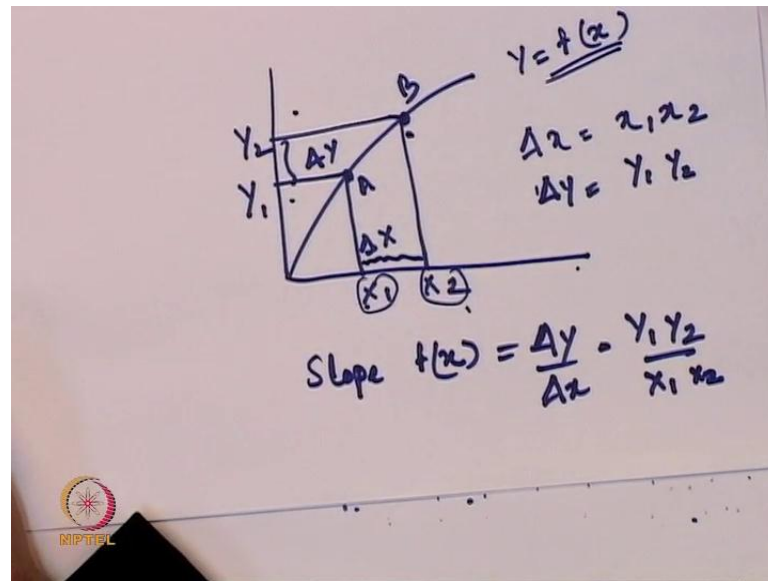
So, when we represent this in a graph, we take a function and that is y is equal to function of x, x 1, and x 2. This is y 1 and this is y 2. Now, what is the change in the x? That is, what is change in x from x 1 to x 2? This is the change in the y from y 1 to y 2. So, this is point A and this is point B.

Now, when x increases from x 1 to x 2, y increases from y 1 to y 2. So, demand function shifts from point A to point B. So here, what is del x now? del x is x 1, x 2 and del y is y 1, y 2. How we will identify what is the slope of this function? So, slope of this function is del y by del x, which is y 1, y 2 by x 1, x 2

So, when the change in the dependent and independent variable is very small, the slope can be calculated from the method of the differentiation or by the method of the differential calculus. Now, we will take the same example here. We have just taken a general function. Now, we will take a function specifically to the demand function to understand how differential calculus is being used in order to calculate the marginal change or in order to measure the changes between the two variables, that is dependent variable and the independent variable.

So, let us take a demand function, that is D x is equal to 32 P x to the power minus 2. So, this is a demand function and we will use the differential calculus to find out the slope. When this differential calculus is required or when this differential calculus is helpful? When the change in the independent and dependent variable is very small and it is difficult to find out the value of slope by the formula what we had discussed earlier.

So, if you are taking the first order derivative equal to 0, then it becomes del D x with respect to del P x. So, this comes to minus 2 32 P x minus 3 . So, this comes to minus 64 by P x cube.

So, if we take the reciprocal of the above equation, then this is del P x del D x and this is minus P x cube by 64. So, if you take this point in point B, considering this function is point B, and the graphical representation that we did earlier and substituting price is equal to 4. So, taking this and substituting price is equal to 4, now what will be the value of this equation? That is, minus 4 cube by 64. So, this is minus 64 by 64, which is cube equal to minus 1. So, the slope of the demand curve using the differential calculus, both at the tangent method and by the differential calculation, this is 1.

So, in this case, what we did is, we took the same demand function what we did earlier by taking the general tangent method to understand the slope and we found the value of the slope is equal to minus 1. Now, we have taken a different formula or may be a different method and that is the rule of differentiation or the difference calculation or

derivative to understand or find out the slope. Following the differential calculus, taking the first order derivative and putting the value of p, we got a value of slope which is equal to minus 1.

So, if you remember in case of a tangent method also, the value of slope is minus 1. So, whether we follow the tangent method or whether we follow the differential calculus method by taking the same demand equation, the slope becomes equal. The only difference here is that we cannot use the tangent method with all changes in the dependent or independent variable. In the dependent and independent variable, the change related to is small, in that case only the differentiation or the differential calculation method can be useful.

So, in the next session, we will look at what are the rules of differentiation. As we discuss that, how we deal with the derivative or a constant function, power function, function of sum and differences of equation, quotient, and then power function, everything we will discuss in the next class related to the rule of derivatives or the rule of differentiation.