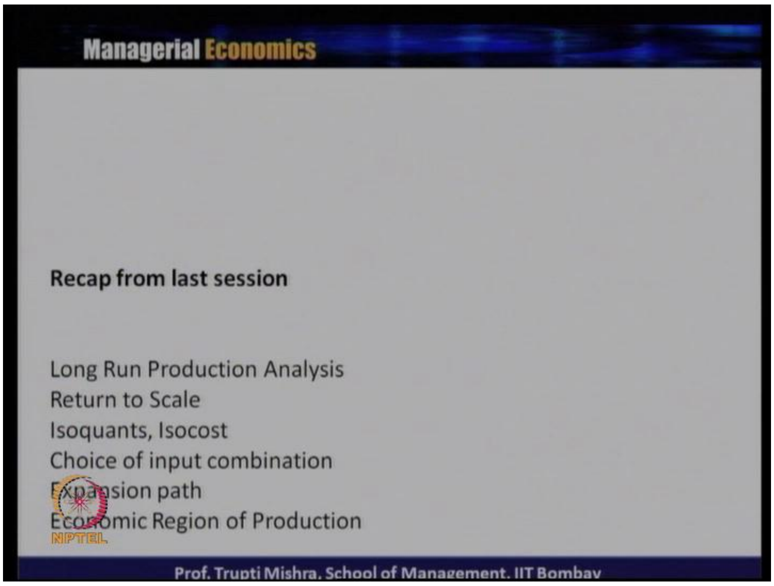
**Managerial Economics**
**Prof. N. Trupti Mishra**
**S. J. M School of Management**
**Indian Institute of Technology, Bombay**

**Lecture -19**
**Theory of Production (Contd…)**

In continuation to our session on theory of production and cost, we are going to cover few more concepts today, in today's session. So, if you remember in the last couple of session we had just discussing about the different type of production analysis. We started with short run production analysis, and we discussed through the law of diminishing return. And then again we started the return to scale that is the long run analysis of production and there we check that how the scale differ with respect to change in the input, and proportionately the change in the output.

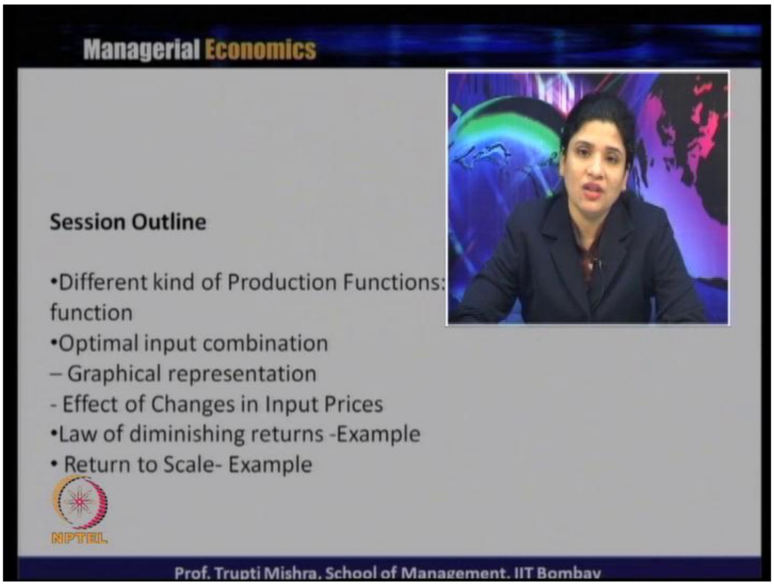
Then we discussed the case of producer equilibrium or so called the least cost input combination, with the help of the two concept; that is isoquant and the isocost, and through which how they reach the how the firms or how the producer they reach the equilibrium.

(Refer Slide Time: 01:21)




Then we discuss about the expansion path and economic region of production which talks about, basically which one is the feasible region where the two inputs can be substituted one to another. And that is the efficient region because by, producing or because by using less of input the producer is producing the desirable output.

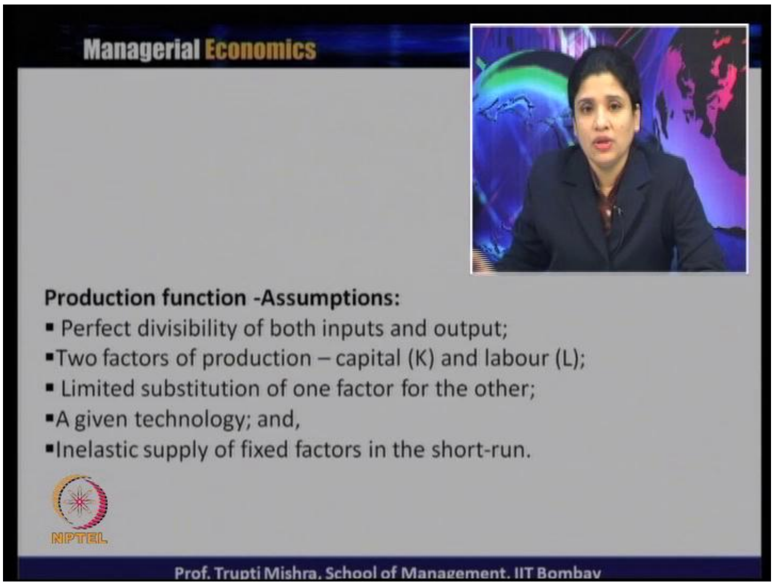
(Refer Slide Time: 01:37)




So, in today's session we will see, what are the different kind of production function? Mainly, and mainly we talk about the Cobb-Douglas production function which is used more in economic analysis. Then again we will continue our discussion optimal input combination through a graphical representation, how the graphical representation in case of a maximization of output and minimization of cost.

And again we will see that when there is a change in the input price whether, it is the input price of the capital or input price of the labor, how it changes. Then we will talk about numerical examples related to the law of diminishing return and the return to scale. How generally this, the firm uses this law of diminishing return and returns to scale empirically. Whether it is really works that marginal product gets decreases and then it reaches the negative. And whether there is a evidence of increasing decreasing and constant return to scale.

(Refer Slide Time: 02:35)



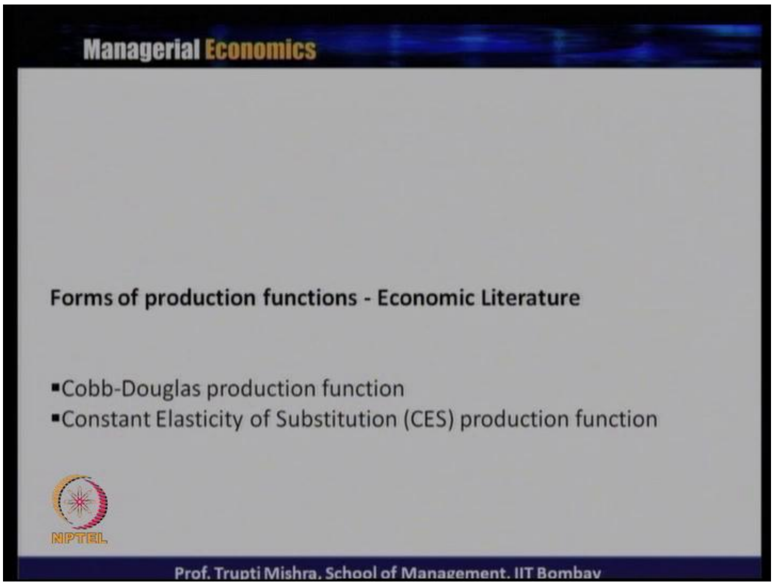

So, to start this today's discussion, we will see that, all the production function they are based on the assumption. It is not that, we can just formulate a production function just taking a functional form, which talks about the relationship between input and output. Rather the production function they, in order to formulate the production function we need to assume certain thing. And what are the general assumptions over here. There is perfect divisibility of both inputs and outputs. So, inputs are divisible and output also divisible.

Two factor of production generally we use, you do not use more than two factors like, if you look at there are number of factor of production, like labor, capital, time, raw material, technology and entrepreneurship. But for all these analysis whether it is short run whether it is long run, we generally use only capital and labor as the input, not any other inputs in the production process.

Then we are assuming that both the factor inputs, that is labor and capital, they are substitute to each other, but they are in a limiting sense. There is no unlimited substitution or they are no closely they are not perfectly substitute to each other. Like if you remember the, if it is perfectly substitute, then the output can be produced either with the help of capital or with the help of labor. But in this case we are assuming that, certain amount of both the inputs are necessary in the production process, the production cannot be run only on the basis of the input.

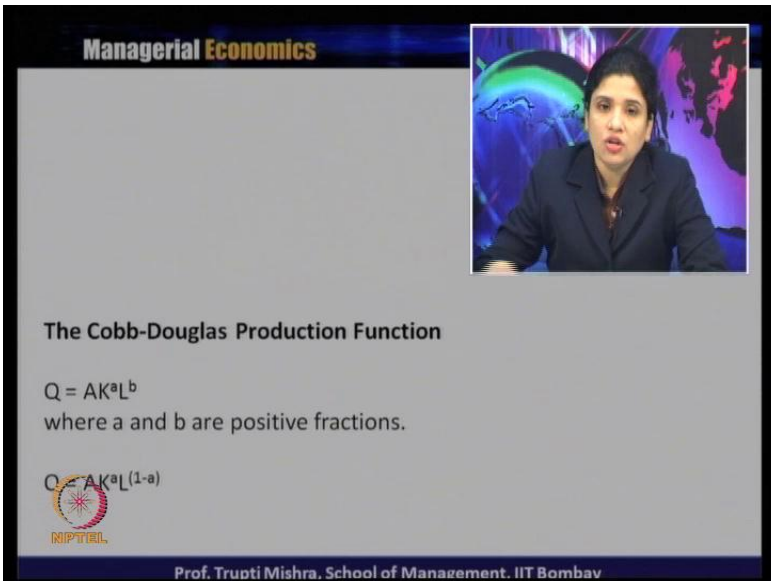
Or only on the basis of the capital then technology is given technology cannot change may be at least in the short run in the long run it can be changed. And also we assume that there is a inelastic supply of fixed factor in the short run. And that is the reason the short run, there are few factors those are considered as fixed. And in specific sense when we are taking the case of two inputs, here generally capital is fixed and there is inelastic supply. And whenever there is a increase in output or whenever there is a need to increase the production output, generally the labor gets changed in order to increase the output, because when there is inelastic supply of fixed factor in the short run.

(Refer Slide Time: 05:01)



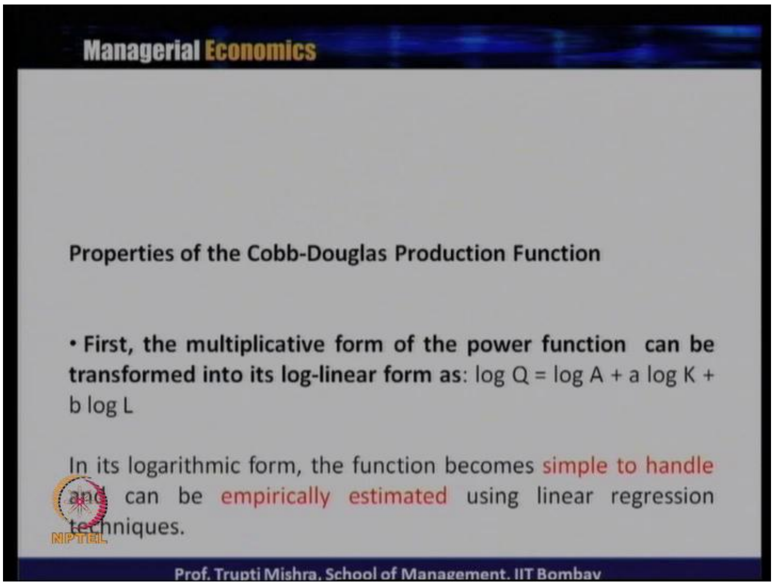

So, in that context, when we in the economic literature there are two main type of production function are used. One, Cobb-Douglas production function and second one is the cost and elasticity of substitution production function that is CES production function. We mainly use, typically in economic literature, either Cobb-Douglas production function or cost and elasticity substitution, popularly known as CES production function.

(Refer Slide Time: 05:21)



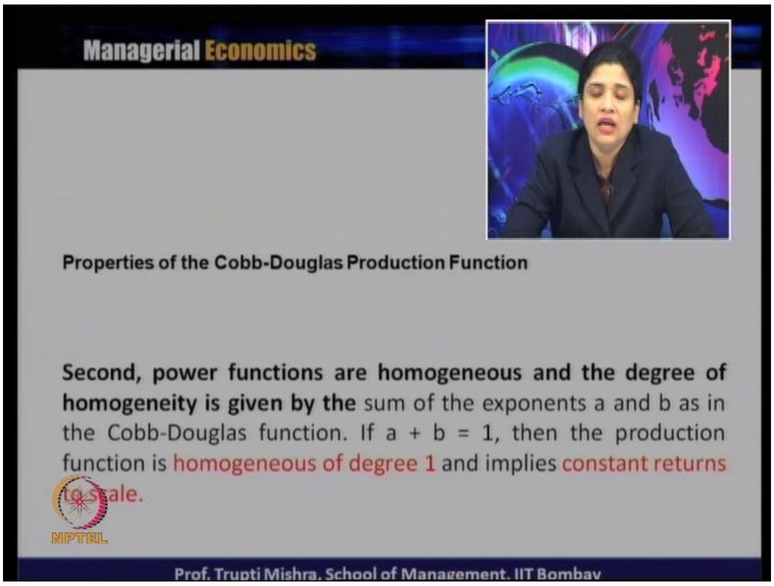

Today we will focus more on the Cobb-Douglas production function, because this is mostly used in case of the economic analysis. Cobb-Douglas production function takes the form of Q that is output, which is a function which is A K to the power a and L to the power b, where a and b they are the positive fractions. And K and L is the, K is the capital and L is the labor over here. So, Q is the A K to power a and alternatively we can take this as L is to the power 1 minus a, because a plus b has to be equal to 1. So, if a plus b is equal to 1 then alternatively we can formulate this production function as Q is equal to capital A K to the power a and L to the power 1 minus a.

(Refer Slide Time: 06:09)



Now what are the properties of Cobb-Douglas production function. Firstly, the multiplicative form of power function can be transformed into a log linear form, like log Q is equal to log A plus small a log K and b log L. so in logarithmic form the function becomes simple to handle and can be empirically estimated using linear regression technique. So, the first property is Cobb-Douglas production function, can be transformed into a log linear form. And why it is generally, what is the benefit if it is getting transferred into a log linear form? It becomes simple to handle, and when we are doing a empirical analysis using the Cobb Douglas production function then this is easy to handle. And using linear regression technique we can empirically estimate the Cobb Douglas production function.

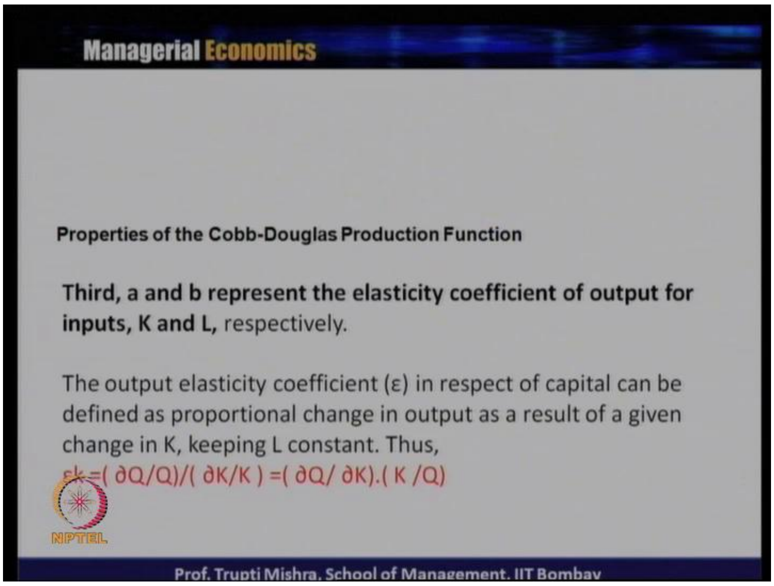
(Refer Slide Time: 06:59)




Then secondly, the second property of Cobb-Douglas production function is that, power function are homogenous, and the degree of homogeneity is given by the sum of exponent, a plus b as in the Cobb Douglas function. So, if a plus b is equal to 1, the production function is homogenous degree 1 and implies a constant return to scale. So, the power functions are homogenous. And the degree of homogeneity is given by the sum of exponents of a and b, as in the Cobb-Douglas function.

So, if a plus b is equal to 1 then this is the production function is homogenous of degree 1, and implies constant return to scale. If a plus b is greater than 1 then it implies a increasing return to scale. And if a plus b is less than 1 again it implies a decreasing return to scale. So, it depends up on the value of the exponent in the Cobb-Douglas production function that is a and b, that determines of what kind of production function it is, and what kind of scale it is bearing on.

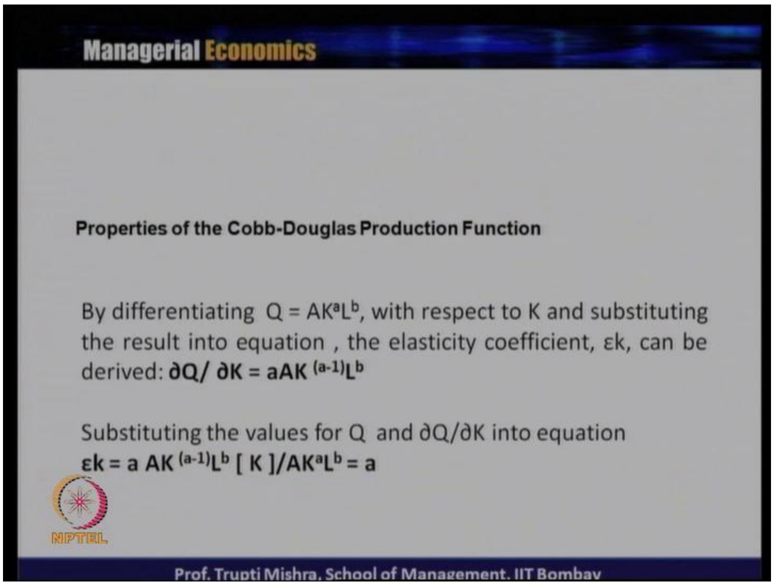
(Refer Slide Time: 08:00)




Thirdly, the a and b represents the elasticity co efficient for output for input, K and L, respectively. So, the output elasticity coefficient E in respect to capital can be defined as the proportional change in the output, as a result of given change in K, keeping L constant. So, if you are keeping L constant, and if you are trying to find out what is the elasticity coefficient of output for input, with respect to capital only then this is del Q by Q that is the change in the output, with respect to change in the capital So, del K by K. And if you simplify this then this del Q by del K multiplied by K by Q. So, this is nothing but the elasticity coefficient with respect to input K, keeping L as the constant.

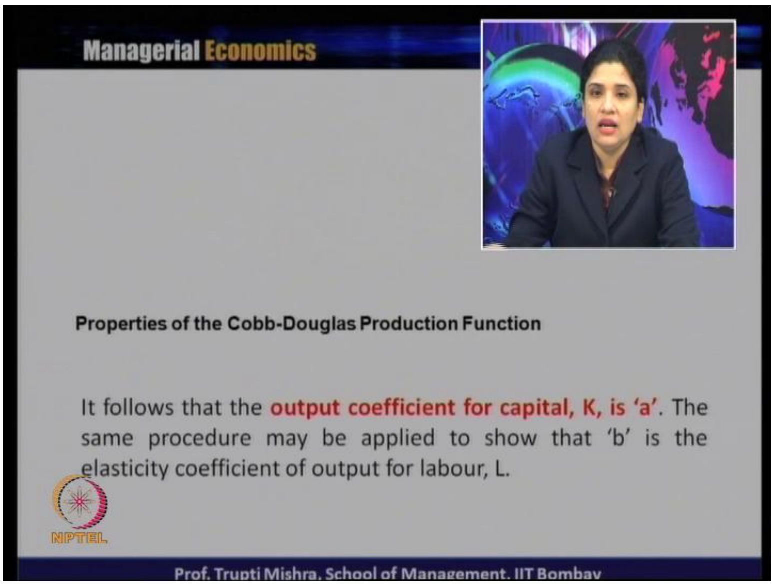
So, partial elasticity of this production function which is a dependent on capital and labor, keeping L as the fixed the elasticity coefficient with respect to capital is del Q by del K, multiplied K by Q.

(Refer Slide Time: 09:04)



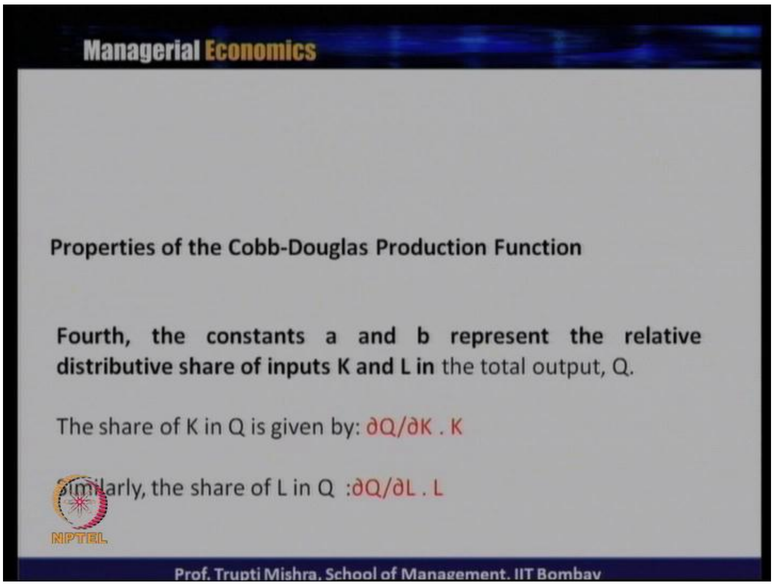

So, taking the specific production function that is Q is equal to A K to the power a L to power b, with respect to K. And substituting the result into equation, the elasticity of coefficient, E K, can be derived as del Q by del K that is a then capital A K to power a minus 1 and L b. So, substituting the value of Q and del Q by del K in the equation elasticity of coefficient with respect to capital says that a A K by a minus 1 L b, and in the input bracketed to have K divided A K a L by b, and when we have simplify this we get it equal to a. So, elasticity coefficient with respect to capital keeping labor as constant the value is equal to a.

(Refer Slide Time: 09:51)



Similarly, when we find out for the b, the same procedure we can follow. And we can find out the output coefficient with respect to, labor capital is constant, and the value of output coefficient with respect to labor is coming out to be b. So, elasticity coefficient of, capital, elasticity coefficient for capital keeping labor as constant it is a. The same procedure can be applied to find out the elasticity of coefficient with respect to labor. And the elasticity coefficient of output for labor is coming to the L. And the value of it will come as the b.

(Refer Slide Time: 10:30)

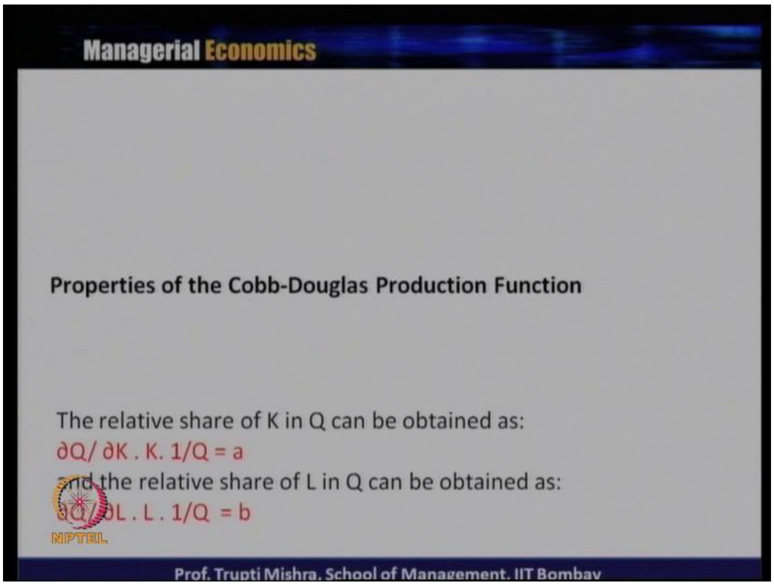



So, properties of Cobb-Douglas production function in continuation with this, we have the fourth property. And here the constant a and b represent the relative distributive share of input K and L in the total output Q. So, fourth property talk about the constant a and b what it represent, so basically constant a and b associated with input capital and labor, represent the relative distributive share in the input K and L in the total output, Q. So, the share of K in Q is given by del Q by del K multiplied by K. And similarly the share of L in Q is del by del L multiplied by L.

So, del Q by del K multiplied by K is the share of K in Q. And share of L in Q is the del Q by del L multiplied by L. So, the, if you look at this del Q by del K multiplied by K, the first part is talks about the change in the Q with respect to change in the K, multiplied by the actual amount of K. And the share of L in Q is that del Q by del L that is the change in the output with respect to labor and multiplied by L.

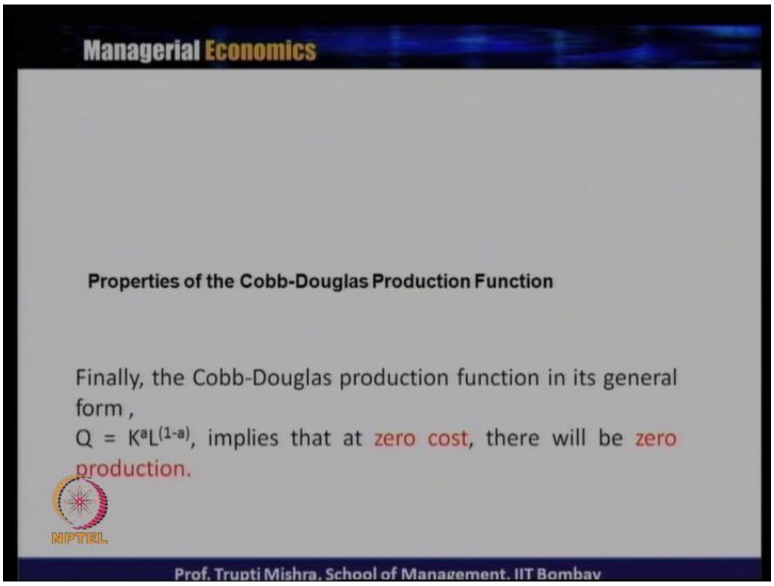
So, four properties, talks about the constant of a, b associated with labor and capital. And they generally represent that relative distributive share about K and L in the total output.

(Refer Slide Time: 11:55)



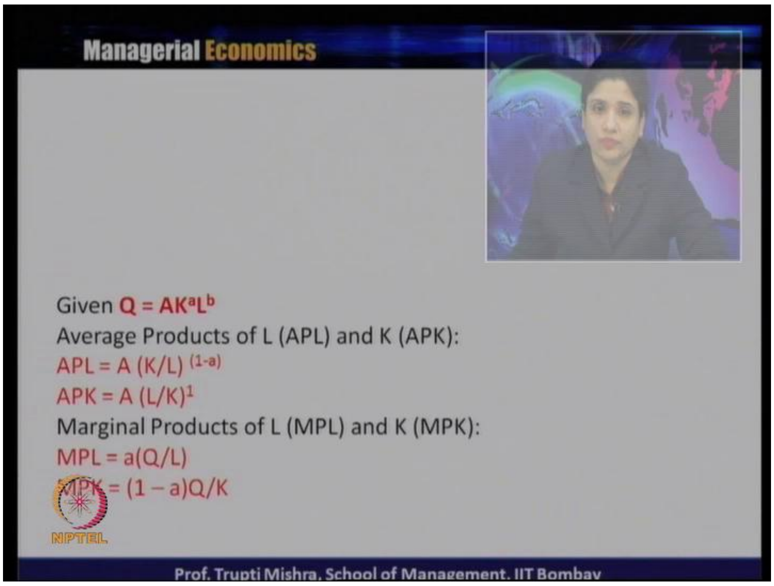

So, in continuation with the fourth property, the relative share of Q and K can be obtained as, del Q by del K multiplied by K multiplied by 1 by Q which comes to a; and the relative share of L in Q can be obtained as del Q by del L multiplied L 1 by Q that comes to b.

(Refer Slide Time: 12:13)



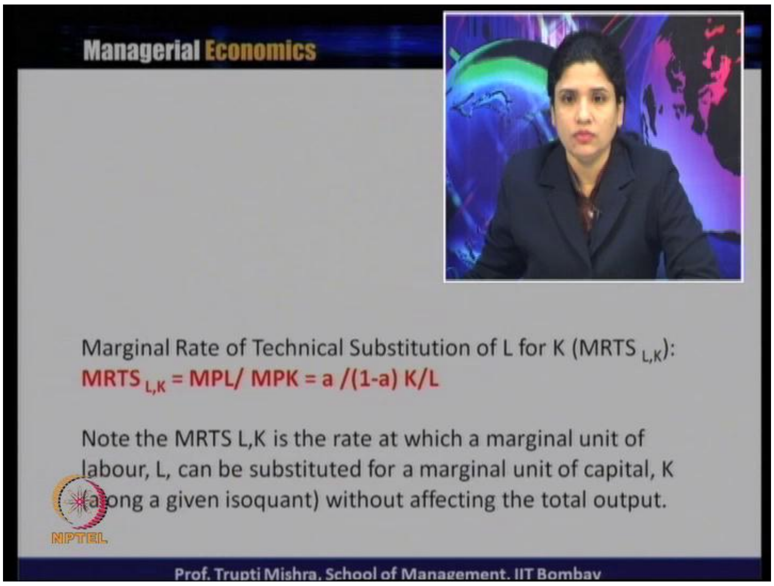
Finally, the Cobb-Douglas production function it is general form, that is Q is equal to K to power a L to the power 1 minus a, implies that at zero cost, there will be zero production, because the value of intercept is or the value of constant is missing here. So, if in the general form if it is Q is equal to K to the power a and L to the power 1 minus a. It implies that at zero cost, there will be zero production because the capital a value is missing over here.

(Refer Slide Time: 12:43)




So, given these Cobb-Douglas production function, if the production function is A K to the power a L to the power b. The average product of L is APL and K is APK. So, APL is A K by L 1 minus a and APK is A L K by 1. And similarly we can find out the marginal product for capital and marginal product for labor, MPL is a Q by L and MPK is 1 minus Q by K. So, considering this is a considering this as a Cobb-Douglas production function, accordingly the value of the average product for labor, average product for capital, marginal product for labor and marginal product of capital will change.

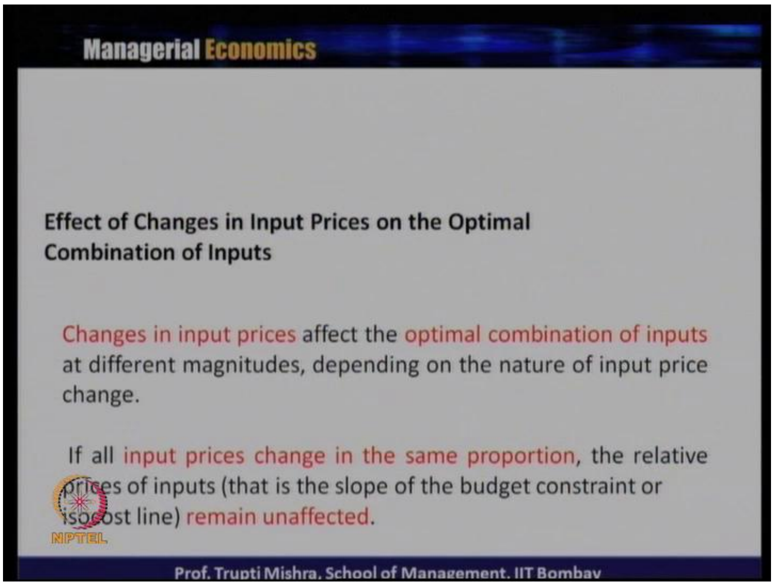
(Refer Slide Time: 13:27)




Similarly, when we are finding the marginal rate of technical substitution of L for K, then taking the specifically this Cobb-Douglas production function. So, as we know there is marginal rate of technical substitution is the, slope of the isoquant. And how the slope of the isoquant can be represent? This is the ratio of the marginal product of both the inputs. So, in this case the ratio of the marginal product of capital and labor, so this is, when we are finding out a marginal rate of technical substitution, specifically for the Cobb-Douglas production function, then marginal rate of technical substitution for, L for K, is MPL by MPK that is a by 1 minus K by L.

Here, we have to note that the marginal rate of technical substitution L by K is the rate, at which, marginal rate of L can be substituted for marginal unit capital K along a given isoquant without affecting the total output. So, it is like rate of substitution between two inputs without and the even if the input level is changing or rather amount of getting used as from labor and capital changing, still it has to be in the same isoquant, So, the level of output is not changing. Similarly if you take a serious production function or any other form of production function, we can, in the similar way, we can derive the basic concept using in the production analysis like average product marginal product and the marginal rate of technical substitution, both for L for K and K for L.

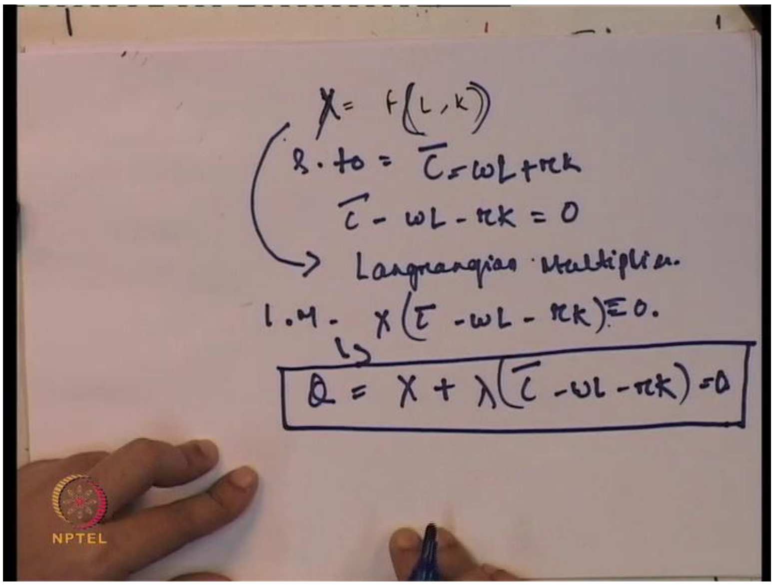
(Refer Slide Time: 15:04)




Now, if you remember in the last class we talk about the least cost input combination. And least cost input combination is one where the slope of the isocost is equal to the slope of the isoquant. And this is the point at which the producer or the firm maximizes the output, looking at the given constant.

So, today we are going to spend, may be another, spend in detail that how the equilibrium conditions are derived, how we can say that the slope of isocost has to be equal to the slope of isoquant, we will see. Then we will look at the graphical representation and then we will come to the point where the input price is changes, and how it generally affects the least cost input combination and how the effects are being captured. So, we will first see that how the equilibrium conditions are derived, or may be, how the precondition for the least cost inputs are derived. And then we will look at into the graphical representation, both for the maximization case and for the minimization case.

(Refer Slide Time: 16:13)



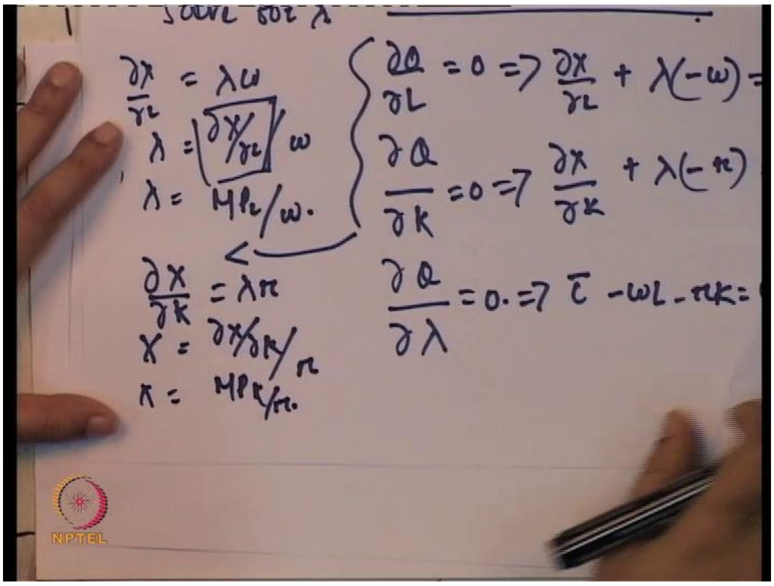

So, let us look at into the equilibrium condition, how the equilibrium condition how the equilibrium conditions are derived. So, this is the, if this is the production function that is for X, for X is the function of labor and capital. In this case, how we can find out, what is the equilibrium condition? Now here there is a constant that is subject to C bar that is w L plus r K.

So, if you remember this is your isocost. Now this, if you can change this constant into this then this is C bar minus w L minus r K which has to be equal to 0. Now we will, whenever we need to maximize something minimize something with a with respect to a constant, in this case we need to use a Lagrangian multiplier. Generally known this as a Lagrangian multiplier method. And here what is the Lagrangian multiplier. Lagrangian multiplier, the Lagrangian multiplier here is X C bar minus w L plus r K which is equal to 0.

Now what is this Lagrangian multiplier? Generally this is the undefined constant or undefined constant which generally use to maximize or minimize a function. Because, if there is a constant associated with this, if there is a constant associated with this, we cannot directly maximize the production function. And that is the reason we need to take the help of the Lagrangian multiplier method. And these are the, the Lagrangian multiplier is the undefined constant which generally use the, to maximize or minimize a function. So, once we get the Lagrangian multiplier method then we will get the composite function. Composite function is X plus lambda C bar minus w L minus r K which has to be equal to 0.

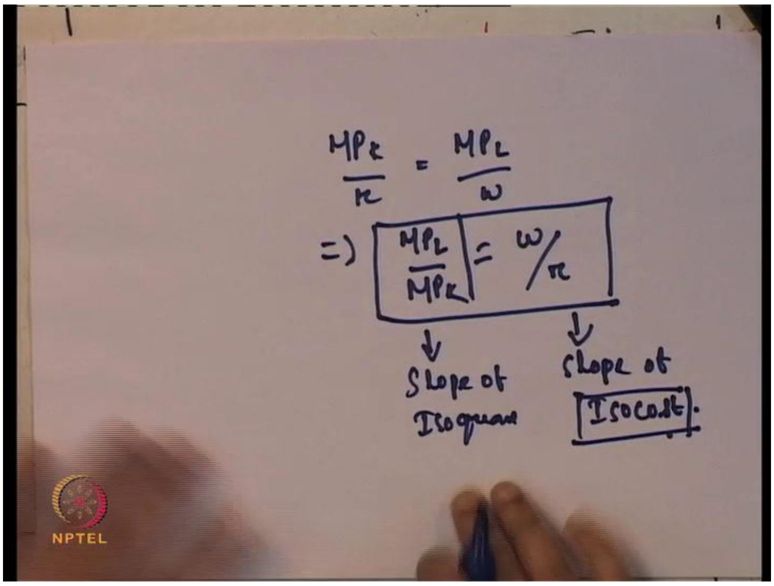
So, this is the composite function, using the lagrangian multiplier method. now what is the next job, next job we need to maximize it; and we will see what should be the first order condition, and what should be the second order condition, in order to maximize in order to minimize. So, given this as the composite function, what should be the first order condition? If you, if you remember, all the first order condition if it is a maximization or may be it is a minimization, the partial derivative has to be equal to 0. So, here we will take the partial derivative with respect to the undefined constant, and we will set then equal to 0 in order to find out the first order condition.

(Refer Slide Time: 19:07)




So, del Q by del L which is to be equal to 0; del Q by del K which has to be equal to 0; and del Q by del lambda which has equal to be 0. So, this implies del X by del L plus lambda minus w equal to 0; then this implies that del X by del K plus del lambda minus r is equal to 0; and this implies c minus w L minus r K which has to be equal to 0. So, from equation, first two, if you solve for lambda then this comes to del X by del L is equal to del w or then this equals to del X by del L by w. And this leads to lambda, this is our marginal product for labor by w. Similarly from equation two, if you find, if you solve for value of lambda then this is del X by del K is lambda r or X is equal to del X by del K by r, and this is since this is MPK by r.

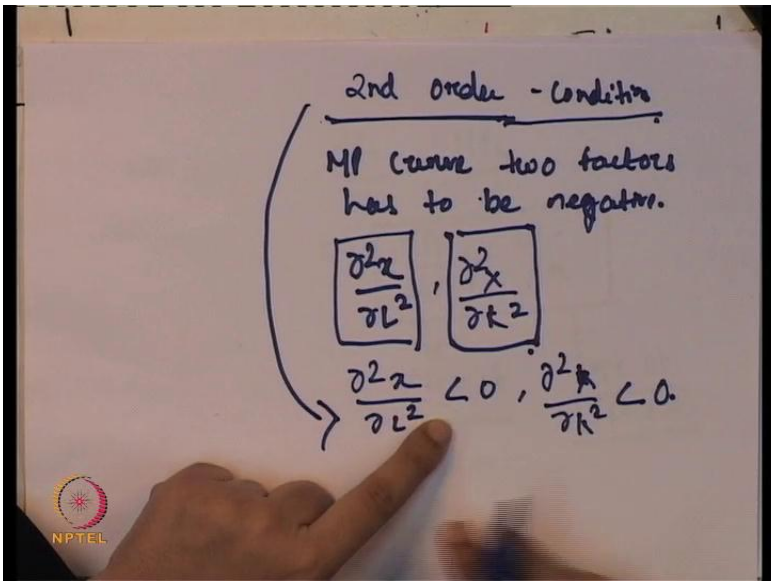
(Refer Slide Time: 21:06)




After solving for both this lambda, then this comes from to MPK by r which is equal to MPL by w leads to MPL by MPK equal to w by r. This is the first order condition for the least input combination because, this represents the slope of isoquant, this represents the slope of isocost.

Since the constant is given in the form of isocost, and the output is given; in this case we can say that the first order condition has to be the point at which slope of the isocost has to be equal to the slope of the isoquant. So, the ratio of marginal product of labor and capital gives us the slope of the isoquant; and the ratio of input prices gives us that is w and r that gives us the slope of the, slope of the isocost. So, first order condition for least cost input combination says that, at the point of equilibrium or at the point of least cost input combination the slope of the isocost has to be equal to the slope of the isoquant.

(Refer Slide Time: 22:35)

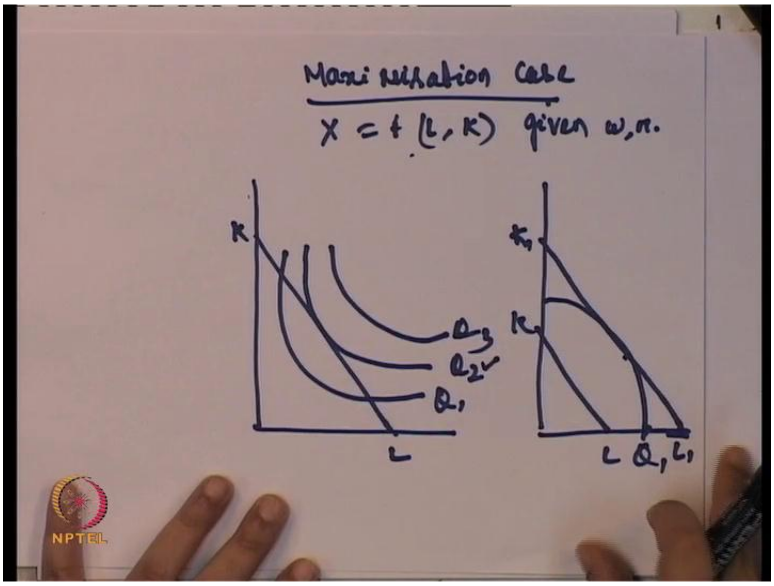



Next we will see, what is the second order condition for this maxima or minima with respect to the least input combination. This requires the marginal product curve for both the factors has to be negative. So, del square X del L square and del square X del K square. So, this is what, in order to find out the slope we need to know the take the second order derivative with respect to labor, and secondary derivative with respect to capital. So, this has to be 0 that is del square X by del L square, has to be less than 0; and del square X and del K square has to be less than 0. So, second order condition for the least input combination requires the marginal product for both the factors that is capital and labor.

The marginal product curve for both the factor has to be negative. And how we will find out the marginal product curve for both the factors from negative? We need to take the second order derivative for the, with respect to capital and with respect to labor of the composite function. And that gives us del square X by del L square for the second order derivative for the labor; del square X by del K square the second order derivative for the capital.

And second order condition says that the it has to be negative, and that is the reason the second order derivative the del square X del L square has to be less than 0; del square X del K square has to be less than 0. Next, we will see how graphically, we look at both the input and the maximization case and the minimization case, in case of the least cost input. So, what is the essential difference between the maximization and the minimization case. In case of maximization case, the cost is given and if taking the cost as constant and the isocost line is the constant, the producer has to maximize the output. Whereas in case of minimization case the output is fixed, and looking at the fixed output, what is the challenge for the producer, the challenge for the firm is to minimize the cost.

(Refer Slide Time: 25:08)

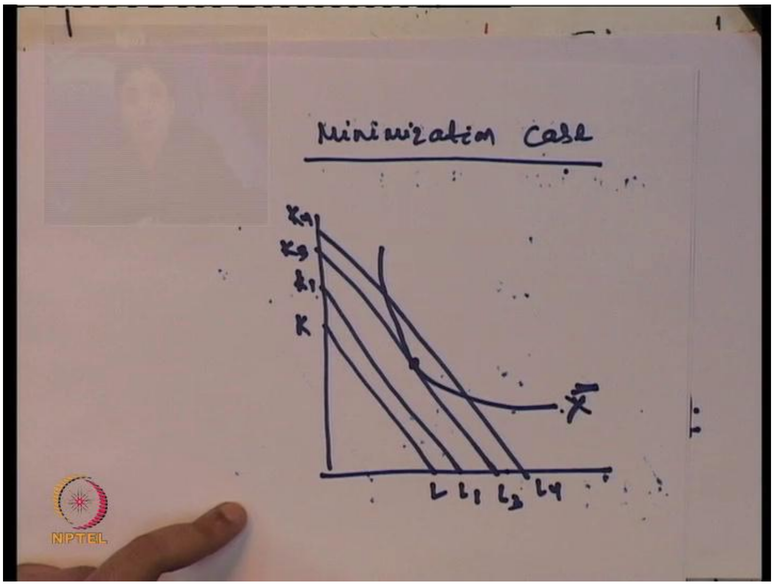



So, let us look at the maximization case first. So, maximize with res[pect], maximization of X which is a function of labor and capital, with respect to the input prices that is w and r. So, if you look at, there are two graphs; graph one is, where there are three isoquant and the isocost is given; and graph two is where the isoquant is there in a different shape, and there are two isocost line. So, in case of maximization case, what happens? This isocost is given, and with this isocost the challenge of the producer is to get the maximum level of output, and looking at this the consumer will always pick up a combination in Q 2 level of output.

Because the Q 2 level of output can be achieved with the isocost K and L which is given. But in case of, second case if you look at, the isoquant is taking a shape of concave which is not possible because, in case of concave it is not following the basic rule of the production analysis, like if you look at basic rule of a isoquant. Because even if, with the same isoquant they are able to achieve the combination it is not giving the same level of, same level of output across all these stages. May be the input combination are different because they are when they are moving from one point to another point.

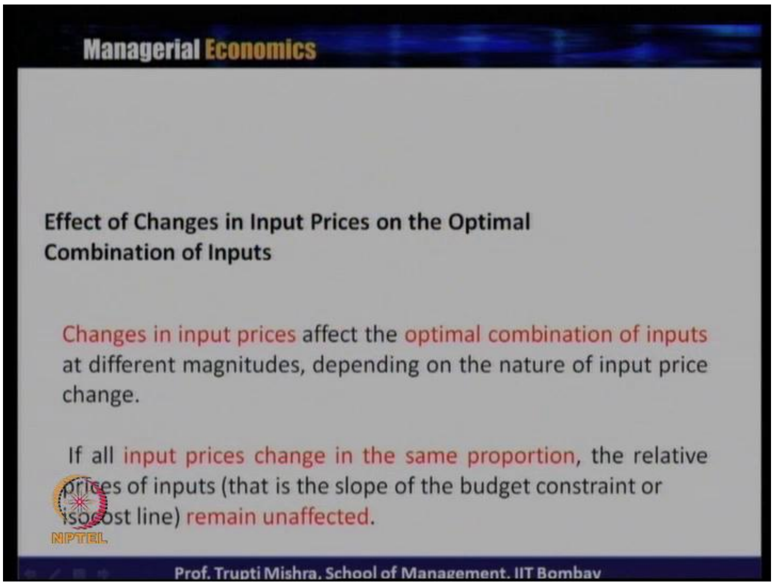
They are using more of the inputs of both, but they are producing the same level of output which is not the cost efficient or which is not the input efficient. That is the reason in case of maximization case, the output level is, can be achieved with the maximum output level can be achieved with the isocost line given or in term of the cost is given. Next, we will see the minimization case where the output is given. The challenge for the producer is to minimize the cost of production or minimize the input prices with respect to the given level of output.

(Refer Slide Time: 28:02)




So, these are all isocost. These are the points which talk about the cost of production. If you are taking any of these combination of input prices. And looking at, this if you look, at if the X bar is the output that is given then in this case the producer will always look for this that, to produce this level of output which one can be the minimum cost. So, in this case to achieve this level of output, K 3, L 3, is the minimum isocost or the minimum cost of production that is the reason they will choose this point as the least cost input combination. Because to produce the input level of output this is the minimum possible cost. So, in case of the minimization case, the challenge of the producer, the challenge of the firm is to minimize. Or they will always look for the combination that gives us the least cost to the producer least cost to the firm for a given level of output.

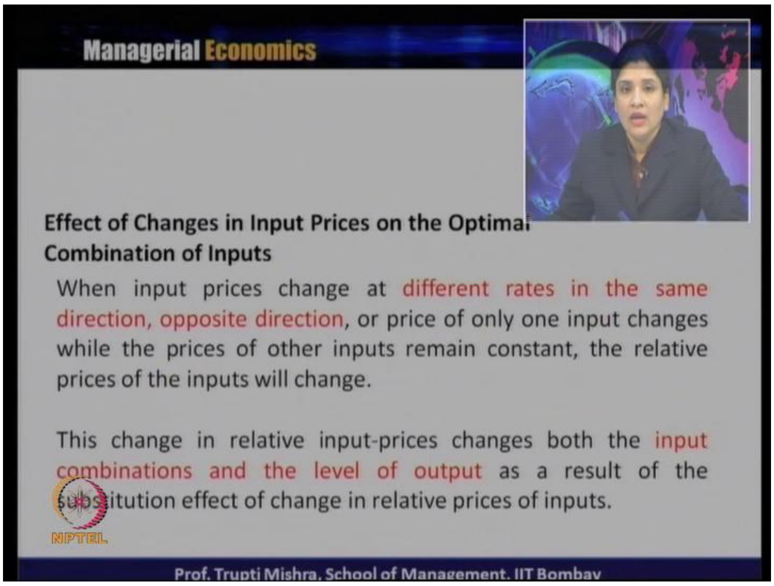
(Refer Slide Time: 29:45)




Let us look at the case where, if there is a change in the input prices, how it affects the least cost input combination? So, change in input price, if you look at it affect the optimal combination of inputs, at different magnitude, depending on the nature of the input price change. So, if all input price change in the same proportion, so it bound to happen the optimal combination of inputs, has to change, if there is a change in the input price.

Either in the different magnitude or same magnitude depends on the nature of the input price change. So, if all input price changes, in the same proportion; the relative price of the input that is the slope of the budget constant or they remain unaffected. So, if all input price changes in the same proportion, the relative prices of inputs also, if you look at they move in the same proportion and those are remain unaffected.

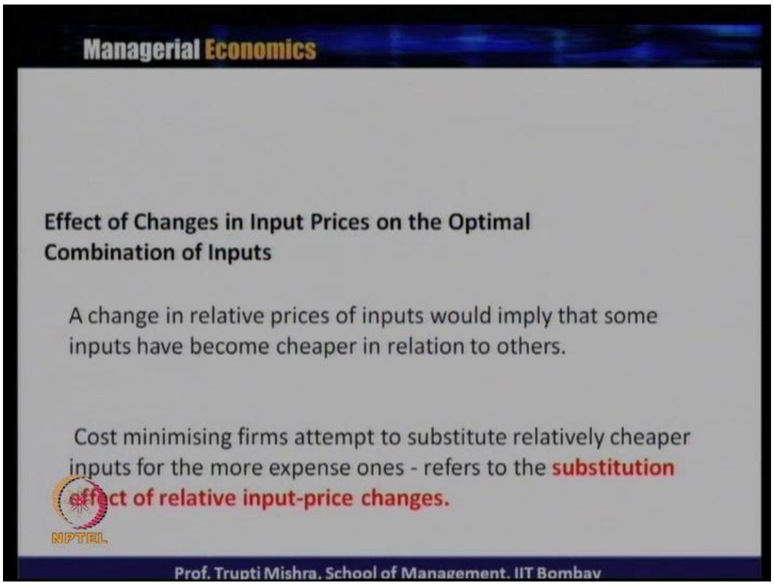
(Refer Slide Time: 30:27)




But when the input price is changes at a different rate in the same direction, or the opposite direction, or the price of only one input changes, while the price of other input remains constant, the relative price of the input will change. Input price when it change in the different rate in the same direction; different rate in the opposite direction. Input price of one changes, other remaining constant the relative price of input will change.

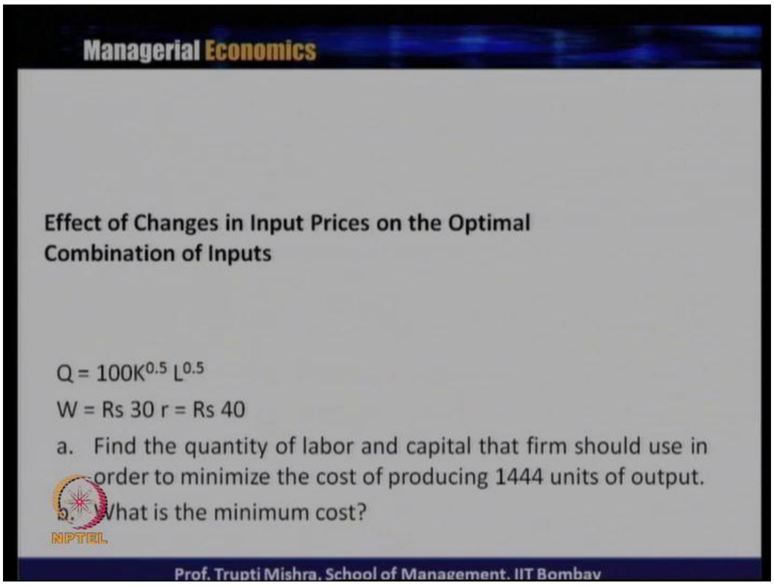
This change in the relative input output price change, if both in the input combination and the level of output, as a result of effect of substitution change in the relative prices of input. This change in the relative input prices, changes both the input combination and the level of output. So, whenever there is a change in the input prices, it affects the input combination and the level of output, as a result of substitution effect of change in the relative price.

(Refer Slide Time: 31:26)



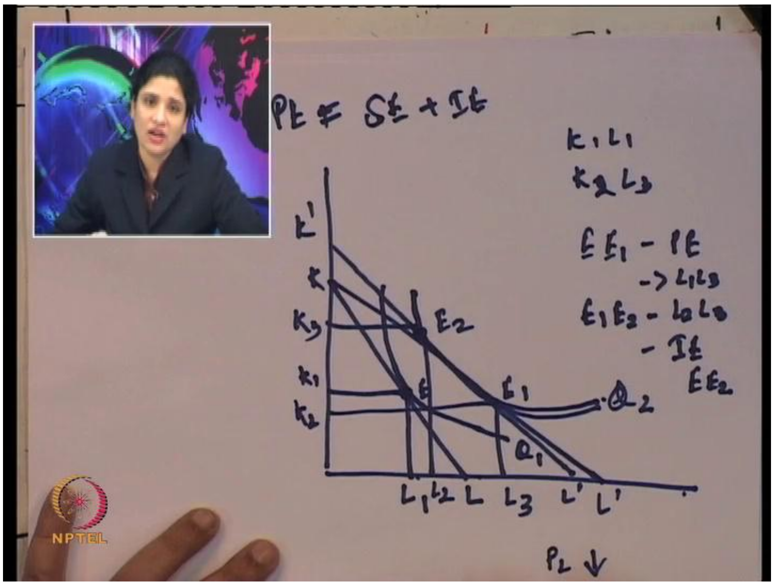

So, if change in the relative price of inputs, either in the same or the opposite, would imply that some input have become cheaper in relation to the others. So, cost minimization firm attempts to substitute relatively cheaper inputs for more expenses one refers to the substitution effect of relative price change. Because whenever the price of one input changes, it becomes cheaper with respect to the other inputs. And what the cost minimizing firms they do it over here? They generally try to replace the expensive input with respect to the cheaper inputs, and this is generally known as the substitution effect of relative input price change.

(Refer Slide Time: 32:07)



So, this numerical we will just look at later that how the quantity of labor and capital changes. Before that we will look at the graphical representation that when there is a change in the input prices, how it affects the least cost input, how it affects the level of output or how it affects the level of input combination.

(Refer Slide Time: 32:32)




So, initially this is the isocost, K L is the isocost; Q is the output, Q 1 is the isoquant and this is the level of output. Now, price of labor input decreases, with the help of that now the quantity of the, or if you look at the isocost will change from K L to K L dash. So, once it changes from K L to K L dash, then in this case, now what is the new input combination? The new input combination is, if you look at, this becomes L 3, that is L 1 and this is K 2 this is K 1. So, with the change in the input price, now the firm will use more of labor and less of capital that is the reason the combination now already it is K 1 L 1, now it is K 3 K 2 and L 3.

Now, to get, to keep it in the same level what ah the producer will do? May be at least at the same level of output, still want to change the input combination, now what they will do? They will try to draw a parallel line which also a tangent at a point which is at this level. So, suppose this is your point E, this is your point E 1, this is your point E 2. So, at this case if you look at, still it uses a higher level of labor as compared to the previous level, but its use a, also a higher level of capital.

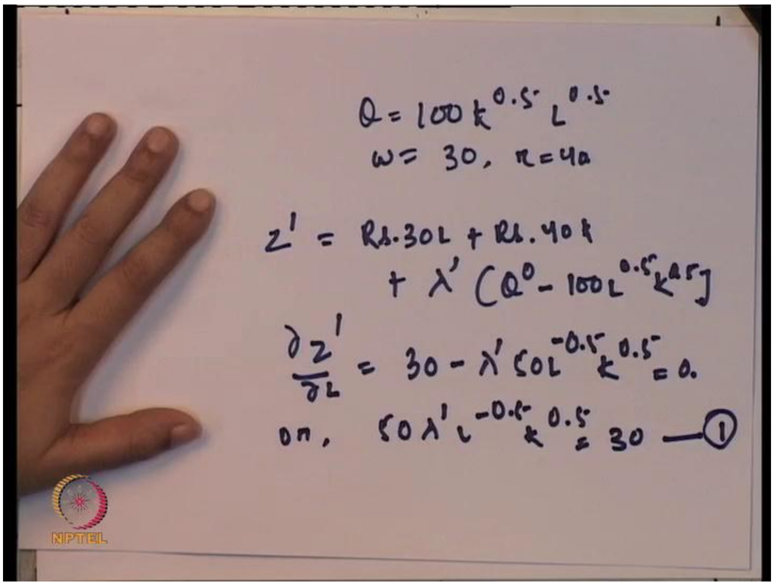
So, the movement from this E to E 1 is the price effect that is in the form of L 1 to L 3. The movement from, may be, E 1 to E 2 is budget defect because the producer is trying to keep the income level, the real income level of the producer at the same level that is the reason, we got a compensated budget line which is may be K dash and L dash which compensate or which may be the reduce the, real income of the producer in term of change in the input prices, and that leads to a different combination that is E 2.

So, E 1 to E 2 E 1 the movement from E to E 1 is the price effect, which in term of the labor input consist of use becomes L 1 and L 3. Movement from E 1 to E 2 is that is L 2 to L 3 is because of the income effect, because the real income is changing. And movement from may be E to E 2 is the substitution effect because of the change in the real income. So, if you look at the price effect is the combination of the substitution effect and the income effect.

So, if you remember your price effect, your substitution effect, and income effect in case of the consumer theory this is nothing but the counter part of the, counter part of the, counterpart of that in the production theory which talks about the change in the input prices. If there is a change in the input prices, generally the producer try to substitute that with a cheaper input, as compared to expensive input, that is the reason they go on using more of that input. So, in this case also the same thing has happened, the producer is, once the price of labor has gone down, the producer has tried to optimize it, and use more of the labor as compared to the capital.

And that leads to the combination of the change in inputs combination or also the change in the level of output. Next we will see the input combination or may be the input combination, how it changes or how to find out numerically when the production function is given price of inputs is given like w and r. And if the production function is given and to produce a specific level of output, how to optimize the cost of production and what should be the minimum cost of production. For that we will just take a example of, we will take a numerical example like Q is equal to 100 K that is to the power 0.5. And L to the power 0.5. This production function is in the form of a Cobb-Douglas production function, where w is 30 and r is 40. So, w is the price of labor that is 30 rupees, r is the price of capital that is 40 rupees, and what we need to do here? We need to find the quantity of labor and capital, that the firm should use in order to minimize the cost of producing a cost of 144 units of output. So, if you look at, this is the minimization case like the, second case where the unit of output is given, we need to minimize the cost, and what is the minimum cost? We need to find out that.

(Refer Slide Time: 38:27)

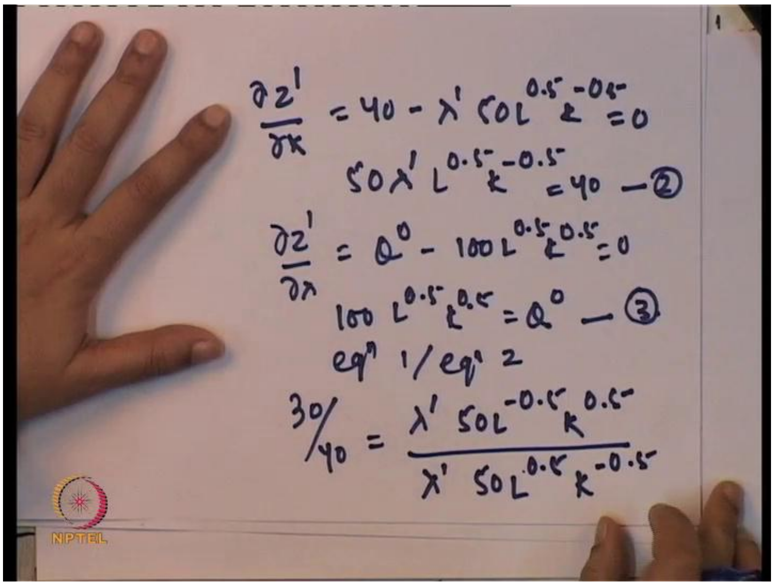



Then how we will go for this, we will use we will take the help of the Lagrangian multiplier to solve this. So, Q is equal to 100, K 0.5 and L 0.5. And here if you look at, then we have w is equal to 30 and r is equal to 40. So, first we will try to find out the composite function. Now composite function is rupees 30 L plus rupees 40 K plus lambda dash that is Q 0 minus 100 L to the power 0.5 K to the power 0.5.

Now, what the, what we need to do? We need to find out the first order condition. we need to find del z by del L that is 3 minus lambda dash fifty L minus 0.5 K 0.5 has to equal to 0 or 50 lambda dash L minus 0.5 K 0.5 has to be equal to 30, suppose this is our equation one.

Now we will need to look at the first order partial derivative or the first order derivative or the partial derivative with respect to the other input. In the first case we have checked it for L, now we will check it for K.

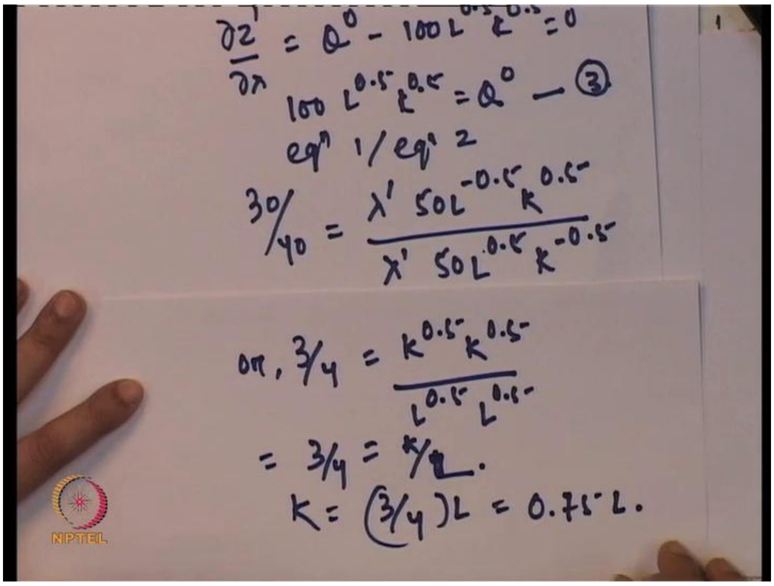
(Refer Slide Time: 39:57)




So, here we need to find out the del partial derivative with respect to K, that gives us 40 minus lambda dash 50 L 0.5 K minus 0.5 which is to be equal to 0; or 50 lambda dash L to the power 0.5 K to the power minus 0.5 which is equal to 40. Let us call it equation 2. Now to find out the partial derivative with respect to the lambda that is Q 0 minus 100 L 0.5 K 0.5 which is equal to 0. So, you can call it 100 L to the power 0.5, K to power 0.5 which is equal to Q 0. That is equation 3.

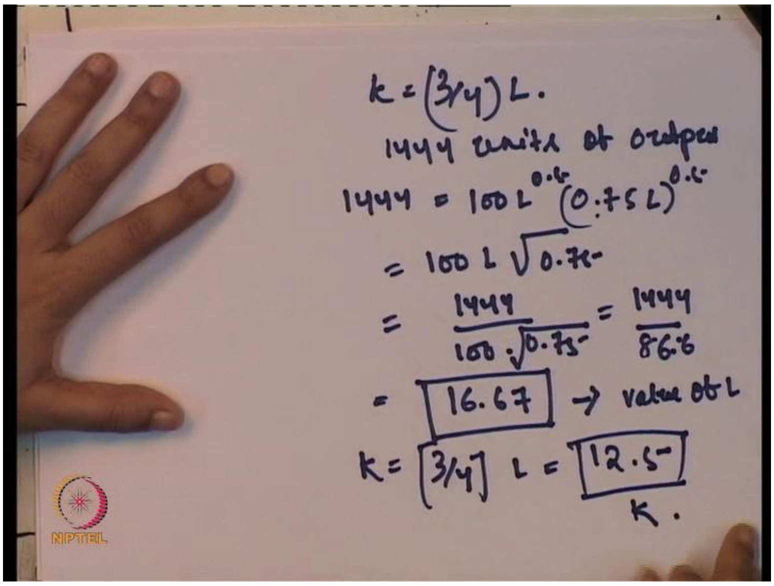
So, now, if you divide equation 1 by equation 2, suppose equation 1 by equation 2, then this 30 by 40 equal to lambda dash that is 50 L minus 0.5 K 0.5, then lambda dash 50 L 0.5 K minus 0.5.

(Refer Slide Time: 41:27)




Simplifying this again, so if you simplify this, then this is 3 by 4 equal to K 0.5 K 0.5 L 0.5 and L 0.5 that comes to 3 by 4 by K by L. So, K is equal to we can say 3 by 4 L or 0.75 L. So, in order to take the first order partial derivative, once we get the partial derivative the first order derivative with respect to L, then the first order derivative with respect to K, and then the first order derivative with respect to lambda. And then we solve for the value K in term of L.

(Refer Slide Time: 42:23)

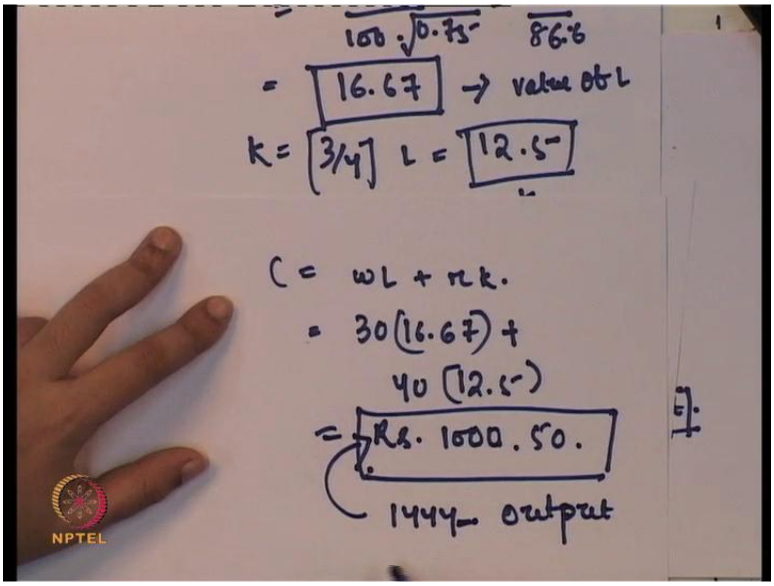


Now, what we have got from all this calculation, that is K is equal to 3 by 4 L. Now we will see how we can substitute the value of K into the equation, and find out the value of K and L. because ultimately, what we need to find out? We need to find out ultimately, to produce 144 units of output, what is the minimum cost because the minimum cost, the because the firm has to incur or what should be the minimum cost or on which isocost, they have to plan.

So, if you substitute the value of K into the given production function for 144 units of output that is 144 that is 100 L 0.5 and 0.75 L because K is equal to 0.75 L. So, that comes to 100 L 0.75 which comes to 144 100 0.75. Then it is comes to 144 86.6 that comes to 16.67 67. So, 16.67 is the value of L.

Now we need to find out K is equal to 3 by 4 L. So, that comes to 12.5. So, 12.5 is the capital, and 16.67 is the labor. So, capital and labor we [nee/need] got the value of capital and labor. Next we need to find out, what is the cost value, when the capital is labor is 16.67 and labor is capital is 12.5. Because ultimately, again let me remind it, ultimately we need to find out what is the minimum cost of producing this given level of output.

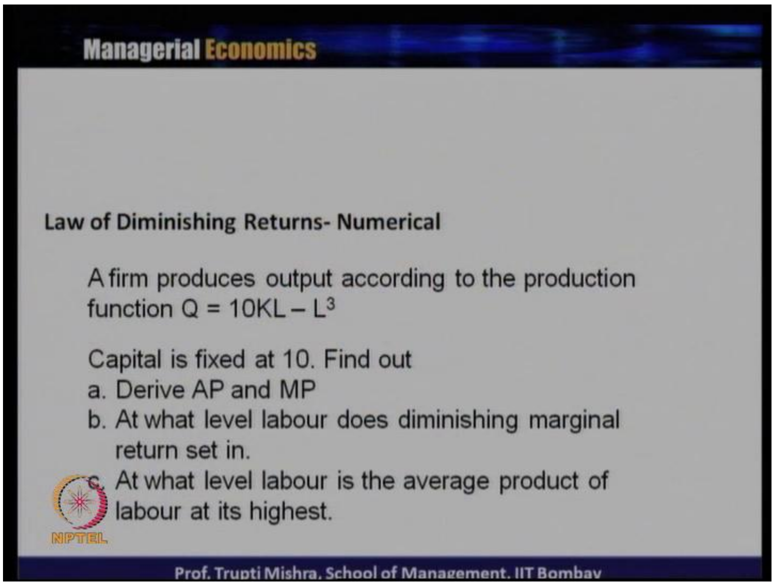
(Refer Slide Time: 44:38)




So, C is equal to w L plus r k. So, w is 30, L is 16.67 plus r is 40 and K is 12.5. So, that comes to rupees 1000.50. So, in order to produce 1444 output, this is the minimum cost. So, this is the given level of output and this is the minimum cost. So, whether it is a minimization case or maximization case, how generally we solve it, numerically, we solve it numerically, with the help of this Lagrangian multiplier method, where we take into the, we take the constant in the form of the Lagrangian multiplier.

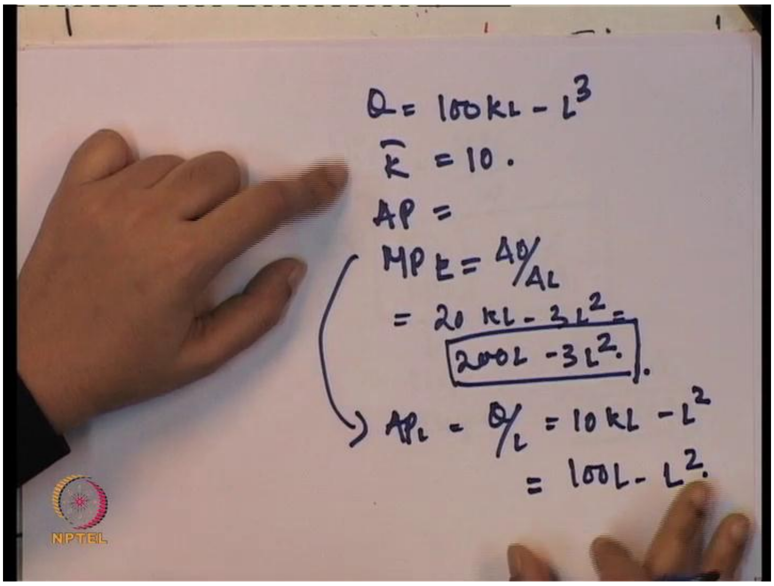
We formulate a composite function then we take the first order partial derivative with respect to 0. Simplifying this that gives us the value of capital and labor, we put the value of capital and labor in the production function equation, we get the exact value of capital and labor. Use this in the cost function and that gives us the minimum cost of producing the given level of output.

(Refer Slide Time: 46:13)




Now let us see, if you remember in the last class we talk about the short run production function that is in term of the law of diminishing return. Next we will see that numerically how we get the value of, the three stages, the different stages of production; and how we find out the value of average product, marginal product; and at what level generally the diminishing marginal set in; and at what level is the average product of the labor is the highest. So, the firm produces the output according to the production function that is Q is equal to100 K minus L q.

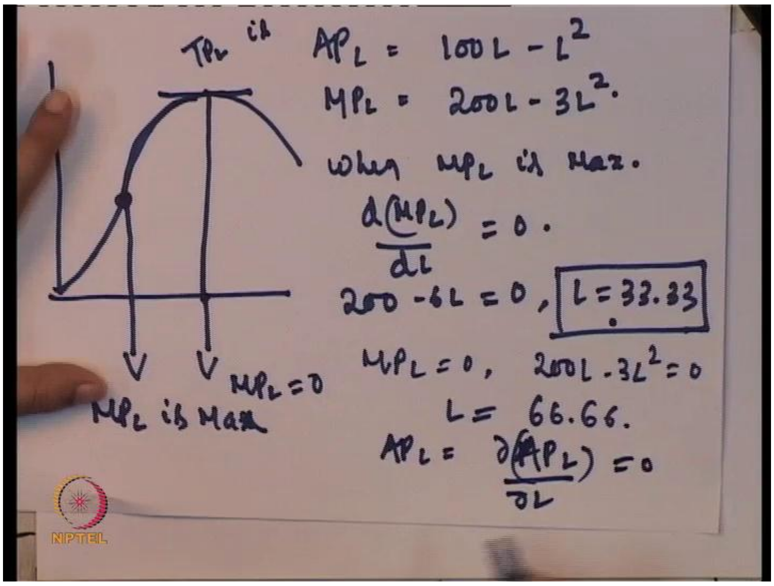
(Refer Slide Time: 46:40)




So, the production function is 100 K L minus L to the power cube. Capital is fixed at ten because this is a short run production function. Now what we need to do? We need to find out the value of average product; and we need to find out the value of marginal product. And then maybe, we can find out what is the different stages or what is the level of output or level of labor where the firm, where the producer has achieved the different level of output.

So, first we will find out the marginal product of labor. Now, marginal product of labor is del Q by del l. So, that comes to 20, that comes to there is a 20 K L minus 3 L square. So, that comes to 200 L minus three L square. Now to, so this is the marginal product of labor. Now we will find out the average product of labor. Average product of labor is Q by l. So, that comes to 10 K L minus L square which is equal to, since K is equal to, so this is 100 L minus L square.

(Refer Slide Time: 48:15)



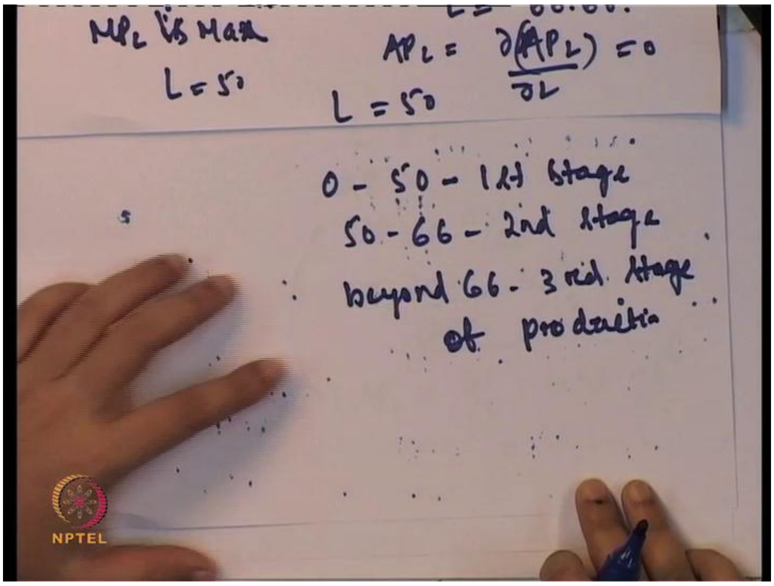

So, average product for, average product is 100 L minus L square. Marginal product for labor is 200 L minus three L square. Now, second one is, we need to find out when m P L is maximum. M P L is maximum where the first order partial derivative with respect to L is equal to 0. So, that comes to 200 minus 6 L which is equal to 0 L is equal to 33.33.

Now, what is the significance of this level of labor? The significance of this level of labor is that, at this level since the marginal product of labor is maximum, beyond this generally, the law of diminishing set in. If you remember you three stages of production function, like how the total product curve, so initially it takes convex, then it is convex, then it is decreasing. So, this corresponding to this our marginal product of labor is maximum.

Because after this the total product of labor is increasing at a decreasing rate; and beyond this, this M P L is maximum and beyond this if you look at then the total product is increasing at the decreasing rate and marginal product is decreasing. And we can say, so corresponding to this point N P L is maximum and this is the point where the law of diminishing return set in. Then the second point we will look at is, when m p L is equal to 0; m p L is equal to 0, when this 200 minus three L square is equal to 0. So, in this case may be L is equal to, if you find out this comes to 66.66.

And where this value of L comes, the value of L comes at this point, because corresponding to this N P L is 0, and T p L is maximum. So, if you remember from, till this point, this is your point, this is your point; beyond which there is the law of diminishing return set in. then when the average product of labor is highest, average product of labor is highest when D A P L with respect to d L is 0, that comes to L is equal to 50. So, this comes somehow here, the average product of labor is maximum when it is intersecting the marginal product of labor.

(Refer Slide Time: 51:19)




So, corresponding to this, L is equal to50. So, now we can say that from 0 to 50; the first stage of production from 50 to 66 unit of labor; second stage of production, and beyond 66 unit of labor we have third stage of production. And how we have identified this first stage, second stage, and third stage.

First stage ends, till the point the marginal product is equal to the average product. And the average product is maximum at that point where marginal product is equal to the average product. Where the second stage ends; second stage ends, when the total product level is maximum or the marginal product is 0. So, that achieve at the labor unit at 66; beyond 66 we have the third stages of production. So, depends up on the labor unit, we can find out at which level generally the stages of production are decided; whether it is the first stage, whether it is the second stage and the third stage.

So, first stage is up to a point where the marginal product of labor is equal to the average product of labor. Second stage is the point where total product of the labor is maximum, marginal product of labor is 0 and third point is beyond this. So, once we find out the labor where marginal product of labor is equal to 0 that is the definition of the second stage. Once we find out the maximum level of a p L that gives us the, may be beginning of the second stage. And when we find the marginal product of labor is maximum that gives us the point beyond which the law of diminishing return set in.

So, in the next session we will talk about the cost of production, different types of cost. How the cost function is formulated; and what is the logic of different shape of the cost function, in the short run and in the long run. So, these are the session references. Generally, these are the references that are being used for the preparation of this particular session.