Time value of money-Concepts and Calculations
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Lecture – 08
Present Value

Welcome to the lecture series on Time value of money-Concepts and Calculations. The topic of the present lecture is Present Value. What is a present value? Present value it is given by PV is the current worth of a future sum of money or stream of cash flows under a specified rate of return.

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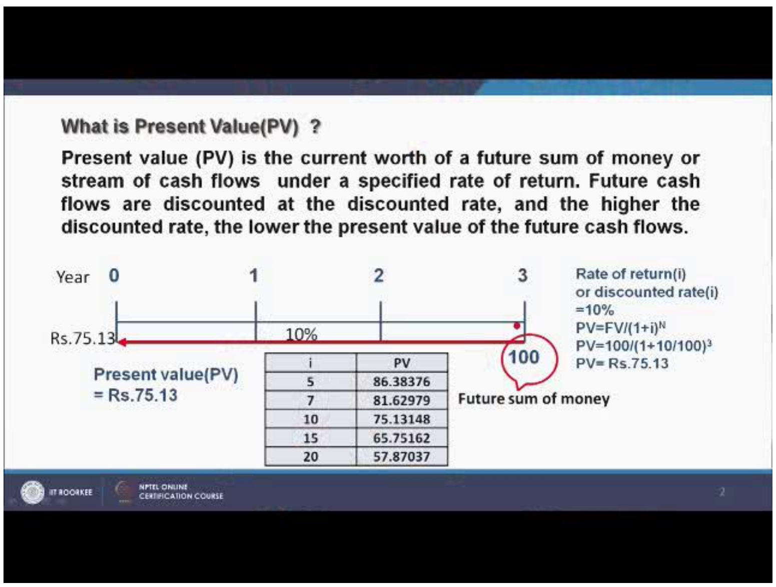

Future cash flows are discounted at the discounted rate and the higher the discounted rate the lower is the present value of the future cash flows. Now this shows a Time line where after 3 years Rupees 100 some is available and the discounter rate is 10 percent.

Now, what will the present value of these 100 Rupees at year 0? At year 0, when we transfer this money to year 0 we get the present value. The present value is 75.13; that means if we invest 73.13 at t equal to 0, at 10 percent interest rate, I will be able to get 100 Rupees after 3 years. Now this is given by PV equal to FV divided 1 plus i to the power N and in this case FV is 100 i is 10 percent. So, 1 plus

10 divided by 100 whole cubes and this get 75.13 Rupees.

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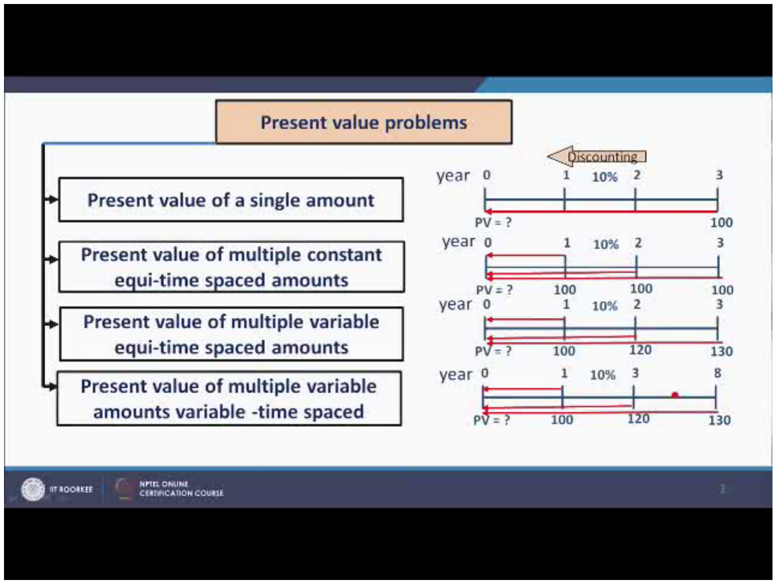

Now, present value problems can be divided into 4 types of problems. The first problem is present value of the single amount here if we see in the time line at the end of third year, we have 100 Rupees and we want to find out what is the present value of it here. The second type of problem is present value of multiple constant equi-time spaced amounts. Here we see that after the end of first year 100 Rupees is invented and at the end of second year again 100 Rupees is invested, at the end of third year again 100 Rupees is invested. So, we want to find out the present value for all these investments at PV equal to 0.

The third type of problem is present value of multiple variable equi-time spaced amounts. Here we are investing money at equi space time intervals, but the amount of money are different. Like at the end of third year I am investing 130 Rupees at the end of second year I am investing 120 Rupees and at the end of first year I am investing 100 Rupees. All this 3 amounts are different, but they are invested at equi-time spaced. The fourth type of problem is present value of multiple variable amounts at variable time spaced. Now here we see that at the end of first year we are investing 100 Rupees, at the end of third year we are investing 120 Rupees and at the end of 8 year we are investing 130 Rupees. So, 130,120 and 100 are different amounts and they are invested at different time line. The first one is after

1 year, second one after 3rd year and third one after 8 year.

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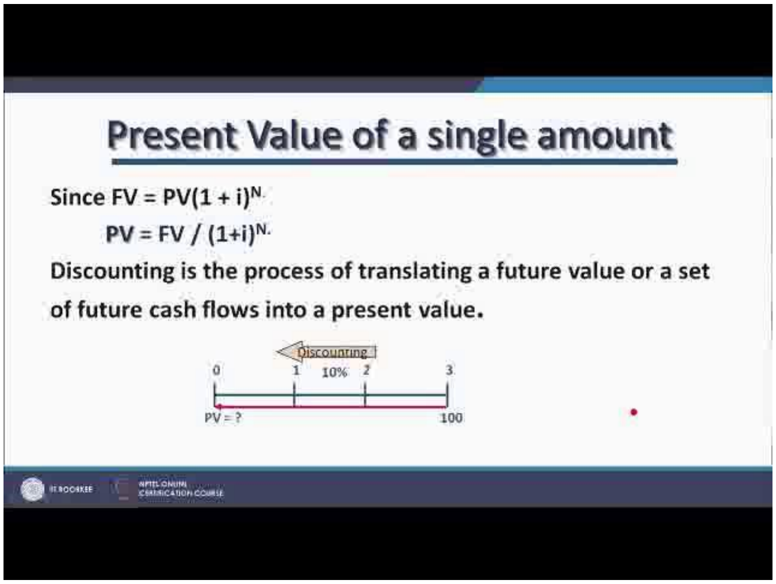

Present value of a single amount; present value of a single amount can be found out from this equation FV is equal to PV into 1 plus i to the power N. From where you can calculate PV is equal to FV divided by 1 plus i to the power N; discounting this is called discounting. Discounting is the process of translating a future value or a set of future cash flows into a present value. So, we are moving in this direction of time and this is called Discounting.

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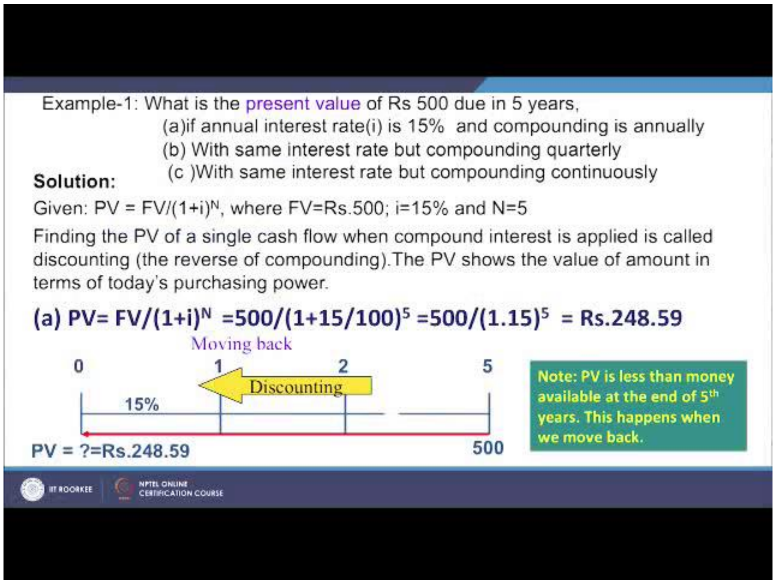

Example 1: what is the present value of Rupees 500 due in 5 years; part a, if annual interest is 15 percent and compounding is annually; part b, with same interest rate but compounding quarterly; c with same interest rate but compounding continuously.

So, my formula is PV is equal to FV divided by 1 plus i to the power N. Where, FV is equal to Rupees 500, i is equal to 15 percent and N is equal to 5. Finding the PV of a single cash flow when compound interest is applied is called discounting, that is reverse of compounding, the PV source, and the value of amount in terms of today's purchasing power. So, you can calculate. PV from the above equation PV is equal to FV divided by 1 plus i to the power n is equal to 500 divided by 1 plus 15 this is the i value divided by 100 to the power 5 comes out to be 248.59.

That means, if I am investing 500 Rupees at the end of 5 years. So it is value at t equal to 0, the present value will be 248.59, when mine discounting rate is 15 percent. Note; PV is less than money available at the end of 5th year. This happens when we move that.

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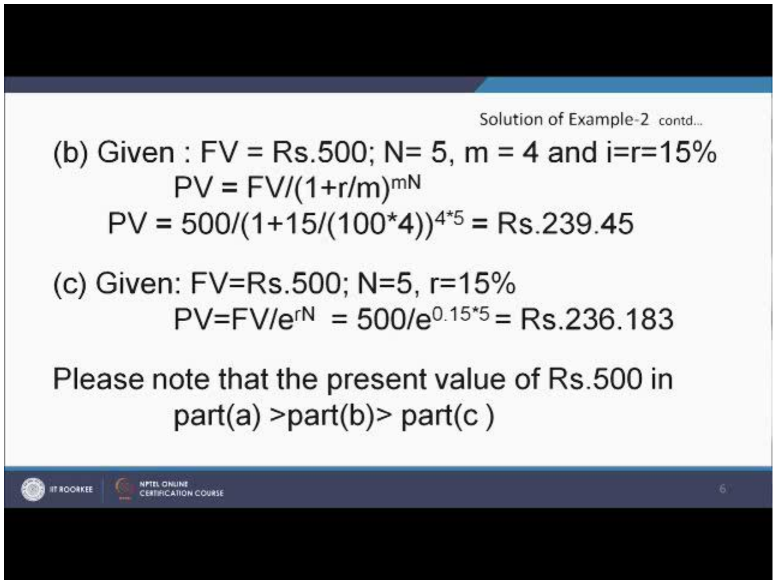

Now part b, given FV is equal to 500, N is equal to 5 m is equal to 4 because it is restrictly compounding and it is quarterly compounding that is why m is equal to 4 i or r is equal to 15 percent. So, my formula for this cases PV is equal to FV divided by 1 plus r divided by m to the power mN, m is 4 and N is 5. So, the multiplication is 20. So, 500 divided by 1 plus 15 divided by 100 divided by 4 to the power 20 that comes out to be Rupees 249.45.

Part c; given FV is equal to 500, N is equal to 5, r is equal to 15 percent. Now it is a continuous interest forming. So, PV is equal to FV divided by e to the power rN is equal to 500 divided by e to the power 0.15 into 5 comes out with 236.183. Please note down that present value of Rupees 500 in part a is greater than part b is greater than part c.

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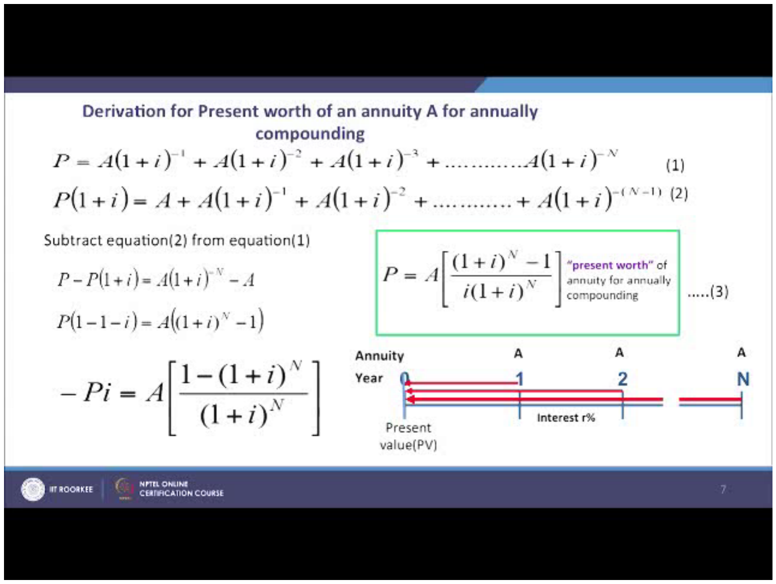

Now, let us see the derivation of present worth of an annuity A for annually compounding. Now P will be equal to A, If here we see this is the time line, at t equal to 0 we want the present value at t equal to 1 we are investing Rupees A, t equal to 2, again A amount is invested and t equal to N and another A amount is invested.

So, continuously we are investing A amount each year up to Nth year and our interest rate is half set. So, the present value I have to transport all this a values to this time line A equal to 0 to this and this to this. So, I can write down this equation P is equal to a 1 plus i to the power minus 1 plus A1 plus i to the power minus 2 like this up to A into 1 plus i to the power minus N.

Now, I both the side I multiply it with 1 plus I, then this A1 plus I to the power minus becomes A and this becomes minus N minus 1 in brackets. So, subtracting equation 2 from equation 1 we have P minus P, in brackets 1 plus i is equal to A, in bracket 1 plus i minus N minus A. So if we solve this we can write down minus Pi is equal to A in brackets 1 minus 1 plus i to the power N divided by 1 plus i to the power N. Now if I take this negative into consideration, I can write down this equation this reverse is A equal to 1 plus i to the power N minus 1 divided by i into 1 plus i to the power minus 1. This is called Present worth of Annuity of annually compounding.

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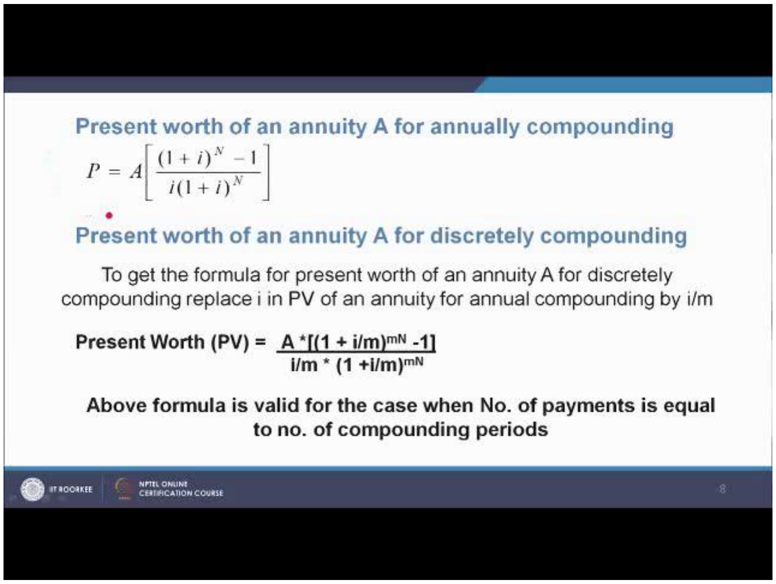

Now you can find out based on this value, we can find out the present worth of annuity A for discretely compounding. The starting point is present worth of annuity A for annually compounding. This equation we have seen P is equal to A into 1 plus i to the power minus 1 whole divided by i into 1 plus i to the power N. To get the formula for present worth of annuity a for discretely compounding replace i in PV of an annuity for annual compounding by i by m. So, if i replace this i by i by m i can get the formula for present worth of an annuity for discretely compounding.

So you will see that here I have changed i with i by m and N with m into N. So, if it is done, So I get this equations. The above formula is valid for the case when number of payments is equal to number of compounding periods, this has to be remembered. This is only valid for the case when number of payments is equal to the number of compounding periods. Now let see the derivation of present worth of annuity A for continuous compounding. His is our time line at the end of first year I am investing A money, end of second year again A money and at the end of Nth year again A money. So, if I take this money to time line t equal to 0, I can find out the present value, but here the compounding is continuous.

So when this A is converted into present value. So, this is A into e to the power minus r. Similarly, if I write for all n values, then the last value is A into e to the power minus rN. Now if I multiply the left hand side and right hand side with e to the power r, then this becomes A, this becomes A into e to the power minus r and the last one becomes Ae to the power minus r in brackets N minus 1. Now if you subtract equation 8 from equation 7, then this is P minus Pe to the power r is equal to Ae to the power minus rN minus A. So you can write down P is equal to A in brackets 1 minus e to the power rN divided by e to the power rN in brackets 1 minus e to the power r. So, this equation links the annuity with the present value. This is the present value and this is the value of the annuity.

So present worth of annuity for continuously compounding, we will be using this equation. Now let us see the different problems which can be worked out based on this information.

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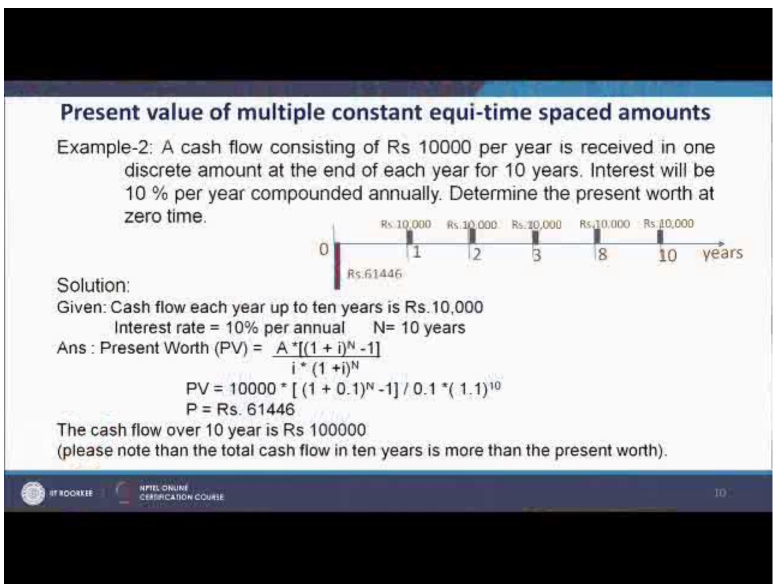

Now this example to a cash flow consisting of 1 10000 per year is received in one discrete amount at the end of each years for 10 years. So, for 10 years we are getting 10000 Rupees each year. Interest will be 10 percent per year compounded annually determine the present worth that is at the 0 time. The solution is cash flow for each year up to 10 years is Rupees 10000 interest at 10 percent per annum, N is equal to 10 years and what is required is the PV value that is present value.

So, my equation is PV equal to A in brackets 1 plus i to the power N minus 1 and divided by i into 1 plus 1 plus i to the power N. So, when we put values into this equation the value of P comes out to be

61446. So the cash flow over the 10 years is 1000000. Now let us please note that the total cash flow in 10 years is more than the present worth and this is obvious because we are discounting the values; now present value of multiple constant equi-time spaced amounts.

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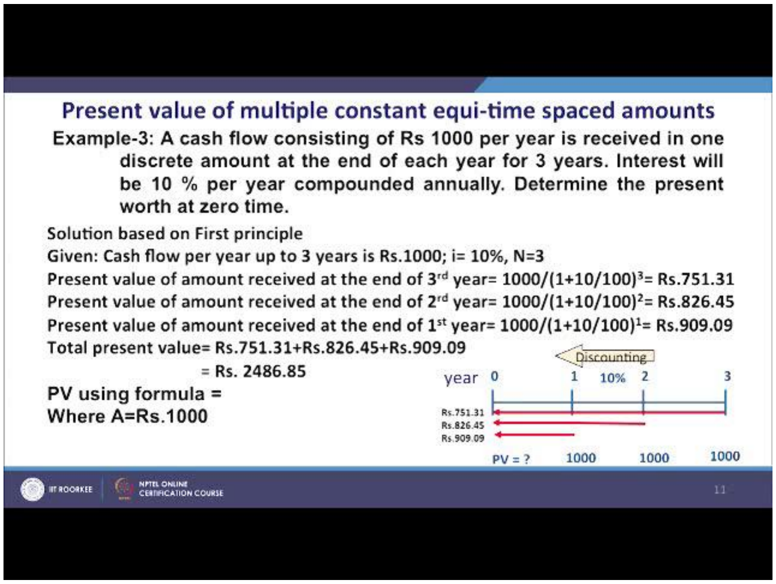

So my amount is not changing. Let us see the problem, example 3 a cash flow consisting of rupee 1000 per year is received in one discrete amount at the end of each year for 3 years; that means, at the end of first year I am getting 1000 at the end of second year again 1000, at the end of third year again 1000. Interest will be 10 percent per year compounded annually, determine the present worth at time equal to 10. So, this I am solving from the first principle as well as using the equation. So, cash flow per year up to 3 years is equal to 1000, i is equal to 10 percent, N is equal to 3. Now if I take this value that is 1000, which is given at the end of third year. Transport it to 0 time line and this can be done by finding out the present value of this 1000 Rupees.

So this will be 1000 divided by 1 plus 10 divided by 10 to the power 3, which comes out to be 751.31, that means, if this money 1000 present value will be calculated it comes out to be 751.31 as written here. Now similarly the 1000 Rupees which we have received at the end of second year, its present value is calculated, it comes out to be 826.45, this is equal to 1000 divided by 1 plus 10 divided by 100 to the power 2. Similarly 1000 which is available at the end of first year it is present value will be calculated it will be 909.09.

Now, let see here, the present value of this 1000 which will available at the end of third year is 751 the present value of this is 826, the present value of this 1000 is 909.09. Have you observed a trend this value is less, this value is bit more and this value is more than this value this will happen always. Now when I add these three values here I get 2486.85. So what I have done, I have added the present worth of this value, present worth of this value and present worth of this value and this 3 values comes out to be 2486.85. So, this is our answer because you are interested in finding out the worth of these 3 values. This can be done by a formula which is PV is equal to A within brackets 1 plus I to the power N minus 1 divided by i 1 plus i to the power, if I use this formula and put my numbers, it comes out to be 2486.85 the same value.

Now, let us take a problem which is little bit of miss miscellaneous in nature. Mr. and Mrs. Sharma wish to have an annuity for their daughter, when she goes to university. They wish to invest into an annuity that will pay their daughter Rupees 2000 per month for 4 years. What is the present value of that annuity given that current interest rate is 9 percent per annum?

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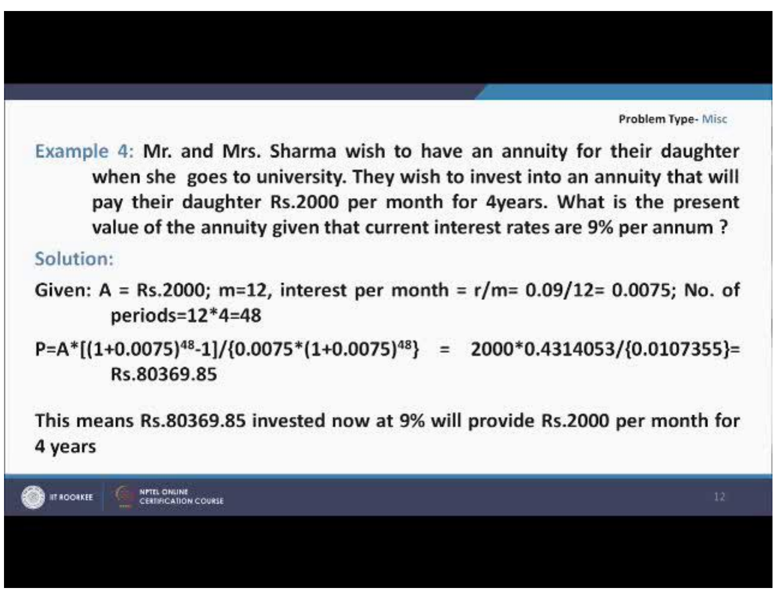

Here you will note that the payments are per month and not per year, whereas my interest rate is per year. So, this is a case where the discounting is done per period which is month. So, for this case A is equal to 2000 m is equal to 12 because there are 12 months in a year interest per month is r by m which is 0.09 divided by 12, which comes out to be 0.0075 and number of periods is equal to 12 into 4 which is 48. So, I use this equation P is equal to A in brackets 1 plus 0.007 to the power 48 minus 1 divided by 0.0075 into 1 plus 0.0075 to the power 48. This comes out to be 80369.85.

So, this is a different problem and uses a compounding method which is not annually. So, this means that if I invest 80369.85 invested now at 9 percent will provide Rupees 2000 per month for 4 years.

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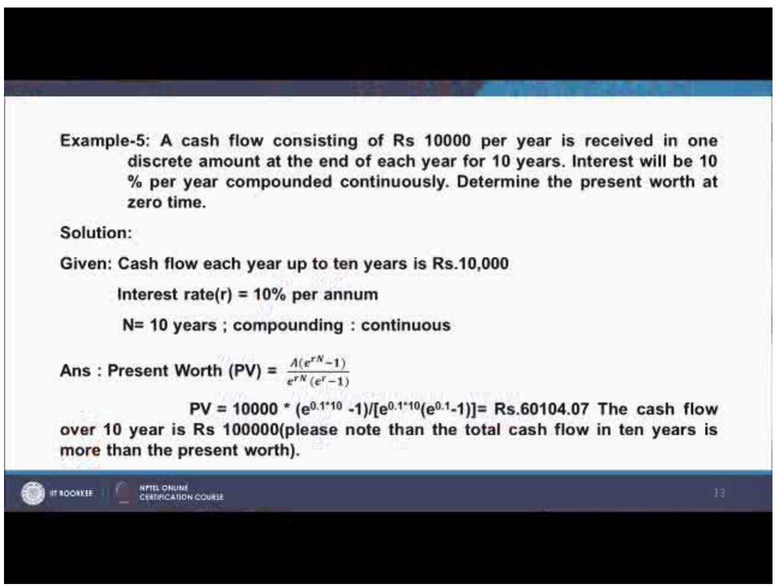

Now, let us take another example, which is based on continuous compounding. A cash flow consisting of Rupees 10000 per year is received in one discrete amount at the end of each year for 10 years; interest will be 10 percent per year, compounded continuously determine the present worth at zero time. The solution is given that cash flow each year up to 10 years is 10000; that means, each year will have 10000 Rupees that means, in first year I will have 10000 Rupees, in the second year I will have 10000 Rupees, at the end of third year I will have 10000 Rupees in so and so for, up to the 10 year interest rate r is 10 percent per annum N is equal to 10 years and compounding method is continuous.

So, we use the equation present worth PV is equal to A in brackets e to the power rN minus 1 in divided by e to the power rN into e to the power r minus 1, when we put our values into this equation then the value of the PV which comes out to be 60104.07. The cash flow over 10 year is 100000. Please note that the total cash flow in 10 years is more than the present worth of it.

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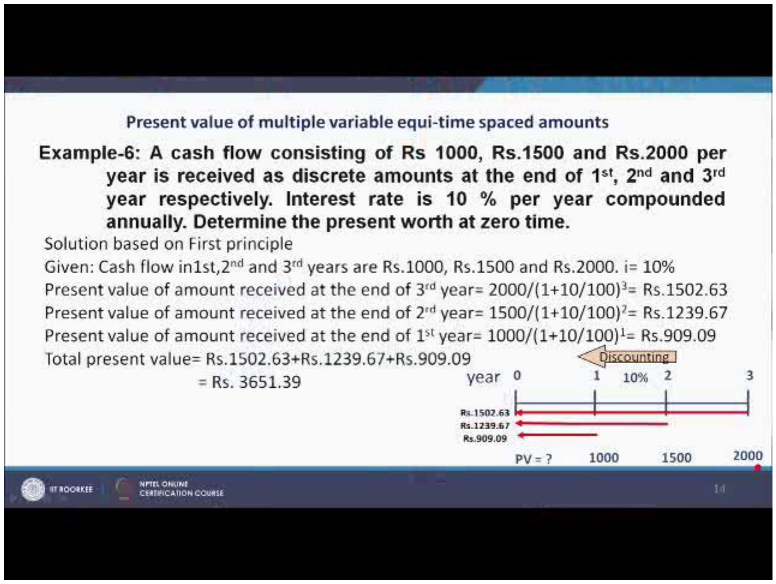

Then another problem present value of multiple variable equi-time spaced amounts. When the values are different; that means, end of the first year value of 1000 end of second year 1500, end of the third year it is 2000. In such problems I cannot use an equation to find out the values and hence has to be solved from the first principle.

So, a cash flow consisting of Rupees 1000, 1500 and 2000 per year is received has discrete amounts, at the end of first year, second year and third year respectively. Interest rate is 10 percent per year compounded annually; determine the present worth at 0 times. So, we have to find out the present worth of this values which were invested at different period of time. So, from the first principle we see that the cash flows are available 1000, 1500 and 2000 and i is equal to 10 percent. So, let us see this, these value is 2000 invested at the end of third year, when I find out the present value of this, this will be 2000 divided by 1 plus 10 divided by 100 to the power 3, which comes out to be 1502.63.

Similarly for this 1500 which is at the end of second year if I want to find out the present value of it here and the 0th here and it is 1500 divided by 1 plus 10 divided by 100 to the power 2 which comes out to Rupees 1239.67. Now, the value which is 1000 which is available at the end of first year, it is present value is found out at time equal to 0 and this is 10000, 1000 divided by 1 plus 10 divided by 100 to the power 1 is 909.09. So, when I add these 3 values here I get a value Rupees 3651.39. So, the present value of all this 3 amounts 1000, 1500 and two 2000 invested at different interval of time is equal to 3651.39.

Now, let us take another example, here cash flow consisting of 1000, 2500 and 5000 per year is received as discrete amount at the end of 1st year, 6th year and 8th year respectively. Interest rate is 10 percent per year compounded annually determine the present worth at zero time. This is a problem, where present value of a multiple variable amounts with variable time spaced. So time is different and amount is different here. So, it will be solved by using the first principle.

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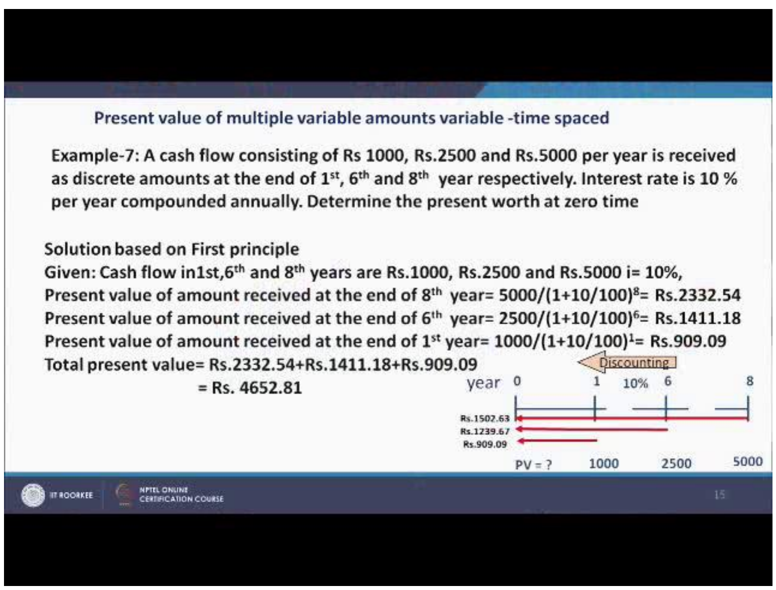

Now, for this what we will do at the end of 8th year the amount available is 5000 to will find out the present value of this 5000 Rupees that means, we will transport this amount from here to this time line and this will be done by 5000 divided by 1 plus 10 divided by 100 to the power 8, which comes out to be 2332.54. Now there is another amount that the end of 6th is which 2500 it is is present value has to be find out by taking it to the 0th here line.

So, this will be 2500 divided by 1 plus 10 divided by 100 to the power 6 is comes out to be Rupees 1411.18 and at the end of 1st year we have 1000 amount and this has to be brought down to the present value. This is 1000 divided by 1 plus 10 divided by 100 to the power 1, is comes out to be Rupees 909.09. Now if we add these 3 values, which is the present worth of these investments 5000, 2500 and 1000 they is added hero, to find out the present value of the cash flow which is 4652.81.

Thank you.