**Time value of money-Concepts and Calculations**
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**Lecture – 09**
**Future Value**

Welcome to the lecture series on Time value of money-Concept and Calculations. In this lecture we will cover Future Value. The Future value which is designated as FV is the value of a current asset, at a specified date in the future based on an assumed rate of growth over time. The FV calculation allows investor to predict with varying degree of accuracy, the amount of profit that can be generated by different investments.

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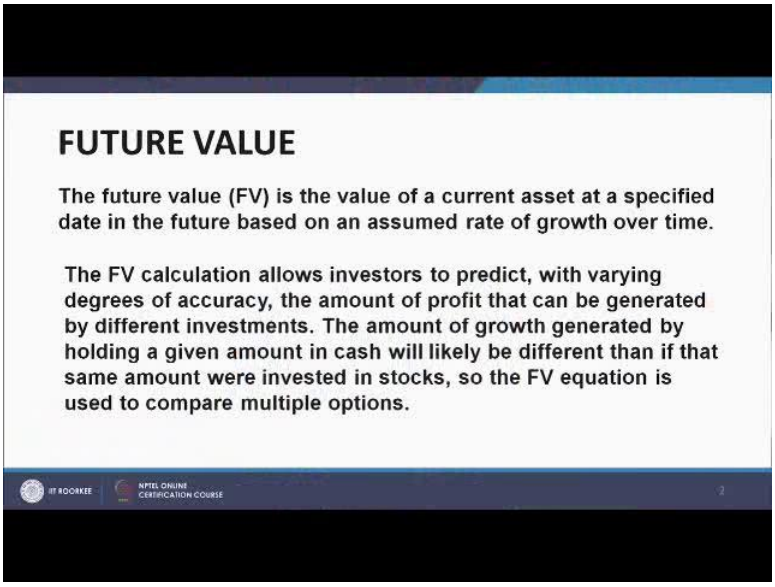


The amount of growth, generated by holding a given amount of cash will likely be different than if that same amount were invested in stocks. So the F V equation is used to compare multiple options. What is Future value? Future value is the value of an asset at a specific date. It measures the nominal future sum of money that a given sum of money is worth at a specified time in the future assuming a certain interest rate, or more generally rate of return. Future cash flows are compounded at the rate of return, and the higher the rate of return, the higher the Future value of the cash flows.

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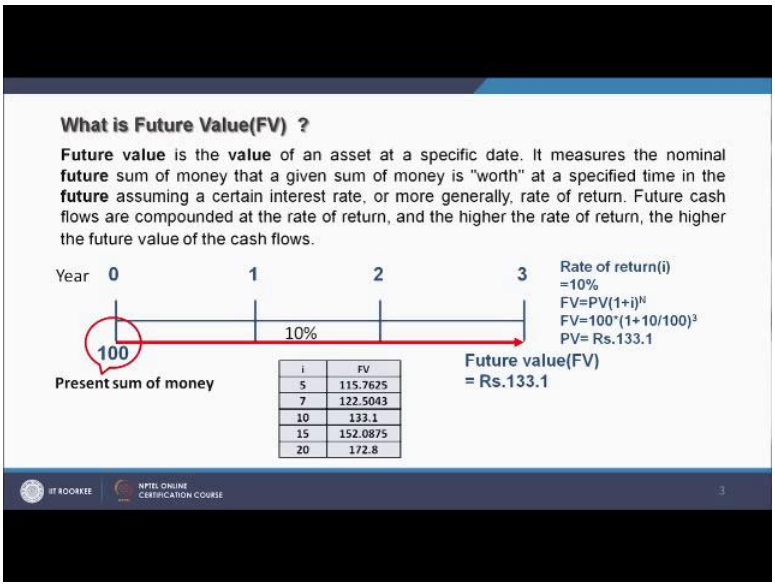


Now, let us see this time line which shows up to 3 years at 0 times Rupees 100 is invested. Now we want to find out what will be it is value at the end of third year. Now at the end of third year, here the Future value of these sum 100 Rupees will be 133.1. This value is dependent on the rate of return. If rate of return will be different, this value will be different. This table shows if the rate of return is 5 percent the value is 115.25, if the value is 7 percent this is 122.5043. The Future value if it is 10 percent it is 133.1, if it is 20 percent it is 172.8.

So, this Future value is a function of this rate of return, as well as the time where we are calculating the Future value. For this the equation used is F V is equal to PV into 1 plus I to the power N. And for 10 percent return if I calculate this value this is 101 plus 10 by 100 to the power 3 is equal to 133.1. Now the Future value problems can be divided into 4 types of problems.

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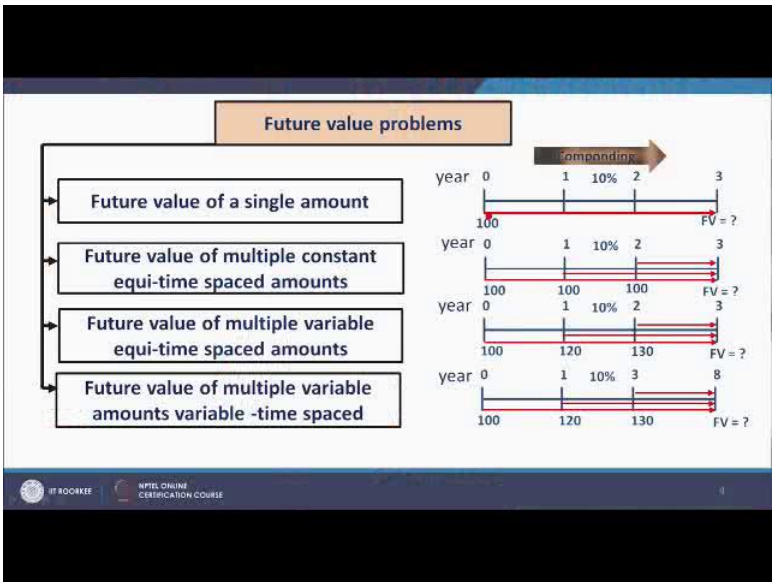


The Future value of a single amount, here in the timeline at 0 100 Rupees invested. This is a single amount. I want to find out the Future value of this as the end of third year. So, here PV is what the second is Future value of a multiple constant equi-time spaced amounts. Here we see that at 0 am I am investing 100 Rupees, at the end of first year I am again investing 100 Rupees, at the end of second year again I am investing 100 Rupees and at the end of third year I want to know what is the Future value of all this 3 amount.

So, the Future value of this amount as to be found out, Future value of this amount at the end of three years as to be found out, and Future value of this amount at the end of third year he as to be found out and when I add this three values the total Future value of this cash flows will come out.

Third type of problem is the Future value of multiple variable equi-time spaced amounts. Here the amount the value of amounts are changing, like at t is equal to 0, the investment is 100 Rupees at the end of first year, the investment is 120 Rupees at the end of second year, the investment is 130 Rupees. So, though time space is same that is one year, but the values are different, one is 100 another is 120 and another is 130. Such type of problems as to be solved by the original technique and not by a equation.

The 4th type of problem is Future value of multiple variable amounts variable time spaced. Here the value of the amount is also changing and the value of the time is also

changing, that is here at 0 time I am investing 100 Rupees, at the end of first year an 120 Rupees, at the end of third year I am investing 130 Rupees. It is not the end of second year, but it is end of third year and I want to find out what is the future worth of all this three investments at the end of 8th year. The last two types of problems which we have discussed now, manual would be solved by equations it as to be solved by first principle.

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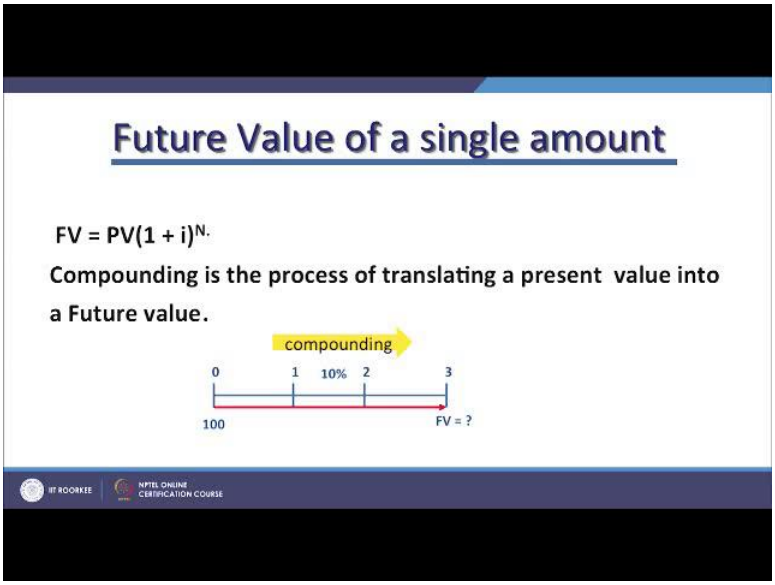


The Future value of a single amount; this is FV is equal to PV into 1 plus I to the power N. Compounding is the process of translating a present value into a future value. So, I am moving from 0th year to third year I am moving in this direction and this is called Compounding.

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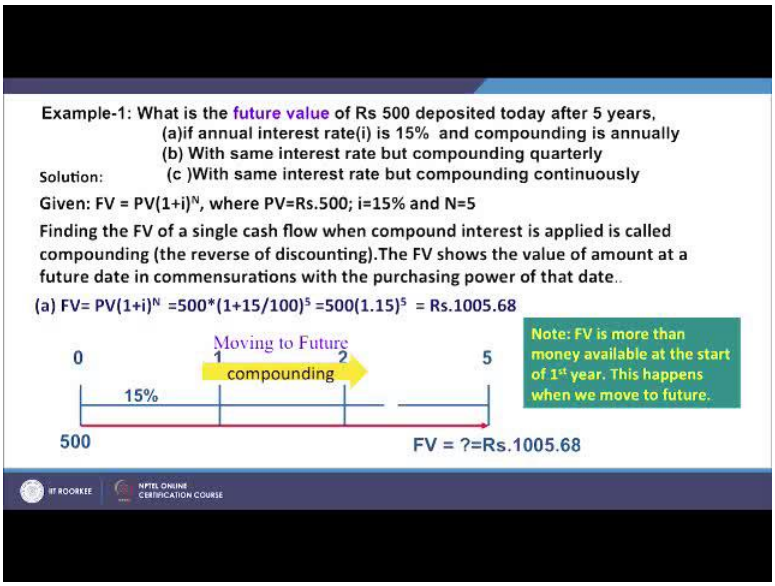


Now let us take an problem small problem. What is the Future value of Rupees 500 deposited today after 5 years? Apart if annual interest rate I is 15 percent and compounding is annually b with the same interest rate, but compounding quarterly with same interest rate, but compounding continuously now the solution is that my formula is FV is equal to PV into 1 plus I to the power N.

This formula is used for when compounding is annually and not compounding quarterly which is discrete compounding or compounding continuously, where PV is equal to Rupees 500 I is equal to 15 percent N is equal to 5 and I have to calculate what is the FV, that is final worth or final value. Finding the final value of a single cash flow, when compounding interest is applied is called compounding and this is the reverse of the discounting.

The FV shows the value of amount at a future date in commensuration with the purchasing power of the date. Now FV is equal to PV into 1 plus I to the power N is equal to 500 into 1 plus 15 divided by 100 to the power 5 that comes out to be 1005.68, that means, if I am spending 500 Rupees today 15 percent and interest rate than at the end of 5th year I will have a sum which will be 1005.68.

Note FV is more than money available at the start of first year or the start or of 0th year basically this happens when we move to future.

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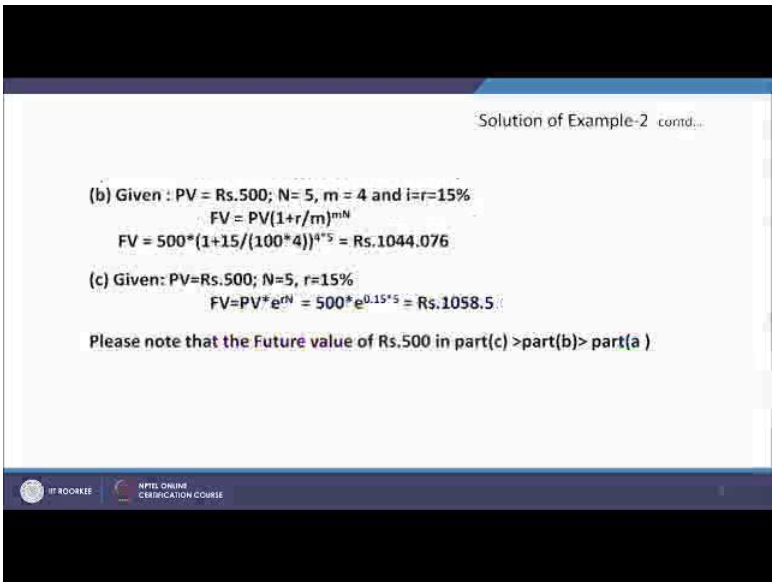


The part two is PV is equal to 500, N is equal to five m is equal to four and I equal to r equal to 15 percent. Now this is a discrete compounding problem. So my equation changes FV is equal to PV 1 plus r divided by m to the power mN, this is the equation from discrete compounding. So FV is equal to 500 into 1 plus 15 by 100 into 4 in brackets and then overall I have 4 into 5 the power is 4 into 5 that is 20 which comes out be one 1044.076.

Now, in the c part the compounding is continuous. So again the equation changes PV is equal to Rupees 500, N is equal to 5, r is equal to 15 percent. So, FV is equal to PV into e to the power rN, which is 500 into e to the power 0.15 into 5, which comes out to be 1058.5. Now from here it is very clear that annual compounding if say FV value which is lower than the discrete compounding and discrete compounding if say FV value which is lower than continuous compounding. So, please note that the Future value of Rupees 500 in part c is greater than part, b is greater than part a.

Now derivation of future worth of an annuity for annually compounding; now this derivation I am not gone for detailed derivation because this is derivation is a small one if I use the Future value is equal to present value 1 plus I to the power N.

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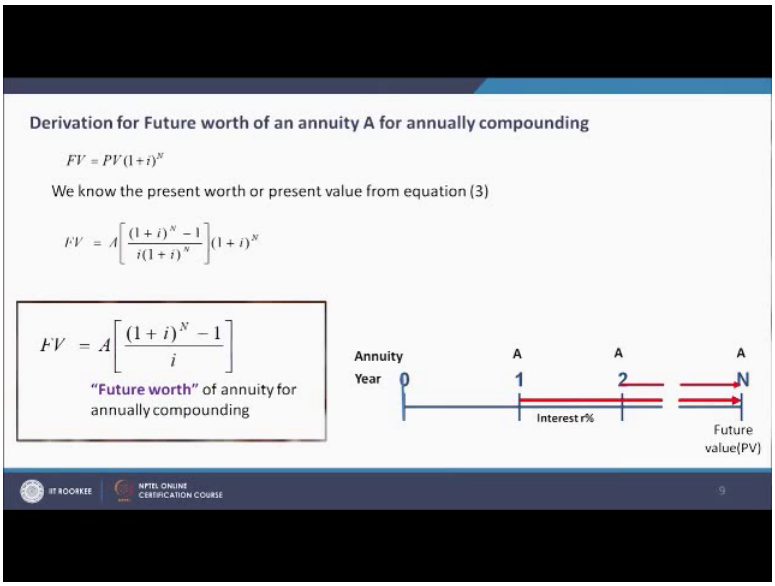


And if I know the present value equation then I can put this and find out the future value. So, FV is equal to a into 1 plus I to the power N minus 1 divided by I in the brackets 1 plus I to the power n, this when you to be multiplied by 1 plus I to the power N this gives you Future value. Because this equation is for present value and Future value is equal to present value into this. So, this is for present value, when I multiply this with this part this gives future value.

So, Future value is equal to a in brackets in small bracket 1 plus I to the power N minus 1 divided by i. So this is future worth of annuity for annually compounding. Now here the investments are like this in the first year I have A at the end of first year at the end. So, the second year I have again A and N at the end of the nth year I have again A. So, you will remember this one for this type of investment this is valid, I am not investing at t equal to 0 year, so this equation as been derived for such type of investment.

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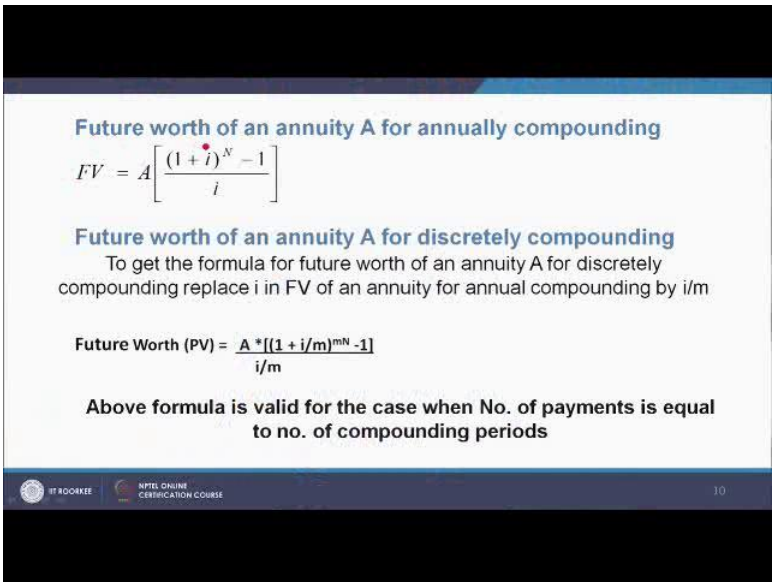


Future worth of annuity A for annually compounding is this and future worth of annuity A for discretely compounding. Than the calculated based on this to get the formula for future worth of an annuity A for discretely compounding replace i in FV equation this of an annuity for annual compounding by i by m and replace N by m into N.

So, if I do that the future worth FV is equal to this into in brackets 1 plus i by m to the power mN minus 1 bracket close divided by i by m. The above formula is valid for the case when number payments is equal to number of compounding periods this as to be remembered. This formula is derived based on this assumption is valid for the case when number of payments is equal to the number of compounding periods. Now let us see the derivation of the Future value of an annuity A for continuous compounding.

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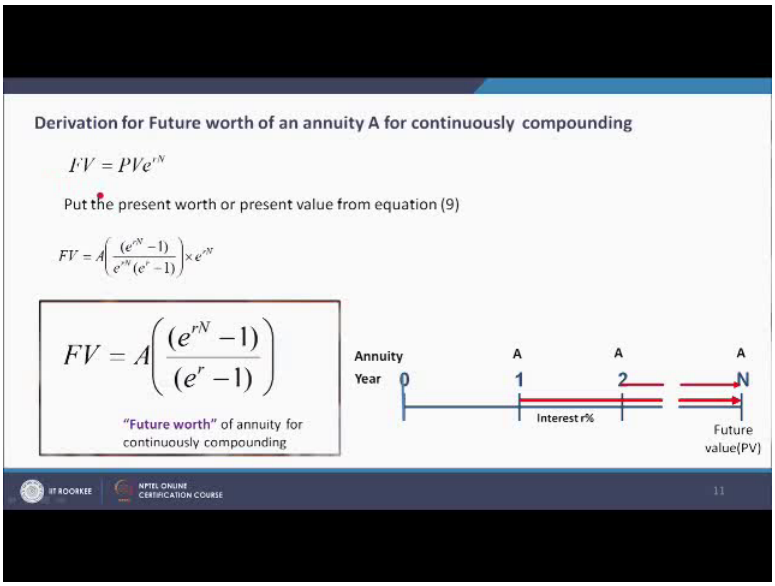


Obliviously we are not deriving it from first principles we are using the equation this FV is equal to PV e to the power rN, now this is for PV part of it. So, PV can be written a into e to the power rN minus 1 divided by e to the power rN in brackets e to the power r minus 1 and we only multiplied this part with this and this cancels out. So, FV is equal to A e to the power rN minus 1 divided by e to the power r minus 1, here also you see that the investment is done like this, at the end of first year the first investment, end of the second year the second d investment and eight of the Nth year that is another investment A.

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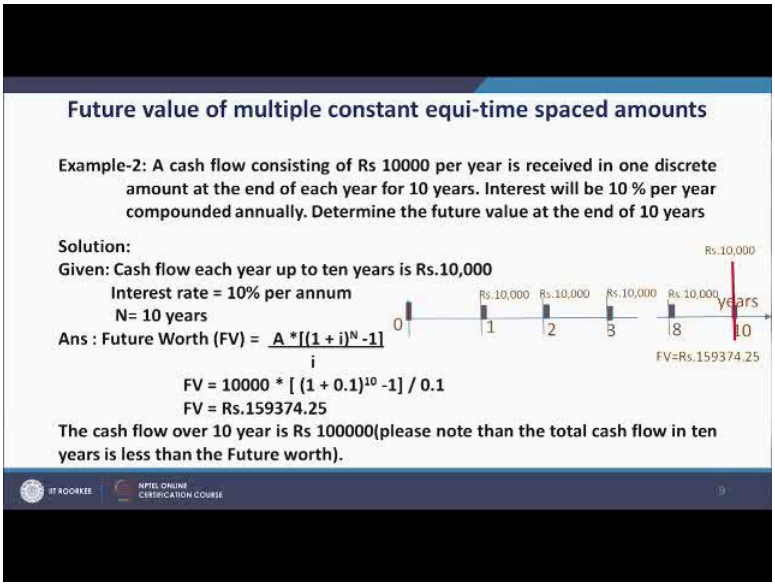

This is a problem which is given in example 2. It is related to Future value of multiple constant equi-time spaced amounts. Here basically the Future value of a annuity is been calculated the example two is a cash flow consisting of Rupees 10000 per year is received in one discrete amount at the end of each year for 10 years, interest will be 10 percent per year compounded annually.

Determine the Future value at the 10 years. So, What is demanding that the at the end of first year 1000 is invested, at the end of second year 10000, is invested at the end of third year 10000 is invested so on so for up to the end of for 10th year and at the end of 10th year we want the future value.

So, the Future value of this Future value of, this Future value of, this Future value of, this and Future value of this amounts are to be added up to find out the total Future value for. This can be done using equation also. So here the cash flow each year up to 10 years is Rupees 10000 interest rate is 10 percent per annum, N is ten years. So, Future value is equal to A into in brackets 1 plus i to the power N minus 1 divided by i ,when I put values into this equation this is a is ten thousand one plus i is 10 percent to the power N. N is 10 minus 1 divided by 0.1 this is the value of i. Then it becomes equal to 159374.25. The cash flow over 10 years is about 100000, please note that the total cash flow in 10 years is less than the future worth. Future value of multiple constant equi-time spaced amounts.

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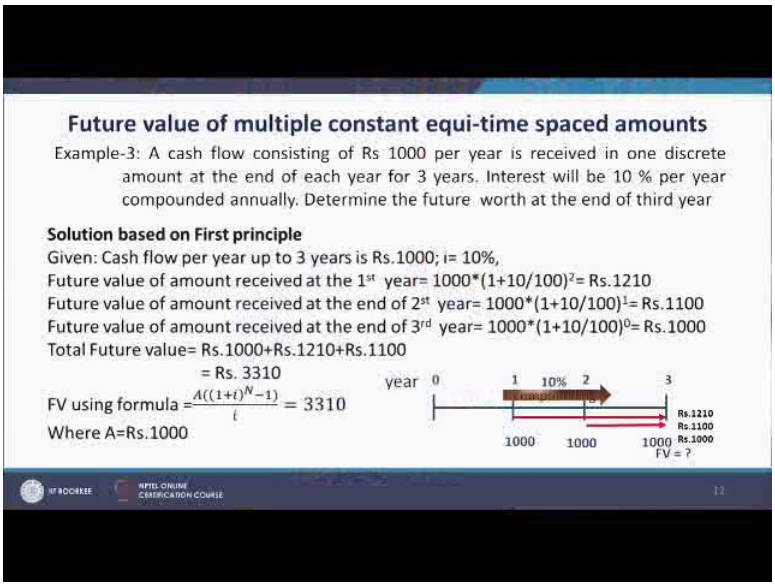

Question 3, a cash flow consisting of 1000 per year is received in one discrete amount at the end of each year for 3 years; interest will be 10 percent per year compounded annually. Determine the future worth at the end of the third year.

Now solution based on first principle will do both solution based on the first principle and then is in the equation. Solution based on the first principle gives you inside of the problem you solved. Now what is with ask is that cash flow per year up to 3 years is 1000, that means, at the end of first year 1000 is paid, end of second year another 1000 is paid and the third year another 1000 is paid. So, if I find out the Future value of this 1000 which is paid at the end of second year it is there for 2 years. So, the Future value at the end of first year is 1000, 1 plus 100 by 100 to the power 2.

This 2 is because it is invested for 2 years not for 3 years. If I invest here then it will remain for 3 years, but investment at the end of first year it remain for 2 years. So, this is 1210 and which has been invested at the end of second year it remains for only 1 year. So, this is 1001 plus 10 divided by 100 to the power 1 it is 1100 and what is been invested in the third year it remains for 0 year and hence this is 1000 only.

When we add this 3 half then it becomes Rupees 3310. So, what we have done, we have found out the Future value of this amount, we have found out the Future value of this amount, we have found out the Future value of this amount and we have added them together to find out the total Future value the same can be done through this formula. The Future value is equal to A, in brackets 1 plus i to the power N minus 1 divided by i, when I put my values here, then it comes out to be 3310.

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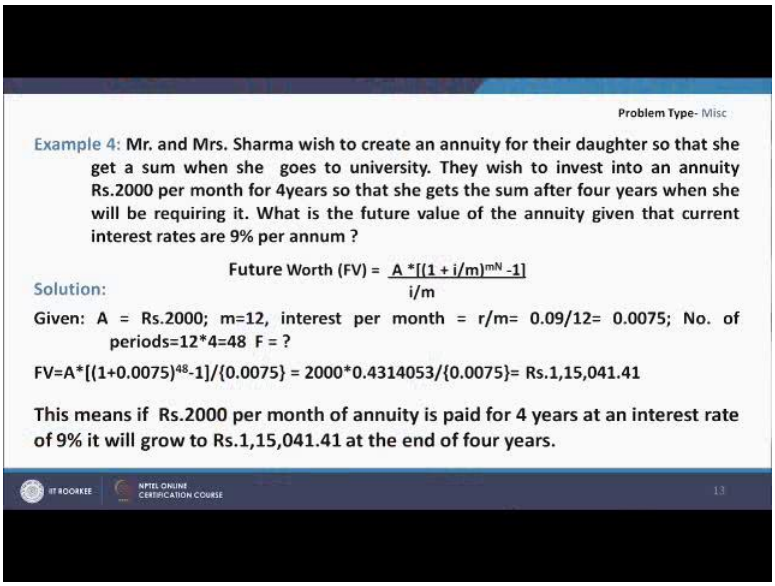


Let us taken mix problem, which is example number 4. Mr. and Mrs. Sharma wish to create an annuity for their daughter. So, that she gets a sum when she goes to university. They wish to invest into an annuity of Rupees 2000 per month for 4 years. So, that she gets the sum after four years when she will be requiring it. What is the Future value of the annuity, given that the current interest rates are nine percent per annum?

Now, here basically he is investing per month and interest rates are in annum. So, this is a problem of discrete compounding and hence the formula which will be used for this case is this. This is a formula for converting annuities to Future value when there is a discrete compounding. So, here A that is annuity, is 2000 per month, m is equal to 12 because there are 12 months per year, interest rate per month is r by m is equal to 0.09, which is the value of r divided by m which is 12 it comes out to be 0075. Number of periods is equal to 12 into 4.

This is 48 because there are 4 years and m is 12. So, m into N is equal to 48 and what is the value of F is demanded. So, if I use this equation and put my values then it comes out to be 115041.41. So, this means if 2000 per month of annuity is paid for 4 years at an interest rate of 9 percent it will grow to 115041.41 at the end of 4 years.

Example number 5, this is related to continuous compounding. A cash flow consisting of 10000 per year is received in one discrete amount at the end of each year for 10 years; that means, at the end of first year I am getting 10000 at the end of second year I am

again getting 10000 at the end of third year again getting 10000 likewise I am getting up to 10 years interest will be 10 percent per year compounded continuously, determine the future worth at the end of 10th year.

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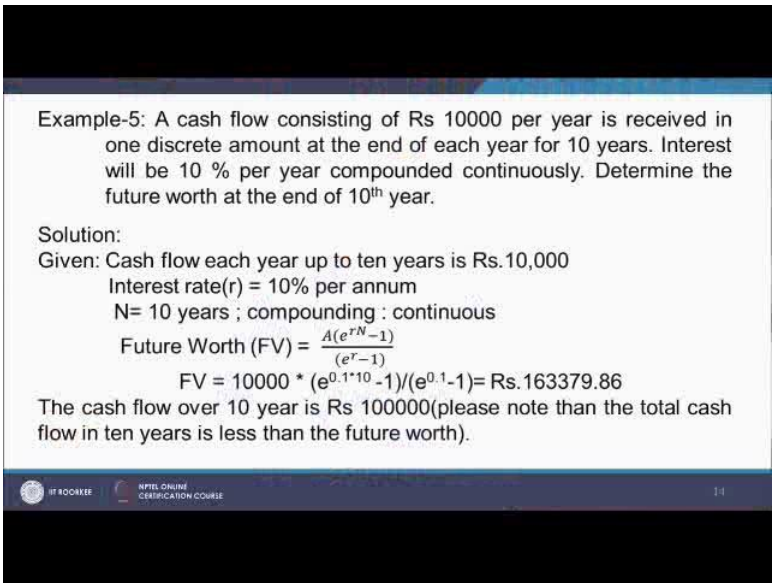


Here the cash flow each year up to ten years is 10000, interest rate r is 10 percent per annum, N is 10 years compounding is continuous. So, future worth for continuous compounding is this value, that is A in brackets e to the power rN minus 1 divided by e to the power r minus 1. So, when we put value on this equation this is 10000 the value of A is 10000 and this is e to the power 0.1 into N is equal to 10 minus 1 and divided by e to the power 0.1 minus 1 it comes out to be 163379.86. The cash flow over 10 year is only 100000, Please note that the total cash flow in 10 years is less than the future worth.

Now this is a problem for Future value of multiple variable equi-time spaced amounts, here the amount is changing. So, it is example number 6 and such type examples are to be solved from first principle. A cash flow consisting of 1000, 1500 and 2000 per year is received as discrete amount at the end of first year, second year and third year respectively. Interest rate is 10 percent per year compounded annually determine the future worth at the end of the third year.

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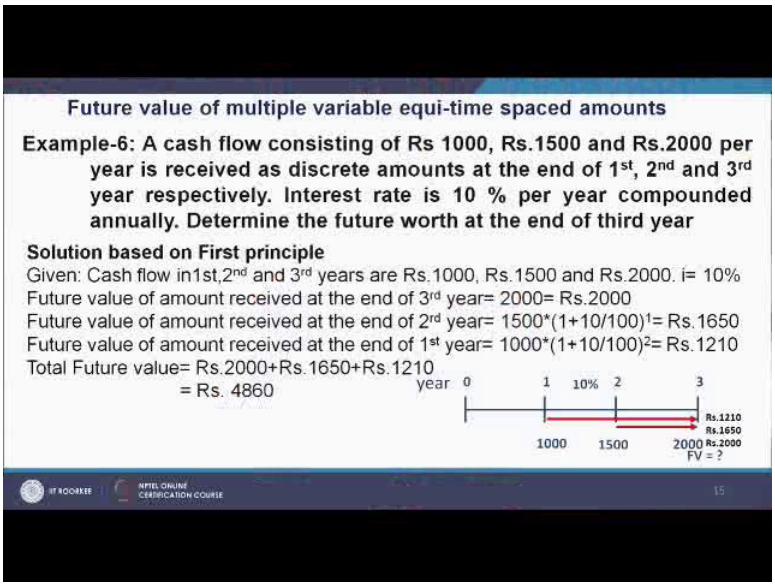


So, what we have to do this is the timeline in which investment is own at the end of first year this is 10000, at the end of second year this is 1500 and the end of third year it is 2000 and what I am suppose to find what are the Future value of this 3 investments at the end of third year.

So, the Future value of the amount received at the end of third year is 2000. So, it remains 2000 because there is it will not earn any interest because at the end of third year this amount is there. Now at the end of second year this will earn interest for only one year. So, this is 1501 plus 10 divided by 100 to the power 1 is equal to 1650 and the investment which is done at the end of first year it will earn interest for 2 years. So, this is 1000 into 1 plus 10 divided by 100 to the power 2. This is 1210. So if I add of this 3, 2000, 1650 and 1210 here it comes out to be Rupees 4860. So, the combined Future value of all this 3 investments is 4860.

Now, let us take Future value of multiple variable amounts in variable time spaced. So, here the investments are different here the investment is 1000, 2500 and 5000 and that to also invested at different time, it is at the end of first year, this is at the end of 6 year and this is the end of 8 year. So, they are not uniform. So such types of problems are done using first principle.

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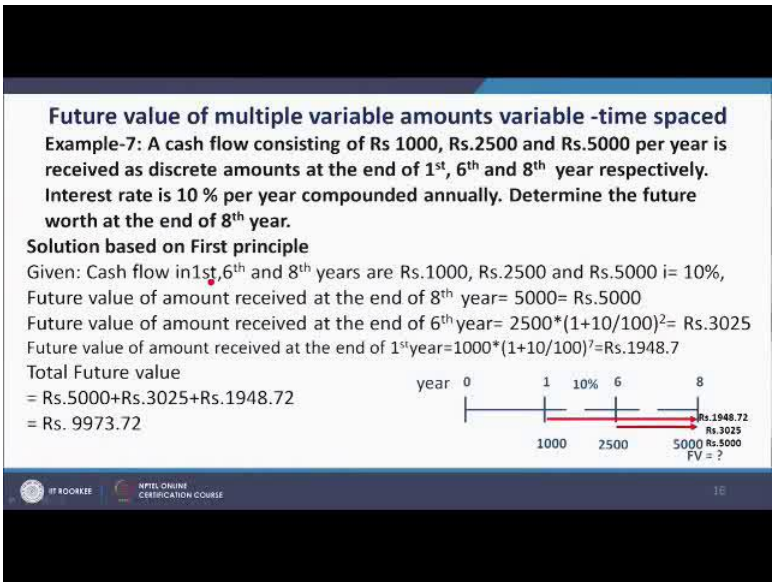


So given cash flow in first 6th and 8th year are 1000, 2500 and 5000, i is equal to 10 percent. So, the Future value of the amount received at the end of 8th year is 5000, because I am finding out the Future value at the end of 8th year.

So, whatever investment which is done at the end of 8th year will not draw any interest and that is why that value 5000 remains 5000. At the end of 6th year, my investment is 2500. So it will draw interest for 2 years 627 and 728 and that is why the Future value will be 2500 into 1 plus 10 by 100 to the power 2 which comes out to be 3025 and the value which has been invested at the end of first year will draw interest rate up to 7 years. So, this is 1000 into 1 plus 10 divided by 100 to the power 7 which comes out to be 1948.72. So, when I add up the Future value of all these 3 investments that is 1000, 2005 and 5000. This is the value I get is Rupees 9973.72.

Thank you.