Managerial Economics
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Lecture - 18
Theory of Production (Contd…)

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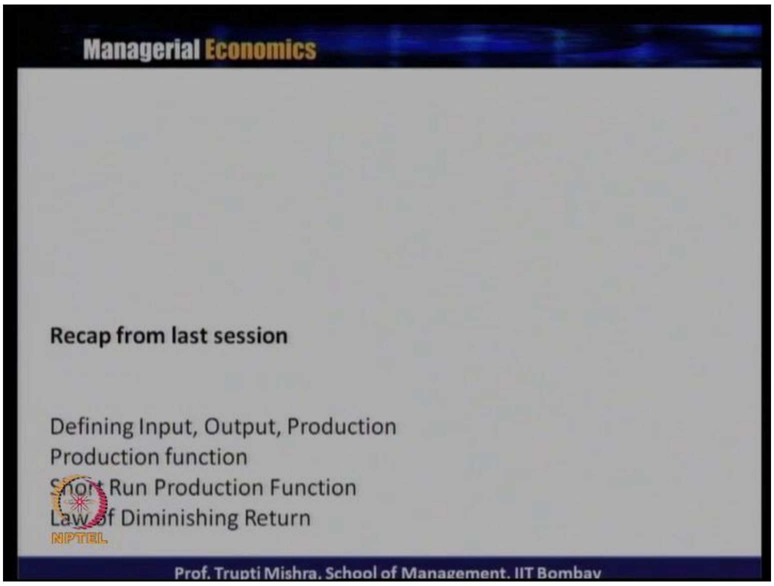

In today's session we will start few more topics on theory of production and cost. So, if you remember in the last session when we are introducing the different concept of production theory, we defined the input output and production. Then we defined what is a production function; which are the dependent variable, which are the independent variable; and then we segregated the production analysis into two way: one is the short run, other is the long run.

Then in case of short run production analysis we understood the law of diminishing return, generally how the total product decreases, when you are keeping one input fixed, when you are going on adding or increasing the other inputs. So, after a certain threshold point, generally the total product decreases, average product decreases and also marginal product leads to a negative segment.

In today's class we are going to discuss about the long run analysis of production, and through specifically the return to scale. So, if you remember in the last class we discuss essential difference between the short run and long run. Apart from the time dimension, it is about the usage of the inputs. In case of short run at least there is one input has to be fixed, typically if the production function is consist of two production, two inputs that is labor and capital. But in case of long run analysis, whenever there is a need to increase the output, whenever there is need to increase the production, generally the producer has to change the input combination, and in that way he has to change both the usage of inputs like labor and capital in order to increase the output.

And when the increase in the inputs takes place, whether the change in the output is proportional, more than that, less than that, that we will understand through the return to scale. So, mainly the long run analysis of production function will be explains through the return to scale.

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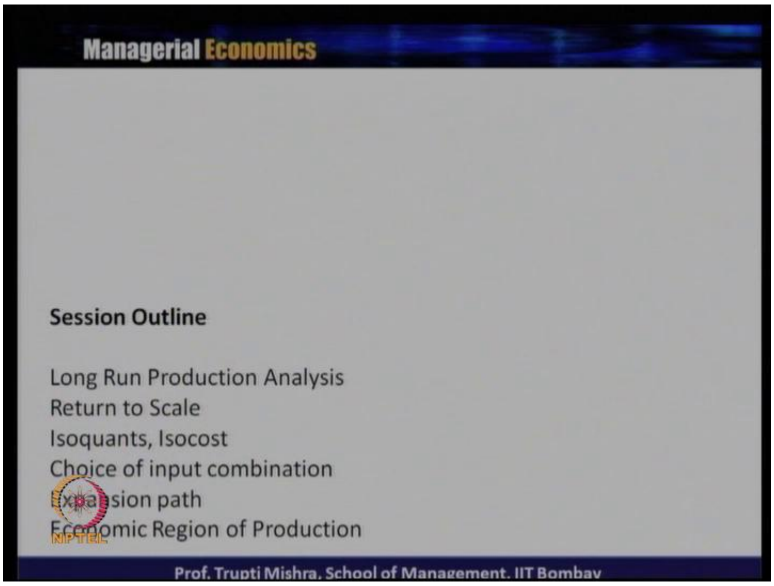

So, in today's class we will talk about the long run production analysis, mainly about the return to scale, then we will introduce the concept of isoquant and isocost, mainly to understand that how the choice of input combinations are being made. Then we will talk about the expansion path or typical if you call it different point of producers equilibrium. And then we discuss the economic region of production, what is the feasible region of production for the producer, given the number of isoquants or may be given the level of isocost line, what should be the economic region of production.

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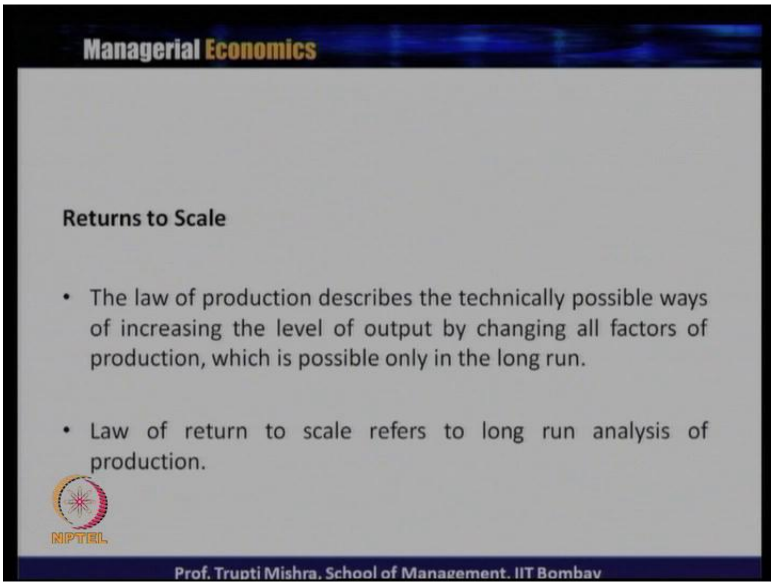

So, to start with, we will discuss the long run production analysis through the return to scale. And if you remember the law of production describes technically possible ways of increasing the level of output, by changing all factor of production which is possible in the long run. So, basically law of production focuses, the technically possible ways of increasing the level of output, by changing all factor of production basically by changing the input combination, and which is possible only in the long run because in the short run if you look at, you cannot do many combination of input because one input has to be fixed.

But in the long run since the output can be changed by changing all the inputs, in that scale number of factor combination can be developed. And that is the reason the law of production describes the technically possible way of increasing the level of output, by doing a ideal mix of both the input that is labor and capital.

So, law of return to scale refers to the long run analysis of production. And we talk about the scale relationship here, because all the inputs are changing and it brings either a proportionate change in the output less than proportionate change in the output, or more than proportionate change in the output.

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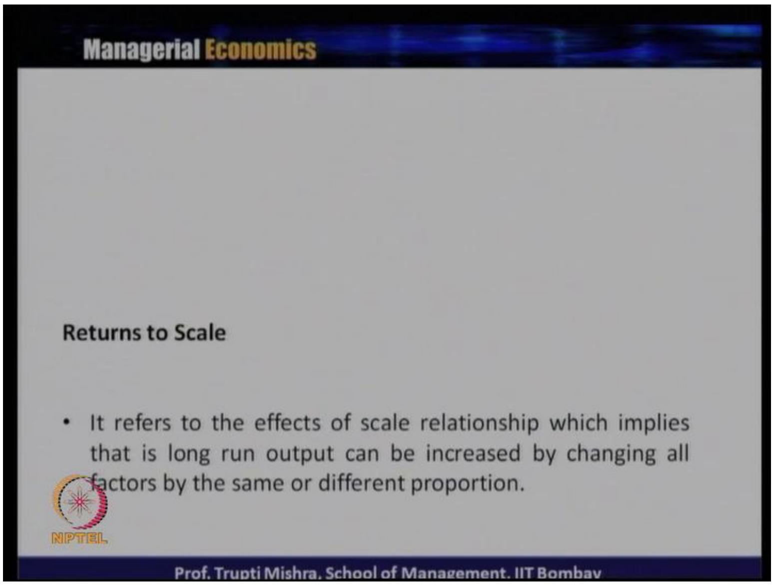

It refers to the effect of scale relationship which implies that the long run output can be increased by changing all the factors by the same proportion or different proportion. So, if there is one unit increase in the output, either that can be changed by half unit change in both the inputs, or may be one unit change in the output, that is one way to understand this. And the other way to understand this that if both the inputs increases by the same proportion, what is the proportionate change in the output? And when both the inputs change in different proportion, what is the different, what is the different outcome on the output? So, basically this is scale relationship which implies that along run output can be increased by changing all the factors by the same, or the different proportion.

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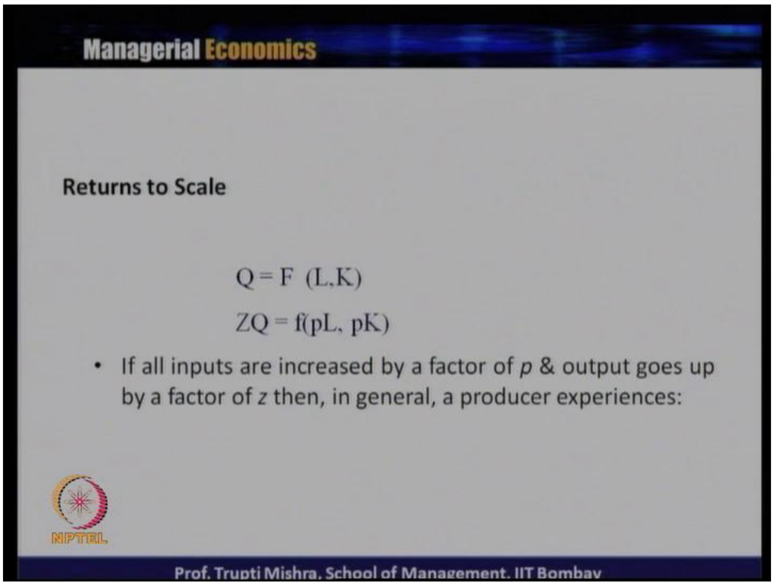

So, last class if you remember we took a production function which is, where Q is the dependent variable, where L and K is the independent variable. So, Q is output here, L and K is the, L is the labor and K is the capital. We simplified the production function by taking just only two variables, although there are number of other inputs are there which influence the output. Like if you remember, the technology, the time, the entrepreneurship, then we have land, then we have the other variable like technology, but here we have considered only two variables, two inputs that is labor and capital, to understand the relationship with the output.

Suppose if the, all the output increase by p and the output by factor Z then in general the producer experience like, now in order to understand scale relationship, let us understand that whenever we need to increase the Q we need to increase the input combination or we need to increase the inputs labor and capital. So, suppose in order to increase the output Q, if L is changed by p amount, and K is changed by p amount it means both the input that is change in same proportion, and if the output goes off by factor Z, then in general a producer experience three type of scale relationship. So, input getting changed in the proportion of p, and output is getting changed, with respect to change in the input that is in the proportion Z.

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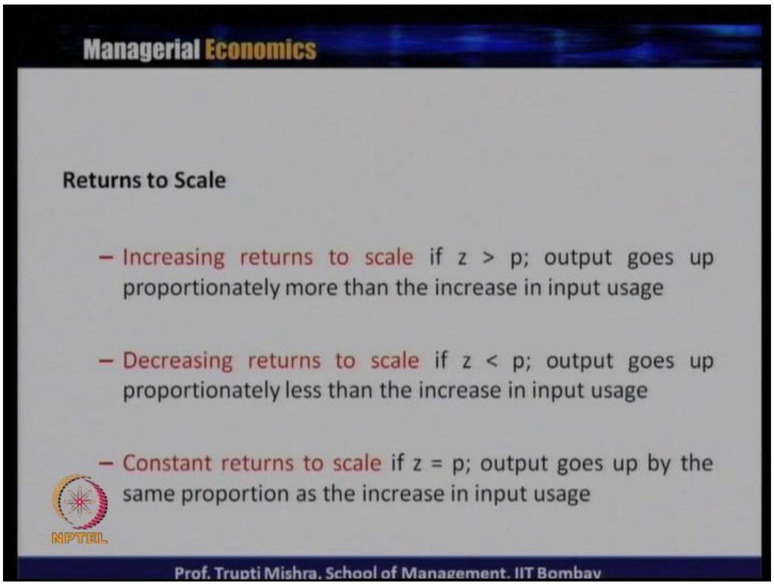

So, in this case the producer experience three types of scale relationships. One, increasing return to scale; two, decreasing return to scale, and last it is the constant return to scale. Depends on the value of z and p generally the return the different type of return to scale is defined. Like in case of increasing return to scale, the value of z will greater than the value of p. So, output goes up proportionately more than the increase in the input usage. So, p is the increase in the input usage, z is the increase in output.

So, in case of increasing return to scale how it happens? In case of increasing return to scale, if z is greater than p, then output goes proportionately more than the increase in the input usage. And in case of decreasing return to scale, if z less than p, output goes up proportionately less than the increase in the input usage.

And in case of constant return to scale, if z is equal to p, then output goes up by the same proportion as the increase in the input usage. So, numerically just to give a number, if input usage increases by 2 percent, and the output increases by 4 percent, this is the case of increasing return to scale, if input usage goes up by 2 percent, and output just increases by 1 percent that is decreasing return to scale; and in case of constant return to scale 2 percent increase in the input usage lead to 2 percent increase in the output and that is the reason this is the constant return to scale.

So, if you look at, the value of change in the input usage and the value of change in the output due to change in the input usage, that decides the scale relationship. If the increase in the output is more than increase in the input then this is the case of the increasing return to scale. If the increase in the output is less than increase in the input usage, this is decreasing return to scale; and if both the changes are proportional, both the changes are equal then this is the case of the constant return to scale.

Then let us understand, since we are going on adding a particular term over here that whether input goes by fixed proportion, or whether input goes by the different proportion both the things it is having some effect on the output. So, let us understand the concept of homogeneity over here that which one is the homogeneity homogenous production function.

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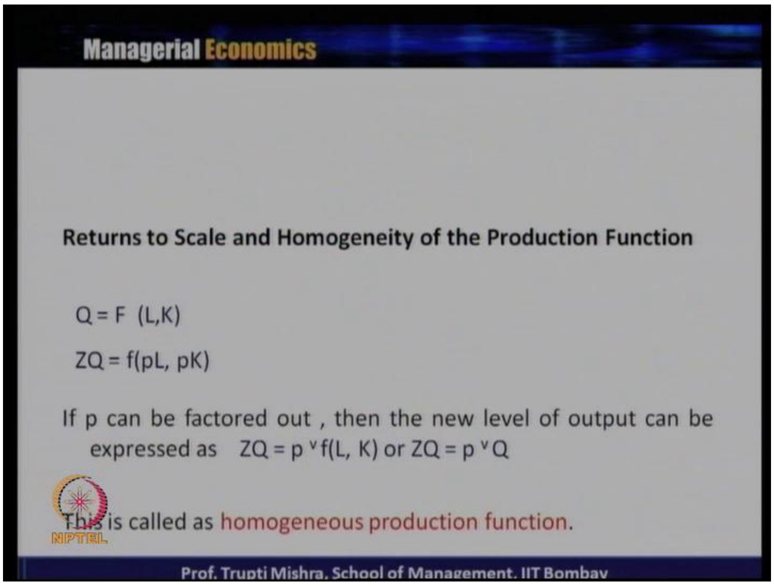

So, taking the same production function where Q is the function of labor and capital, and both the inputs are getting changed by the proportion p, and p, with respect to both L and K, and the output goes by the change by the proportion Z then in this case the value of Z and p that decides the return to scale. But here we will understand that on this basis how to find out a homogenous production function. So, if p can be factored out then the new level of output can be expressed as Z Q that is p to the power v which is a function of labor and capital, and Z Q that is p to the power v and that is to the Q which is the output.

Now to understand simply what is a homogenous production function? If mathematically we can take out the proportionate change in the input, then this is the case of your homogenous production function. So, if it is labor is changing by 2, capital is changing by 2, we can factor out; labor is changing by 2, capital is changing by 4, we can factor out; labor is changing by 3, capital is changing by 3, we can factor out; for example, labor is changing by 3 capital is factored by 6 we can factor out.

So, if the value of p can be factor out mathematically, then the new level of output will be p to the power v Q and this is a homogenous production function. So, degree of homogeneity or homogenous production function is 1, where the parameter associated with the variable is having a, having a, value which can be factor out. So, in this case if you remember, when the both the inputs are getting changed at the rate of p, and the final outcome that is Z Q which can also be also represented as p to the power v Q, in this case the value of v will decide that what is the degree of the homogeneity.

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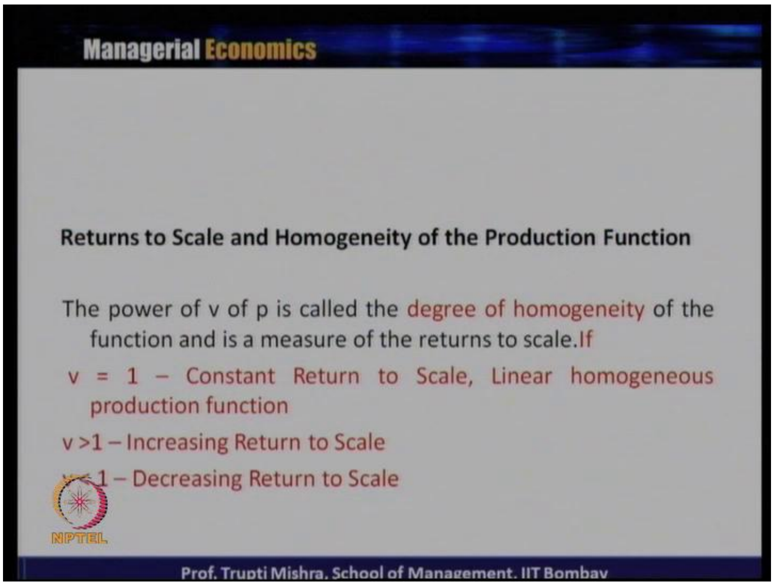

So, the power of v is generally called the degree of homogeneity of the function, and it is a measure of return to scale if v is taking a value one constant return to scale or generally we call it a linear homogenous production function, if v is greater than 1 this is the case of the, increasing return to scale and if v is less than 1 then this is the case of a decreasing return to scale. So, we have reached to the output, which is like p to the power v and Q and the input changes by the proportion p.

Now this power of v, through the value of v, we can find out what is the degree of homogeneity, in the production function. So, if v is equal to one, this is the case of constant return to scale, and we also call it as linear homogenous production function. If it is greater than one increasing return to scale because the proportionate change in the output is more than the proportionate change in the input, and v is less than one this is the case of a decreasing return to scale.

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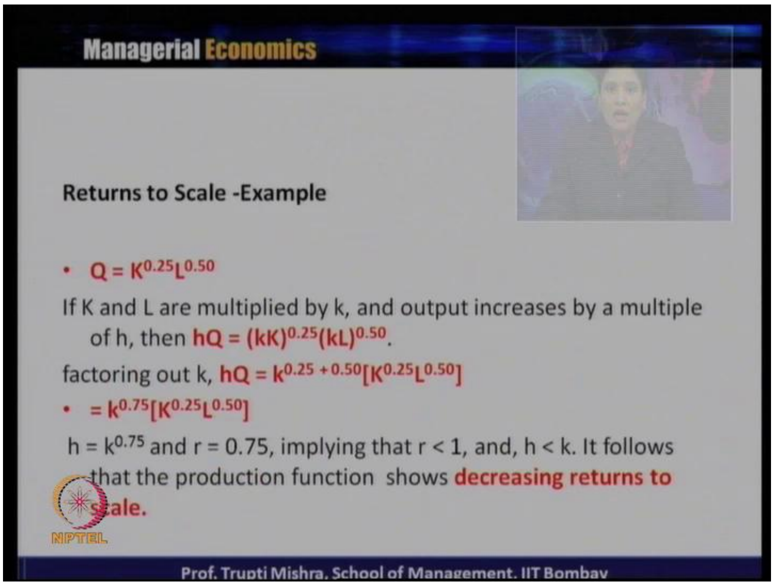

Now we will just take a example to understand this constant return to scale, increasing return to scale, and decreasing return to scale. Given a production function, how to identify whether it is following a constant return to scale, increasing return to scale or a decreasing return to scale.

Suppose the production function is Q which is K to the power 0.25, L to the power 0.50, now how to identify whether the production function is showing, which return to scale to understand. This let assume that K and L are multiplied by the factor K, and output increases by multiple of h; then the Q will be h q whereas the, both the input that is capital and labor, they will change by proportion k. So, that leads to K k to power 0.25 small k L to the power 0.5. Now if you are taking out, factoring out K because if you remember in case of homogenous production function also if you can take out the factor then it is the case of the homogenous production function. So, in this case when you are factoring out K then it comes as h Q which is equal to K to power 0.25 plus 0.50 and again this K to the power 0.25 and 0.50 L to the power 0.50.

So, that again if you simplify this then K to power 0.75 and K to power 0.25 and L to the power 0.50 within the bracket. So, here what is the value of h or what is the value of may be, the, by which proportion the output is changing, h is equal to K 0.75 and r is equal to 0.75, implying that r is less than 1 and h is less than k. So, it follows that the production function shows decreasing return to scale, if you remember, if v is talking a value less than 1 which is here actually the r, we are considering this as r, this case if r is taking a value less than equal to less than 1 then in this case it is the case of a decreasing return to scale. When r will take the value which is equal to 1 then this is the case of the constant return to scale; and when r which is taking a value greater than 1 this will take as the increasing return to scale.

So, how the entire problem has been solved to understand the return to scale, initially the production function which is the power 0.25 and L to the power 0.5, in order to understand return to scale relationship we have multiplied, we have changed the input curve, input proportion by the amount K which leads to change in the output proportion by h. If you simplify this then the K has been factored out K takes the power K to the power of 0.75, and here if you look at then h is equal to K 0.75 which is a power of 0.75 so there is calls about the degree of production function which is 0.75. So, since 0.5 is less than 1 then in this case we can call it this is a decreasing return to scale.

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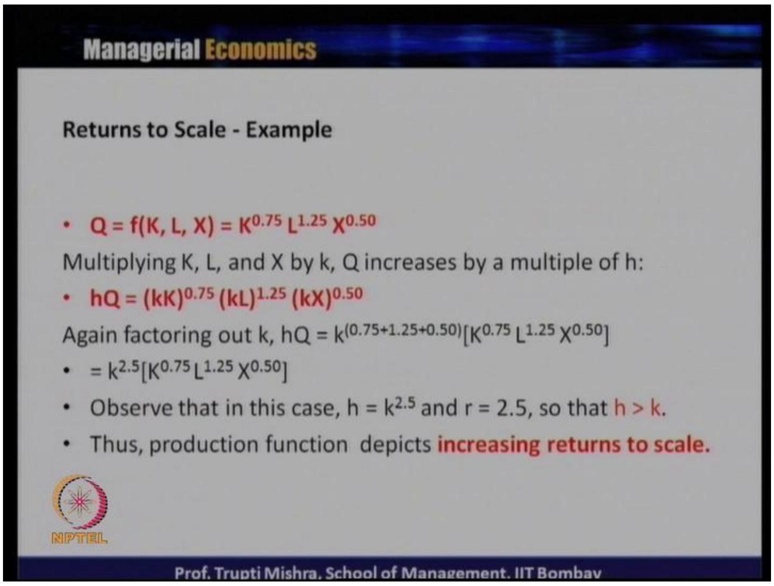

Now, similarly we will take one more example to understand what is the return to scale? Here if you look at, this production function is different from the previous production function. Now, what is the difference over here? The difference over here is, here the production function the Q is not only the function of capital and labor, rather this is a function of three inputs that is capital, labor and X.

So, there are three inputs which decide the output. So, Q is a function of capital, labor and x or we can say that Q is a factor of K, L and x. Now Q is equal to K to the power of 0.75, L to the power 1.25, and X to the power 0.5. In order to understand the scale relationship, we will multiply K, L and X by the amount small k, and Q increases by the multiple of h.

So, when change the input proportion by the small K that is K k to the power 0.75, small k L to the power 1.25, small k X to power 0.5. So, when all the inputs changes in the proportion of K, then the output gets change in the proportion of h. If you can factoring out K then h Q is become K to the power 0.75 plus 1.25 plus 0.5 and in bracket; again this is the same production function that is K is equal to 0.75 L is equal to 1.25 and x is equal to 0.25. Simplify this, we get a value of K to the power 2.5 and the production function. So, in this case h is equal to K 0.25, r is equal to 2.5. Since r takes a value greater than 1, because r is having a value of 2.5; and h takes a value which is greater than K, then the production function depicts increasing return to scale.

Because if you remember the power of v, the v if it is equal to one it is constant return to scale, if it is greater than 1 it is a increasing return to scale, and if it is less than 1 this is a decreasing return to scale. Since in this case it is taking a value which is greater than 1, this is the case of the increasing return to scale.

So, if you look at whenever in through the return to production function, if you want to know that what it is showing, what kind of scale relationship it is showing, generally the best way to find out is to change the input proportion; and the similar way, what is the effect on the output; then factoring out the change in the input proportion and finding out what is the degree of homogeneity. If the degree of homogeneity is equal to 1 then it is a case of constant return to scale, if degree of homogeneity is greater than 1 it is increasing return to scale, and if the degree of homogeneity is less than 1 it is a case of decreasing return to scale.

So, in the short run analysis we generally understand the relationship between the input and output through the law of diminishing return; and in case of long run we understand the relationship between the input and output, in case of the, by taking the help of return to scale.

Now we will understand, or now we will get into the optimum, optimization problem of the producer, where the producer is always wish to optimize the output, with the minimum cost or the minimum input combination. And to understand the optimum input combination or to understand the maximization of output we need the help of the isoquant and the isomap. So, first we will introduce the concept of isoquant, isomap and then we will see how to achieve the lowest input combination, or how to achieve the lowest possible cost in order to maximize the output.

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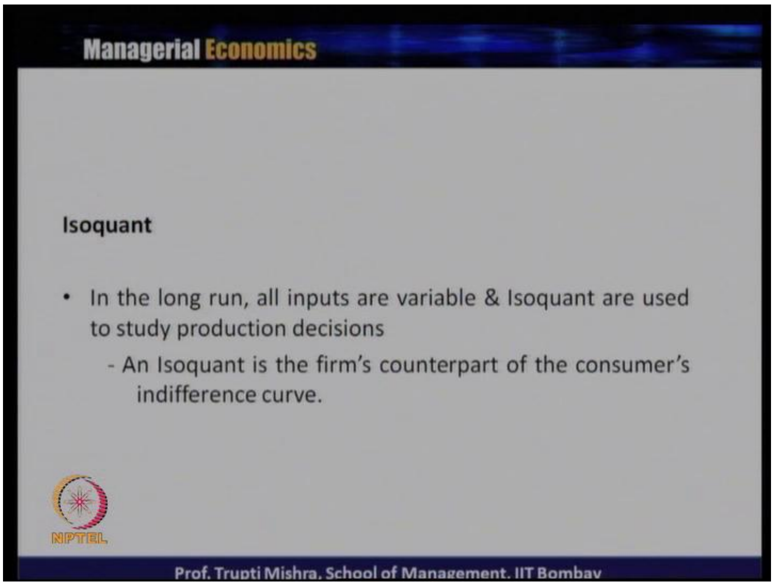

So, if you remember your indifference curve what we discuss in the case of your consumer theory. So, in case of production analysis, it is just like the indifference curve what we discussed in the consumer theory, the isoquant serve the same kind of utility in case of the production analysis. So, in the long run, all inputs are variable and isoquant are used to study production decisions. Now, what is an isoquant? An isoquant is the firms counterpart of the consumer indifference curve. So, if you remember the indifference curve, it is nothing but the producer indifference curve, in case of theory of production or in case of the production analysis.

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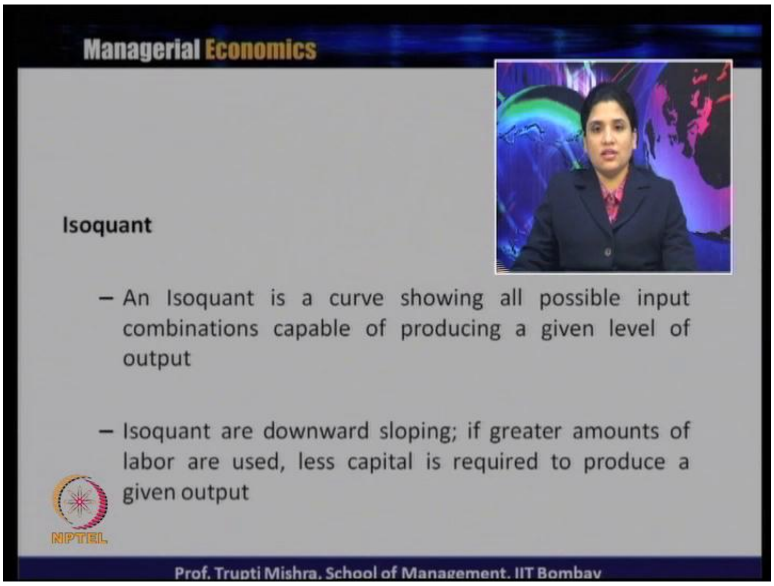

So, isoquant is a curve, showing all possible input combinations capable of producing a given level of output. And isoquant are downward sloping, if greater amounts of labor are used less capital is required to produce the given output.

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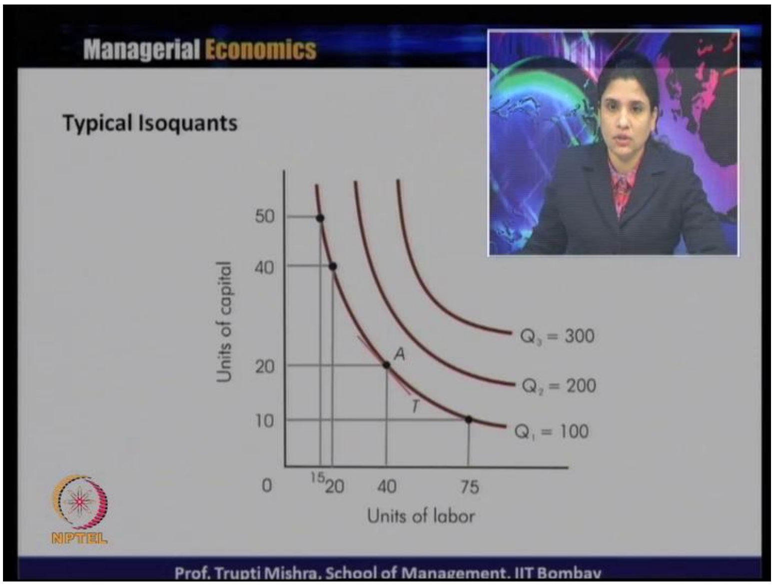

So, now let’s find out how the isoquant is being developed. So, suppose our production function is Q which is a function of capital and labor.

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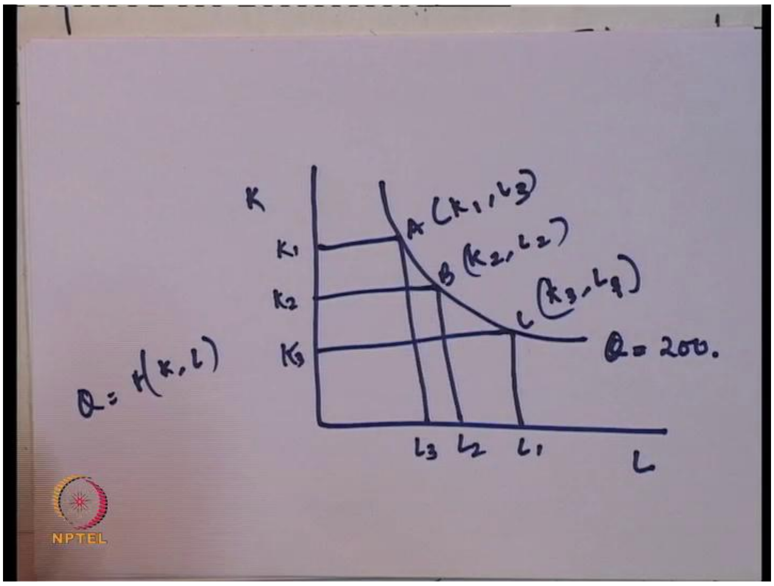

Let us take labor here, and capital here. Now what is a isoquant? Isoquant is the level of, or isoquant is the locus of different combination of capital and labor, which keeps the same level of output. So, if Q is equal to 200, and in this case isoquant represents the different combinations of capital and labor, that will gives the output which is equal to 200 unit. So, suppose this is K 1, this is K 2, this is K 3, then this is L 3, this is L 2, this is L 1. We have 3 points A, B and C. So, point A gives a combination of K 1, L 1, point B gives us a combination of K 2, L 2, point C gives the combination of K 3 and L 1. This one is L 3 K 1, this is K 2 L 2 and this is K 3 and L 1. So, irrespective of the combination whether it is point A, point B, or point C they gives the, they produce the same level of output.

So, A is, it gives a combination of more of capital, less of labor; B gives a moderate amount of capital and labor; and C gives more of labor and less of capital. So, you can say that A is the capital internship production process because if it uses more of capital and less of labor. And C is the labor internship production process because it uses more of labor and less of capital. So, isoquant is one, where the different combination of the input that is capital and labor they gives the equal level of output.

So, irrespective of the capital and labor combination, they give the, producer produce the same level of output. So, one thing we need to assume here is, since the different combination of capital and labor gives the same level of output, it means the capital and labor are closely substitute to each other. Both the inputs they are closely substitute to each other, and that is the reason if you look at, irrespective of changing the output level they are just changing the input combination still they are producing the same level of output. So, whether a producer chooses a production process A, chooses a production process B, chooses a production process C only the input combinations are getting changed, otherwise the output produced become same.

So, isoquant is the locus of combination of different quantities of capital and labor which gives the same level of output. Now if you look at the graph over here, Q 1 is equal to 100 units of output, Q 2 is equal to 200 units of output, and Q 3 is equal to 300 units of output. In the y axis we are taking capital, in the x axis we are taking labor. So, what is the difference between Q 1, Q 2 and Q 3 over here.

When we use combination of more of labor and more of capital then you produce a higher level of output. And if it is a higher level of output, it is a higher level of isoquant. Similarly, if you still uses more of capital more of labor then more than 200 units of output, then it leads to again more level of output, the producer produces more output because they are using more of capital and more of labor, and that is the reason the output level is 300 units. So, Q 1 is 100 units, Q 2 is 200 units and Q 3 is 300 units, and all these different level of output represent different isoquants.

So, Q 1 is 1 isoquant, Q 2 is the other isoquant, and Q 3 is the third isoquant; and the essential difference between these 3 isoquants is that, in case of higher isoquant, higher amount of capital labor being used to produce the output. So, higher isoquant always gives a higher level of production, and lower isoquant always give a lower level of production. Now what is marginal rate of technical substitution. As we know, that capital and labor they are closely substitute to each other. So, whenever the producer changes the production process from one level to another level, generally they do changes with a input combination. And when the change in the input combination takes place when the producer is increasing amount of one input, he has to reduce the amount of the other input.

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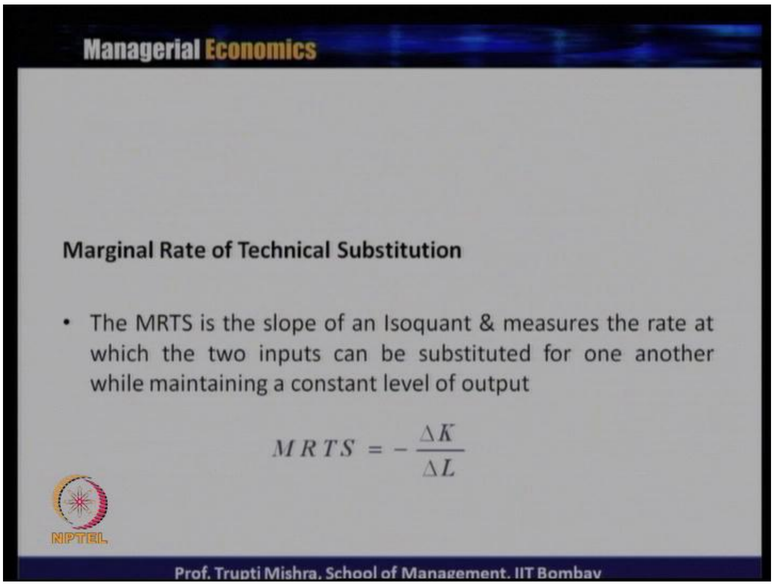

So, marginal rate of technical substitution is 1, this is the rate at which two inputs can be substituted for one another while maintaining a constant level of output. And this is also the slope of the isoquant. So, if you remember the concept of marginal rate of substitution, what we use in case of consumer theory; the counter part of this marginal rate of substitution is, marginal rate of technical substitution in case of production analysis.

So, marginal rate of technical substitution is nothing but the slope of isoquants and it is the rate at which the two inputs can be substituted of for one another while maintaining a constant level of output. So, marginal rate of technical substitution, the change in the K with respect to change in the L, and why this is negative? Because whenever we have to increase the amount of one input, we have to reduce the amount of the other input. So, if you look at the graph now, how to find out the marginal rate of technical substitution that is from the slope of the isoquant.

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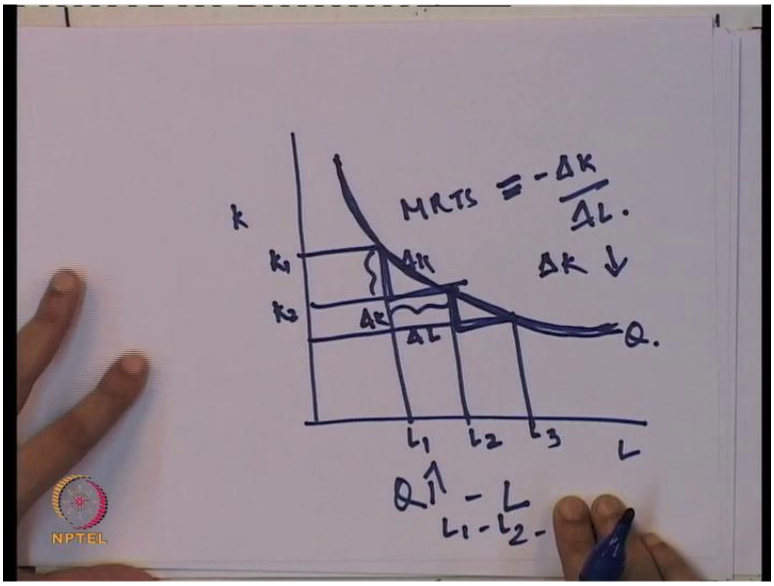

So, in the X axis we take L, in the Y axis we can take K. This is our indifference curve. And the marginal rate of technical substitution is nothing but the slope, that is change in the K, and this is the change in the L. So, marginal rate of technical substitution is equal to minus del K by del L. This is leads to one more properties of the isoquant, which leads to the fact that, marginal, this isoquant is always downward sloping, because whenever we have to increase one input suppose from L 1 to L 2 then there is a decrease from the other input that is K 1 to K 2. Or whenever you are increasing from K 2 to K 1, you have to decrease the labor amount that is used from L 2 to L 1. So, you cannot increase the quantity of or amount of one input, without keeping the fixed, the other has to be decreased then only you can increase it. So, marginal rate of technical substitution is the slope of the isoquant.

And if you look at, this slope goes on decreasing when you, goes on producing may be increasing, producing more by increasing one of the input. So, if you are increasing the quantity, just by changing L, initially from L 1 to L 2 and again L 2 to L 3. The amount what the producer ready to sacrifice to increase this L that goes on decreasing. So, the change in the K goes on decreasing, and that is the reason if you look at, the marginal rate of technical substitution is decreasing, and the isoquant follows a follows a shape of a convex isoquant, because the slope is decreasing. The producer till the time they are going on adding the going on, going on adding amount of one input in order to increase the output generally the rate at which its get exchanged, the producer is no more ready to sacrifice the other input in order to increase the one input on a constant basis.

That is the reason the marginal rate of technical substitution or the slope goes on decreasing, and the isoquant follows a convex shape, now in which case generally the isoquant follows a different kind of shape.

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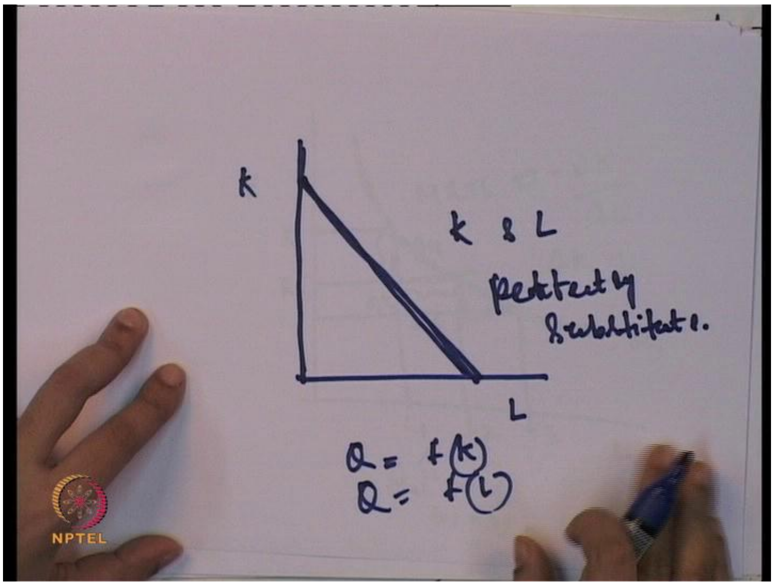

So, if both the inputs they are closely substitute, like we are taking labor and capital here, if both the inputs are closely substitute then isoquant will be a, downward sloping line which touches both the axis, because Q can be produced only with the help of capital, and Q can be produced only with the help of labor. So, indifference curve is takes this shape, if K and L they are perfectly substitute. So, in case of perfectly substitute inputs, the isoquant follows a straight line and it touches both the axis, because output can be produced with the help of either capital or labor.

Now let us understand if capital and labor both are complimentary to each other, you cannot produce output at least by some amount of the other, other inputs. So, it is not cannot be produced only with the help of capital or only with the help of the labor. And both capital and labor they are perfectly complimentary to each other.

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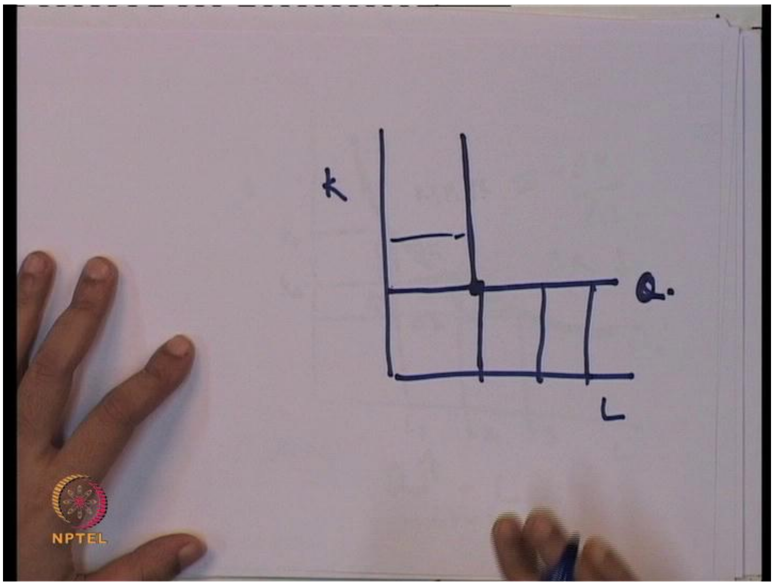

So, in this case the isoquant follows a L shaped curve. And why it follows a L shaped curve because if you look at it is a point rather than a L shaped curve because at this point the level of input is such, that it inverse the combination of capital and labor. Apart from it, whether you use more of labor, or you use more of capital, you cannot produce more amount of output, because this is exactly, perfectly complimentary to each other. Like if you take the example of a monitor and a keyboard, you cannot use only a monitor because the output is nil, you cannot only use the keyboard because the output is nil. So, monitor and keyboard, they are perfectly complimentary input, you cannot produce any level of output if you use only input, or only input, that is there in the form of monitor or in the form of the keyboard.

But interestingly when you have more of the other input also still cannot increase the output. In order to increase the output, you need to increase proportionately both the inputs, like if you have two key boards and one monitor still the output level is not going to increase. In order to increase the output level at least you have to two monitors and two keyboards then only the total output will increase. So, perfectly complimentary, in case of perfectly complimentary inputs, the isoquant, the isoqunat takes the shape of the L, L shaped isoquant. Because it is perfectly complimentary to each other and the equal units of inputs are equal in order to increase the output.

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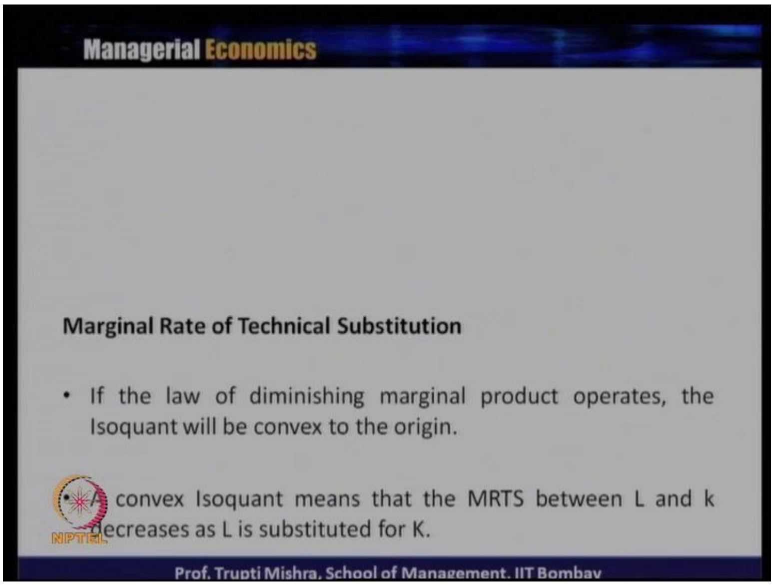

Then next, we will see some, more the points on the marginal rate of technical substitution. And if the law of diminishing marginal product operates, the isoquant will be convex to the origin as we just explained in case of the graphical. A convex isoquant means, that the marginal rate of technical substitution between L and K decreases, as L is substituted for K, what we have already explained through the graph. If you go on substituting the capital for labor, eventually the slope decreases and the isoquant follows a, that is the reason the isoquant follows a convex shape.

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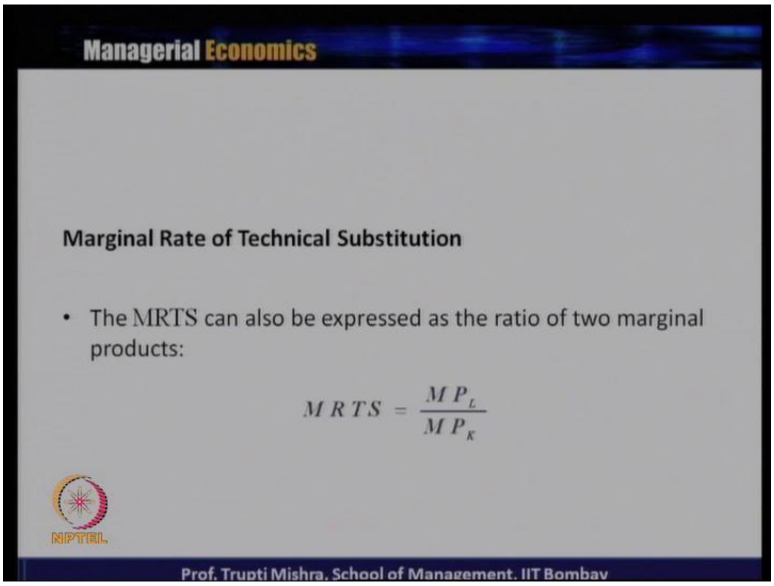

The marginal rate of technical substitution can also be expressed as the ratio of 2 marginal products that is ratio of marginal product of labor and capital.

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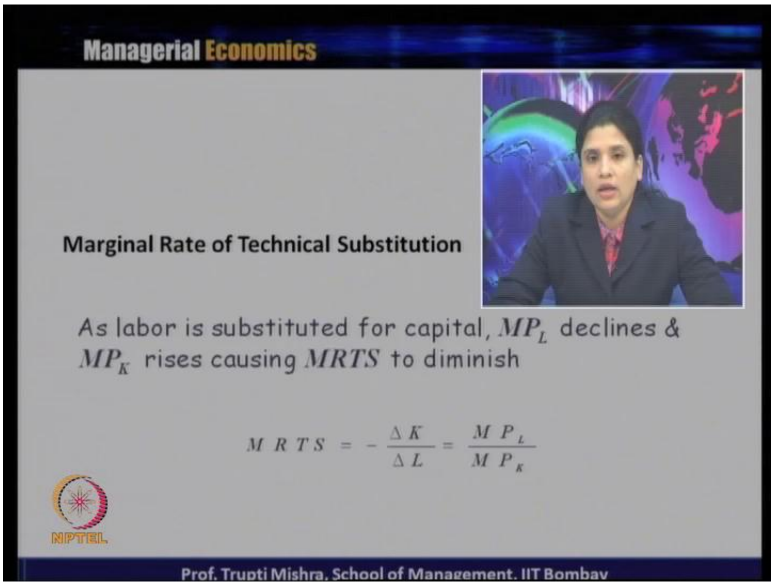

As labor is substituted for capital, generally the marginal product for labor declines, and marginal product for capital increases causing the marginal rate of technical substitution to diminish. So, marginal rate of technical substitution is the change in the K with respect to change in the L or this is just the ratio of marginal product of labor and capital.

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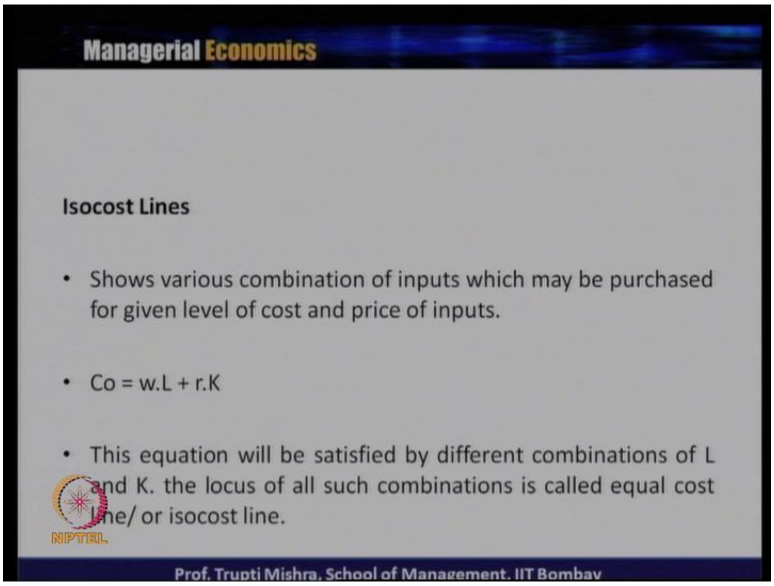

Now we will introduce the constraint over here. Producer can increase the output, by changing any level of inputs. But what is the constant over here, the constant over here is that, whatever the cost of the input, whether the firm can buy the inputs or not, there is always a constant in term of the fund available or the money available to the firm or the industry, that is the reason we introduce the concept of isocost line which is one way the budget constant for the firm, and that restrict them to produce any level of output, by using any level of the inputs. So, isocost lines show a various combination of inputs which may be purchased for given level of cost and the price of inputs.

And generally isocost lines takes the form of C 0 which is equal to w L plus r K where L is the labor and K is the capital, w is the cost of L that is typically the wages and salary, and r is the interest that we generally pay for taking the capital. So, C 0 is a combination of w L plus r K nothing but the, nothing but the price associated or the cost associated with the inputs. The equation will be satisfied by different combination of labor and capital. And locus of such combination is called the equal cost line or the isocost line.

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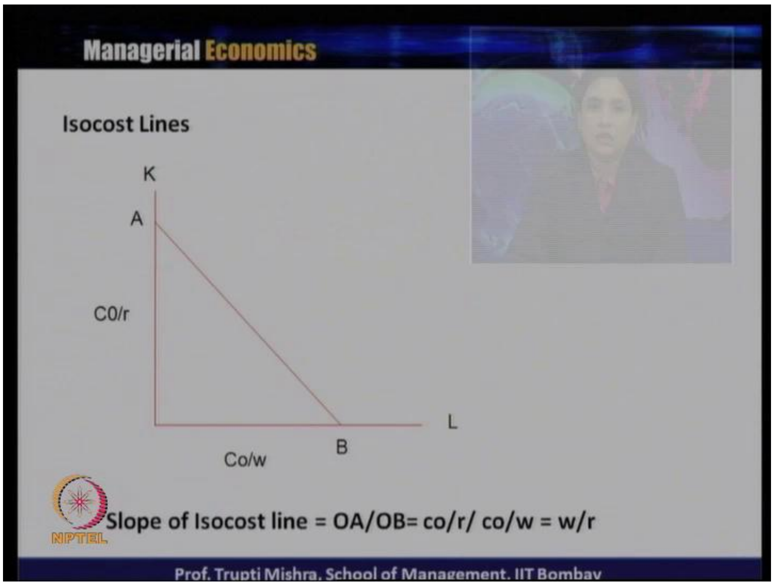

So, C 0 is equals to w L plus r K. assuming that the firm is spending entire fund only on the, only on the labor or firm is producing the output only with the help of the labor, then this becomes 0.

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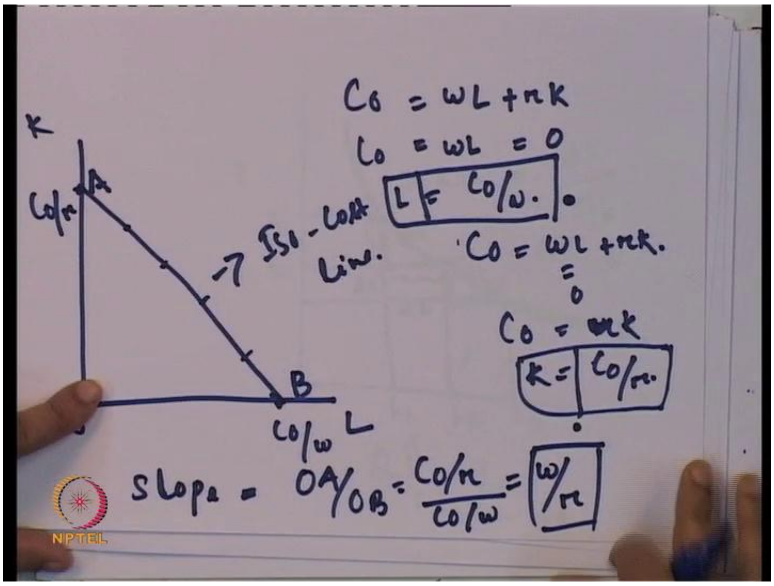

So, C 0 is equal to w L. And if you solve for then this is C 0 by w. Similarly if the firm is producing the output only by using capital, then this becomes 0. So, C 0 is equal to just r K, and K is equal to C 0 by r. So, if you plot this now, with the help of this, we got two extreme points. So, if the firm is just spending the entire money on, just using labor or just using the capital. So, in this case, the firm is producing the output Q only with the help of labor, in this case the firm is producing the output only with the help of capital.

So, that is the reason we get a value here that is C 0 r and here C 0 w. If you join this two point we get the isocost line. And in case of isocost line point A and point B are two extreme where the entire output is just, or the entire money available to the firm is just getting spent on the labor. Here the entire money available to the firm is just getting spent on capital. In between we have different combination of labor and capital, or the firm is just using different combination of labor and capital, to produce the output level Q. And what will be the slope of the isocost line over here.

The slope will be OA by OB which is C 0 r by C 0 w which leads to w by r. Now, what is this w by r, this is nothing but the input price; w is the price for labor, r is the price for capital. The slope of the isocost line is the ratio of the input prices, because w is the input price for labor and r is the input price for the capital.

So, we, we know isoquant which gives the level of production with the different combination of capital and labor. Isocost which is the budget constant of the firm, because firm cannot go on producing the output by changing the input level, because there is, always a budget constraint. They cannot just go on adding the input, because they also have to bear the cost of inputs. So, that is represented in term of the isocost line. So, with the help of isocost and isoquant, let us see how to get into the optimal input combination.

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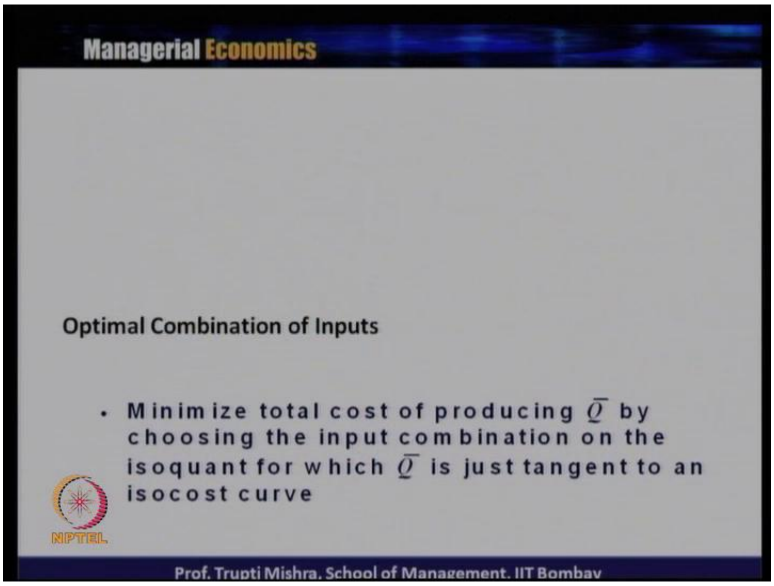

Now what is a optimal input combination? This minimizes the total cost of production Q by choosing the input combination on the isoquant for which Q is just tangent to an isocost curve. So, Q is level of output which is given, and the optimal combination of inputs generally minimize the total cost of producing Q. And how they generally do this, this optimal combination of input? By choosing the input combination on the isoquant, for which the isoquant is just tangent to the isocost curve. So, Q star, Q bar is given level of output, and optimal combination of input will help to minimize the total cost of producing this Q bar. And how to achieve that choosing a combination or picking up a combination on the isoquant, where is tangent to the isocost curve.

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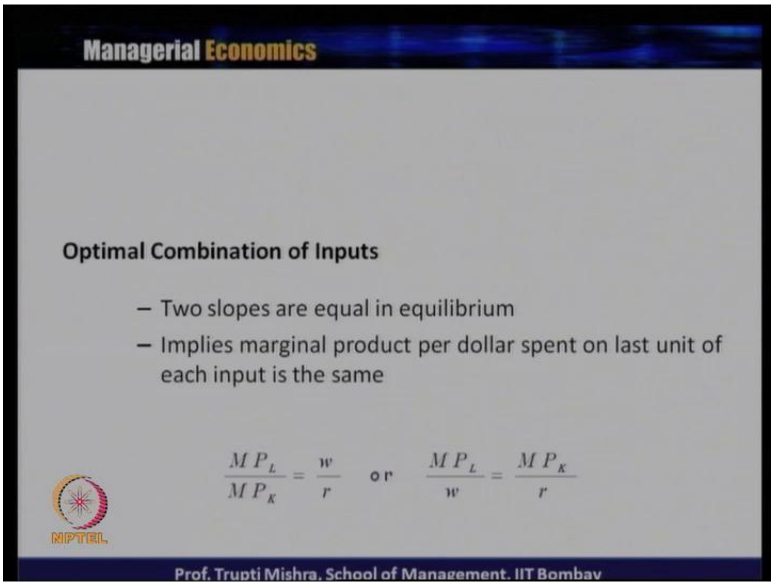

So, the conditions for optimal combination of the inputs are, two slopes are equal in equilibrium, means the slope of the isocost and the slope of the isoquant, which implies marginal product per dollar spent on the last unit of each input is same. So, what is the slope of the isoquant? That is ratio of marginal product of labor and marginal product of capital. And what is the slope of the isocost? That is the ratio of input price that is w and r. So, if you simplify this, then this is M P L by w and M P K by r the first left hand side gives us the ratio of marginal product and input price for labor; and right hand side gives us the ratio of marginal product and input price for the capital. And if the equality is maintained, then in this case we can say this is the optimal combination of input to produce a given level of output.

So, the, what are the conditions for this optimal combination of input? Two slopes are in equilibrium. So, basically the ratio of the marginal product of both capital and labor, should be equal to the input prices associated with the capital and the labor.

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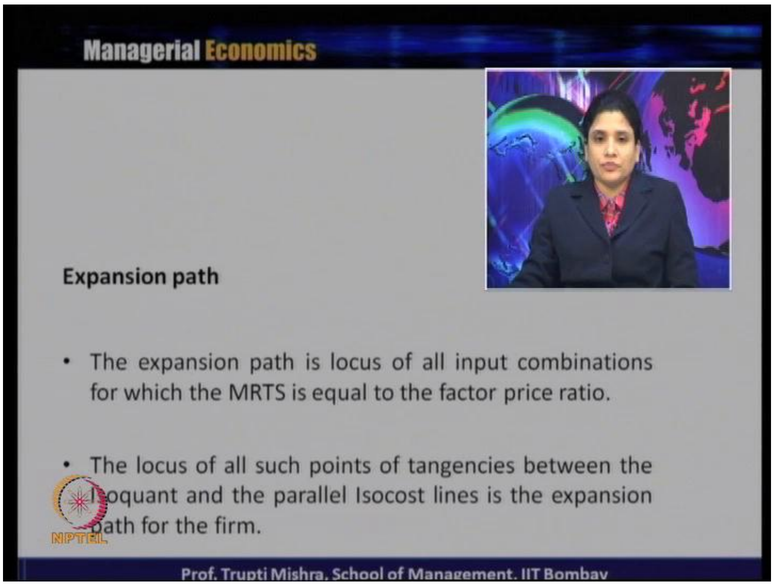

So, now, we will see this graphical representation of this optimal combination of input.

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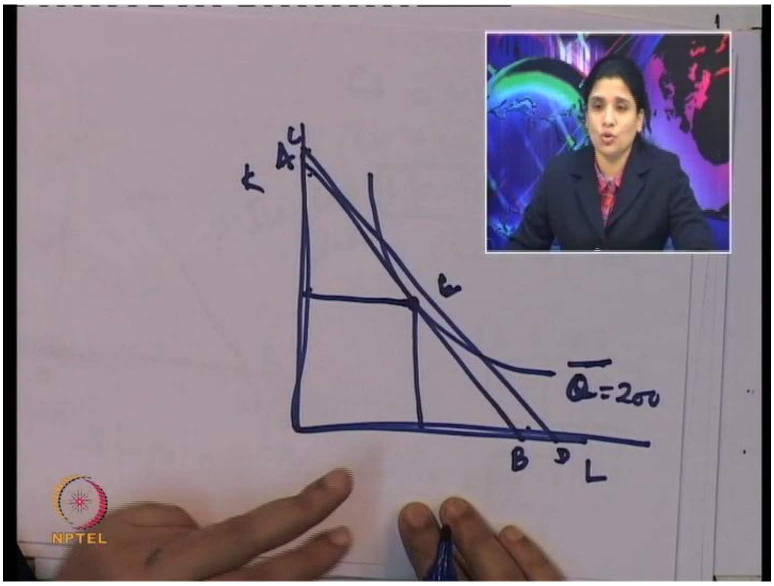

So, Q is 200 suppose this is given. And how to find out the optimal combination of input over here; may be choosing a point, which is just tangent to the isocost line, if the isocost line is A, B, and there is one more isocost line that is C, D. Why C D will not be chosen? Even if it is at the same isoquant crossing the isoquant still it is not tangent to the isoquant. And it is that is the reason it is not going to be chosen as the optimal combination of the input. So, if Q bar is the isoquant which produce 200 unit of level,

level of output; and A B is the isocost, in this case point E will be chosen or this will be chosen as the optimal combination of the inputs. Because at this point the slope of the isocost is just equal to slope of the isoquant, or we can say that the, slopes are equal, or may be the isocost is just tangent to the isoquant.

Then we will understand the concept of the expansion path. And what is expansion path? Basically this is the optimal combination, input combination for the different level of output, and with the different isocost line.

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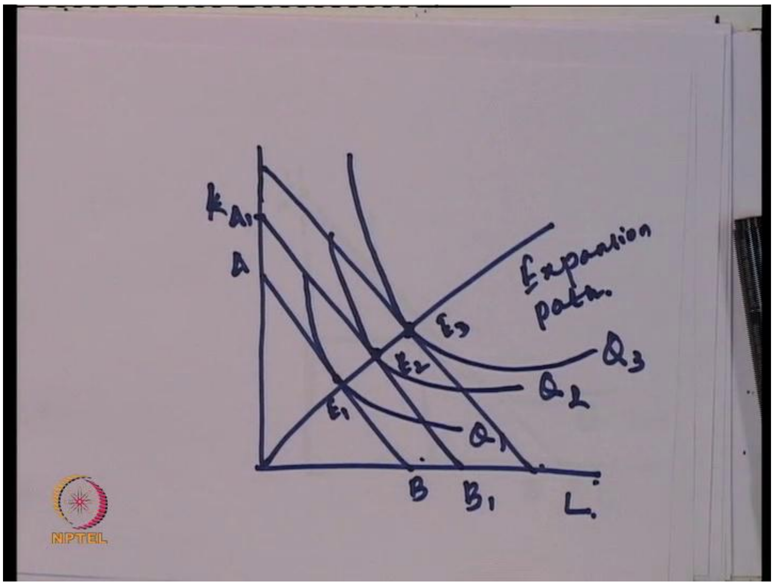

We will take K over here we will take L over here. So, we have Q 1, then we have one input. So, with the increase in the budget constant that is may be A 1 B 1, the producer will always try to get a higher level of output. And the optimal level input combination will be again the same level or the same condition at the at this point where both the slopes are the in equilibrium.

Now, suppose there is again increase in the budget constant, the producer will always try to produce at a higher level of output. And the producer equilibrium, also this point can be called as the producer equilibrium. And if join this, the three point then this is the case of the producer expansion path.

So, Q 1, Q 2, Q 3 is the different level of isoquant. And in order to produce this Q 1, Q 2, Q 3 the producer has to take the help of the different isocost whatever the budget constant given by the firm. And identifying the isocost and the corresponding isoquant, we have three different level of the optimal combination of input or we call it is a producer equilibrium point. Joining these three points it will gives us the expansion path.

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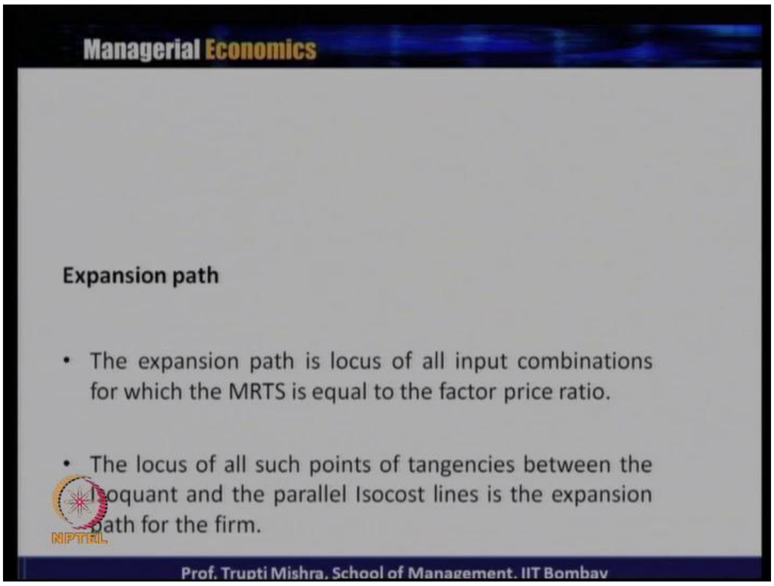

So, expansion path is the locus of all input combination, for which the marginal rate of technical substitution is equal to the factor price ratio. So, if you look at in the graph also, at each point the slopes are in equilibrium, there is a equality in the slope which also implies that marginal rate of technical substitution is the slope of the isoquant. And factor price ratio is the slope of the isocost. So, expansion path is the locus of all input combination, for which the marginal rate of technical substitution is equal to the factor price ratio. The locus of all such points of the tangents between the isoquant and parallel isocost line is the expansion path for the firm.

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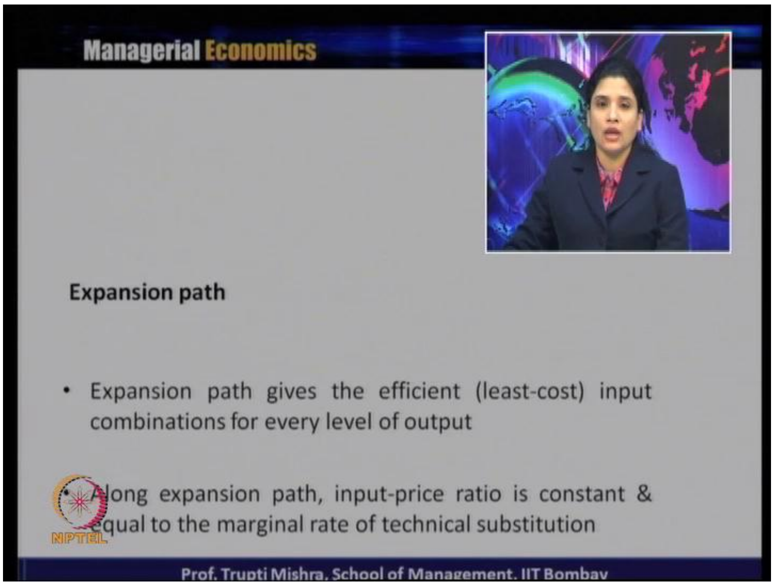

Then expansion path gives a efficient that is the least cost input combination for every level of output, because this is the locus of all optimal combination of input, at the point where the slope of isocost is just equal to the slope of the isoquant. And along the expansion path, the input price ratio is constant and equal to the marginal rate of the technical substitution.

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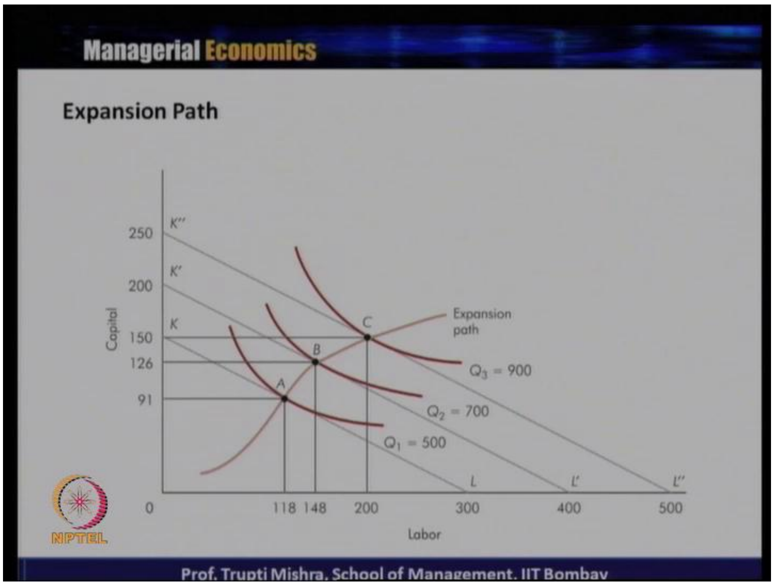

So, this is a typical expansion path, where may be this Q 1 is 500, Q 2 is 700, Q 3 is 900. And Q 1, Q 2 and Q 3 are different isoquants with a different level of outputs. K L is 1 isocost, K dash L dash is another isocost, K double dash and L double dash is another isocost; and A, B, C are three different points which talks about three different level of output, with three optimal input combination. And if you join these three points you get a expansion path which is the locus of the least input combination. So, this expansion path takes the shape on the basis of the relationship between both the inputs the labor and the capital.

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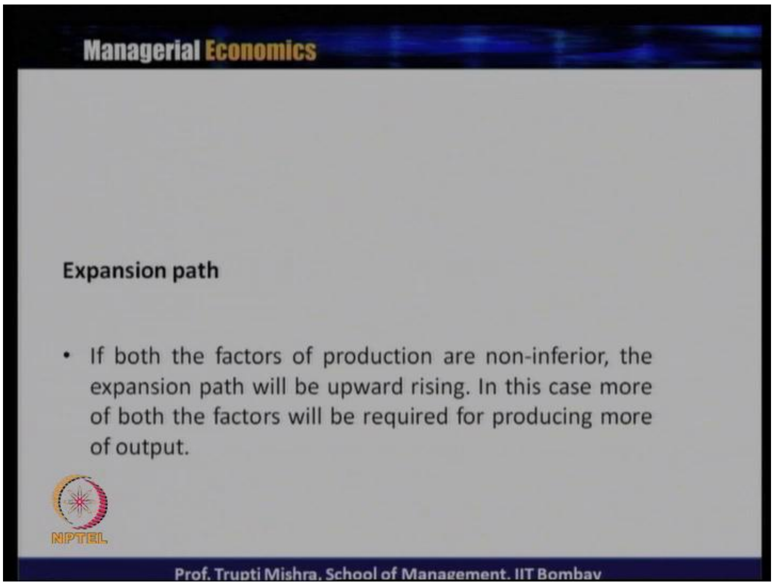

How both the inputs they are related to each other? Whether they are substitute, whether they are complimentary, or whether they are, may be perfectly substitute to each other. And also if both the inputs are not inferior, the expansion path will be in upward rising. In this case more of both the factor will be required for producing the more output.

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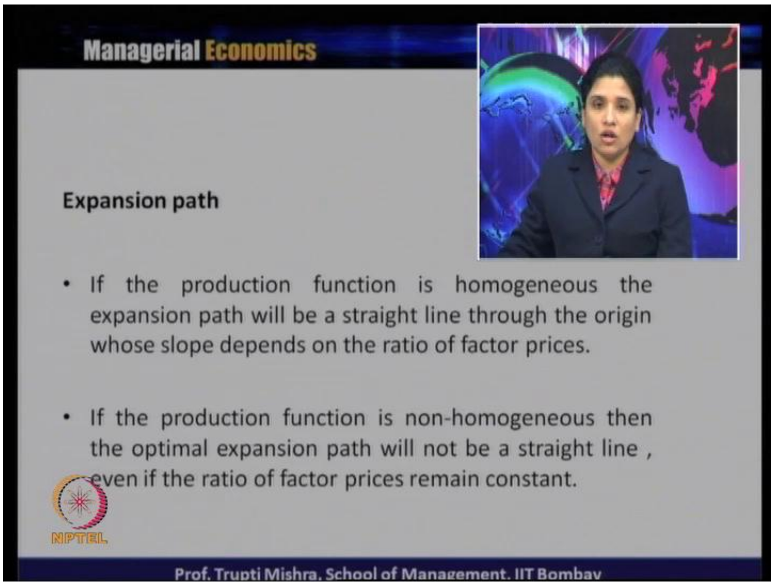

But if it is homogenous, if the production function is homogenous; the expansion path will be a straight line through the origin whose slope depends on the ratio of the factor price. So, if it is non inferior, it is upward sloping; if both the input they are non inferior then it is upward sloping; if the production function is homogenous then expansion path will be straight line, through the origin whose slope depends up on the ratio of factor prices.

And if the production function is non homogenous then the optimal expansion path will not be a straight line. Because even if the ratio of factor prices remain constant, in case of non homogenous, it is not a straight line, it is a somehow like a zigzag, even if the ratio of factor price remain constant. Because it is not, it is not homogenous. So, whenever there is a engine, it is not changing by the fixed proportion, rather it is not changing by the different proportion.

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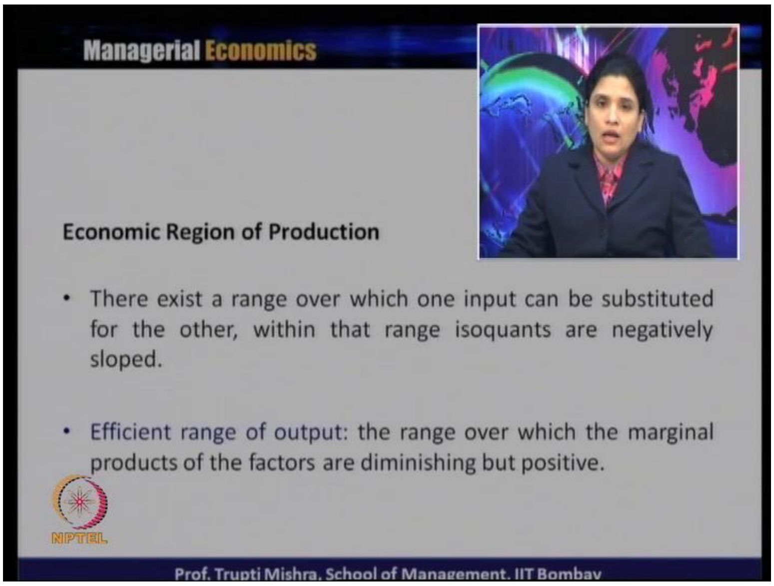

Then we will talk about the economic region of production. How this economic region of production comes in to picture over here, because if you remember two inputs they are closely substitute to each other, they, the producer goes substituting one input for the another input. In order to, may be change the input combination, or sometimes just to see where what is the availability of the resources, availability of the inputs are there. But the question comes here that, how long one input can be substituted to the another inputs.

Because, if it is closely substitute, then only it goes on to the extreme which is x axis or the y axis; otherwise it is just there is a limit with which the inputs can be substituted to one to another. And the typical region is generally called as the economic region of production. So, there exist a range over which one input can be substituted for the other within the range of isoquant that are negatively sloped. And this is also the efficient range of output, because this is the range over which the marginal product of factors are positive, and it is not negative.

When the marginal products for the factors are not negative, they cannot be substituted to one into, one input to another input. We will understand this relationship between the input substitution, and we will identify the economic region of production in a graphical manner.

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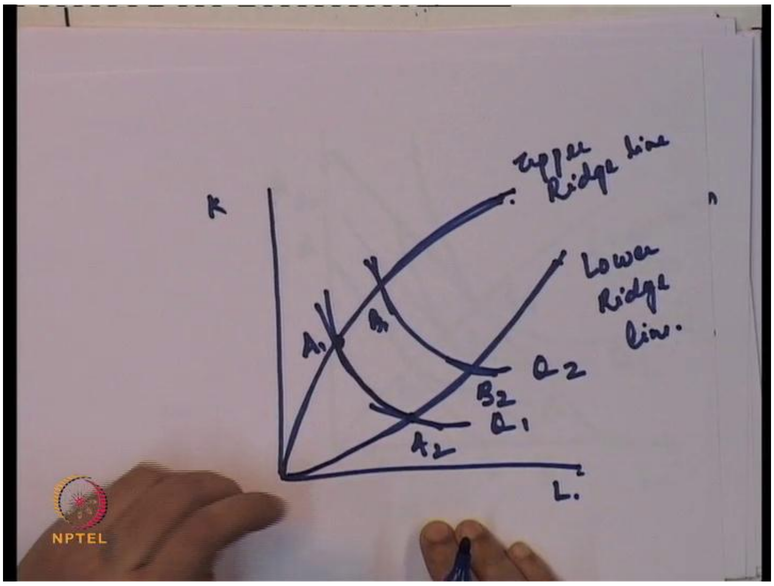

So, we have 2 isoquant, Q 1 and Q 2. And on the basis of, we have two concept here one is upper ridge line and one is the lower ridge line. Production does not takes place when the marginal product of the factor is negative. The locus of points isoquants where marginal products is 0, this is generally known as the ridge line.

So, whether it's upper ridge line or whether it's lower ridge line, the locus of all this, the ridge line is the locus of the points where the marginal products are 0. So, in this case if you look at, the upper ridge line is the point is where the marginal product of capital is 0.

And the lower ridge line is one where the marginal product of ridge is 0. And within this ridge line, these are the efficient range where the one input can be substituted for the other input. So, production techniques are technically efficient, if you look at the technically efficient, if inputs are substituted for one another where the marginal product is not negative. It is between the range where it is positive.

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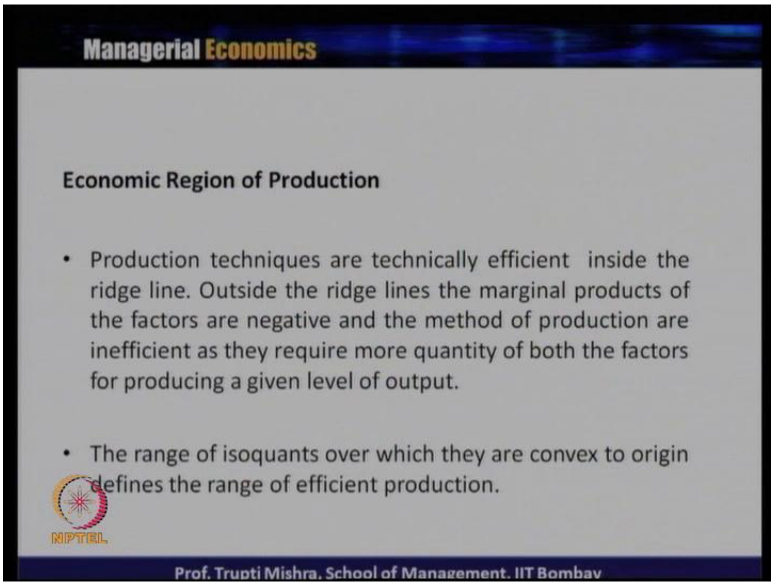

So, production techniques are technically efficient in techniques inside the ridge line. Outside the ridge the components of the factors are negative, and the methods of production are inefficient. As they require more of both the factors for producing the given level of output. The range of isoquant over which they are convex to origin, defines the range of efficient production.

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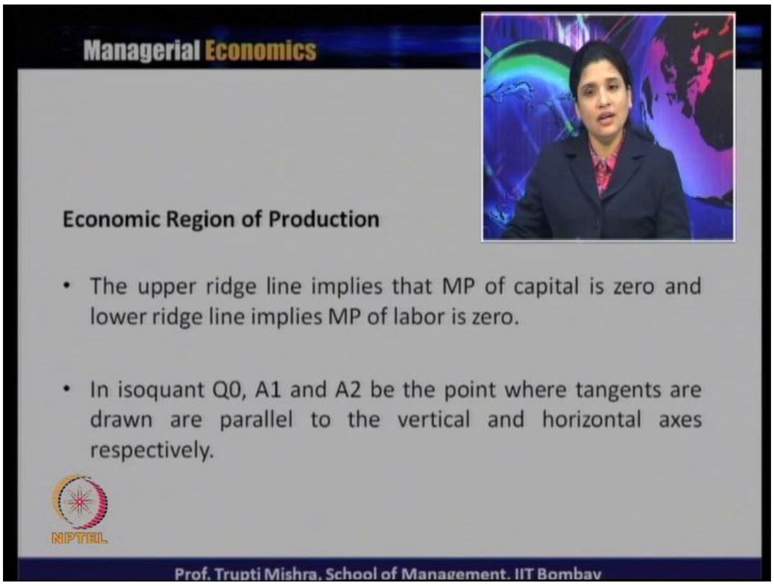

So, in this case, the figure, the upper ridge line the marginal product of the capital is equal to 0. And in case of lower ridge line, the marginal product of labor is equal to 0.

Beyond upper ridge line, marginal product of the capital is negative; beyond lower ridge line, marginal product of lower ridge line is negative. For isoquant Q1 and Q 0, A 1 and A 2 is the range where inputs can be substituted to one another. Similarly for isoquant two, the between point B 1 and B 2, input can be substituted to one to another. And this range is generally known as the efficient range of production, because beyond this point if you look at you are using more of input, but still we are getting the same level of output.

So, generally this is known as the efficient range of production at the different input level. Like suppose, if you introduce one more level of output then this is Q 3; and again you get a point may be this is C 1 and C 2, where the input substitution can takes place. And this can be called as the efficient range of production.

So, the basis of economic region of production or the efficient range of production, is a range, where the input substitution can be done efficiently or may be usage of input combination can be done effectively. And beyond which, marginal product of capital or marginal product of labor goes in a negative direction. So, in this case, even if you are choosing a point beyond this, you are using more of the input, but still you are producing the same level of output. So economic region of production which talks about efficient range of production where two inputs can be substituted for one another; and they produce a efficient level of output.

So, in the next class we will take some numerical, we will try to some examples of the short run and long run production analysis. And we will see what are the different kind of production function generally gets in used of the economic analysis or the economic theory. And these are few of the session references, that is generally used for preparation for this session.